CBSE Class 12 - Mathematics
Sample Paper 12

Maximum Marks: 80
Time Allowed: 3 hours

General Instructions:

- All the questions are compulsory.
- The question paper consists of 36 questions divided into 4 sections A, B, C, and D.
- Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 6 questions of 4 marks each. Section D comprises of 4 questions of 6 marks each.
- There is no overall choice. However, an internal choice has been provided in three questions of 1 mark each, two questions of 2 marks each, two questions of 4 marks each, and two questions of 6 marks each. You have to attempt only one of the alternatives in all such questions.
- Use of calculators is not permitted.

Section A

1. The number of all possible matrices of order 3×3 with each entry 0 or 1 is

   a. 81

   b. none of these

   c. 512

   d. 18

2. If the value of a third order determinant is 11, then the value of the square of the determinant formed by the cofactors will be

a. 1331

b. 14641

c. 121

d. 11

3. $\underset{x \to \infty}{Lt} \left(1 + \frac{3}{x}\right)^x$ is equal to

a. 3 e

b. None of these

c. e^3

d. $e^{1/3}$

4. Differential coefficient of a function f(g(x)) w.r.t. the function g (x) is

a. f‘(g (x))

b. None of these

c. $\frac{f'(g(x))}{g'(x)}$

d. f ‘(g (x)) g’ (x)

5. General solution of $\frac{dy}{dx} + \frac{y}{x} = x^2$ is

a. $xy = \frac{x^7}{4} + C$

b. $xy = \frac{x^5}{4} + C$

c. $xy = \frac{x^4}{4} + C$

d. $xy = \frac{x^6}{4} + C$

6. Domain of f(x) = $\sin^{-1} x - sec^{-1} x$ is

a. None of these

b. { 0, 1}

c. { -1 ,1}

d. 0 or 1

7. Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of E(X) is

a. $\frac{3}{13}$

b. $\frac{2}{13}$

c. $\frac{41}{131}$

d. $\frac{1}{13}$

8. $\int_{1}^{e} \frac{1+\ln x}{2x} dx$ is equal to

a. e

b. $\frac{1}{4}$

c. $\frac{3}{4}$

d. $\frac{1}{e}$

9. If $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_1}{l_2} = \frac{y-y_1}{m_2} = \frac{z-z_1}{n_2}$ are the equations of the two lines, then express acute angle between the two lines in terms of the direction cosines of the two lines.

a. $\cot\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|\ l_1^2 + m_1^2 + n_1^2 = 1; l_2^2 + m_2^2 + n_2^2 = 1$

b. $\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|\ l_1^2 + m_1^2 + n_1^2 = 1; l_2^2 + m_2^2 + n_2^2 = 1$

c. $\tan\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|\ l_1^2 + m_1^2 + n_1^2 = 1; l_2^2 + m_2^2 + n_2^2 = 1$

d. $\sin\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|\ l_1^2 + m_1^2 + n_1^2 = 1; l_2^2 + m_2^2 + n_2^2 = 1$

10. Vector has

   a. direction

   b. None of these

   c. magnitude

   d. magnitude as well as direction

11. Fill in the blanks:

    A binary operation * on a set X is said to be ________, if a * b = b * a, where a, b $\in$ X.

12. Fill in the blanks:

    The possibility of having 53 Thursdays in a non-leap year is ________.

13. Fill in the blanks:

    If A and B are symmetric matrices, then AB - BA is a ________ matrix.

14. Fill in the blanks:

    The indefinite integral of $x^{\frac{1}{3}}$ is ________.

OR

Fill in the blanks:

The indefinite integral of $2x^{\frac{1}{2}}$ is ________.

15. Fill in the blanks:

    A linear function Z = px + qy (p and q are constants) which has to be maximised or minimised, is called an _______ function.

OR

Fill in the blanks:

In a LPP, the linear function which has to be maximised or minimised is called a linear ________ function.

16. Let $\begin{vmatrix} 3 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$. Find possible values of x, y natural numbers.

17. Find the angle between the vector having direction ratios (3,4,5) and (4, -3, 5).

18. Evaluate $\int \sin^3 x dx$.

OR

$\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$

19. Find the equation of the tangent to the curve $y = x + \frac{4}{x^2}$ which is parallel to the X-axis.

20. Find the value of p, if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = \vec{0}$.

Section B

21. Let A = {a, b, c} and the relation R be defined on A as follows:
R = {(a, a), (b, c), (a, b)}.
Then, write minimum number of ordered pairs to be added in R to make R reflexive and transitive.

22. Differentiate the following function with respect to x : $\cos(\sqrt{x})$

OR

Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$, $\frac{-1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

23. Find the sum of the vectors: $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}, \vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$

24. Find the set of values of x for which log(1+x)<x

OR

Find the approximate value of $(1.999)^5$.

25. Find the equation of the line parallel to x - axis and passing through the origin.

26. Find the probability of getting a multiple of 3 aleast 8 times in throwing a die 10 times.

Section C

27. $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$.

28. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$

OR

If x = asin pt, y = b cospt. find the value of $\frac{d^2y}{dx^2}$ at t = 0

29. A discrete random variable X has the probability distribution given as below:

X	0.5	1	1.5	2
P(X)	k	k^2	$2k^2$	k

i. Find the value of k
ii. Determine the mean of the distribution.

30. Maximise the function Z = 11x + 7y, subject to the constraints:
$x \leqslant 3, y \leqslant 2, x \geqslant 0, y \geqslant 0.$

31. Solve $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + xdy = 0; y = \frac{\pi}{4}$ when x = 1

OR

Find the particular solution of the differential equation $2ye^{x/y}dx + (y - 2xe^{x/y})dy = 0$, given that x = 0, when y = 1.

32. $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$

Section D

33. Solve the system of the following equations: (Using matrices):

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4; \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1; \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2;$

OR

If X - Y = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and X + Y = $\begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$,find X and Y.

34. Find the equations of tangents to the curve $3x^2 - y^2 = 8$, which passes through the point $\left(\frac{4}{3}, 0\right)$.

35. Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

OR

Using integration, find the area of region bounded by the triangle whose vertices are (-2, 1), (0, 4) and (2, 3).

36. A variable plane which remains at a constant distance 3p from the origin cut the coordinate axes at A, B, C. Show that the locus of the centroid of triangle ABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.

Solution
Section A

1. (c) 512

   Explanation:

   2^{3x3} = 2^9 = 512.

   The number of elements in a 3 $\times$ 3 matrix is the product 3 $\times$ 3 = 9.

   Each element can either be a 0 or a 1.

   Given this, the total possible matrices that can be selected is 2^9 = 512

2. (b) 14641

   Explanation:

   Since the matrix formed by the cofactors is adj(A) and we know that $|adj(A)|=|A|^2$ for third order square matrix.

   So, $|adj(A)|^2 = |A|^4$ and $|A|=11 \Rightarrow |adj(A)|^2=(11)^4=14641$

3. (c) e^3

   Explanation:

   $$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^x=\lim_{t\to0^+}(1+3t)^{\frac{1}{t}}=\lim_{3t\to0^+}\left[(1+3t)^{\frac{1}{3t}}\right]^3=e^3$$

4. (a) f'(g (x))

   Explanation:

   $$\frac{d}{d(g(x))}(f(g(x))=f'(g(x))$$

5. (c)

$xy = \frac{x^4}{4} + C$

Explanation:

$\frac{dy}{dx} + \frac{y}{x} = x^2$
$Here\ P = \frac{1}{x}, Q = x^2$
$\Rightarrow I.F. = e^{\int P.dx} = e^{\int \frac{1}{x}.dx}$
$= e^{\log|x|} = x$
$\Rightarrow y.x = \int x.x^2 dx \Rightarrow xy$
$= \frac{x^4}{4} + C$

6. (c) { -1 ,1}

   Explanation:

   Since, Domain of $\sin^{-1} x$ is [-1,1] and domain of $\sec^{-1} x$ is R - (-1,1), D_f = { -1,1 }.

7. (b)

   $\frac{2}{13}$

   Explanation:

   The possible values of X are 0 , 1 and 2 .

   As we know that pack of cards contains 4 aces.

   $P(X=0) = \frac{^{48}C_2}{^{52}C_2} = \frac{48\times47}{52\times51} = \frac{2256}{2652}$

   $P(X=1) = \frac{^{4}C_1 \times ^{48}C_1}{^{52}C_2} = \frac{4\times48\times2}{52\times51} = \frac{384}{2652}$

   $P(X=2) = \frac{^{4}C_2}{^{52}C_2} = \frac{4\times3}{52\times51} = \frac{12}{2652}$

   Therefore , the probability distribution of X is :

X	0	1	2
P(X)	$\frac{2256}{2652}$	$\frac{384}{2652}$	$\frac{12}{2652}$
XP(X)	0	$\frac{384}{2652}$	$\frac{24}{2652}$

Therefore required value of

E(X) is = 0+ 384/2652 + 24/2652 = 408/2652 = 2/13.

8. (c)

   $\frac{3}{4}$

   Explanation:

   $\frac{1}{2}\int\limits_1^e \frac{1}{x}dx + \int\limits_1^e \frac{\ln x}{x}dx = \frac{1}{2}\left[\log x + \frac{(logx)^2}{2}\right]_1^e$

9. (b)

   $\cos\theta = |l_1l_2 + m_1m_2 + n_1n_2|\ \ l_1^2 + m_1^2 + n_1^2 = 1; l_2^2 + m_2^2 + n_2^2 = 1$

   Explanation:

   If $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_1}{l_2} = \frac{y-y_1}{m_2} = \frac{z-z_1}{n_2}$ are the equations of the two lines, then the acute angle between the two lines is given by :
   $\cos\theta = |l_1l_2 + m_1m_2 + n_1n_2|$. and $l_1^2 + m_1^2 + n_1^2 = 1; l_2^2 + m_2^2 + n_2^2 = 1$

10. (d) magnitude as well as direction

    Explanation:

    A vector has both magnitude as well as direction.

11. commutative

12. $\frac{1}{7}$

13. skew symmetric

14. $\frac{3}{4}x^{\frac{4}{3}} + c$

    OR

    $\frac{4}{3}x^{\frac{3}{2}} + c$

15. objective

OR

objective

16. We have,

$$\begin{vmatrix} 3 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$

$\Rightarrow$ 3 - xy = 3 - 8 $\Rightarrow$ xy = 8

$\Rightarrow$ x = 1, y = 8 or x = 2, y = 4 or x = 4, y = 2 or x = 8, y = 1

17. Let a_1 = 3, b_1 = 4, c_1 = 5 and a_2 = 4, b_2 = -3, c_2 = 5

$$\cos\theta = \left(\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right)$$

$$= \left(\frac{(3)(4)+(4)(-3)+(5)(5)}{\sqrt{3^2+4^2+5^2}\sqrt{4^2+(-3)^2+5^2}}\right)$$

$$= \frac{25}{\sqrt{50}\sqrt{50}} = \frac{25}{50} = \frac{1}{2}$$

$$\theta = 60^0$$

18. Let $I = \int \sin^3 x dx = \int \frac{3\sin x - \sin 3x}{4} dx \ [\because \sin 3x = 3\sin x - 4\sin^3 x]$

$$= \frac{1}{4}\int 3\sin x dx - \frac{1}{4}\int \sin 3x dx$$

$$= \frac{1}{4}\left(-3\cos x + \frac{\cos 3x}{3}\right) + C$$

OR

$$\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha} dx$$

$$= \int \frac{(2\cos^2 x - 1) - (2\cos^2\alpha - 1)}{(\cos x - \cos\alpha)} dx$$

$$= \int \frac{2(\cos x + \cos\alpha)(\cos x - \cos\alpha)}{(\cos x - \cos\alpha)} dx$$

$$= 2(\sin x + \cos\alpha . x) + c$$

19. We have, $y = x + \frac{4}{x^2}$ ....(1)

$\Rightarrow y = x + 4x^{-2}$

On differentiating w.r.t x, we get,

$\frac{dy}{dx} = 1 + 4 \times (-2x^{-3}) = 1 - 8x^{-3} = 1 - \frac{8}{x^3}$

$\frac{dy}{dx} = 1 - \frac{8}{x^3}$

Since the tangent is parallel to X-axis, therefore

$\frac{dy}{dx} = 0$

$\Rightarrow 1 - \frac{8}{x^3} = 0$

$\Rightarrow x^3 = 8$

$\Rightarrow x = 2$

From (1), when x = 2 , we get , y=$2 + \frac{4}{4} = 2 + 1 = 3$

Therefore, y=3 is required equation.

20. Vector $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = \vec{0}$

$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & 3 & p \end{vmatrix} = \vec{0}$

$\Rightarrow (6p - 81)\hat{i} - (2p - 27)\hat{j} + 0\hat{k} = \vec{0}$

$\Rightarrow 6p - 81 = 0$

$\Rightarrow p = \frac{81}{6} = \frac{27}{2}$

Section B

21. Given relation, R = {(a, a), (b, c), (a, b)}.
To make R reflexive we must add (b, b) and (c, c) to R. Also, to make R transitive we must add (a, c) to R.
So, minimum number of ordered pairs to be added is 3.

22. Let $y = \cos(\sqrt{x})$

$\therefore \frac{dy}{dx} = -\sin\sqrt{x}\,.\,\frac{d}{dx}\sqrt{x}$

$= -\sin\sqrt{x}\,.\,\frac{1}{2}(x)^{\frac{-1}{2}}$

$= \frac{-\sin\sqrt{x}}{2\sqrt{x}}$

OR

Given: $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), \frac{-1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$
Putting $x = \sin\theta$
$y = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$
$= \sin^{-1}\left(2\sin\theta\sqrt{\cos^2\theta}\right)$
$= \sin^{-1}(2\sin\theta\cos\theta)$
$= \sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1}x$
$\therefore \frac{dy}{dx} = 2.\frac{1}{\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}$

23. Given: $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}, \vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$

Adding, $\vec{a} + \vec{b} + \vec{c} = \hat{i} - 2\hat{j} + \hat{k} - 2\hat{i} + 4\hat{j} + 5\hat{k} + \hat{i} - 6\hat{j} - 7\hat{k}$

$= 0\hat{i} - 4\hat{j} - \hat{k} = -4\hat{j} - \hat{k}$

24. Let f(x)=log(1+x)-x

$f'(x) = \frac{1}{1+x} - 1 = -\frac{x}{1+x}$
$f'(x) < 0, for\ x > 0$
$f(x)\ is\ decreasing\ for\ x > 0$

$\Rightarrow$ f(x)<f(0),for x>0

$\Rightarrow$ log(1+x)-x<0,for x>0

i.e, log(1+x)<x, for x>0

OR

Let x = 2

and $\Delta x = -0.001$ $[\because 2 - 0.001 = 1.999]$

let $y = x^5$

On differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} = 5x^4$

Now, $\Delta y = \frac{dy}{dx} . \Delta x = 5x^4 \times \Delta x$

$= 5 \times 2^4 \times [-0.001]$

$= -80 \times 0.001 = -0.080$

$\therefore (1.999)^5 = y + \Delta y$

$= 2^5 + (-0.080)$

$= 32 - 0.080 = 31.920$

25. We know that a unit vector along x - axis is $\hat{i} = \hat{i} + 0\hat{j} + 0\hat{k}$

$\therefore$ Direction cosines of x - axis are coefficients of $\hat{i}, \hat{j}, \hat{k}$ in the unit vector

i.e., 1, 0, 0 $= l, m, n$

$\therefore$Equation of the required line passing through the origin (0, 0, 0) and parallel to x - axis is $\frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0} \Rightarrow \frac{x}{1} = \frac{y}{0} = \frac{z}{0}$

Vector equation of the required line is $\vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow \vec{r} = \vec{0} + \lambda\hat{i}$ [$\vec{a} = \vec{0}$ and $\vec{b} = \hat{i}$]

$\Rightarrow \vec{r} = \lambda\hat{i}$

26. Here success is getting a multiple of 3 i.e., 3 or 6.
Therefore, p(3 or 6) $= \frac{2}{6} = \frac{1}{3}$
The probability of r success in 10 throws is given by
$p(r) = {}^{10}C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{10-r}$

Now P (at least 8 successes) = P (8) + P (9) + P (10)
$={}^{10}C_8\left(\frac{1}{3}\right)^8\left(\frac{2}{3}\right)^2+{}^{10}C_9\left(\frac{1}{3}\right)^9\left(\frac{2}{3}\right)^1+{}^{10}C_{10}\left(\frac{1}{3}\right)^{10}$
$=\frac{1}{3^{10}}[45\times4+10\times2+1]=\frac{201}{3^{10}}$

Section C

27. $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$
$\implies \tan^{-1}\left(\frac{2x+3x}{1-2x.3x}\right)=\frac{\pi}{4}$
$\implies \frac{5x}{1-6x^2}=\tan\frac{\pi}{4}$
$\implies \frac{5x}{1-6x^2}=\frac{1}{1}$
$\implies$ 1 - 6x^2 = 5x
$\implies$ 6x^2 + 5x -1 = 0
$\implies$ (6x - 1)(x - 1) = 0
$x=\frac{1}{6}$, x = -1 (Ignore x=-1 as it does not satisfy the given equation)
Therefore, $x=\frac{1}{6}$

28. $x\sqrt{1+y}+y\sqrt{1+x}=0$

$x\sqrt{1+y}=-y\sqrt{1+x}$

Squaring both sides, we get

$x^2(1+y)=y^2(1+x)$

$x^2+x^2y=y^2+xy^2$

$x^2-y^2+x^2y-xy^2=0$

$(x-y)(x+y)+xy(x-y)=0$

$(x-y)[x+y+xy]=0$

$x+y+xy=0[\because x\neq y]$

$y(1+x)=-x$

$y=\frac{-x}{1+x}$

$\frac{dy}{dx} = -\left[\frac{(1+x)(1)-(x)(1)}{(1+x)^2}\right]$

$= -\left[\frac{1+x-x}{(1+x)^2}\right]$

$\frac{dy}{dx} = \frac{-1}{(1+x)^2}$

OR

x = a sin pt

$\frac{dx}{dt} = a\cos pt.p$ ...(1)

y = b cos pt

$\frac{dy}{dt} = -b\sin pt.p$ ...(2)

$\frac{dy}{dx} = \frac{-b}{a}\tan pt$ ...[(2) divide by (1)]

$\frac{d^2y}{dx^2} = \frac{-b}{a}.\frac{d}{dt}(\tan pt).\frac{dt}{dx}$

$= \frac{-b}{a}.\sec^2 pt.p.\frac{1}{a\cos pt.p}$

$= \frac{-b}{a^2}\sec^3 pt$

$\left[\frac{d^2y}{dx^2}\right]_{t=0} = \frac{-b}{a^2}\sec^3(p.0)$

$= \frac{-b}{a^2}(1)$

$= \frac{-b}{a^2}$

29. We have,

X	0.5	1	1.5	2
P(X)	k	k^2	$2k^2$	k

i. We know that, $\sum_{i=1}^{n} p_i = 1$, where $P_i \geqslant 0$

$\Rightarrow$ p1 + p2 + p3 + p4 = 1

$\Rightarrow$ k + k^2 + $2k^2$ + k = 1

$\Rightarrow$ $3k^2$ + 2k - 1 = 0

$\Rightarrow$ (3k - 1) (k + 1) = 0

$\Rightarrow$ either $k = \frac{1}{3}$ or k = -1

k=-1 not possible

because k is $\geqslant 0 \Rightarrow k = \frac{1}{3}$

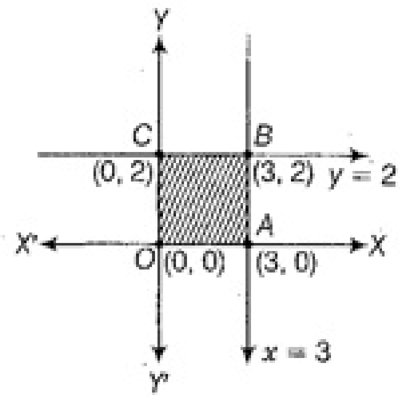
ii. Mean of the distribution $(\mu) = E(X) = \sum_{i=1}^{n} x_i p_i$

= 0.5(k) + 1(k^2) + 1.5($2k^2$) + 2k = $4k^2$ + 2.5k

$= 4 \cdot \frac{1}{9} + 2.5 \cdot \frac{1}{3} \left[\because k = \frac{1}{3}\right]$

$= \frac{4+7.5}{9} = \frac{23}{18}$

30. Maximise Z = 11x + 7y, subject to the constraints $x \leqslant 3, y \leqslant 2, x \geqslant 0, y \geqslant 0$.



The shaded region as shown in the figure as OABC is bounded and the coordinates of corner points are (0, 0), (3, 0), (3, 2), and (0, 2), respectively.

Corner Points	**Corresponding value of Z**
(0, 0)	0
(3, 0)	33
(3, 2)	47 (Maximum)
(0, 2)	14

Hence, Z is maximise at (3, 2) and its maximum value is 47.

31. $\left\{x\sin^2\left(\frac{y}{x}\right)-y\right\}dx+xdy=0$

$\Rightarrow \sin^2\left(\frac{y}{x}\right)-\frac{y}{x}+\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\frac{y}{x}-\sin^2\left(\frac{y}{x}\right)$ .....(i)

Let y = vx, then

$\frac{dy}{dx}=v+x\frac{dy}{dx}$

Put $\frac{dy}{dx}$ in eq (i),we get,

$v+x\frac{dv}{dx}=v-\sin^2 v$

$\Rightarrow \int \cos ec^2 v\,dv=\int-\frac{dx}{x}$

$\Rightarrow -\cot v=-\log x+c$

$\Rightarrow \log x-\cot\left(\frac{y}{x}\right)=c$

When x = 1, $y=\frac{\pi}{4}$, we get,

c = -1

$\therefore \log x-\cot\left(\frac{y}{x}\right)=-1$

OR

According to the question,

Given differential equation is,

$2ye^{x/y}dx+\left(y-2xe^{x/y}\right)dy=0$

$\Rightarrow \frac{dx}{dy}=\frac{2xe^{x/y}-y}{2y^{x/y}}$ ...(i)

Let $F(x,y)=\frac{\left(2xe^{\frac{x}{y}}-y\right)}{\left(2ye^{\frac{x}{y}}\right)}$

On replacing x by λx and y by λy both sides, we get

$$F(\lambda x, \lambda y)=\frac{\left(2 \lambda x e^{\frac{\lambda x}{\lambda y}}-\lambda y\right)}{\left(2 a y e^{\frac{\partial x}{2 y}}\right)}$$

$$\Rightarrow \quad F(\lambda x, \lambda y)=\frac{\lambda\left(2 x e^{x / y}-y\right)}{\lambda\left(2 y e^{x / y}\right)}=\lambda^0[F(x, y)]$$

Thus, F (x, y) is a homogeneous function of degree zero.

Therefore, the given differential equation is a homogeneous differential equation.

put $x=v y$,

$$\Rightarrow \quad \frac{d x}{d y}=v+y \frac{d v}{d y}$$

$$v+y \frac{d v}{d y}=\frac{2 v e^v-1}{2 e^v}$$

$$\Rightarrow \quad y \frac{d v}{d y}=\frac{2 \nu e^v-1}{2 e^v}-v$$

$$=\frac{2 v e^v-1-2 v e^v}{2 e^v}$$

$$\Rightarrow \quad 2 e^v d v=\frac{-d y}{y}$$

On integrating both sides, we get

$$\int 2 e^v d v=-\int \frac{d y}{y}$$

$$\Rightarrow \quad 2 e^v=-\log |y|+C$$

$$\Rightarrow \quad 2 e^{x / y}+\log |y|=C\left[\operatorname{put} v=\frac{x}{y}\right] \text{ ...(ii)}$$

On substituting x = 0 and y = 1 in Eq. (ii), we get

$$2 e^0+\log |1|=C \Rightarrow C=2$$

On substituting the value of C in Eq. (ii), we get

$$2 e^{x / y}+\log |y|=2$$

which is the required particular solution of the given differential equation.

32. $I=\int_0^\pi \frac{x \sin x}{1+\cos ^2 x} d x$ ...(1)

$$=\int_0^\pi \frac{(\pi-x) \sin (\pi-x)}{1+\cos ^2(\pi-x)} d x \;[by\, P_4$$

$$=\int_0^\pi \frac{(\pi-x) . \sin x}{1+\cos ^2 x} d x$$

$$I=\int_0^\pi \frac{\pi . \sin x}{1+\cos ^2 x} d x+\int_0^\pi \frac{-x \sin x}{1+\cos ^2 x} d x$$

$I=\int_0^{\pi} \frac{\pi.\sin x}{1+\cos^2 x} dx - I$

$2I=\pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx$

put cos x = t

sin x dx = -dt

When x = 0

t = 1

when $x=\pi$

t = -1

$=\pi\int_1^{-1} \frac{dt}{1+t^2}$

$=\pi\left[\tan^{-1} t\right]_1^{-1}$

$=\pi\left[\tan^{-1}(-1) - tan^{-1}(1)\right]_1^{-1}$
$=\pi\left[-\frac{\pi}{4}-\frac{\pi}{4}\right]=\frac{\pi^2}{2}$

$\Rightarrow I=\frac{\pi^2}{4}$

Section D

33. Putting $\frac{1}{x}=u$, $\frac{1}{y}=v$ and $\frac{1}{z}=w$ in the given equations,

$2u+3v+10w=4;\ 4u-6v+5w=1;\ 6u+9v-20w=2$

$\therefore$ The matrix form of given equations is $\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} x \\ v \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$ [AX= B]

Here, $A=\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}$, $X\begin{bmatrix} x \\ v \\ z \end{bmatrix}$ and $B=\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$$\therefore |A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$$

$$= 2(120 - 45) - 3(-80 - 30) + 10(36 + 36)$$

$$= 150 + 330 + 750 = 1200 \neq 0$$

$\therefore A^{-1}$ exists and unique solution is $X = A^{-1}B$....(i)

Now $A_{11} = 75, A_{12} = 110, A_{13} = 72$ and $A_{21} = 150, A_{22} = -100, A_{23} = 0$ and $A_{31} = 75, A_{32} = 30, A_{33} = -24$

$$\therefore adj.A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}' = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

And $A^{-1} = \frac{adj.A}{|A|} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$

$\therefore$ From eq. (i),

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 300 + 150 + 150 \\ 440 - 100 + 60 \\ 288 + 0 - 48 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$\therefore u=\frac{1}{2}, v=\frac{1}{3}, w=\frac{1}{5}$

$\Rightarrow x=\frac{1}{u}=2, y=\frac{1}{v}=3, z=\frac{1}{w}=5$

OR

Given,

X - Y = $\begin{bmatrix}1 & 1 & 1\\1 & 1 & 0\\1 & 0 & 0\end{bmatrix}$

X + Y = $\begin{bmatrix}3 & 5 & 1\\-1 & 1 & 4\\11 & 8 & 0\end{bmatrix}$

Now,

(X - Y) + (X + Y) = $\begin{bmatrix}1 & 1 & 1\\1 & 1 & 0\\1 & 0 & 0\end{bmatrix}$ + $\begin{bmatrix}3 & 5 & 1\\-1 & 1 & 4\\11 & 8 & 0\end{bmatrix}$

$\Rightarrow$ 2X = $\begin{bmatrix}1+3 & 1+5 & 1+1\\1-1 & 1+1 & 0+4\\1+11 & 0+8 & 0+0\end{bmatrix}$

$\Rightarrow$ 2X = $\begin{bmatrix}4 & 6 & 2\\0 & 2 & 4\\12 & 8 & 0\end{bmatrix}$

$\Rightarrow$ 2X = 2$\begin{bmatrix}2 & 3 & 1\\0 & 1 & 2\\6 & 4 & 0\end{bmatrix}$

$\Rightarrow$ X = $\begin{bmatrix}2 & 3 & 1\\0 & 1 & 2\\6 & 4 & 0\end{bmatrix}$

Now,

(X - Y) - (X + Y) = $\begin{bmatrix}1 & 1 & 1\\1 & 1 & 0\\1 & 0 & 0\end{bmatrix}$ - $\begin{bmatrix}3 & 5 & 1\\-1 & 1 & 4\\11 & 8 & 0\end{bmatrix}$

$\Rightarrow$ X - Y - X - Y = $\begin{bmatrix}1-3 & 1-5 & 1-1\\1+1 & 1-1 & 0-4\\1-11 & 0-8 & 0-0\end{bmatrix}$

$\Rightarrow$ -2Y = $\begin{bmatrix} -2 & -4 & 0 \\ 2 & 0 & -4 \\ -10 & -8 & 0 \end{bmatrix}$

$\Rightarrow$ -2Y = -2 $\begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$

$\Rightarrow$ Y = $\begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$

Hence,

X = $\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}$, Y = $\begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$

34. Given equation of curve is $3x^2 - y^2 = 8$.........(i)

Therefore,on differentiating both sides w.r.t. x, we get

$6x - 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{3x}{y}$

Equation of tangent at point (h, k) is

$y - k = \left(\frac{dy}{dx}\right)_{(h,k)}(x - h)$

$\Rightarrow \quad y - k = \frac{3h}{k}(x - h)$...............(ii)

Since, it passes through the point $\left(\frac{4}{3}, 0\right)$

$\therefore \quad 0 - k = \frac{3h}{k}\left(\frac{4}{3} - h\right) \Rightarrow -k^2 = 3h\frac{(4-3h)}{3}$

$\Rightarrow 3h^2 - k^2 - 4h = 0$.....................(iii)

Also, the point (h, k) satisfy the Eq. (i), so we get

$3h^2 - k^2 = 8$..........(iv)

Therefore, on solving Eqs. (iii) and (iv), we get

4h= 8

$\Rightarrow$ h = 2

Therefore,on putting h = 2 in Eq. (iv), we get

$3(2)^2 - k^2 = 8$

$\Rightarrow k^2 = 4$

$\Rightarrow k = \pm 2$

Therefore, putting the values of h and k in Eq. (ii), we get,

$y-(\pm 2)=\frac{3(2)}{\pm 2}(x-2)$

$\Rightarrow$y $\mp$2 = $\pm$3 (x - 2)

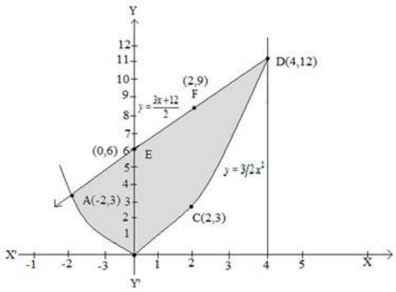
$\Rightarrow$y = ± 3(x - 2) ± 2

$\Rightarrow$ y = ± {3(x - 2) + 2}

$\Rightarrow$ y = ± (3x - 6 + 2)

$\Rightarrow$ y = ±(3x - 4)

Hence, y = - 3x + 4 and y = 3x - 4 are two required equations of tangent.

35. 

$4y=3x^2$......(1)

2y = 3x + 12......(2)

From (2), $y=\frac{3x+12}{2}$

Using this value of y in (1), we get,

x^2 - 6x - 8 = 0

$\Rightarrow$ (x + 2) (x - 4) = 0

$\Rightarrow$ x = -2, 4

From (2),

When, x = -2, y = 3

When, x = 4, y = 12

Thus, points of intersection are, (-2, 3) and (4, 12).

Area $=\int_{-2}^{4}\frac{3x+12}{2}dx-\int_{-2}^{4}\frac{3}{4}x^2dx$

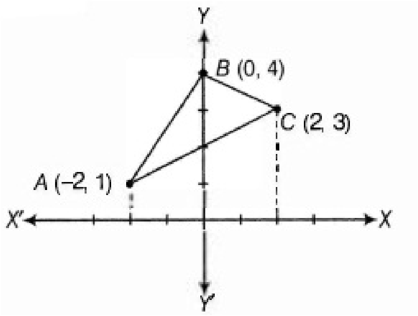
$=\frac{1}{2}[\frac{3x^2}{2}+12x]_{-2}^{4}-\frac{3}{4}[\frac{x^3}{3}]_{-2}^{4}$

$\frac{1}{2}[(24+48)-(6-24)]-\frac{1}{4}[64-(-8)]$

= 45 - 18 = 27 sq units.

OR

Let the vertices of a $\triangle$ABC are A(-2,1) , B(0,4) and C(2,3).



Equation of line AB is given by ,

y - 1 = $\frac{4-1}{0+2}$ (x + 2)

$\Rightarrow$y - 1 = $\frac{3}{2}$ (x + 2)

$\Rightarrow$y = $\frac{3}{2}$ (x + 2) + 1 = $\frac{3}{2}$ x + 4

Equation of line BC is given by ,

y - 4 = $\frac{3-4}{2-0}$ (x - 0)

$\Rightarrow$ y - 4 = -$\frac{1}{2}$ (x)

$\Rightarrow$ y - 4 = $-\frac{x}{2}$

$\Rightarrow$ y = $-\frac{x}{2}$ + 4

Equation of line AC is given by ,

y - 1 =$\frac{3-1}{2+2}$ (x + 2)

$\Rightarrow$y - 1 = $\frac{2}{4}$ (x + 2)

$\Rightarrow$y - 1 = $\frac{1}{2}$ (x + 2)

$\Rightarrow y = \frac{x}{2} + 2$

Required area = (Area under line segment AB) + (Area under line segment BC) - (Area under line segment AC)

Area under line segment AB = $\int_{-2}^{0} \left(\frac{3}{2}x + 4\right) dx$

Area under line segment BC = $\int_{0}^{2} \left(-\frac{x}{2} + 4\right) dx$

Area under line segment AC =$\int_{-2}^{2} \left(\frac{x}{2} + 2\right) dx$

$= \int_{-2}^{0} \left(\frac{3}{2}x + 4\right) dx + \int_{0}^{2} \left(-\frac{x}{2} + 4\right) dx - \int_{-2}^{2} \left(\frac{x}{2} + 2\right) dx$

$= \left[\frac{3}{2} \cdot \frac{x^2}{2} + 4x\right]_{-2}^{0} + \left[-\frac{x^2}{4} + 4x\right]_{0}^{2} - \left[\frac{x^2}{4} + 2x\right]_{-2}^{2}$

$= \left[0 - \left\{\frac{3}{4}(-2)^2 + 4(-2)\right\}\right] + \left[-\frac{(2)^2}{4} + 4(2)\right]$

$- \left[\left(\frac{(2)^2}{4} + 2.2\right) - \left(\frac{(-2)^2}{4} + 2(-2)\right)\right]$

= -(3 - 8) + (-1 + 8) - (1+ 4 - 1 + 4)

= 5 + 7 - 8

= 4 sq units.

36. Let the equation of the plane be

$\frac{x}{a} + \frac{y}{b} + \frac{z}{c}$ = 1 ...(i)

where a, b, c are variables.

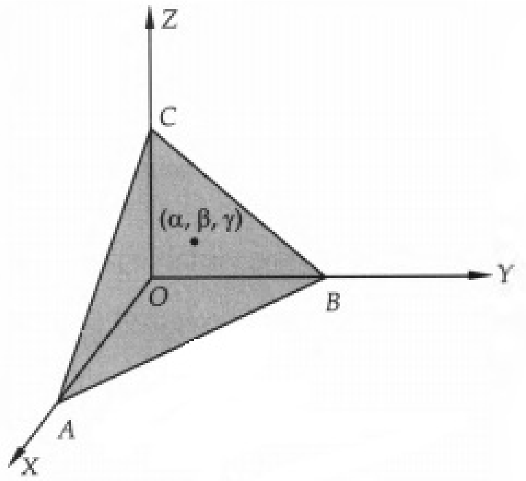
This meets X, Y and Z axes at A (a, 0, 0), B (0, b, 0) and C (0, 0, c).

Let (α, β, γ) be the coordinates of the centroid of triangle ABC. Then,

$\alpha = \frac{a+0+0}{3} = \frac{a}{3}, \beta = \frac{0+b+0}{3} = \frac{b}{3}, \gamma = \frac{0+0+c}{3} = \frac{c}{3}$ ...(ii)

The plane (i) is at a distance 3p from the origin.

∴ 3p = Length of perpendicular from (0,0,0) to the plane (i)



$$\Rightarrow \quad 3p = \frac{\left|\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1\right|}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}$$

$$\Rightarrow \quad 3p = \frac{1}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}$$

$$\Rightarrow \quad \frac{1}{9p^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \text{ ...(iii)}$$

From (ii), we have a = 3α, b = 3β and c = 3γ.

Substituting the values of a, b, c in (iii), we obtain

$$\frac{1}{9p^2} = \frac{1}{9\alpha^2} + \frac{1}{9\beta^2} + \frac{1}{9\gamma^2}$$

$$\Rightarrow \quad \frac{1}{p^2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$$

Hence, the locus of (α, β, γ) is

$\frac{1}{p^2} = \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}$ or, $x^{-2} + y^{-2} + z^{-2} = p^{-2}$.