

Mathematics

Single correct answer type:

1. The value of $\frac{1}{\sqrt{2} + \sqrt{3}}$ is _____

- (A) $\frac{1}{\sqrt{2}}$
- (B) $\frac{1}{\sqrt{3}}$
- (C) $\frac{1}{\sqrt{2} + \sqrt{3}}$
- (D) $\frac{1}{\sqrt{2} - \sqrt{3}}$
- (E) $\frac{1}{\sqrt{2} + \sqrt{3}}$

Solution: (B)

$$\frac{1}{\sqrt{2} + \sqrt{3}} \times \frac{\sqrt{2} - \sqrt{3}}{\sqrt{2} - \sqrt{3}} = \frac{\sqrt{2} - \sqrt{3}}{2 - 3} = \frac{\sqrt{2} - \sqrt{3}}{-1} = \sqrt{3} - \sqrt{2}$$

$$\frac{1}{\sqrt{2} + \sqrt{3}} = \sqrt{3} - \sqrt{2}$$

$$\frac{1}{\sqrt{2} + \sqrt{3}} = \sqrt{3} - \sqrt{2}$$

2. If the conjugate of a complex number is $a - bi$, then $a + bi$ is

- (A) $a - bi$
- (B) $a + bi$
- (C) $a - bi$
- (D) $a + bi$
- (E) $a - bi$

Solution: (D)

$$\frac{1}{\sqrt{2} + \sqrt{3}} = \sqrt{3} - \sqrt{2}$$

$$\frac{\frac{(-)}{(-)(-)}}{-(-)(-)} \frac{(-)}{(-)}$$

3. The value of (...../) is equal to

- (A) —
- (B)
- (C) —
- (D) $\sqrt{\sqrt{-}}$
- (E)

Solution: (E)

$$8(-9) = 8(-9) \rightarrow -9$$

$$[(-)] -$$

$$[+]$$

$$-(-) -$$

4. The modulus of is—

- (A)
- (B) $\sqrt{-}$
- (C)
- (D)
- (E)

Solution: (A)

$$\frac{(-)(-)}{(-)(-)} =$$

Modulus of $\frac{1+i}{1-i}$ is —
 $\sqrt{\frac{1^2+1^2}{1^2+1^2}}$ —

5. If $z = \frac{1+i}{1-i}$, then \overline{z} is equal to

- (A) $\frac{1+i}{1-i}$
- (B) $\frac{1-i}{1+i}$
- (C) $\frac{1-i}{1-i}$
- (D) $\frac{1+i}{1+i}$
- (E) $\frac{1-i}{1-i}$

Solution: (B)

$$z = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)^2}{1-i^2} = \frac{1+i^2+2i}{1-(-1)} = \frac{1-1+2i}{1+1} = \frac{2i}{2} = i$$

$\overline{z} = \overline{i} = -i$
 $\frac{1-i}{1+i}$

()
 ()

6. If a and b are real numbers and $z = \frac{a+bi}{1-i}$, then \overline{z} is equal to

- (A) $\frac{a+bi}{1-i}$
- (B) $\frac{a-bi}{1-i}$
- (C) $\frac{a+bi}{1+i}$
- (D) $\frac{a-bi}{1+i}$
- (E) $\frac{a+bi}{1-i}$

Solution: (E)

Given, $z = \frac{a+bi}{1-i}$

So, $\overline{z} = \frac{a-bi}{1+i}$ (i)

Then, $\overline{z} = \frac{a-bi}{1+i} \times \frac{1-i}{1-i} = \frac{(a-bi)(1-i)}{1-i^2} = \frac{a-bi-ai+bi^2}{1-(-1)} = \frac{a-bi-ai-b}{2} = \frac{a-b-i(a+b)}{2}$

$\frac{a-b-i(a+b)}{2}$

From Equation (i), we get

()

7. If α, β are the roots of the equation $x^2 - 2x + 1 = 0$, then the equation having α^2, β^2 as its roots is

- (A) $x^2 - 2x + 1 = 0$
 (B) $x^2 - 4x + 2 = 0$
 (C) $x^2 - 2x + 2 = 0$
 (D) $x^2 - 4x + 4 = 0$
 (E) $x^2 - 2x + 4 = 0$

Solution: (E)

Given, $\alpha + \beta = 2$ and $\alpha\beta = 1$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 2^2 - 2(1) = 4 - 2 = 2$$

Similarly, $\alpha^2\beta^2 = (\alpha\beta)^2 = 1^2 = 1$

$$\alpha^2 + \beta^2 = 2 \quad \alpha^2\beta^2 = 1$$

Now, addition of roots

$$\frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{2}{1} \Rightarrow \alpha^2 + \beta^2 = 2$$

Multiplication of roots $\alpha^2\beta^2 = 1$

$$\therefore \text{The equation is } x^2 - 2x + 4 = 0$$

8. The focus of the parabola $y^2 = 4x$ is

- (A) $(-1, 0)$
 (B) $(1, 0)$
 (C) $(0, 1)$
 (D) $(0, -1)$
 (E) $(1, 1)$

Solution: (D)

$$y^2 = 4x \Rightarrow y^2 = 4 \cdot 1 \cdot x$$

$$4a = 4 \Rightarrow a = 1$$

$$\therefore \text{Focus is } (0, -1)$$

Let(i)
 Here, () ()
 Vertices () ()
 Equation (i) comparing on

Focus - / . - /

9. If () is an increasing function and if $\frac{()}{()} \rightarrow$ then $\frac{()}{()}$
 is equal to
 (A) -
 (B) -
 (C)
 (D)
 (E)

Solution: (E)
 Given () is an increasing function.
 And $\frac{()}{()}$
 So, () ()
 () constant.
 Therefore, $\frac{()}{()}$

10. If is differentiable at , then $\frac{() ()}{}$ is
 (A) ()
 (B) () ()
 (C) () ()
 (D) () ()
 (E) () ()

Solution: (C)
 $\frac{() ()}{}$

By Rule,
 $\frac{() ()}{() ()}$

11. Eccentricity of the ellipse is

(A) $\sqrt{-}$

(B) $\frac{\sqrt{a^2 - b^2}}{a}$

(C) $\frac{\sqrt{a^2 - b^2}}{b}$

(D) $\frac{\sqrt{a^2 - b^2}}{c}$

(E) $\frac{\sqrt{a^2 - b^2}}{d}$

Solution: (A)

Equation is an ellipse.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

where

Eccentricity $e = \frac{c}{a}$

$$\sqrt{a^2 - b^2} = \sqrt{a^2 - \frac{a^2(c^2 - b^2)}{a^2}} = \sqrt{a^2 - c^2 + b^2} = \sqrt{b^2} = b$$

12. The focus of the parabola $y^2 = 4ax$ is (A)

(A) $(a, 0)$

(B) $(-a, 0)$

(C) $(0, a)$

(D) $(0, -a)$

(E) $(0, 0)$

Solution: (A)

Given, $y^2 = 4ax$

Here, $a = a$

Vertices $(0, 0)$

Comparing Equation (i) from

Focus $(a, 0)$

$(a, 0)$

13. Which of the following is the equation of a hyperbola?

(A) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(B) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(D) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$

(C) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = -1$

(E) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

Solution: (D)

$$(\quad -) (\quad) \quad -$$

$$(\quad -) (\quad) \quad \text{---}$$

$$\frac{-}{\text{---}} / \frac{(\quad)}{\text{---}}$$

It is hyperbola equation.

14. Let (\quad) , where are constants and If $(\quad) (\quad)$ and (\quad) , then the other root of is

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (A)

(\quad)

(\quad)

..... (i)

One root is

and $(\quad) (\quad)$

(\quad)

.....(ii)

Equation (ii) - Equation (i), we get

Then, equation is

Roots

Sum of roots $-$

So, another root

15. Let satisfy $(\quad) (\quad) (\quad)$ for all real numbers and . If (\quad) , then

$- /$

(A)

(B) $-$

(C) $-$

(D)

(E) Solution:

(B) Given

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \dots + (-1)^{n-1} \binom{n}{n-1} + (-1)^n \binom{n}{n} \dots (i)$$

On taking

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \dots + (-1)^{n-1} \binom{n}{n-1} + (-1)^n \binom{n}{n}$$

Now, $x=1$, then from equation (i)

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \dots + (-1)^{n-1} \binom{n}{n-1} + (-1)^n \binom{n}{n} = 0$$

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \dots + (-1)^{n-1} \binom{n}{n-1} + (-1)^n \binom{n}{n} = 0$$

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \dots + (-1)^{n-1} \binom{n}{n-1} + (-1)^n \binom{n}{n} = 0$$

On putting the value of x ,

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \dots + (-1)^{n-1} \binom{n}{n-1} + (-1)^n \binom{n}{n} = 0$$

16. Sum of last coefficients in the binomial expansion of $(1-x)^n$ is

- (A) $2^n - 1$
- (B) 2^n
- (C) $2^n - 1$
- (D) 2^n
- (E) $2^n - 1$

Solution: (C)

We have, $(1-x)^n$

Sum of last coefficient of the binomial expansion

We know that,

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \dots + (-1)^{n-1} \binom{n}{n-1} + (-1)^n \binom{n}{n} = 0$$

Sum of last coefficient of the binomial expansion $(1-x)^n$ is $2^n - 1$.

$$17. (\sqrt{x})^n (\sqrt{x})^n = x^n$$

- (A) \sqrt{x}
- (B) \sqrt{x}
- (C) \sqrt{x}
- (D) \sqrt{x}
- (E) \sqrt{x}

Solution: (D)

Take,

$$(1-x)^n = \binom{n}{0} - \binom{n}{1}x + \binom{n}{2}x^2 - \dots + (-1)^{n-1} \binom{n}{n-1}x^{n-1} + (-1)^n \binom{n}{n}x^n$$

(\quad) (i)
 Similarly, (\quad) (ii)

On subtracting Equation (ii) from Equation (i), we get

$$\begin{array}{r}
 (\quad)(\quad)(\quad) \\
 \text{Now, putting } \sqrt{\quad} \text{ and } \sqrt{\quad} \\
 (\sqrt{\quad}\sqrt{\quad})(\sqrt{\quad}\sqrt{\quad})\sqrt{\quad}\sqrt{\quad}0(\sqrt{\quad})(\sqrt{\quad})1 \\
 \sqrt{\quad}(\quad)\sqrt{\quad}\sqrt{\quad}
 \end{array}$$

18. Three players and play a game. The probability that and will finish the game are respectively and . The probability that the game is finished is.

- (A) -
- (B)
- (C) -
- (D) -
- (E) -

Solution: (D)

We have, $(\quad)(\quad)$ and (\quad) - -

Required probability

$$\begin{array}{r}
 (\quad)(\quad)(\quad) \\
 - - - \\
 - -
 \end{array}$$

19. If and , then $|+|$ is —

- (A)
- (B) $\sqrt{\quad}$ -
- (C)
- (D) $\sqrt{\quad}$ -
- (E)

Solution: (B)

Given, and

Then, $|+|+|$ —

$$\begin{array}{r}
 (\quad) \\
 | \quad | \quad | \\
 (\quad)(\quad)
 \end{array}$$

$$\frac{(\quad)}{\frac{(\quad)}{\frac{(\quad)}{\sqrt{(\quad)(\quad)}}}}$$

20. If $(\quad)\sqrt{\quad}$, then (\quad) is equal to

- (A)
- (B)
- (C) -
- (D)
- (E)

Solution: (A)

Given, $(\quad)\sqrt{\quad}$

Now, $(\quad)\sqrt{\quad}$

$$\sqrt{\quad}$$

$$\sqrt{\quad} \quad 4 \quad \quad \quad 5$$

21. The value of is

- (A) $\sqrt{\quad}$
- (B) $\frac{\quad}{\sqrt{\quad}}$
- (C) $\frac{\sqrt{\quad}}{\quad}$
- (D) $\frac{\quad}{\sqrt{\quad}}$
- (E) -

Solution: (A)

$$\frac{\quad}{\quad} \cdot \frac{\quad}{\sqrt{\quad}}$$

22. The sum of odd integers from to is

- (A) ()
- (B) ()
- (C) ()
- (D) ()
- (E) ()

Solution: (C)

Sum of odd integers ()

23. If _____, then () is equal to

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (C)

$$\frac{\frac{1}{2} + \frac{1}{2}}{\frac{1}{2} + \frac{1}{2}} = \frac{1}{1} = 1$$

24. The foci of the hyperbola are

- (A) - $\frac{\sqrt{5}}{2}$ /
- (B) - $\frac{\sqrt{5}}{2}$ /
- (C) - $\frac{\sqrt{5}}{2}$ /
- (D) - $\frac{\sqrt{5}}{2}$ /
- (E) - $\frac{\sqrt{5}}{2}$ /

Solution: (A)

() ()

$$\frac{\begin{pmatrix} \\ \end{pmatrix} \begin{pmatrix} \\ \end{pmatrix}}{\begin{pmatrix} \\ \end{pmatrix} \begin{pmatrix} \\ \end{pmatrix}} \dots\dots\dots (i)$$

We know that,..... (ii)

Foci ()

On comparing Equations (i) and (ii), we get

$$\sqrt{} \quad \sqrt{} \quad -$$

$$\begin{array}{c} \\ \sqrt{} \quad - \\ \\ \sqrt{} \\ \\ \sqrt{} \\ \\ \sqrt{} \end{array}$$

Hence, - $\frac{}{}$ /

25. If the sum of the coefficients in the expansions of () is zero, then is equal to

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (B)

()

For sum of coefficients put

()

$$*() +$$

26. The mean deviation of the data from the mean is

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (C)

Mean of the given data is

– _____

The deviations of the respective observations from the mean \bar{x} , i.e. $x_i - \bar{x}$ are

The absolute values of the deviations, i.e. $|x_i - \bar{x}|$ are

The required mean deviation about the mean is

$$\left(\frac{\sum |x_i - \bar{x}|}{n} \right)$$

—

27. The mean and variance of a binomial distribution are and respectively. What is ()?

(A) –

(B) —

(C) –

(D) –

(E) –

Solution: (B)

Let n and p be the parameters of the binomial distribution.

Mean and variance

and

– and

Required probability ()

$$\binom{n}{r} p^r (1-p)^{n-r} - \binom{n}{r} p^r (1-p)^{n-r}$$

— —

28. The number of diagonals of a polygon with sides is

(A)

(B)

(C)

(D)

(E)

Solution: (A)

The number of diagonals of a polygon with sides is

29. In a class, of students study Maths and Science and of students study Maths. What is the probability of a students studying Science given the student is already studying Maths?

(A) -

(B) -

(C) -

(D) -

(E) -

Solution: (C)

Probability of Maths and Science students — -

Probability of maths students — -

$P(\text{Science/Maths}) = \frac{(\quad)}{(\quad)} = \frac{\quad}{\quad}$

30. The eccentricity of the conic is

(A)

(B) \sqrt{e}

(C) -

(D) \sqrt{e}

(E)

Solution: (B)

$(\quad) = (\quad - \quad - \quad)$

$(\quad) = (\quad - \quad) -$

$(\quad) = (\quad - \quad) -$

$\frac{(\quad)}{\quad} = \frac{\quad}{\quad}$

$$\text{Ellipse } \sqrt{\frac{a^2}{b^2} - \frac{y^2}{b^2}} = \sqrt{\frac{a^2}{b^2} - \frac{y^2}{b^2}}$$

$$\sqrt{\frac{a^2}{b^2} - \frac{y^2}{b^2}} = \sqrt{\frac{a^2}{b^2} - \frac{y^2}{b^2}}$$

31. If the mean of a set of observations is \bar{x} , then the mean of $\frac{1}{\bar{x}}$ is

- (A) $\frac{1}{\bar{x}}$
- (B) $\frac{\bar{x}}{n}$
- (C) $\frac{1}{n\bar{x}}$
- (D) $\frac{n}{\bar{x}}$
- (E) $\frac{\bar{x}}{n^2}$

Solution: (C)

Mean of a set of observations

Then, according to question,

$$\frac{1}{\bar{x}} = \frac{1}{\frac{1}{n} \sum_{i=1}^n x_i} = \frac{n}{\sum_{i=1}^n x_i}$$

32. A letter is taken at random from the word "STATISTICS" and another letter is taken at random from the word "ASSISTANT". The probability that they are same letters is

- (A) $\frac{1}{10}$
- (B) $\frac{1}{11}$
- (C) $\frac{1}{12}$
- (D) $\frac{1}{13}$
- (E) $\frac{1}{14}$

Solution: (C) Probability of take a random from the word STATISTICS

Probability of take a random from the word ASSISTANT

The probability is that they are same letters

$$\frac{1}{12} = \frac{1}{12}$$

33. If α and β are the roots of the equation $x^2 - 5x + 6 = 0$, then (A)

(B) $\alpha + \beta = 5$

(C) $\alpha\beta = 6$ (D)

(E)

Solution: (A)

Roots are α and β

$$\alpha + \beta = 5 \quad \text{..... (i)}$$

and $\alpha\beta = 6$ (ii)

$$\alpha = \frac{5 \pm \sqrt{5^2 - 4 \cdot 6}}{2} = \frac{5 \pm \sqrt{1}}{2}$$

Using Equation (i), we get

$$\alpha = \frac{5 + 1}{2} = 3$$

34. If the sides of triangle are 3, 4 and 5. Then the area (in sq cm) of triangle is

(A) 6

(B) $\frac{1}{2}$

(C) 3

(D) $\frac{1}{2}$

(E) $\frac{1}{2}$

Solution: (E)

Given, triangle of sides 3, 4 and 5

$$a = 3, b = 4, c = 5$$

Then, area of triangle

$$= \frac{1}{2} \times 3 \times 4 = 6$$

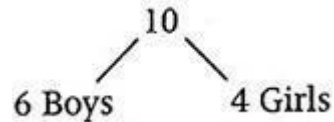
$$= \frac{1}{2} \times 3 \times 4 = 6$$

$$= \frac{1}{2} \times 3 \times 4 = 6$$

35. In a group of boys and girls, a team consisting of four children is formed such that the team has atleast one boy. The number of ways of forming a team like this is

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (B)



The team has atleast one boy
 = Total case – No anyone boy

36. A password is set with distinct letters from the word LOGARITHMS. How many such passwords can be formed?

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (B)

LOGARITHMS letters are .

A password is set with distinct letters

37. If is divided by , the remainder obtained is

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (B) We know that,
 , where is a positive integer.

() ()
 () () () ()
 theorem)

(by binomial

[]

()
 On multiplying both sides by , we get
 ()

Hence, the required remainder is .

38. A quadratic equation , with distinct coefficients is formed. It
 are chosen from the numbers then the probability that the equation has real roots
 is

- (A) –
- (B) –
- (C) –
- (D) –
- (E) –

Solution: (A)

Total number of ways of assigning values to

Now, for quadratic equation to have real roots
 possible only when or

This is

Required probability – –

39. _____ is equal to

- (A) –
- (B) –
- (C) —
- (D) –
- (E) –

Solution: (D)

$$\begin{array}{r} \text{_____} \\ 0 \quad - \quad - \quad -1 \\ \hline 0 \quad - \quad -1 \end{array}$$

On putting , we get

$$\begin{array}{r} , - \\ \hline , - \end{array}$$

40. The minimum value of () * + is

- (A) –

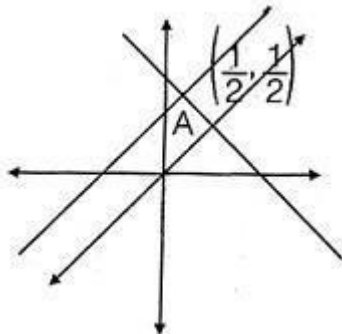
- (B) –
 (C)
 (D)
 (E)

Solution: (B)

we have,

$()^* +$

The graph of $()$ is



Clearly from graph minimum value of $()$ at point $_{-} - /$.

Minimum value of $()$ is $_{-}$

41. The equations of the asymptotes of the hyperbola

are

- (A)
 (B)
 (C)
 (D)
 (E)

Solution: (B)

We have equation of hyperbola is

$$(\quad) (\quad)$$

We know that asymptote of hyperbola
 is and .

Asymptote of hyperbola

$() ()$ is

42. If $()$ then $()$ is equal to

- (A)
 (B)
 (C) $()$
 (D)

(E)

Solution: (C)

()

() () 0 () 1 —

[() ()]

43. The standard deviation of the data is

(A) $\sqrt{\quad}$

(B) —

(C) $\sqrt{\quad}$

(D) —

(E)

Solution: (A)

Given data Mean of the given data (

—

The deviation of the respective data from the mean i.e. (are

() ()

(

Σ (

Standard deviation ()

$\sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$ —

44.44. —

(A) —

(B) —

(C)

(D)

(E)

Solution: (A)

$$\{ \text{---} \}$$

$$\left[\overline{\{-\overline{\quad}\}} \quad \overline{\quad} \quad \overline{\quad} \quad \overline{\{-\overline{\quad}\}} \right]$$

$$\begin{array}{r} 45.4 \\ 5 \\ \cdot \end{array} \quad \begin{array}{r} (\sqrt{} \\ \hline \end{array}$$

- (A)
(B)
(C) —
(D)
(E) —

Solution: (D)

$$\sqrt{\quad}$$

Using Rule,

$$\sqrt{\quad}$$

Using limit, we get

$$\sqrt{}$$

46. Let f and g be differentiable functions such that $f'(x) = g(x)$ and $g'(x) = f(x)$. If $f(0) = 1$ and $g(0) = 0$, then $f(x)$ is
- (A) $\cos x$
(B) $\sin x$
(C) e^x
(D) e^{-x}
(E) $\cos x$

Solution: (B)

$$() (())$$

$$\begin{array}{ccccccc} & (&) & (&) & (&) \\ (&) & & (&) & (&) & [& (&) &] \\ & & & & & & & & (&) \end{array}$$

47. $\frac{\sqrt{-}}{() } \quad \frac{}{() }$

(A)

(B) $\sqrt{}$

(C)

(D)

(E)

Solution: (D)

$$\begin{array}{r} \sqrt{-} \\ \hline () () \\ \sqrt{() ()} \\ \hline \sqrt{-} () () \\ 6 \quad () \quad \frac{()}{7} \\ \hline () () () \\ , \\ () \\ \hline () \\ \hline , () - \\ \hline \end{array}$$

48. A poison variate satisfies

$() () ()$ is equal to

(A) —

(B) —

(C) -

(D) -

(E) —

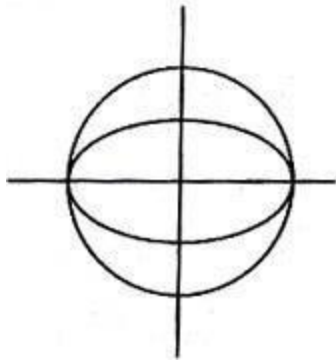
Solution: (A)

Given that,

$() ()$

Solution: (B) –
 Given, $\sqrt{\quad}$ –

$$\frac{\quad}{\quad} = \quad \dots\dots(i)$$



$$(\quad) = \quad$$

$$\frac{\quad}{\quad} = \quad \text{[using Equation (i)]}$$

$$(\quad) = \quad$$

51. If the vectors $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{e}$ and $\hat{a} + \hat{b} + \hat{c}$ are coplanar, then \hat{d} is equal to

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (C)

Given vectors $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{e}$ and $\hat{a} + \hat{b} + \hat{c}$ are coplanar.

Then, $|\quad|$

$$(\quad)(\quad)(\quad)$$

52. Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ and $\vec{a} + \vec{b} + \vec{c}$. Then, the area of the parallelogram with diagonals $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$ is

- (A) $\sqrt{\quad}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{1}{\sqrt{2}}$

(E) $\frac{1}{\sqrt{2}}$

Solution: (A) Given,

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{b} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{c} = \hat{i} + \hat{j} + \hat{k}$$

Diagonals: $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$

Area of parallelogram = $\frac{1}{2} |\vec{a} + \vec{b}| |\vec{b} + \vec{c}| \sin \theta$... (i)

$$|\vec{a} + \vec{b}| = \sqrt{(\hat{i} + \hat{j} + \hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k})} = \sqrt{3}$$

From Equation (i),

$$|\vec{a} + \vec{b}| |\vec{b} + \vec{c}| \sin \theta = 2$$

$$|\vec{a} + \vec{b}| |\vec{b} + \vec{c}| \sin \theta = 2$$

$$|\vec{a} + \vec{b}| |\vec{b} + \vec{c}| \sin \theta = 2$$

$$\sin \theta = \frac{2}{\sqrt{3} \sqrt{3}} = \frac{2}{3}$$

53. If $|\vec{a}| = |\vec{b}| = |\vec{c}|$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is equal to

(A)

(B)

(C)

(D)

(E)

Solution: (D)

$$|\vec{a}| = |\vec{b}| = |\vec{c}|, \quad \vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$$(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = (\vec{c}) \cdot (\vec{c})$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{c}|^2$$

$$2|\vec{a}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{c}|^2$$

54. If $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$, then the angle between \vec{a} and \vec{b} is equal to

- (A) –
 (B) —
 (C) –
 (D)
 (E)

Solution: (A)

$$\begin{aligned} & \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta \\ & \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos 90^\circ \\ & \vec{a} \cdot \vec{b} = 0 \end{aligned}$$

55. If the vectors \vec{a} , \vec{b} , \vec{c} and \vec{d} are mutually orthogonal, then $\vec{a} \cdot \vec{b} + \vec{c} \cdot \vec{d}$ is equal to

- (A)
 (B)
 (C)
 (D)
 (E)

Solution: (B)

Given,

$$\vec{a} \cdot \vec{b} = 0, \vec{a} \cdot \vec{c} = 0, \vec{a} \cdot \vec{d} = 0, \vec{b} \cdot \vec{c} = 0, \vec{b} \cdot \vec{d} = 0, \vec{c} \cdot \vec{d} = 0$$

Vectors are mutually orthogonal, so

$$\vec{a} \cdot \vec{b} = 0, \vec{a} \cdot \vec{c} = 0, \vec{a} \cdot \vec{d} = 0, \vec{b} \cdot \vec{c} = 0, \vec{b} \cdot \vec{d} = 0, \vec{c} \cdot \vec{d} = 0$$

$$\dots (i)$$

$$\dots (ii)$$

On solving Equations (i) and (ii), we get

56. The solution of $\vec{a} \cdot \vec{b} = 0$ and $\vec{c} \cdot \vec{d} = 0$ are

- (A)
 (B)
 (C)
 (D)
 (E)

Solution: (D)

We have, $x^2 - 3x + 2 = 0$

Let $x = \frac{p}{q}$

$$\frac{p^2}{q^2} - 3\frac{p}{q} + 2 = 0$$

$$p^2 - 3pq + 2q^2 = 0$$

$$(p - q)(p - 2q) = 0$$

$$p = q \text{ or } p = 2q$$

$$\frac{p}{q} = 1 \text{ or } \frac{p}{q} = 2$$

$$x = 1 \text{ or } x = 2$$

57. If the equations $x^2 - 3x + 2 = 0$ and $x^2 - 5x + 6 = 0$ have a real common root, then the value of is equal to

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (C)

Given equations,

$x^2 - 3x + 2 = 0$ is common root, so satisfied both equations.

$$(x - 1)(x - 2) = 0$$

58. If $\frac{p}{q} = \frac{r}{s}$, then the value of is equal to

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (A)

Given,

$$\frac{p}{q} = \frac{r}{s}$$

$$(ps - qr) = 0 \quad \dots (i)$$

On squaring both sides,

$$(ps - qr)^2 = 0$$

()

..... (ii)

From Equations (ii) and (i), we get

59. Two dice of different colours are thrown at a time. The probability that the sum is either or is

(A) —

(B) —

(C) —

(D) —

(E) —

Solution: (B)

Probability of sum of

() () () () () () and probability of sum of

() ()

() — —

— —

60. — — — — — Is equal to

(A) —

(B) —

(C) —

(D) —

(E) —

Solution: (A)

— — — — —

— [— — — — —]

— [— — — — —]

— —

61. The order and degree of the differential equation () () () is

- (A) and
- (B) and
- (C) and
- (D) and
- (E) and

Solution: (A)

The given differential equation is () () ()

Clearly, its order is and degree is . Hence, option and is correct.

62. $\int ||$ is equal to

- (A)
- (B)
- (C)
- (D)
- (E) -

Solution: (D) Given that,

$$\int ||$$

$$\int \int$$

$$6_7 \quad 6_7$$

$$() ()$$

63. \int _____ is equal to

- (A)
- (B) -
- (C) —
- (D) -
- (E) —

Solution: (B)

Given that,

$$\int \text{—————}$$

$$\int \text{—} \quad , ()-$$

$$, () ()- \quad \text{—}$$

64. If $f(x)$ and $g(x)$, then $h(x)$ is

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (E)

We know, $f(x)$ -

$$f(x)$$

$$g(x)$$

$$h(x)$$

$$f(x)$$

$$f(x) \dots \dots \dots (i)$$

Now, $f(x)$

$$f(x) f(x)$$

$$f(x) \quad \text{[from Equation (i)]}$$

$$f(x)$$

65. $f(x)$

- (A) —
- (B) —
- (C) —
- (D) —
- (E) —

Solution: (A)

Given that,

$$\begin{aligned} f(x) &= f(x) \\ f(x) &= f(x) \end{aligned}$$

66. The remainder when is divided by is

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (A)

()

() ()

When divided by , then remainder ()

Hence, remainder

67. The coefficient of in the expansion of () is (A)

- (B)
- (C)
- (D)
- (E)

Solution: (B)

For the coefficient of ,

Coefficient of

68. The maximum value of

- / is

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (C)

- / -

, -

$$6 \frac{\sqrt{-}}{\sqrt{-}} 7$$

Let $\sqrt{-}$

Then expression becomes,

Maximum value of this type of expression is equal to, ----- Maximum value

After putting values of and , we get , --Max value
Max value

69. The area of the triangle in the complex plane formed by and is

- (A) $||$
- (B) $| |$
- (C) $\downarrow |$
- (D) $\downarrow |$
- (E) $| |$

Solution: (C)

Let $()$ $()$ and . If is the area of triangle formed by and , then

$$- | |$$

Applying

$$- | |$$

$$- () | | -$$

70. Let $() ()$ be a differentiable function. If is even, then $()$ is equal to

- (A)
- (B)
- (C)
- (D)
- (E) -

Solution: (C)

$$\begin{aligned} & () () \\ & () () () () \\ & () () \end{aligned}$$

71. The coordinate of the point dividing internally the line joining the points () and () in the ratio is

- (A) ()
- (B) ()
- (C) $\frac{1}{2}$ - $\frac{1}{2}$
- (D) $\frac{1}{2}$ - $\frac{1}{2}$
- (E) ()

Solution: (C)
Here, and

$$\frac{x_1 + \lambda x_2}{1 + \lambda} = \frac{1 + \lambda \cdot 2}{1 + \lambda}$$

and $\frac{y_1 + \lambda y_2}{1 + \lambda} = \frac{1 + \lambda \cdot 2}{1 + \lambda}$

$$\frac{1 + 2\lambda}{1 + \lambda} = \frac{1 + 2\lambda}{1 + \lambda}$$

$$(1) (1 - 2)$$

72. The area of the triangle formed by the points () () () is

- (A)
- (B)
- (C)
- (D)
- (E) ()

Solution: (D)

Area of triangle = $\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$$= \frac{1}{2} |1(2 - 1) + 2(1 - 1) + 1(1 - 2)|$$

, -

$$= \frac{1}{2} |1 - 1 - 1|$$

73. If () is equidistant from () and (), then

- (A)
- (B)
- (C)

- (D)
(E)

Solution: (D)

According to question,

$$*() + *() +$$

$$*() + *() +$$

$$\begin{pmatrix} (&) \\ (&) \end{pmatrix} \quad \begin{pmatrix} (&) \\ (&) \end{pmatrix} \quad \begin{pmatrix} (&) \\ (&) \end{pmatrix} \quad \begin{pmatrix} (&) \\ (&) \end{pmatrix}$$

By solving, we get

74. The equation of the line passing through () and parallel to the line – – is

- (A) – –
(B) – –
(C) – –
(D) – –
(E) – –

Solution: (B)

Given equation of line is

$$- - \dots\dots(i)$$

So, equation of line passing through () and parallel to Equation (i) is

$$-()$$

$$\begin{aligned} - & - \\ - & - \end{aligned}$$

75. If the points () () and () enclose a triangle of area units, then the centroid of the triangle is equal to

- (A) ()
(B) ()
(C) ()
(D) $(\sqrt{7}) -$
(E) ()

Solution: (B)

Given, that Area of triangle

$$-\frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$(\frac{1}{2}) (\frac{1}{2}) (\frac{1}{2})$$

Now, centroid of the given triangle will be

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

When centroid is $\left(\frac{1}{3}, \frac{1}{3} \right)$

76. The area of a triangle is 1 unit. Two of its vertices are $(1, 1)$ and $(2, 2)$. The third vertex lies on the line $x + y = 3$. The coordinates of the third vertex can be

(A) $\left(\frac{1}{2}, \frac{1}{2} \right)$

(B) $\left(\frac{1}{3}, \frac{1}{3} \right)$

(C) $\left(\frac{1}{2}, \frac{1}{2} \right)$

(D) $\left(\frac{1}{3}, \frac{1}{3} \right)$

(E) $\left(\frac{1}{2}, \frac{1}{2} \right)$

Solution: (C)

Let the coordinates of third vertex be (x, y) . Given that, area of a triangle

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$(\frac{1}{2}) (\frac{1}{2}) (\frac{1}{2})$$

.....(i)

Since, third vertex lies on the line $x + y = 3$(ii)

By solving Equations (i) and (ii), we get

$$x = \frac{1}{2}, y = \frac{1}{2}$$

77. If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ represents a pair of straight lines, then $\frac{a^2}{b^2}$ is equal to

(A) $\frac{a^2}{b^2}$

(B) $\frac{b^2}{a^2}$

- (C)
(D)
(E)

Solution: (B)

Given equation of pair of straight lines is

Since, the necessary and sufficient condition for pair of straight lines is

$$| \quad |$$

$$| \quad |$$

$$()()$$

78. If θ is the angle between the pair of straight lines, then $\tan \theta$ is equal to

- (A) —
(B) —
(C) —
(D) —
(E) —

Solution: (C)

Given equation of straight line is

$$\frac{x^2 + y^2 + 2x + 2y + 1}{\sqrt{2}} = 0$$

$$\frac{x^2 + y^2 + 2x + 2y + 1}{\sqrt{2}} = 0 \quad \text{---} \quad \sqrt{2} \quad \text{---} \quad \text{---} \quad \text{---}$$

$$\text{---}$$

79. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $(\vec{a} \cdot \vec{b}) \vec{c} = (\vec{a} \cdot \vec{c}) \vec{b}$, then

- (A)
(B)
(C)
(D)

(E)

Solution: (C)

Given that,

$$\begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \end{pmatrix} \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \end{pmatrix} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \end{pmatrix} \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \end{pmatrix}$$

By equating the coefficients of \hat{i} and \hat{j} , we get

$$\dots(i)$$

$$\dots(ii)$$

$$\dots(iii)$$

By solving Equations (i), (ii) and (iii), we get

80.

(A) $\frac{\sqrt{2}}{\sqrt{2}}$

(B) $\frac{\sqrt{2}}{\sqrt{2}}$

(C) $\frac{\sqrt{2}}{\sqrt{2}}$

(D) $\frac{\sqrt{2}}{\sqrt{2}}$

(E) $\frac{(\sqrt{2})}{\sqrt{2}}$

Solution: (A)

$$()$$

$$\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}}$$

81. If \vec{a} and \vec{b} are collinear and \vec{c} is equal to

(A) $(\hat{i} \hat{j} \hat{k})$

(B) $\hat{i} \hat{j} \hat{k}$

(C) $\hat{i} \hat{j} \hat{k}$

(D) $\hat{i} \hat{j} \hat{k}$

(E) $\hat{i} \hat{j} \hat{k}$

Solution: (B)

Since, \vec{a} and \vec{b} are collinear vector. Therefore,

$$\dots(i)$$

$$\vec{a} \cdot \vec{a}$$

$$\frac{|\vec{a}| |\vec{a}| |\vec{a}|}{|\vec{a}|} \sqrt{\quad}$$

$$|\vec{a}| \quad \text{---}$$

By Equation (i),

$$\vec{a} \cdot \vec{a}$$

$$|\vec{a}| |\vec{a}| |\vec{a}|$$

$$|\vec{a}|$$

$$|\vec{a}| \quad \text{---}$$

$$\vec{a} \cdot \vec{a} = (|\vec{a}|^2)$$

$$\vec{a} \cdot \vec{a} = (|\vec{a}|^2)$$

82. If $|\vec{a}| = |\vec{b}|$ and $\vec{a} \cdot \vec{b}$, then $|\vec{a} + \vec{b}|$ is equal to

(A)

(B) $\sqrt{2} \quad \text{---}$

(C) $\sqrt{2} \quad \text{---}$

(D) $\sqrt{2} \quad \text{---}$

(E) $\sqrt{2} \quad \text{---}$

Solution: (E)

Given that,

$$|\vec{a}| = |\vec{b}| \text{ and } \vec{a} \cdot \vec{b}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$\quad \quad \quad \text{---} \quad \text{---}$$

$$\quad \quad \quad \text{---}$$

$$\quad \quad \quad \text{---}$$

$$\sqrt{\quad}$$

$$|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$$

$$\sqrt{\quad}$$

$$\quad \sqrt{\quad}$$

83. If , then is equal to

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (B)

Given that,

$$\frac{(\quad)}{(\quad)} = \frac{(\quad)}{(\quad)}$$

84. Distance between two parallel lines is

- (A)
- (B) $\sqrt{\quad}$
- (C) $\sqrt{\quad}$
- (D) \quad
- (E) $\sqrt{\quad}$

Solution: (C)

Distance between parallel lines

or and

or is

$$\frac{\sqrt{(\quad)} - \sqrt{(\quad)}}{\sqrt{\quad}} = \sqrt{\quad}$$

85. $(\quad)(\quad)(\quad)$

- (A) (B)
- (C)
- (D)
- (E)

Solution: (C)

$(\quad)(\quad)(\quad)$

, -

86. The coefficient of in the expansion of $(\quad)(\quad)$ is

- (A)
- (B)
- (C)

- (D)
(E)

Solution: (D)

() ()

()

()

() ()

Coefficient of

87. The equation of the circle with centre () which passes through () is

(A)

(B)

(C)

(D)

(E)

Solution: (B)

Radius of circle is $\sqrt{(\quad)^2 + (\quad)^2}$ So, equation of circle is

() ()

88. The point in the plane which is equidistant from () () and () is

(A) ()

(B) ()

(C) ()

(D) ()

(E) ()

Solution: (D)

Let the points are () () and ().

We know that -coordinate of every point on -plane is zero so let () be a point on -plane such that

Now,

$$(\quad)(\quad)(\quad)(\quad)(\quad)(\quad)$$

.....(i)

and,

$$(\quad)(\quad)(\quad)(\quad)(\quad)(\quad)$$

..... (ii)

Putting in Equation (i), we get

Hence, the required point is ()

89. Let be such that (n) and $(n)(n)$ for all natural numbers n and n . If $\sum_{n=1}^{\infty} (n)(n)$, then is equal to

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (A)

We have,

(1) and $(1)(1)$

Now, $(2)(2)(2)(2)$

$(3)(3)(3)(3)$

and so on

$(n) \dots \dots \dots (i)$

Now, we have

$$\sum_{n=1}^{\infty} (n)(n)$$

$$(1)(1)(1)(1)$$

$$(2)(2)(2)(2)(2)(2)$$

$$(3), (3)(3)(3) - (3)$$

$$(4), \dots \dots \dots - (4) - (4)$$

$$(5) 6 \dots \dots \dots 7(5)$$

$$(6)(6)(6)(6)$$

$$, (7)(7) -$$

90. If (n) and $(n)(n)$, then

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (D)

Given that,

and

Here, $\dots \dots \dots$ and $\dots \dots \dots$

and

By solving these equations, we get

91. Let $f(x)$ be continuous, $f'(x)$ for all x and $f(0) = 1$, then the value of $\int_0^1 f(x) dx$ is

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (B)
 $f(x)$ is continuous
 $f'(x)$ for all x
 $f(0) = 1$

$$f'(x) = f(x) - 1$$

Given, $f'(x) = f(x) - 1$

$$f'(x) - f(x) = -1 \quad \dots (i)$$

Therefore, $f'(x) = f(x) - 1$

- [using Equation (i)]

$$92. \int_0^1 \frac{1}{\sqrt{1-x^2}} dx$$

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (C)

$$\int_0^1 \frac{1}{\sqrt{1-x^2}} dx = \int_0^1 \frac{1}{\sqrt{(1-x)(1+x)}} dx$$

$$= \int_0^1 \frac{1}{\sqrt{1-x^2}} dx \quad (\text{by rationalization})$$

$$= \int_0^1 \frac{1}{\sqrt{1-x^2}} dx = \int_0^1 \frac{1}{\sqrt{1-x^2}} dx$$

$$= \int_0^1 \frac{1}{\sqrt{1-x^2}} dx = \int_0^1 \frac{1}{\sqrt{1-x^2}} dx$$

$$= 4\sqrt{1-x^2}$$

93. If $f(x)$ is differentiable at $x = a$ and $f(a) = 1$

- (A)

- (B)
(C)
(D)
(E)

Solution: (E)

() $\frac{f'(x)}{f(x)}$; function is differentiable.

() and $\frac{f'(x)}{f(x)}$; Given function is continuous.

Hence, () $\frac{f'(x)}{f(x)}$

94. The maximum value of the function is attained at

- (A)
(B)
(C)
(D)
(E)

Solution: (D)

We have, ()

()

and ()

At point of local maximum as minimum, we must have

() ()

Clearly, ()

and ()

So, () has local maximum at .

95. If $\int (f(x) - g(x)) dx = 0$, then..... \neq is

- (A)
(B) –
(C)
(D)
(E)

Solution: (C)

We have,

$$\int (f(x) - g(x)) dx = 0$$

On differentiating both sides, we get

() () ()

()

()

$\therefore f(x) = g(x)$

(A) $(\sqrt{\quad})$
 (B) $(\sqrt{\quad})$
 (C) $(\sqrt{\quad})$
 (D) $(\sqrt{\quad})$
 (E) $\frac{\quad}{\sqrt{\quad}}$

- Solution: (E)

$$\int \frac{\overline{\psi} \gamma^\mu \psi}{\overline{\psi} \gamma^\mu \psi} \dots (ii)$$

By adding Equations (i) and (ii), we get

$$\int_{-\infty}^{\infty} \delta(x) dx = 1$$

$$\int_{-}$$

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$$\frac{(\sqrt{x})^{[\sqrt{x}]}}{(\sqrt{x})}(\sqrt{x})$$

$$\frac{1}{\sqrt{x}}$$

97. $\int \frac{1}{\sqrt{x}} dx$

- (A) $\frac{1}{2}\sqrt{x}$
 (B) $\frac{1}{2}\sqrt{x} + C$
 (C) $-\frac{1}{2}\sqrt{x}$
 (D) $-\frac{1}{2}\sqrt{x} + C$
 (E) $\frac{1}{2}\sqrt{x} + C$

Solution: (C)

Let $\int \frac{1}{\sqrt{x}} dx = \frac{1}{2}\sqrt{x} + C$ (i)

$$\int \frac{1}{\sqrt{x}} dx = \frac{1}{2}\sqrt{x} + C$$

$$\int \frac{1}{\sqrt{x}} dx = \frac{1}{2}\sqrt{x} + C$$
(ii)

By adding Equations (i) and (ii), we get

$$\int \frac{1}{\sqrt{x}} dx = \frac{1}{2}\sqrt{x} + C$$

$$\int \frac{1}{\sqrt{x}} dx = \frac{1}{2}\sqrt{x} + C$$

—

—

98. $\int_4^5 \frac{1}{\sqrt{x}} dx$

- (A) $\frac{1}{2}\sqrt{x}$
 (B) $\frac{1}{2}\sqrt{x} + C$
 (C) $-\frac{1}{2}\sqrt{x}$
 (D) $-\frac{1}{2}\sqrt{x} + C$
 (E) $\frac{1}{2}\sqrt{x} + C$

Solution: (D)

$$\int \sqrt{x} \, dx$$

Where, $() ()$

$$\frac{()}{()}$$

Where, $() \sqrt{()}$ — —

————

99. The area bounded by and is —

(A) —

(B) —

(C) —

(D) —

(E)

Solution: (B)

Required area \int

$$\int [\text{————}]$$

—

$$- \int ()$$

—

$$- [\text{————}]$$

$$- 0() \cdot -/1$$

$$- 04 \text{ —}$$

100. The differential equation of the family of curves $()$, where and are arbitrary constants is

(A)

(B) (C)

(D)

(E)

Solution: (A)

Given, system of equation is

$$\begin{aligned} & \frac{1}{x} + \frac{1}{y} = \frac{1}{2} \quad \text{.....(i)} \\ & \frac{1}{x} - \frac{1}{y} = \frac{1}{3} \end{aligned}$$

$$\begin{aligned} & \frac{1}{x} + \frac{1}{y} = \frac{1}{2} \quad \text{.....(i)} \\ & \frac{1}{x} - \frac{1}{y} = \frac{1}{3} \quad \text{.....(ii)} \end{aligned}$$

This is required differential equation.

101. The real part of $(\sqrt{7})^{\frac{1}{2}}$ is

- (A) $\frac{1}{2}$
- (B) $\frac{1}{3}$
- (C) $\frac{1}{4}$
- (D) $\frac{1}{5}$
- (E) $\frac{1}{6}$

Solution: (B)

$$\begin{aligned} & (\sqrt{7})^{\frac{1}{2}} = \frac{7^{\frac{1}{2}}}{7^{\frac{1}{2}}} \\ & = \frac{7^{\frac{1}{2}}}{7^{\frac{1}{2}}} = \frac{7^{\frac{1}{2}}}{7^{\frac{1}{2}}} \\ & = \frac{7^{\frac{1}{2}}}{7^{\frac{1}{2}}} = \frac{7^{\frac{1}{2}}}{7^{\frac{1}{2}}} \end{aligned}$$

Hence, real part is $\frac{1}{2}$.

102. ()

- (A) $\frac{1}{2}$
- (B) $\frac{1}{3}$
- (C) $\frac{1}{4}$
- (D) $\frac{1}{5}$
- (E) $\frac{1}{6}$

Solution: (B) _____

_____ [by Hospital's rule]

_____ - -

103. $\int \frac{(\)(\)}{\text{_____}}$

(A) _____

(B) _____

(C) _____

(D) _____

_____ (E)

Solution: (B)

We have,

$$\int \frac{(\)(\)}{\text{_____}}$$

Put (_____)

and (_____)

$$\int \frac{((\))}{(\)}$$

$$\int \frac{(\)(\)}{\text{_____}}$$

$$\int \text{_____}$$

$$\int \frac{\text{_____}}{\text{_____}}$$

$$\int \frac{\text{_____}}{\text{_____}}$$

Put

$$\frac{\text{_____}}{\text{_____}} / \frac{\text{_____}}{\text{_____}}$$

$$\int \frac{\text{_____}}{\text{_____}}$$

$$\frac{\text{_____}}{\text{_____}}$$

$$\text{_____}$$

104. A plane is at a distance of units from the origin and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$. The equation of the plane is

- (A) $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$
 (B) $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$
 (C) $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 3$
 (D) $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$
 (E) $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 5$

Solution: (C)

Equation of plane whose distance from origin is 1 and normal is $\hat{i} + \hat{j} + \hat{k}$ is

Given that,

$$\frac{\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k})}{\sqrt{1^2 + 1^2 + 1^2}} = 1$$

By formula,

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 3$$

105. is equal to

- (A) $\frac{1}{\sqrt{2}}$
 (B) $\frac{1}{2}$
 (C) $\frac{1}{\sqrt{3}}$
 (D) $\frac{1}{3}$
 (E) $\frac{1}{4}$

Solution: (D)

Given that,

$$\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = \frac{1}{2}$$

106. If , then _____

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (B)

Given that,

$$\dots\dots(i)$$

Differentiating to ,

$$\text{---} \quad ()$$

$$\text{---} \quad , -$$

Again differentiating to ,

$$\text{---} \quad , () -$$

$$\quad , -$$

$$\text{---} \quad \text{[by Equation (i)]}$$

107. The arithmetic mean of _____ is

- (A) _____
- (B) —
- (C) _____
- (D) _____
- (E) _____

Solution: (A)

$$()$$

Take

$$() \quad () \quad () \quad ()$$

Now, arithmetic mean

$$\Sigma \quad , \text{ where } ()$$

$$\text{_____}$$

$$\text{_____}$$

108. The variance of first natural numbers is

- (A) —
- (B) —
- (C) —
- (D) —
- (E) —

Solution: (D)

Since, variance of first natural number is

() —

Variance of first natural number is

() —

—

— —

109. If is a set with elements and $\ast() +$, then the number of elements in is

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (B)

Total numbers of elements in the set The selection of two distinct elements from given elements.

()

110. A coin is tossed and a die is rolled. The probability that the coin shows head and the die shows is

- (A) —
- (B) —
- (C) —
- (D) —
- (E) —

Solution: (B)

() and () —

So, required probability = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

111. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$, then the sum of all the diagonal entries of A^{-1} is

- (A) $\frac{1}{10}$
- (B) $\frac{1}{5}$
- (C) $\frac{1}{2}$
- (D) $\frac{3}{10}$
- (E) $\frac{1}{4}$

Solution: (E)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 1(15-12) - 2(10-12) + 3(10-9) = 3 + 4 + 3 = 10$$

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

$$\text{adj}(A) = \begin{pmatrix} 1 & -2 & 3 \\ -2 & 1 & -4 \\ 3 & -4 & 1 \end{pmatrix}$$

$$\text{Sum of all diagonal entries of } A^{-1} = \frac{1}{10} (1 - 2 + 1) = \frac{0}{10} = 0$$

112. Let $z = \cos \theta + i \sin \theta$. If z is a root of

- (A) $z^2 + z + 1 = 0$
- (B) $z^2 - z + 1 = 0$
- (C) $z^2 + 1 = 0$

(D) and
(E) and

Solution: (A)

Given, $(x^2 - 1) \mid (x^3 - 1)$

$\mid (x^3 - 1)$

, -

$(x^2 - 1) \mid (x^3 - 1)$

$(x^2 - 1) \mid (x^3 - 1)$

, -

$(x^2 - 1) \mid (x^3 - 1)$

$(x^2 - 1) \mid (x^3 - 1)$

$(x^2 - 1) \mid (x^3 - 1)$

at $(x^2 - 1)$

$(x^2 - 1) \mid (x^3 - 1)$

Hence, other roots are and .

113. If, $\frac{1}{x^2 - 1} = \frac{A}{x - 1} + \frac{B}{x + 1}$, then can be

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (D)

Given that, $\frac{1}{x^2 - 1} = \frac{A}{x - 1} + \frac{B}{x + 1}$

$\frac{1}{x^2 - 1} = \frac{A}{x - 1} + \frac{B}{x + 1}$

$\frac{1}{x^2 - 1} = \frac{A}{x - 1} + \frac{B}{x + 1}$

$(x^2 - 1) \mid (x^3 - 1)$

$\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$
 $\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

114. If $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, then

- (A)
- (B) -
- (C)
- (D)
- (E)

Solution: (B)

We have,

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\left\{ \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Now, it is given that

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

115. If $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, then is equal to

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (B)

Given that, $| \quad |$,

$() () ()$

On equating the coefficient of both sides, we get

$() () ()$

116. $| \quad |$

- (A)
- (B)
- (C) $() () ()$
- (D)
- (E) $()$

Solution: (B)

Given that,

$| \quad | \quad || \quad ||$

, -

$(\quad) | \quad |$

$(\quad) \quad , -$

117. If $() | \quad (\quad) (\quad) (\quad) (\quad)$, then $()$

- (A)
- (B)
- (C)
- (D)
- (E)

Solution: (A)

Given,

$$\begin{aligned}
 & \left(\frac{1}{x} \right)' = -\frac{1}{x^2} = -x^{-2} \\
 & \left(\frac{1}{x^2} \right)' = -2x^{-3} = -\frac{2}{x^3} \\
 & \left(\frac{1}{x^3} \right)' = -3x^{-4} = -\frac{3}{x^4} \\
 & \left(\frac{1}{x^4} \right)' = -4x^{-5} = -\frac{4}{x^5} \\
 & \left(\frac{1}{x^5} \right)' = -5x^{-6} = -\frac{5}{x^6} \\
 & \left(\frac{1}{x^6} \right)' = -6x^{-7} = -\frac{6}{x^7} \\
 & \left(\frac{1}{x^7} \right)' = -7x^{-8} = -\frac{7}{x^8} \\
 & \left(\frac{1}{x^8} \right)' = -8x^{-9} = -\frac{8}{x^9} \\
 & \left(\frac{1}{x^9} \right)' = -9x^{-10} = -\frac{9}{x^{10}} \\
 & \left(\frac{1}{x^{10}} \right)' = -10x^{-11} = -\frac{10}{x^{11}}
 \end{aligned}$$

118. If $f(x) = \frac{1}{x^{10}}$, then $f'(x) =$

- (A) $-\frac{1}{x^{11}}$
- (B) $-\frac{1}{x^{10}}$
- (C) $-\frac{1}{x^9}$
- (D) $-\frac{1}{x^8}$
- (E) $-\frac{1}{x^7}$

Solution: (A)
Given,

$$\begin{aligned}
 f(x) &= \frac{1}{x^{10}} = x^{-10} \\
 f'(x) &= -10x^{-10-1} = -10x^{-11} \\
 &= -\frac{10}{x^{11}}
 \end{aligned}$$

$$f^{-1}(x) = -f^{-1}(x)$$

$$\begin{aligned}
 & -[\quad -] \\
 & - , - \\
 & - , - \\
 & - \quad -
 \end{aligned}$$

119. The equation of the plane passing through the points $(\quad)(\quad)$ and (\quad) is
- (A) \quad
- (B) \quad
- (C) \quad
- (D) \quad
- (E) \quad

Solution: (B)
Given that,

and
Equation of plane passing through these points is

$$\begin{vmatrix}
 \quad & \quad & \quad \\
 \quad & \quad & \quad \\
 \quad & \quad & \quad
 \end{vmatrix}$$

120. In an arithmetic progression, if the n th term is \quad , then the sum of first n terms is
- (A) \quad
- (B) \quad
- (C) \quad
- (D) \quad
- (E) \quad

Solution: (A)
Let a be the first term of an AP and d is the common difference.
 \quad
Since,
 \quad

$() ()$
 Equating both sides, we get
 and

$$\rightarrow () -$$

$$\text{---} , -$$

$$, - ()$$