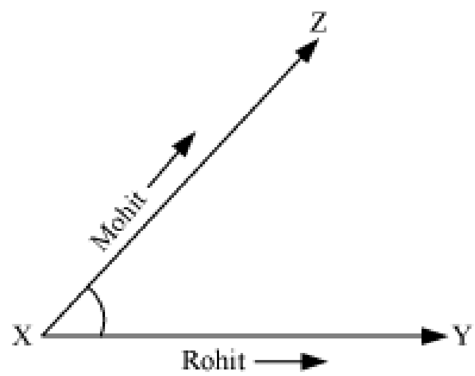
Angles and Lines

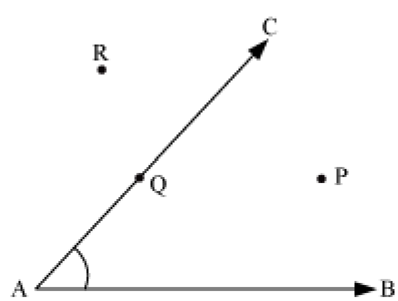
Identification of Regions and Points That Lie Inside, Outside and on the Angles

Rohit and Mohit start walking from the same point in different directions. Let Rohit move towards Y and Mohit towards Z as shown below.

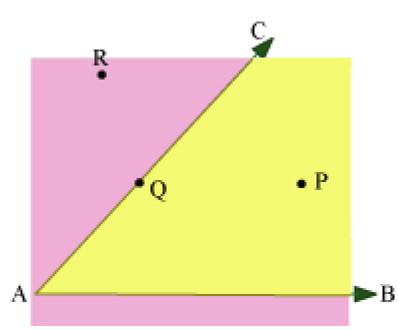


Here, we can say that the rays $\overrightarrow{XY}$ and $\overrightarrow{XZ}$ form an angle ZXY. These rays, i.e. $\overrightarrow{XY}$ and $\overrightarrow{XZ}$, are known as the arms of ∠ZXY. The point, i.e. X, that is common to these arms is called the vertex of ∠ZXY.

Let us consider the following ∠CAB with some points P, Q, and R.



Let us shade the different parts of the angle as shown below.



Here, we can see that the region of the angle shaded by yellow colour lies between the two arms of the angle. This region is called the **interior region** of the angle. It extends indefinitely, since the two arms also extends indefinitely. Every point in this region is said to lie in the **interior** of the angle**.** In this figure, point P lies in the interior of ∠CAB.

The region of the angle shaded pink lies outside the two arms of the angle. This region is called the **exterior region** of the angle. Like the interior region of an angle, the exterior region also extends indefinitely. Every point in this region is said to lie in the **exterior of the angle.** In this figure, point R lies in the exterior of ∠CAB.

The boundary of ∠CAB is formed by its arms $\overrightarrow{AB}$ and $\overrightarrow{AC}$. These arms are called the **boundary** of the angle. Every point lying on the arms is said to lie on the boundary of the angle, or simply, **on the angle**. In this figure, points A, B, C, and Q lie on the angle.

Using this concept, we can say that an angle has three regions. They are interior region, exterior region, and boundary region. Using this idea, we can easily identify whether a point lies inside, outside, or on the given angle. Let us discuss one more example to understand the concept better.

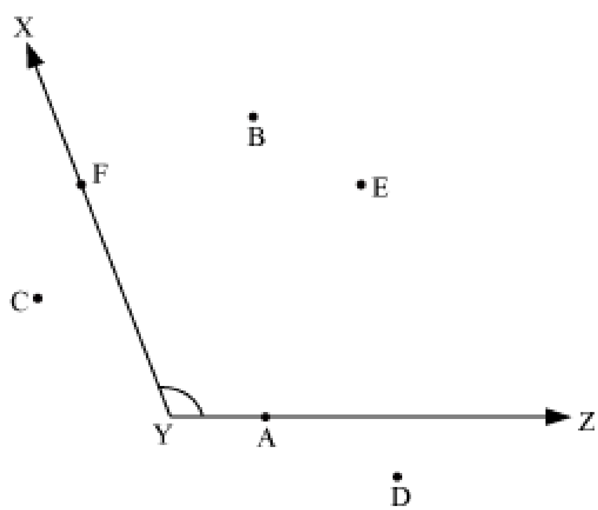
Example 1:

In the figure given below, name the point or points that lie

(i) in the interior of the angle

(ii) in the exterior of the angle

(iii) on the angle



Solution:

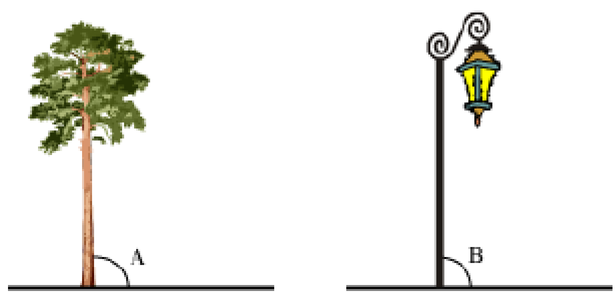
(i) Points B and E lie in the interior of the angle.

(ii) Points C and D lie in the exterior of the angle.

(iii) Points A, F, X, Y, and Z lie on the angle.

Arms and Vertices of Angles

We have seen lamp posts and trees on the street side. These lamp posts and trees stand straight on the road as shown in the figure below.

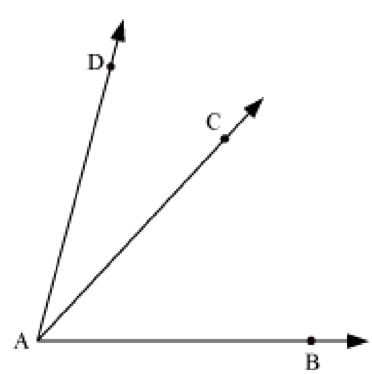


What do we observe in the above figures?

We observe that there is a small curve denoted by A between the tree and the ground. Similarly, in the case of lamp post, a curve is denoted by B. These curves are known as **angles.** We say that the tree and the lamp post are making angles A and B respectively with the ground.

Watch this video to learn more about angles.

In some cases, it is difficult to specify an angle by its vertex. For example, in the figure given below, ∠A may denote any of the angles among ∠BAC, ∠CAD, or ∠BAD.



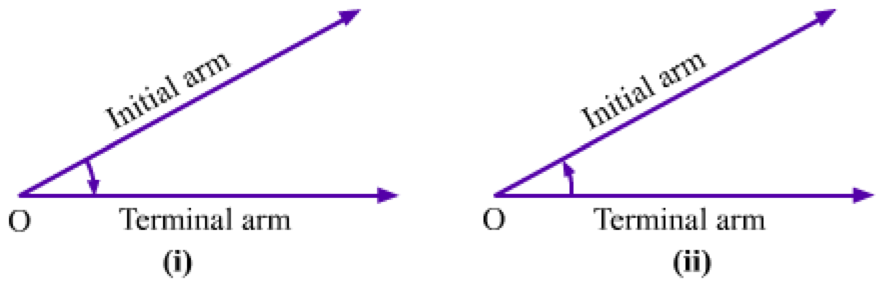
Therefore, it is desirable to represent an angle by three letters (which make the angle) and not just by the vertex letter.

Angles in terms of rotation:

An angle is obtained when a ray is rotated about its end point. The ray can be rotated in two ways such as clockwise (direction in which the hands of a clock move) and anticlockwise (direction opposite to clockwise direction).

When a ray is rotated clockwise, the obtained angle is regarded as negative while the ray is rotated anticlockwise, the angle obtained is regarded as positive.

Observe the following figures.



In figure (i), the angle is obtained by rotating the initial arm clockwise, thus this angle is negative. On the other hand, the angle obtained in figure (ii) is positive as it is obtained by rotating the initial arm anticlockwise.

The amount of rotation of the ray from its initial position to terminal position is known as the measure of the angle.

In figures (i) and (ii), the angles are not same, even if their measures are equal because one of them is positive and the other is negative.

Directed angle:

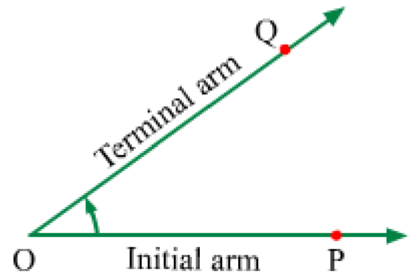
When a ray rotates about its origin point to occupy the position of another ray having the same origin point, it forms an angle which is known as the directed angle. Name of the directed angle is written according to the direction of rotation of initial arm.

For directed angle, rays can be represented in the form of ordered pair as **(ray acting as initial arm, ray acting as terminal arm).**

If we have two rays OP and OQ, then there can be two possibilities for directed angle. These are as follows:

(i) Ray OP is initial arm and ray OQ is terminal arm:

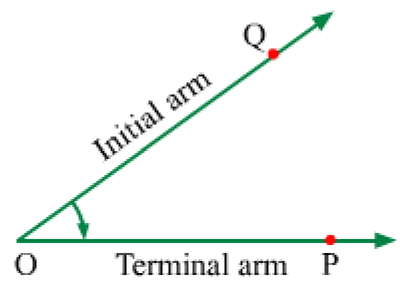
In this case, we get the ordered pair as ray OP, and ray OQ, which represents that the directed angle is obtained by the rotation of ray OP to occupy the position of ray OQ. Thus, obtained angle is called directed angle POQ.



Directed angle POQ is denoted as ∡POQ.

(ii) Ray OQ is initial arm and ray OP is terminal arm:

In this case, we get the ordered pair as ray OQ, and ray OP, which represents that the directed angle is obtained by the rotation of ray OQ to occupy the position of ray OP. Thus, obtained angle is called directed angle QOP.



Directed angle QOP is denoted as ∡QOP.

So, it can easily be concluded that the directed angle POQ and directed angle QOP are different.

i.e., ∡POQ ≠ ∡QOP

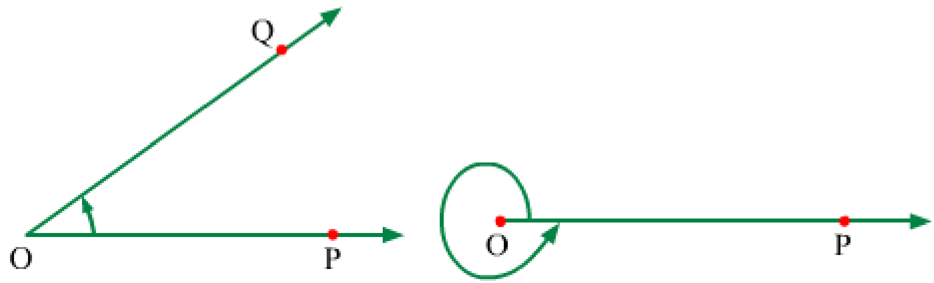
Positive and negative angles:

Angle obtained on anticlockwise rotation of initial arm is regarded as positive angle. For example, ∡POQ in the above shown figure is positive angle.

Angle obtained on clockwise rotation of initial arm is regarded as negative angle. For example, ∡QOP in the above shown figure is negative angle.

One complete rotation:

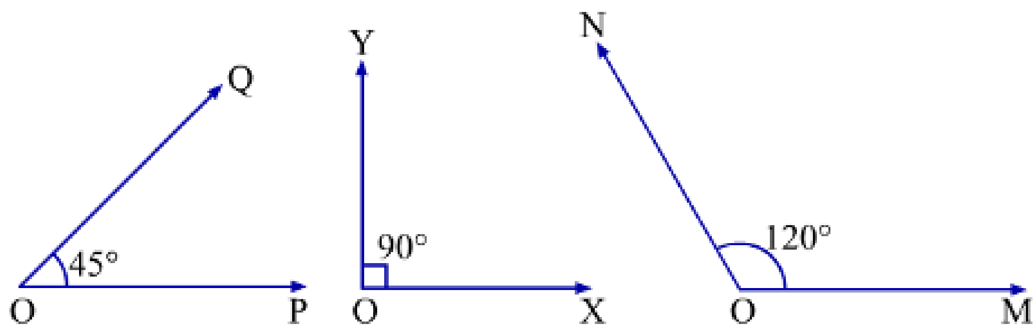
If the initial ray OP is rotated about its end point O in anticlockwise direction such that it comes back to the position OP again for the first time, then it is said that the ray OP has formed one complete rotation.



The measure of the angle traced during one complete rotation in anticlockwise direction is 360°. Similarly, the measure of the angle traced during two complete rotations in anticlockwise direction is just double i.e., 720° and so on.

Writing the measure of an angle:

Observe the following angles.



It can be seen that ∠POQ is a 45° angle, or we can say that ∠POQ measures 45°.

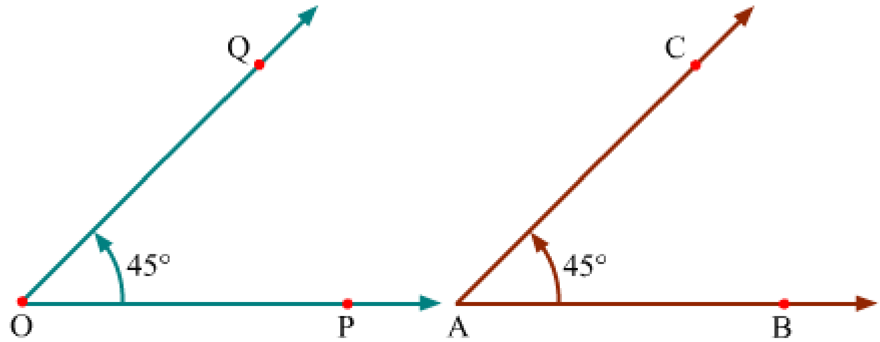
Mathematically, it is denoted as $m\angle POQ = 45°$.

Similarly, $m\angle XOY = 90°$, and $m\angle MON = 120°$.

This is how we denote the measure of an angle.

Congruent angles:

Two angles are said to be congruent if their measures are equal.



From the above figures, it can be observed that ∠POQ = 45° = ∠BAC.

Thus, ∠POQ ≅ ∠BAC.

Properties of congruent angles:

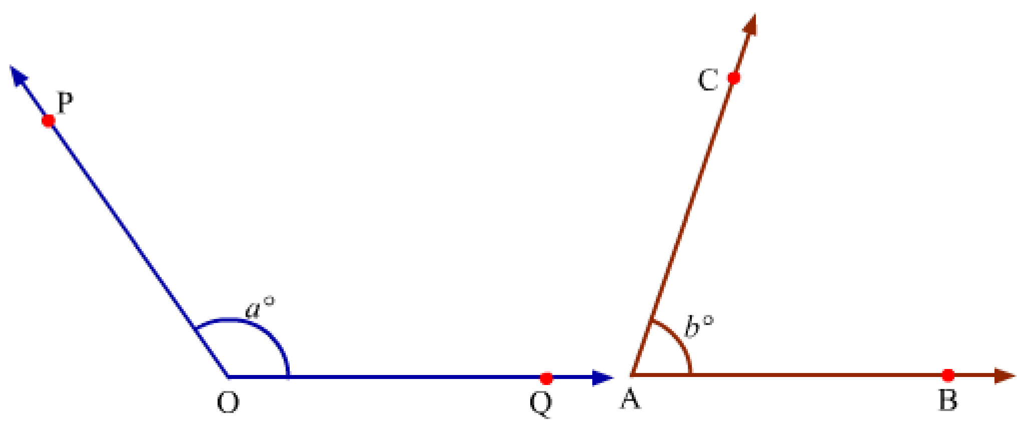
(i) Reflexivity: Every angle is congruent to itself i.e., ∠PQR ≅ ∠PQR.

(ii) Symmetry: If ∠PQR ≅ ∠ABC, then ∠ABC ≅ ∠PQR.

(iii) Transitivity: If ∠PQR ≅ ∠ABC, and ∠ABC ≅ ∠XYZ, then ∠PQR ≅∠XYZ.

Inequality of angles:

Out of two angles, the angle with greater measure is said to be greater than the angle with smaller measure.



In the above figures, we have

∠POQ = *a*° and ∠CAB = *b*°

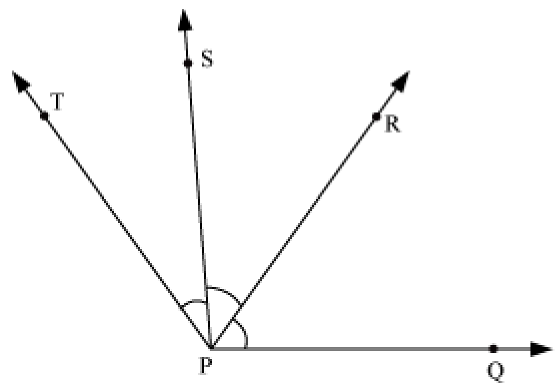
If *a* >*b,* then ∠POQ > ∠CAB

Now, let us discuss some examples based on the concept of angle.

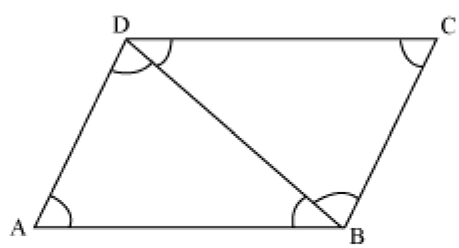
Example 1:

Name the angles marked in each of the following figures. Name the vertices and arms associated with these angles.

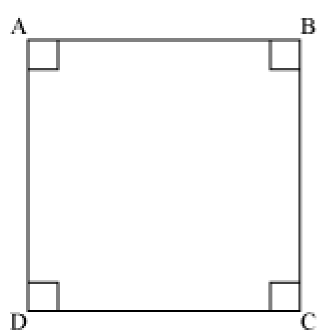
(i)



(ii)



(iii)



Solution:

(i) The angles marked in the figure are ∠QPR, ∠RPS, and ∠SPT.

The vertices and arms of each of these angles are listed as:

Angle	Vertex	Arms
∠QPR	P	$\overrightarrow{PQ}$ and $\overrightarrow{PR}$
∠RPS	P	$\overrightarrow{PR}$ and $\overrightarrow{PS}$
∠SPT	P	$\overrightarrow{PS}$ and $\overrightarrow{PT}$

(ii) The angles marked in the figure are ∠BAD, ∠ADB, ∠BDC, ∠BCD, ∠CBD, and ∠ABD.

The vertices and arms of each of these angles are listed as:

Angle	Vertex	Arms
∠BAD	A	$\overrightarrow{AB}$ and $\overrightarrow{AD}$
∠ADB	D	$\overrightarrow{DA}$ and $\overrightarrow{DB}$
∠BDC	D	$\overrightarrow{DB}$ and $\overrightarrow{DC}$
∠BCD	C	$\overrightarrow{CB}$ and $\overrightarrow{CD}$
∠CBD	B	$\overrightarrow{BC}$ and $\overrightarrow{BD}$
∠ABD	B	$\overrightarrow{BA}$ and $\overrightarrow{BD}$

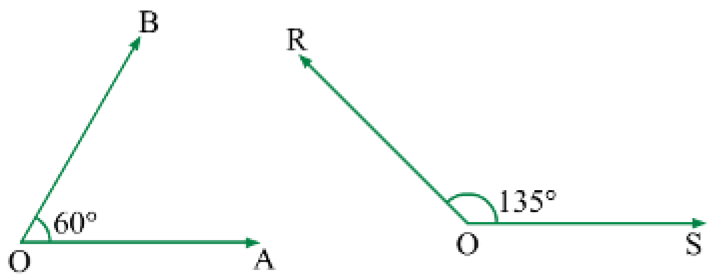
(iii) The angles marked in the figure are ∠ABC, ∠BCD, ∠CDA, and ∠DAB.

The vertices and arms of each of these angles are listed as:

Angle	Vertex	Arms
∠ABC	B	$\overrightarrow{BA}$ and $\overrightarrow{BC}$
∠BCD	C	$\overrightarrow{CB}$ and $\overrightarrow{CD}$
∠CDA	D	$\overrightarrow{DC}$ and $\overrightarrow{DA}$
∠DAB	A	$\overrightarrow{AD}$ and $\overrightarrow{AB}$

Example 2:

Denote the measures of given angles.



Solution:

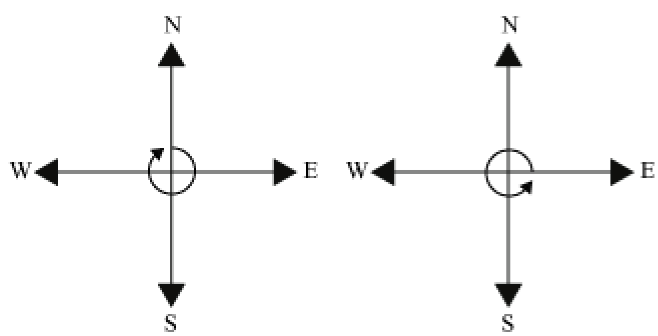
It can be seen that ∠AOB is a 60° angle, while ∠SOR is a 135° angle.

Mathematically, we can denote the measures of given angles as follows:

$m\angle AOB = 60°$ and $m\angle SOR = 135°$.

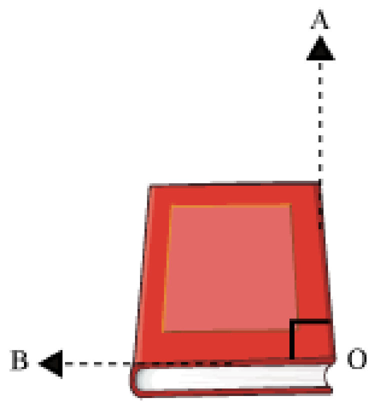
Classification of Angles

Let us consider a situation. Suppose that you are facing North. You turn around making a complete revolution such that you again face North. The angle you turned by is a **complete angle.** Try the same for different directions and you will obtain the same results.



Now, suppose while facing the North direction, if you turn around making a half turn, then the angle you turned by is a **straight angle,** and if you turn around making a quarter turn, then the angle you turned by is a **right angle.**

Now, consider a notebook and look at the angle made by the adjacent sides.



The adjacent sides OB and OA of the notebook form an angle AOB. This angle is a **right angle***.*

Similarly, the sides of the alphabet '**L**' form a right angle.

Now, consider the hour and minute hands of a clock when the time is 8'o clock as shown in the figure below.

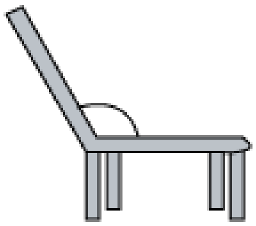
What angle is formed between the hands of the clock?

It is clear from the above discussions that the angle shown in the figure is not among right angle, straight angle, and complete angle. Therefore, how can we classify this angle?

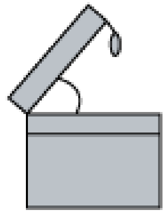
Let us first look at different types of angles.

1. An angle smaller than a right angle is called an **acute angle** i.e., acute angle < 90°.
2. An angle larger than a right angle but smaller than a straight angle is called an **obtuse angle** i.e., 90° < Obtuse angle < 180°.
3. An angle larger than a straight angle but smaller than a complete angle is called a **reflex angle** i.e., 180° < Reflex angle < 360°.

The following figures are examples of the different types of angles.



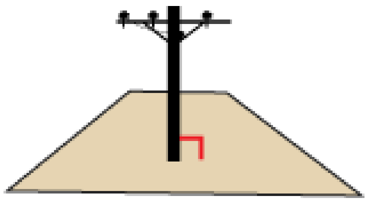
Obtuse angle



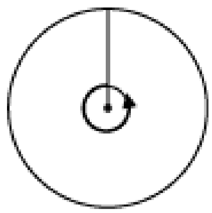
Acute angle

Reflex angle

Straight angle



Right angle



Complete angle

We cannot always say which angle is the greatest out of the given two angles merely by looking at the angles. To compare the angles, we have to measure the angles individually.

An angle is measured in terms of degrees. One complete revolution is equivalent to 360° (° is the symbol of degrees). Thus, the measure of a complete angle is 360°.

The measure of a straight angle is 180° and the measure of a right angle is 90°.

Now, observe the ray OP.

If this ray does not rotate from this position, then the directed angle so formed by the ray is known as the **zero angle**. In other words, it can be said that the amount of rotation of the ray OP about the point O is 0.

The measure of zero angle is 0°.

Now, let us go through the following video to understand the relation between angle measure of a complete angle, straight angle, and a right angle.

Thus, the angle measure of one complete revolution is 360°.

Let us now look at some examples to understand this concept better.

Example 1:
Does the hour hand of a clock make a straight angle when it moves from

1. **12 noon to 6 pm**
2. **5 pm to 10 pm**
3. **2 am to 8 am**
4. **3 pm to 9 pm**
5. **11 am to 7 pm**
6. **3 am to 6 am**

Solution:

We know that two straight angles make a complete angle. A complete angle means a complete revolution. Thus, to make a complete angle, the hour hand has to cross 12 hour marks. To make a straight angle, the hour hand of a clock has to cross 6 hour marks.

1. When the hour hand of a clock moves from 12 noon to 6 pm, it covers 6 hour marks and thus forms a straight angle.
2. When the hour hand of a clock moves from 5 pm to 10 pm, it covers 5 hour marks. Thus, the angle formed is less than a straight angle.
3. When the hour hand of a clock moves from 2 am to 8 am, it covers 6 hour marks and thus forms a straight angle.
4. When the hour hand of the clock moves from 3 pm to 9 pm, it covers 6 hour marks and thus forms a straight angle.
5. When the hour hand of the clock moves from 11 am to 7 pm, it covers 8 hour marks. Thus, the angle formed is more than a straight angle.
6. When the hour hand of the clock moves from 3 am to 6 am, it covers 3 hour marks. Thus, the angle formed measures less than a straight angle.

Example 2:
Does the hour hand of a clock make a complete angle when it goes from

1. **3 am to 3 pm**
2. **9 am to 8 pm**
3. **5 pm to 6 am**
4. **7 pm to 7 am**

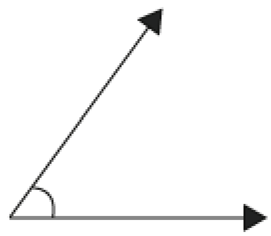
Solution:

We know that a complete angle means a complete revolution. Thus, the hour hand will make a complete angle when it moves from a mark and comes back to the same mark again. By looking at the above four cases, we observe that the hour hand of a clock forms a complete angle in the following cases.

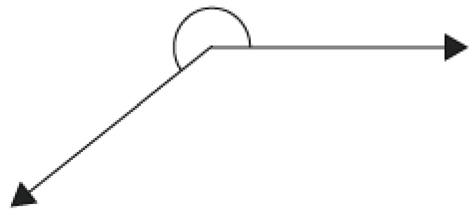
i) 3 am to 3 pm

iv) 7 pm to 7 am

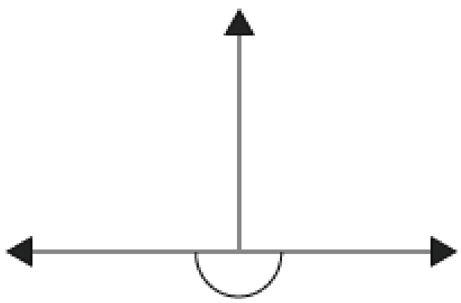
Example 3:
Classify each of the following angles as acute, right, obtuse, straight, or reflex.



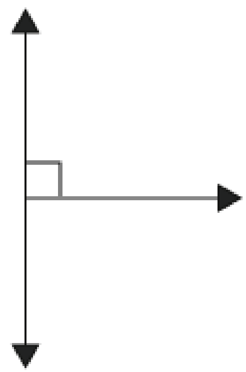
(i)



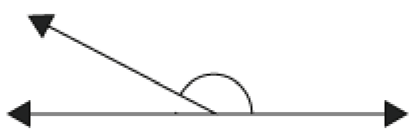
(ii)



(iii)



(iv)



(v)

Solution:

1. The given angle is smaller than a right angle. Hence, it is an acute angle.
2. The given angle is more than a straight angle and less than a complete angle. Hence, it is a reflex angle.
3. The given angle is formed on a straight line and is thus a straight angle.
4. The given angle is a right angle.
5. The given angle is more than a right angle and less than a straight angle. Hence, it is an obtuse angle.

Example 4:
The measures of some angles are given below. Classify these angles based on their measures.

1. **280° (iii) 90° (v) 56°**
2. **180° (iv) 360° (vi) 108°**

Solution:

1. The measure of a reflex angle lies between 180° (straight angle) and 360° (complete angle). Since the given measure is 280°, the angle is a reflex angle.
2. The measure of a straight angle is 180°. Since the given measure is also 180°, the angle is a straight angle.
3. The measure of a right angle is 90°. Since the given measure is also 90°, the angle is a right angle.
4. The measure of a complete angle is 360°. Since the given measure is also 360°, the angle is a complete angle.
5. The measure of an acute angle is less than 90° (right angle). Since the given measure is 56°, the angle is an acute angle.
6. The measure of an obtuse angle lies between 90° (right angle) and 180° (straight angle). Since the given measure is 108°, the angle is an obtuse angle.

Example 5:
Where will the hour hand of a clock stop, if it starts from

1. **6 and turns through two right angles**
2. **5 and turns through two straight angles**
3. **4 and turns through one right angle**

Solution:

1. Since the hour hand turns through two right angles, it crosses 6 hour marks. Therefore, the hour hand will stop at 12.
2. Since the hour hand turns through two straight angles, it makes a complete revolution. Therefore, the hour hand will stop at 5.
3. Since the hour hand turns through one right angle, it crosses 3 hour marks. Therefore, the hour hand will stop at 7.

Example 6:
Find in degrees the angle between the hands of a clock at 4'o clock.

Solution

At 4'o clock, we notice that there are two angles formed by the hands of a clock.

In 4 hours, the hour hand of a clock rotates $\frac{4}{12}$ turn. Hence, the smaller angle between the hands at 4'o clock is $\frac{4}{12}$ turn.

We know that,
1 turn = 360°

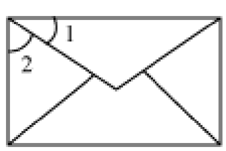
$$\therefore \frac{4}{12}\text{turn} = \frac{4}{12}\times 360^\circ = \frac{1}{3}\times 360^\circ = 120^\circ$$

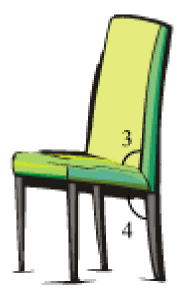
Now, since the larger and the smaller angle add up to 1 complete turn, the larger angle between the hands at 4'o clock is $\left(1-\frac{4}{12}\right)$ turn $=\frac{8}{12}$ turn

$$\frac{8}{12}\text{turn} = \frac{8}{12}\times 360^\circ = \frac{2}{3}\times 360^\circ = 240^\circ$$

Adjacent Angles

Look at the following figures. You will see that some angles are marked in the figures as 1, 2, 3, and 4.





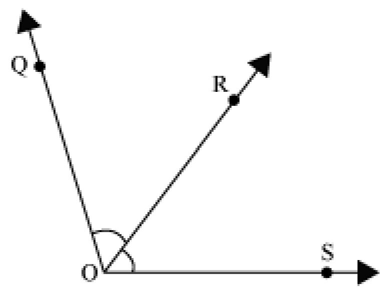
What did you notice?

In the figure of the envelope, ∠1 and ∠2 are adjacent angles. Similarly, in the figure of the chair, ∠3 and ∠4 are adjacent angles.

To understand the conditions in which two angles are adjacent, look at the following video.

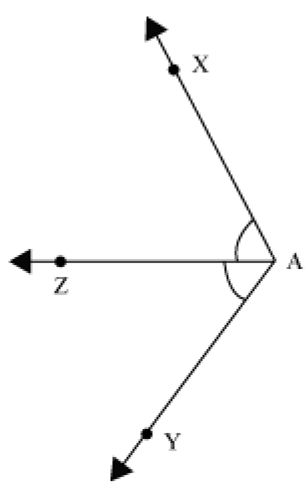
Some more adjacent angles are shown below.

(i)



∠QOR and ∠ROS

(ii)



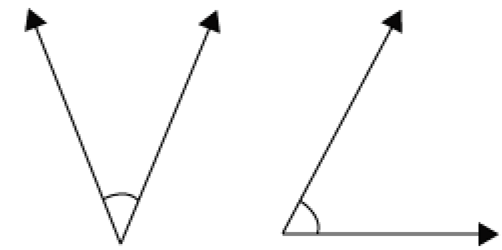
∠XAZ and ∠ZAY

Now, let us discuss some examples based on the above concept.

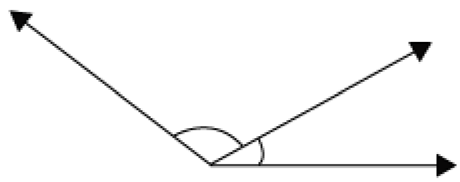
Example 1:

Find out the pairs of adjacent angles from the following pairs of angles.

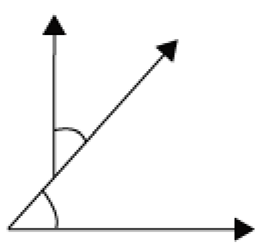
(i)



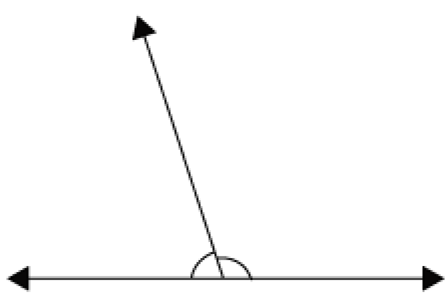
(ii)



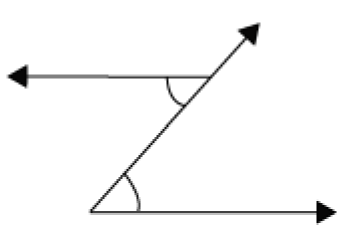
(iii)



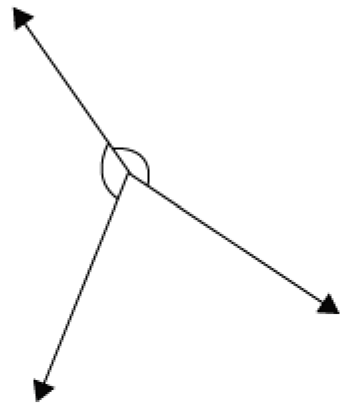
(iv)



(v)



(vi)



Solution:

The pairs of angles in figures (**ii), (iv),** and **(vi)** are adjacent angles. These angles share a common vertex and a common side but no common internal points.

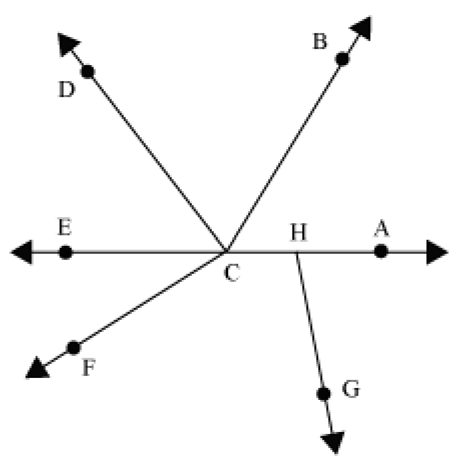
In figure **(i),** the two angles are totally different.

In figures **(iii**) and **(v),** the given angles do not share a common vertex.

Hence, the pairs of angles in figures **(i), (iii),** and **(v)** are not adjacent angles.

Example 2:

Find out whether the following statements are correct or incorrect.



1. **∠ACB is adjacent to ∠DCE.**
2. **∠ECF is adjacent to ∠BCE.**

3. **$\angle$BCF is not adjacent to $\angle$BCA.**

4. **$\angle$GHA is adjacent to $\angle$DCF.**

Solution:

1. False. The angles do not share a common side.

2. True. The angles share a common vertex and a common side but no common internal points.

3. False. $\angle$BCF is adjacent to $\angle$BCA.

4. False. The angles share neither a common side nor a common vertex.

Example 3:

The sum and the difference of two adjacent angles is 99° and 5° respectively. Find the measures of the two angles.

Solution:

Let the measure of one angle be x.

Since the sum of the measures of the angles is 99°, the measure of the other angle will be $99° - x$.

Now, we know that the difference between the angles is 5°.

$\therefore x - (99° - x) = 5°$

$\Rightarrow x - 99° + x = 5°$

$\Rightarrow 2x - 99° = 5°$

By adding 99° to both sides, we get

$2x - 99° + 99° = 5° + 99°$

$\Rightarrow 2x = 104°$

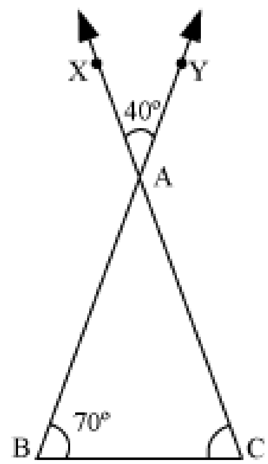
i.e., $x = \frac{104}{2}$

Hence, $x = 52°$

Hence, the measure of one angle is 52° and the measure of other angle is 99° − 52° = 47°.

Vertically Opposite Angles

Let us consider the following figure.



Can you find the measure of ∠ACB?

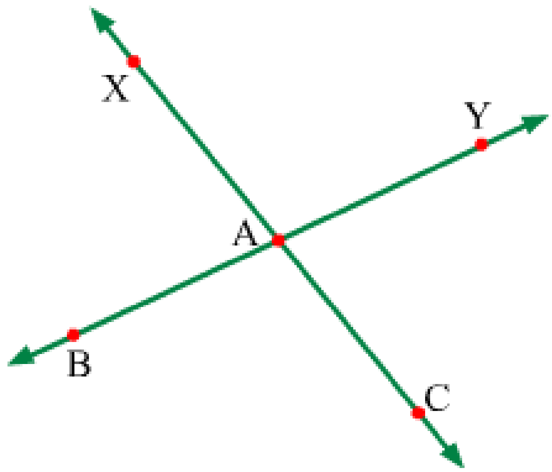
In the given figure, one angle of ΔABC is given as 70° and the other two angles are unknown. We know that the sum of all the three angles of a triangle is 180°. We can find ∠ACB, if we can find ∠BAC.

Therefore, first of all, we have to find ∠BAC. We can easily do so by using the concept of vertically opposite angles.

Therefore, let us know about vertically opposite angles and its property.

Let us prove the property discussed in the video.

Given: Two lines XC and YB intersecting each other at a point A.



To prove: ∠XAY = ∠BAC and ∠XAB = ∠CAY

Proof: Since lines XC and YB intersect each other, we have

∠XAB + ∠CAB = 180° ..(1) (Linear pair)

Also, ∠CAB + ∠CAY = 180° ...(2) (Linear pair)

From (1) and (2), we have

⇒ ∠XAB + ∠CAB = ∠CAB + ∠CAY

⇒ ∠XAB = ∠CAY

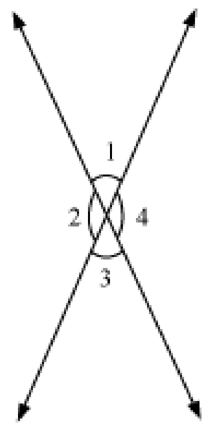
Similarly, we can easily prove that ∠XAY = ∠BAC.

Therefore, if two lines intersect each other then vertically opposite angle are equal.

Let us solve some more examples to understand the concept better.

Example 1:

In the given figure, if ∠1 = 75°, then find ∠2, ∠3, and ∠4.



Solution:

$\angle 1$ and $\angle 4$ form a linear pair of angles.

$\therefore \angle 1 + \angle 4 = 180°$

$\Rightarrow 75° + \angle 4 = 180°$

$\Rightarrow \angle 4 = 180° - 75°$

$\Rightarrow \angle 4 = 105°$

Now, $\angle 1$ and $\angle 3$ are vertically opposite angles,

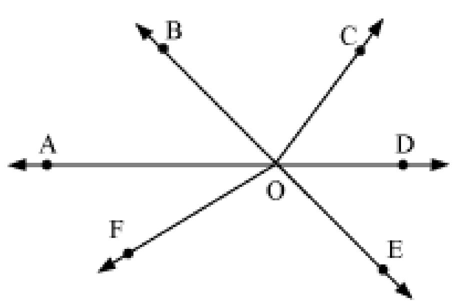
$\therefore \angle 1 = \angle 3 = 75°$

Again, $\angle 2$ and $\angle 4$ are vertically opposite angles,

$\therefore \angle 2 = \angle 4 = 105°$

Example 2:

In the given figure, name the pairs of vertically opposite angles.



Solution:

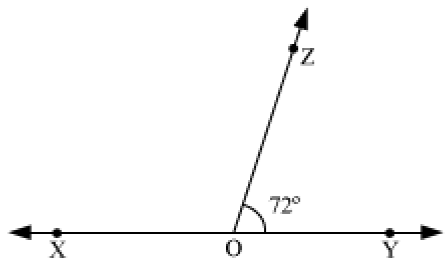
There are two pairs of vertically opposite angles in the given figure.

(1) ∠AOB and ∠DOE

(2) ∠BOD and ∠AOE

Linear Pair of Angles

A ray OZ stands on a line XY such that ∠ZOY = 72°, as shown in the following figure.

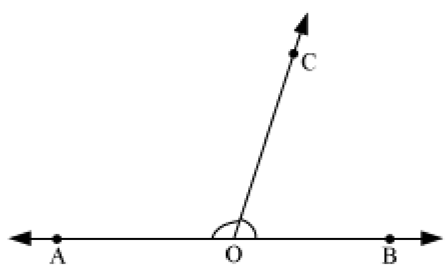


Can we find ∠XOZ?

We can find ∠XOZ using the concept of linear pair.

Therefore, first of all let us know about the linear pair of angles.

"**A linear pair is a pair of adjacent angles and whose non-common sides are opposite rays."**

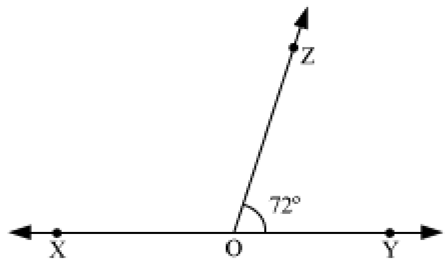


In the above figure, ∠AOC and ∠BOC form a linear pair. Their non-common arms form a straight line.

In other words, we can say, **"When a ray stands on a line, the two angles thus obtained form a linear pair".**

One very important property of a linear pair of angles is that **the sum of measures of linear pair of angles is equal to 180°.**

Now, let us solve the above given example using this property.



Here, we have to find ∠XOZ.

Now, ∠XOZ and ∠ZOY form a linear pair. Therefore,

M ∠XOZ + m ∠ZOY = 180°

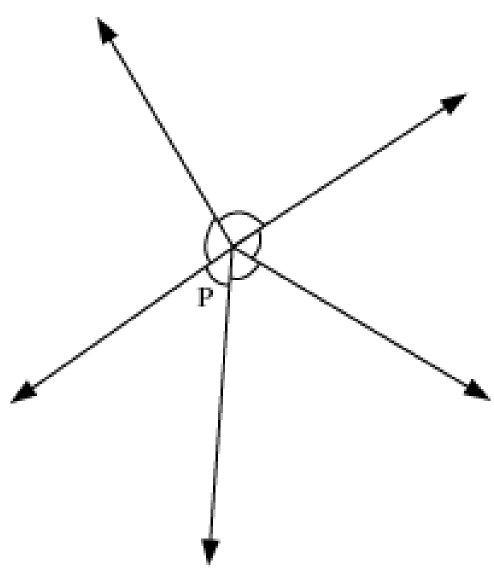
M ∠XOZ + 72° = 180°

M ∠XOZ = 180° − 72°

M ∠XOZ = 108°

We can also state this property as "**The sum of angles lying on a straight line is equal to 180°".**

Now consider the following figure:



In this figure, five angles have a common vertex, which is point P. In other words, the five angles make a complete turn and therefore the sum of these five angles will be equal to 360°. This is true no matter how many angles make a complete turn.

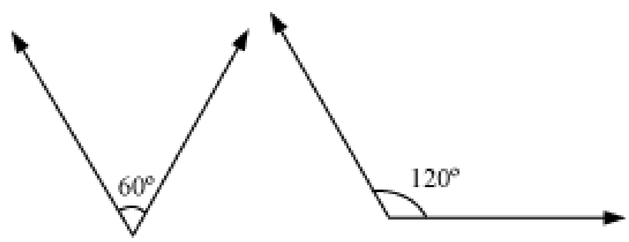
“**The sum of angles around a point is equal to 360°”.**

Let us solve some examples to understand the above discussed concept better.

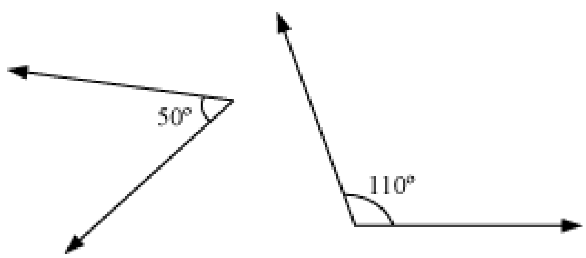
Example 1:

Which of the following pairs of angles forms a linear pair?

(i)



(ii)



Solution:

(i) Sum of measures of angles = 60° + 120°

= 180°

Thus, the given pair forms a linear pair of angles.

(ii) Sum of the measures of angles = 50° + 110°

= 160°

Thus, the given pair does not form a linear pair of angles.

Example 2:

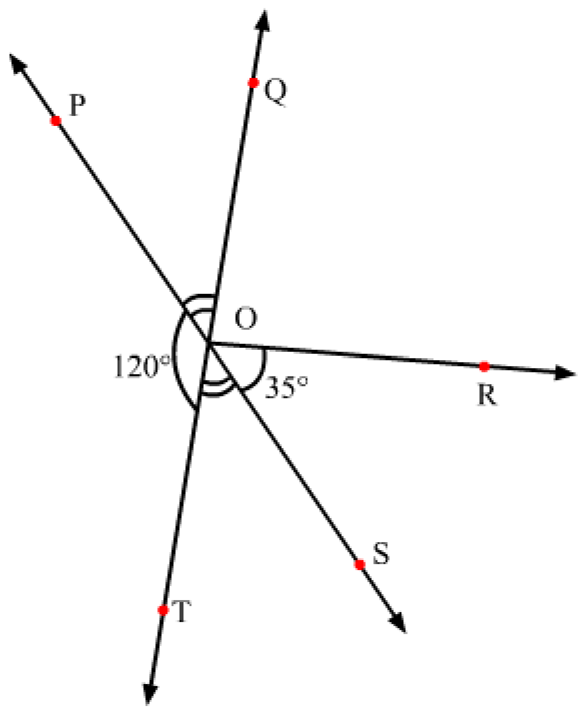
Can two acute angles form a linear pair of angles?

Solution:

Two acute angles cannot be supplementary angles as the sum of the measures of two acute angles is less than 180°. Therefore, two acute angles cannot form linear pair of angles.

Example 3:

In the given figure, ∠POQ = ∠SOT. Find the measure of ∠QOR.



Solution:

In the given figure, ∠POT and ∠SOT form a linear pair.

Therefore,

∠POT + ∠SOT = 180°

⇒ ∠SOT = 180° − 120° = 60°

Now, it is given that ∠SOT = ∠POQ

$\therefore \angle POQ = 60°$

Now, we know that the sum of angles around a point is equal to 360°. Therefore,

$\angle POT + \angle SOT + \angle SOR + \angle QOR + \angle POQ = 360°$

$\Rightarrow 120° + 60° + 35° + \angle QOR + 60° = 360°$

$\Rightarrow \angle QOR = 360° - 275°$

$\Rightarrow \angle QOR = 85°$

Example 4:

Let $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ be three rays such that $\overrightarrow{OC}$ is between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. If $\angle BOC + \angle COA = 180°$, then prove that A, O, B are collinear, that is, they lie on the same straight line.

Solution:

Given: $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$ are three rays. $\angle BOC$ and $\angle COA$ are adjacent angles formed by the rays such that $\angle BOC + \angle COA = 180°$.

To Prove: A, O, B all lie on the same line.
Construction:
Extend $\overline{AO}$ to D such that A, O, D all lie on the same line AD.

Proof:

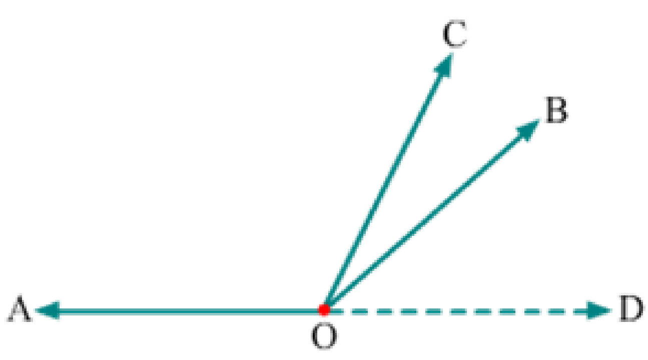


Fig (1)

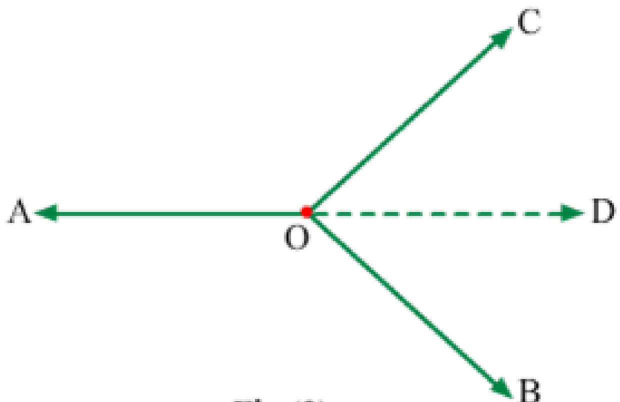


Fig (2)

Since AD is a line,
∴ $\angle DOC + \angle COA = 180°$ ...(1)
Given, $\angle BOC + \angle COA = 180°$ ...(2)

From (1) and (2), we have
$\angle DOC + \angle COA = \angle BOC + \angle COA$
$\Rightarrow \angle DOC = \angle BOC$

Now, there are two possibilities:

1. $\overrightarrow{OB}$ lies between $\overrightarrow{OD}$ and $\overrightarrow{OC}$ (fig 1)

2. $\overrightarrow{OD}$ lies between $\overrightarrow{OB}$ and $\overrightarrow{OC}$ (fig 2)

From the first possibility, we have
$\angle BOC = \angle DOC = \angle DOB + \angle DOC \Rightarrow \angle DOB = 0$

From the second possibility, we have
$\angle DOC = \angle BOC = \angle BOD + \angle DOC \Rightarrow \angle BOD = 0$

Thus, the angle between the rays $\overrightarrow{OB}$ and $\overrightarrow{OD}$ is zero. Hence, the rays $\overrightarrow{OB}$ and $\overrightarrow{OC}$ coincides. This means that the points B and O are on the same line AD. Thus, A, O, B are collinear.

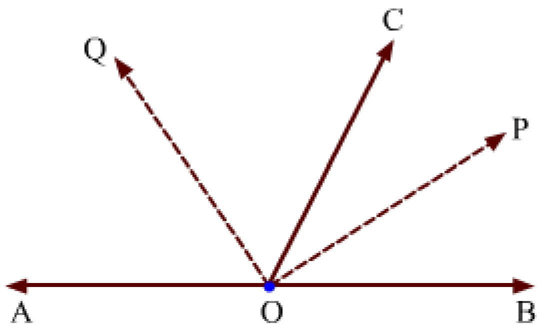
Example 5:

Let AB be a straight line and $\overrightarrow{OC}$ be any ray standing on it. If $\overrightarrow{OP}$ is the bisector of $\angle BOC$ and $\overrightarrow{OQ}$ is the bisector of $\angle COA$, then prove that $\angle POQ = 90°$.

Solution:

Given: $\overrightarrow{OP}$ is the bisector of $\angle BOC$ and $\overrightarrow{OQ}$ is the bisector of $\angle COA$.

To prove: ∠POQ = 90°
Proof:



Since $\overrightarrow{OP}$ is the bisector of ∠BOC, we have

$$\angle POC = \frac{1}{2}\angle BOC \quad ...(1)$$

Since $\overrightarrow{OQ}$ is the bisector of ∠COA, we have

$$\angle COQ = \frac{1}{2}\angle COA \quad ...(2)$$

Adding (1) and (2), we have

$$\angle POQ = \frac{1}{2}(\angle BOC + \angle COA)$$

$$\Rightarrow \angle POQ = \frac{1}{2}\times 180° = 90°$$

Complementary and Supplementary Angles

In some cases, pairs of angles show some special properties. Complementary and supplementary angles are examples of such pairs of angles, whose sum of measures exhibit a relationship. Go through the video to learn more about them.

So, the two important definitions of complementary and supplementary angles are as follows.

“If the sum of the measures of two angles is 90°, then the two angles are said to be complement to each other or complementary angles”.
“If the sum of the measures of two angles is 180°, then the two angles are said to be supplement to each other or supplementary angles.”

Let us solve some examples related to complementary and supplementary angles to understand the concept better.

Example 1:

Find the complement of the following angles.

52° and 75°

Solution:

Complement of 52° = 90° − 52°

= 38°

Complement of 75° = 90° − 75°

= 15°

Example 2:

Find the supplement of the following angles.

100° and 36°

Solution:

Supplement of 100° = 180° − 100°

= 80°

Supplement of 36° = 180° − 36°

= 144°

Example 3:

Can two acute angles be supplementary angles?

Solution:

No, two acute angles cannot be supplementary angles. The measure of an acute angle is less than 90°. Therefore, the sum of the measures of two acute angles is always less than 180°.

Example 4:

Write True or False.

(i) The opposite angles of a square are complementary angles.

(ii) Two obtuse angles can be supplementary angles.

Solution:

(i) False, as each angle of a square is a right angle.

Sum of two opposite angles = 90° + 90°

= 180°

(ii) False, because the measure of an obtuse angle is greater than 90°. Therefore, the sum of the measures of two obtuse angles cannot be 180°.

Example 5:

An angle measures four times its supplementary angle. Find the measures of both the angles.

Solution:

Let the measure of the supplementary angle of the given angle be $x°$ then the measure of the given angle will be $4x°$.
According to the definition of supplementary angles, we obtain

$4x° + x° = 180°$

$\Rightarrow 5x° = 180°$

$\Rightarrow x° = 36°$

$\therefore 4x° = 4 \times 36°$

$\Rightarrow 4x° = 144°$

Thus, the measures of the given angles are 144° and 36°.

Example 6:

The measure of an angle is 6° more than the twice of its complementary angle. Find the measures of both the angles.

Solution:

Let the measure of the complementary angle of the given angle be $x°$ then the measure of the given angle will be $2x° + 6°$.
According to the definition of complementary angles, we obtain

$2x° + 6° + x° = 90°$

$\Rightarrow 3x° + 6° = 90°$

$\Rightarrow 3x° = 84°$

$\Rightarrow x° = 28°$

$\therefore 2x° + 6° = 2 \times 28° + 6°$

$\Rightarrow 2x° + 6° = 56° + 6°$

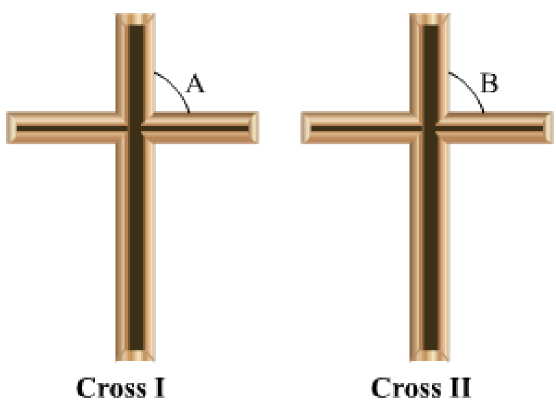
$\Rightarrow 2x° + 6° = 62°$

Thus, the measures of the given angles are 62° and 28°.

Corresponding Angles Axiom

Corresponding Angles in Real Life

Consider the two crosses shown in the given figure.



In cross I, the right arm makes ∠A with the head of the cross; in cross II, the right arm makes ∠B with the head of the cross. Thus, both ∠A and ∠B are at the same position in the two crosses. Such angles are called **corresponding angles**.

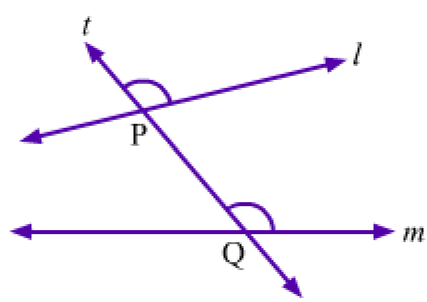
Now, suppose the right arm of cross I is joined to the left arm of cross II. What we obtain is a transversal cutting across a pair of lines to form different pairs of corresponding angles (like ∠A and ∠B). So, we can say that the relation between the angles in the so formed corresponding pairs of angles is decided by the relation between the lines cut by the transversal. This is defined by the **corresponding angles axiom**.

In this lesson, we will discuss the above axiom and its converse. We will also crack some problems based on the same.

Concept Builder

A transversal is a line that intersects two or more lines in the same plane at different points.

Look, for example, at the following figure.



In the figure, line *t* intersects lines *l* and *m* at two different points P and Q respectively; so, *t* is a transversal. The marked angles are corresponding angles.

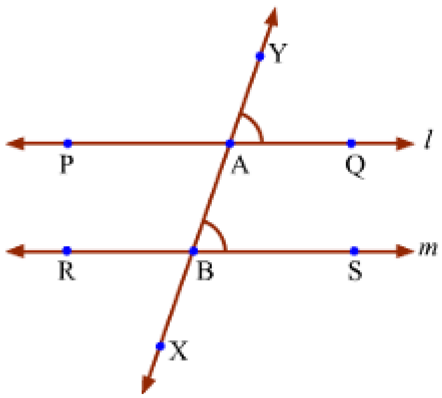
Proof of Corresponding Angle Axiom

When a transversal crosses two parallel lines, each pair of corresponding angles is equal (or congruent).

Given: Transversal XY of coplanar lines *l* and *m* such that *l* || *m*

To prove: ∠YAQ = ∠ABS

Proof:



$l \parallel m$ (Given)

$\therefore \angle PAB = \angle ABS$ ...(1) (Alternate angles)

And, $\angle PAB = \angle YAQ$ ...(2) (Vertically opposite angles)

From (1) and (2), we obtain

$\angle YAQ = \angle ABS$

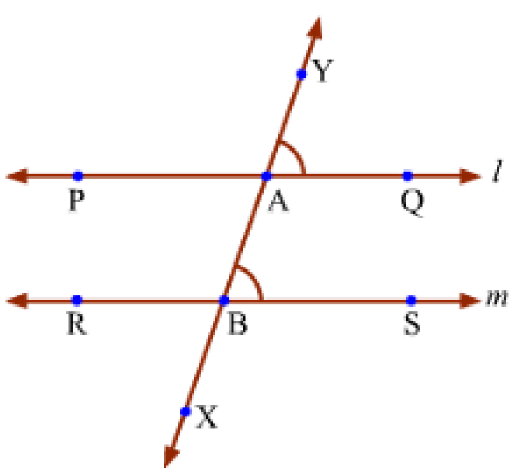
Proof of Converse of Corresponding Angle Axiom

If a pair of corresponding angles is equal (or congruent) then the lines are parallel to each other.

Given: Transversal XY of coplanar lines *l* and *m* such that $\angle YAQ = \angle ABS$

To prove: $l \parallel m$

Proof:



∠YAQ = ∠ABS ...(1) (Given)

And, ∠YAQ = ∠PAB ...(2) (Vertically opposite angles)

From (1) and (2), we obtain

∴ ∠PAB = ∠ABS

∴ *l* || *m* (Using alternate angle axiom)

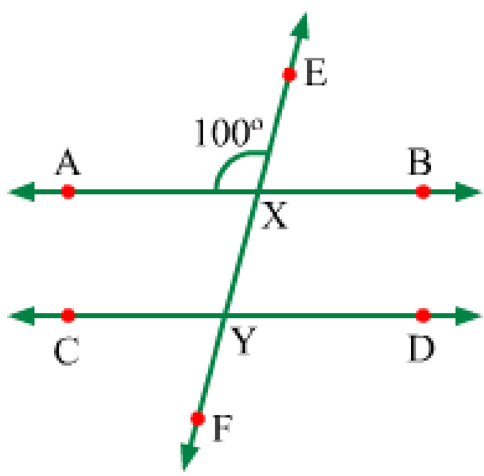
Note: If one pair of corresponding angles is equal (or congruent) then all the pairs of corresponding angles are equal.

Solved Examples

Easy

Example 1:

In the given figure, if AB and CD are parallel lines, then find the measure of ∠CYX.



Solution:

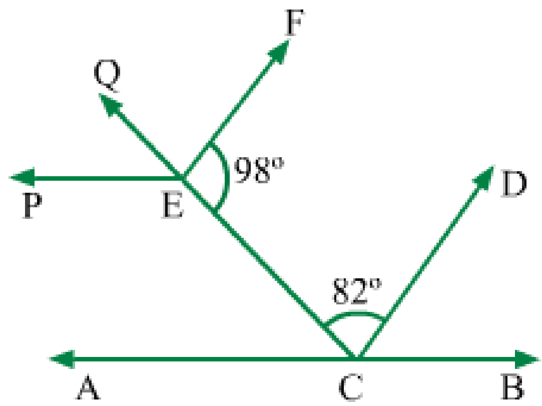
In the figure, ∠AXE and ∠CYX are corresponding angles made by the transversal EF on lines AB and CD respectively.

Using the corresponding angles axiom, we obtain:

∠CYX = ∠AXE = 100°

Example 2:

In the given figure, prove that EF||CD.



Solution:

In the give figure, ∠QEF and ∠FEC form a linear pair.

⇒ ∠QEF + ∠FEC = 180°

⇒ ∠QEF + 98° = 180°

⇒ ∴ ∠QEF = 82°

It can be seen that ∠QEF and ∠ECD are corresponding angles made by the transversal QC on lines EF and CD respectively.

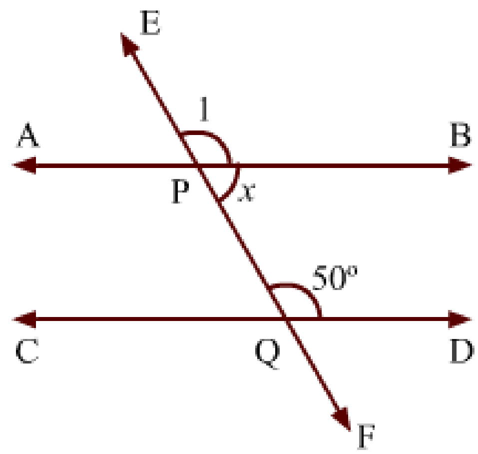
Now, ∠QEF = ∠ECD = 82°

∴ EF||CD (By the converse of the corresponding angles axiom)

Medium

Example 1:

In the given figure, line segments AB and CD are parallel. Find the value of *x*.



Solution:

It is given that AB is parallel to CD and EF is the transversal.

∴ $\angle 1 = \angle DQP = 50°$ (Corresponding angles)

Now, $\angle 1 + x = 180°$ (Angles forming a linear pair)

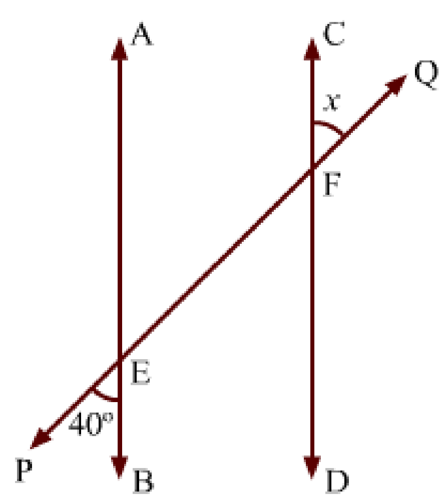
⇒ $50° + x = 180°$

⇒ $x = 180° - 50°$

⇒ ∴ $x = 130°$

Example 2:

In the given figure, line segments AB and CD are parallel. What is the value of *x*?



Solution:

It is given that AB is parallel to CD and PQ is the transversal.

∠PEB = 40° (Given)

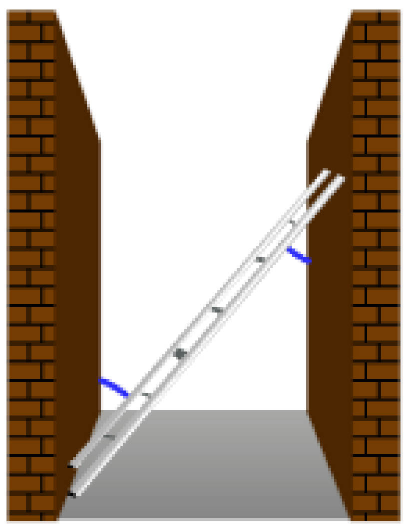
∠AEF = ∠PEB = 40° (Vertically opposite angles)

x = ∠AEF (By the corresponding angles axiom)

⇒ ∴ *x* = 40°

Introduction to Alternate Angles

We can find examples of alternate angles in daily life. These angles play an important role in various situations. Look, for example, at the given figure.

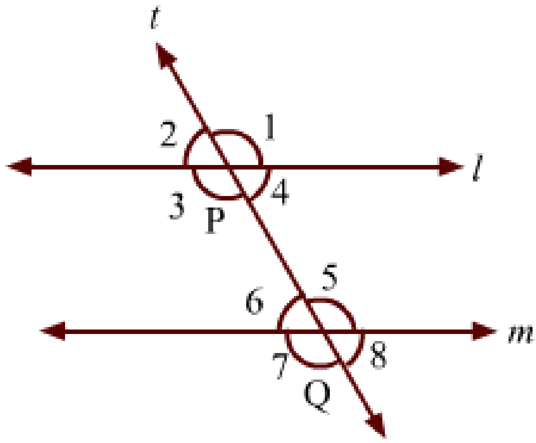


In the figure, the ladder resting between the two walls resembles a transversal joining two lines. The angles marked in the figure are **alternate angles**. These angles exhibit a special property that is defined by the **alternate angles axiom**.

In this lesson, we will study about the above axiom and its converse. We will also solve some examples based on the same.

Alternate Angles Axiom

Consider the given figure in which line *t* is a transversal intersecting two parallel lines *l* and *m* at points P and Q respectively.



In the figure, there are two pairs of alternate angles lying outside the parallel lines, i.e., ∠1 and ∠7, and ∠2 and ∠8. These angles are called **alternate exterior angles**. Also, there are two pairs of alternate angles lying between the parallel lines, i.e., ∠3 and ∠5, and ∠4 and ∠6. These angles are called **alternate interior angles**. The alternate angles made by a transversal on parallel lines have a special property which is stated as follows:

***If a transversal intersects two parallel lines, then the angles in each pair of alternate angles are equal*.**

This property is known as the alternate angles axiom.

So, by using the alternate angles axiom, we can say the following for the given figure.

∠1 = ∠7, ∠2 = ∠8, ∠3 = ∠5 and ∠4 = ∠6

The converse of this axiom is also true. It states that:

***If a transversal intersects two lines such that the angles in a pair of alternate angles are equal, then the two lines are parallel*.**

The alternate angles axiom and its converse are helpful in solving many problems in geometry as well as in real life.

Did You Know?

Playfair's axiom

If *m* is a line and P is any point which does not lie on this line, then there is only one line parallel to line *m* through point P.

For example:

In the given figure, line *l* is the only line parallel to line *m* through point P.

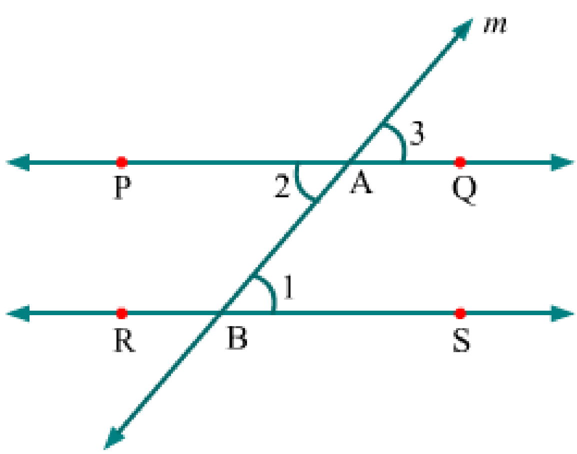
Proof of the Converse of the Alternate Angles Axiom

Statement of the converse of the alternate angles axiom

If a transversal intersects two lines such that the angles in a pair of alternate angles are equal, then the two lines are parallel.

Let us prove this statement.

Given: The transversal *m* intersects lines PQ and RS at points A and B respectively, such that the alternate angles 1 and 2 are equal.



To prove: PQ||RS

Proof: From the figure, we have

$\angle 2 = \angle 3$ (Vertically opposite angles)

$\angle 1 = \angle 2$ (Given)

$\Rightarrow \angle 1 = \angle 3$

∠1 and ∠3 are corresponding angles; so, by using the converse of the corresponding angles axiom, it can be said that PQ and RS are parallel or PQ||RS.

Note: If one pair of alternate angles is equal (or congruent) then the other pair of alternate angles is also equal (or congruent).

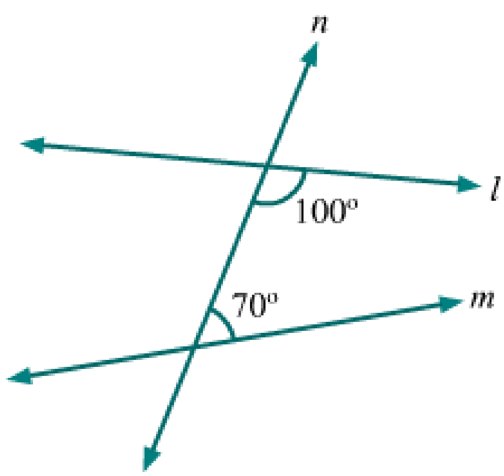
An Example on the Alternate Angles Axiom

Watch this video to understand the alternate angles axiom with the help of an example.

Whiz Kid

The great mathematician Euclid gave a few important results about geometry which are known as Euclid's postulates. The fifth postulate (or the parallel postulate) is about transversals. This postulate states that *if the sum of the interior angles on the same side of a transversal is less than two right angles, then the lines cut by the transversal must intersect when extended indefinitely*.

For example:



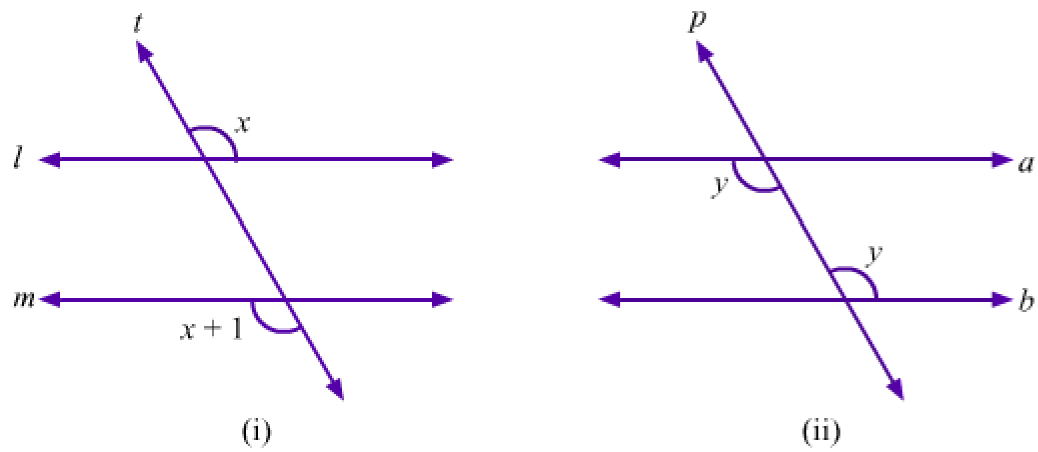
In the given figure, the sum of the interior angles on the right side of the transversal *n* is 170° which is less than two right angles or 180°. So, lines *l* and *m* will intersect when extended indefinitely on the same side as the given interior angles.

Solved Examples

Easy

Example 1:

In which case are the lines cut by the transversal parallel to each other?



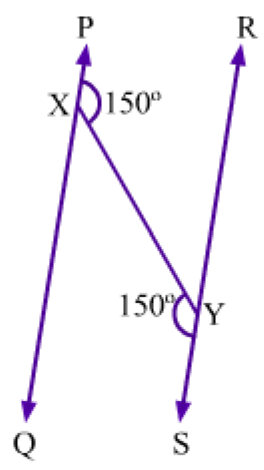
Solution:

In figure (i), the alternate exterior angles formed by the transversal *t* with lines *l* and *m* are not equal as $x \neq x + 1$. Therefore, lines *l* and *m* are not parallel to each other.

In figure (ii), the alternate interior angles formed by the transversal *p* with lines *a* and *b* are equal as $y = y$. Therefore, line *a* is parallel to line *b*.

Example 2:

Is PQ parallel to RS in the given figure?



Solution:

In the figure, we are given ∠PXY and ∠SYX.

These are the alternate interior angles made by PQ and RS with the transversal XY.

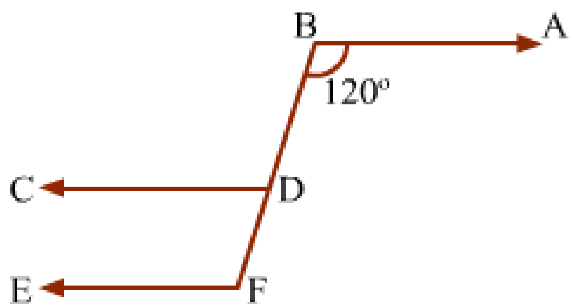
Now, ∠PXY = ∠SYX = 150°

Therefore, by the converse of the alternate angles axiom, we have PQ||RS.

Medium

Example 1:

In the given figure, AB is parallel to CD and CD is parallel to EF. It is given that ∠ABD = 120°. Show that AB is parallel to EF.



Solution:

It is given that ∠ABD = 120°.

Also, AB is parallel to CD.

∴ ∠BDC = ∠ABD = 120° (Pair of alternate interior angles between parallel lines)

We know that CD is parallel to EF.

∴ ∠BFE = ∠BDC = 120° (By the corresponding angles axiom)

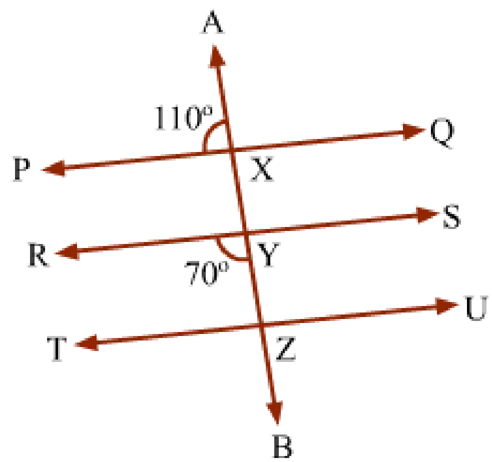
⇒ ∠BFE = ∠ABD

Now, ∠ABD and ∠BFE form a pair of alternate interior angles with respect to lines AB and EF.

Thus, by using the converse of the alternate angles axiom, we obtain AB||EF.

Example 2:

If RS||TU, then prove that PQ||TU.



Solution:

We are given that RS||TU and AB is the transversal.

Now, the interior angles on the same side of the transversal are supplementary.

So, ∠RYB + ∠TZA = 180°

⇒ ∠TZA = 180° − 70°

⇒ ∴ ∠TZA = 110°

It is given that ∠PXA = 110°.

∴ ∠QXB = 110° (because ∠PXA and ∠QXB are vertically opposite angles)

⇒ ∠QXB = ∠TZA

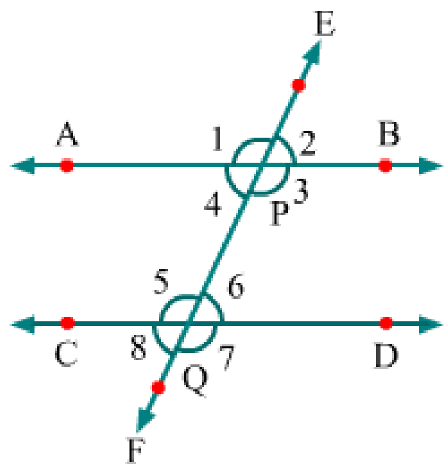
Now, ∠QXB and ∠TZA form a pair of alternate interior angles with respect to lines PQ and TU.

Thus, by using the converse of the alternate angles axiom, we obtain PQ||TU.

Hard

Example 1:

In the following figure, AB and CD are parallel to each other and EF is the transversal intersecting AB and CD at points P and Q respectively. If ∠APE = 110°, then find all the angles formed at points P and Q.



Solution:

In the given figure, we have the angles at points P and Q numbered from 1 to 8. Also, we have $\angle APE = \angle 1 = 110°$.

Now,

$\angle 3 = \angle 1 = 110°$ (Vertically opposite angles)

$\angle 5 = \angle 3 = 110°$ (Alternate interior angles between parallel lines)

$\angle 7 = \angle 5 = 110°$ (Vertically opposite angles)

$\angle 1$ and $\angle 2$ form a linear pair.

So, $\angle 1 + \angle 2 = 180°$

$\Rightarrow 110° + \angle 2 = 180°$

$\Rightarrow \therefore \angle 2 = 70°$

Now,

$\angle 4 = \angle 2 = 70°$ (Vertically opposite angles)

$\angle 6 = \angle 4 = 70°$ (Alternate interior angles between parallel lines)

$\angle 8 = \angle 6 = 70°$ (Vertically opposite angles)

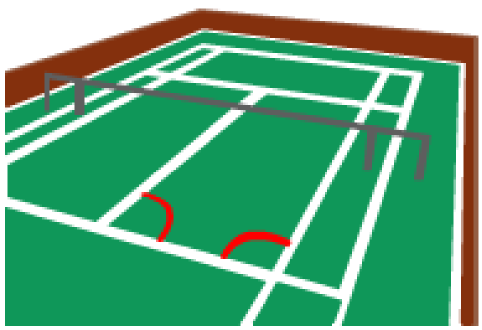
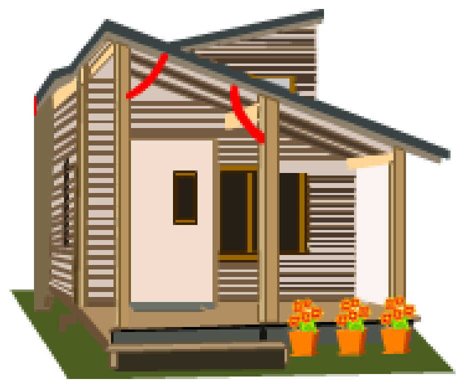
Thus, we have the angles around points P and Q as follows:

$\angle 1 = \angle 3 = \angle 5 = \angle 7 = 110°$ and $\angle 2 = \angle 4 = \angle 6 = \angle 8 = 70°$

Interior Angles on The Same Side of The Transversal

Experiencing Interior Angles on the Same Side of a Transversal

Consider the following figures of a tennis court and a house.

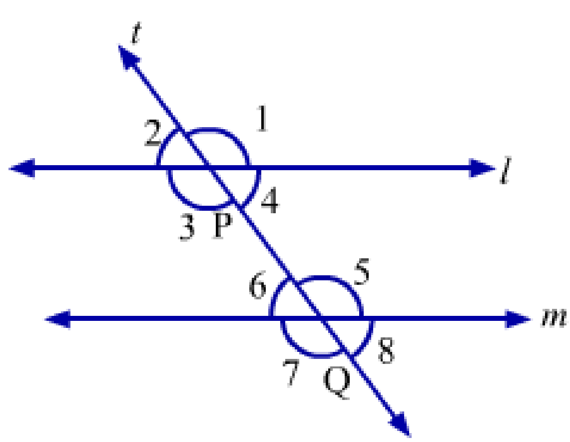



Transversals on parallel lines can be seen easily in the two figures. In each figure, the marked angles are interior angles lying on the same side of a transversal. Examples of such angles can be seen in many of the things that surround us.

In this lesson, we will discuss the property of interior angles on the same side of a transversal and solve some problems based on the same.

Property of Interior Angles on the Same Side of a Transversal

Consider the given figure.



In the figure, the transversal *t* intersects two parallel lines *l* and *m* at points P and Q respectively. Four angles are formed around each point. These angles have been numbered from 1 to 8.

Now, ∠3 and ∠6 form a pair of interior angles lying on the same side of the transversal *t*. ∠4 and ∠5 is another such pair of interior angles. The property exhibited by these types of angles is stated as follows:

If a transversal intersects two parallel lines, then the angles in a pair of interior angles on the same side of the transversal are supplementary.

∴ ∠3 + ∠6 = 180° and ∠4 + ∠5 = 180°

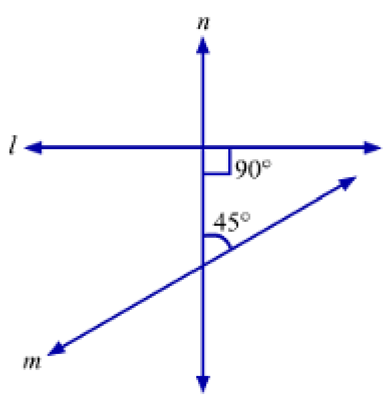
The converse of this property is also true. It states that:

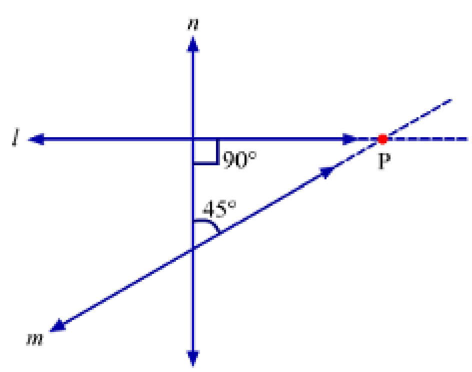
If a transversal intersects two lines such that the interior angles on the same side of the transversal are supplementary, then the lines intersected by the transversal are parallel.

Whiz Kid

If the sum of the interior angles on the same side of a transversal is less than two right angles or 180°, then the lines cut by the transversal must intersect when extended along that side of the transversal.

For example:





In first figure, the sum of the interior angles on the right side of transversal *n* is 90° + 45° = 135° < 180°. So, lines *l* and *m* will intersect when extended along the right side of the transversal *n*, as is shown in second figure.

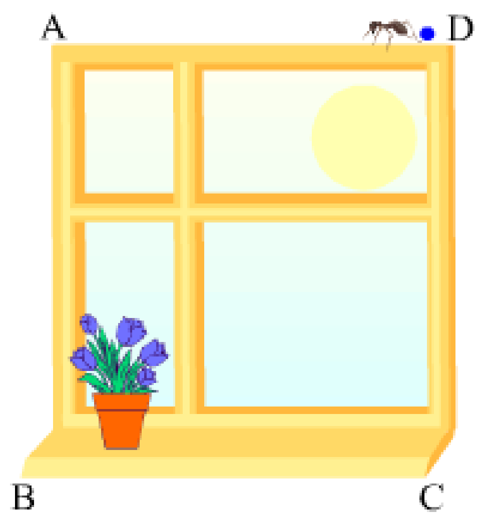
Proving the Property of Interior Angles on the Same Side of a Transversal

Solved Examples

Easy

Example 1:

An ant is moving on a window frame by following the path DABC. If edge AD is parallel to edge BC, then what angle will the ant have to move along in order to reach point C from point D?



Solution:

In the window frame, edges AD and BC are parallel to each other and edge AB is the transversal. So, ∠A and ∠B are interior angles lying on the same side of the transversal AB.

From point D to point C, the ant will move along a total angle that is the sum of ∠A and ∠B.

We know that if a transversal intersects two parallel lines, then the angles in a pair of interior angles on the same side of the transversal are supplementary.

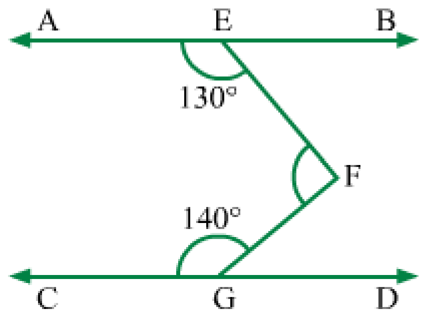
$\therefore \angle A + \angle B = 180°$

Hence, the ant will have to move along an angle of 180° to reach point C from point D.

Medium

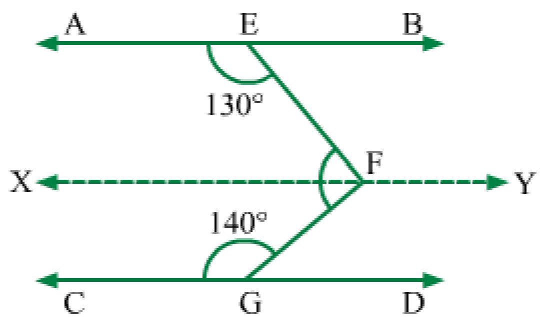
Example 1:

In the given figure, AB and CD are parallel lines. Find the measure of ∠EFG.



Solution:

Construction: Draw a line XY passing through point F and parallel to lines AB and CD.



Now, we have EF as the transversal on the parallel lines AB and XY. Similarly, we have GF as the transversal on the parallel lines CD and XY.

∠AEF and ∠EFX are interior angles on the same side of the transversal EF.

$\therefore \angle AEF + \angle EFX = 180°$

$\Rightarrow 130° + \angle EFX = 180°$

$\Rightarrow \angle EFX = 180° - 130°$

$\Rightarrow \therefore \angle EFX = 50°$

Similarly,

$\angle CGF + \angle GFX = 180°$

$\Rightarrow 140° + \angle GFX = 180°$

$\Rightarrow \angle GFX = 180° - 140°$

$\Rightarrow \therefore \angle GFX = 40°$

From the figure, we have:

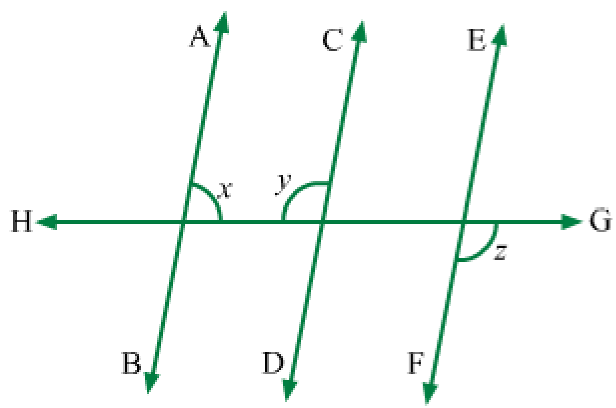
$\angle EFG = \angle EFX + \angle GFX$

$\Rightarrow \angle EFG = 40° + 50°$

$\Rightarrow \therefore \angle EFG = 90°$

Example 2:

In the given figure, AB||CD, CD||EF and *x* : *z* = 2 : 3. What are the measures of *x*, *y* and *z*?



Solution:

It is given that AB||CD, CD||EF and GH is the transversal on these pairs of parallel lines.

Also, $x : z = 2 : 3$

$$\Rightarrow \frac{x}{z} = \frac{2}{3}$$

Let $x = 2a$ and $z = 3a$

Now,

$y = z$ (Exterior alternate angles formed by GH on CD and EF)

$x + y = 180°$ (Interior angles on the same side of GH)

⇒ $x + z = 180°$

⇒ $2a + 3a = 180°$

⇒ $5a = 180°$

⇒ ∴ $a = 36°$

So, we have $x = 2 \times 36° = 72°$ and $z = 3 \times 36° = 108°$

Also, $y = z = 108°$

Activity Time

Follow these steps to verify the property of interior angles lying on the same side of a transversal.

• Take a chart paper.

• Draw two parallel lines.

• Draw four or five transversals, each intersecting the parallel lines at two different points.

• Make a list of the pairs of interior angles formed on the same side of each transversal.

• Measure each angle in the list using a protractor.

• Find the sum of the angles in each listed pair of angles.

• Check whether each pair of angles consists of supplementary angles or not.

• Write the result common to all the transversals.

This activity proves the property that interior angles on the same side of a transversal lying on two parallel lines are supplementary.

Properties of Parallel Lines and Their Transversals with Respect to Intercepts

Properties of Parallel Lines and Their Transversals with Respect to Intercepts

Aline that intersects two (or more) distinct lines at different points is known as transversal the portion of the transversal lying between these two distinct lines is known as **intercept.**

To understand intercept better, let us draw two lines and their transversal in a few different ways.

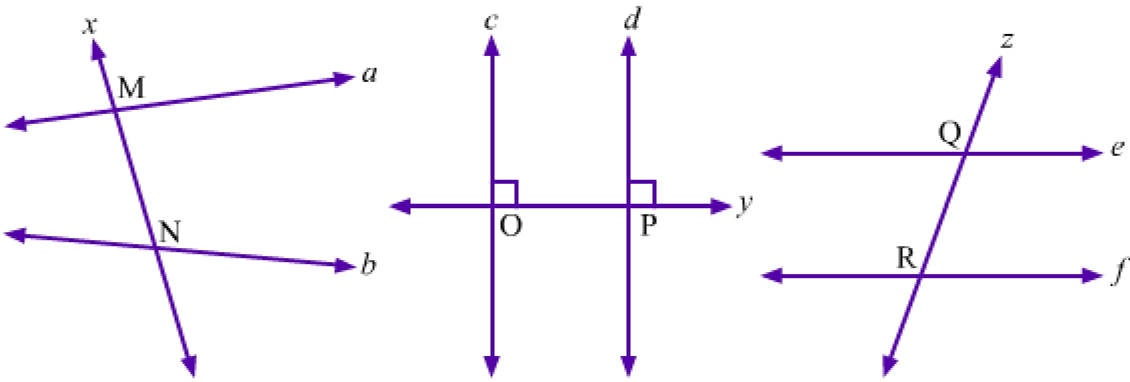


Fig. (1) Fig. (2) Fig. (3)

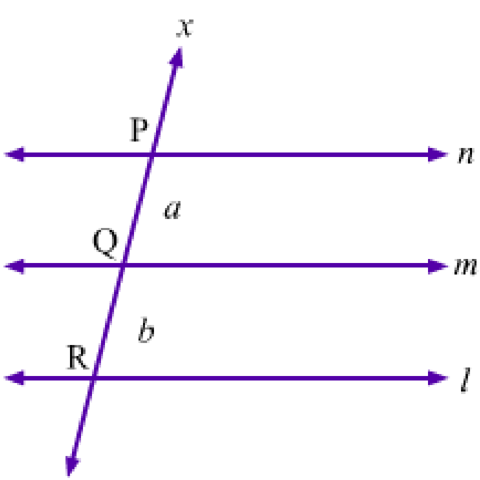
In fig. (1), *x* is the transversal which intersects lines *a* and *b* at points M and N respectively. So, line segment MN is the intercept made by lines *a* and *b* on transversal *x*.

Similarly, in fig. (2), OP is the intercept made by lines *c* and *d* on transversal *y*.

In fig. (3), QR is the intercept made by lines *e* and *f* on transversal *z*.

If two lines are parallel, then the transversal exhibit a few interesting properties related to the intercepts formed on it.

Let us consider the following figure.

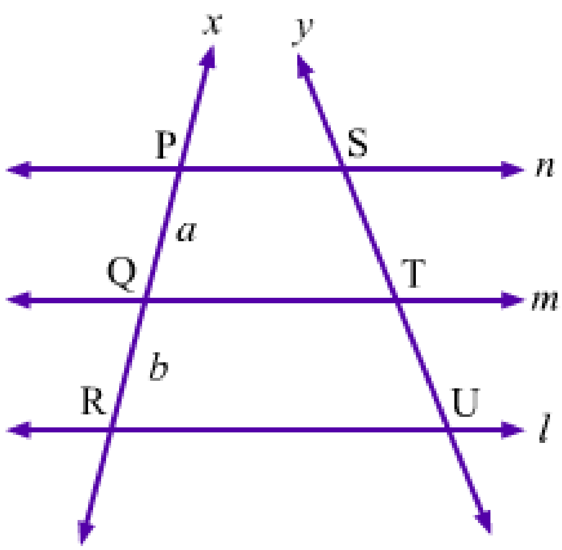


Here, lines *l*, *m* and *n* are parallel to each other, and line *x* is the transversal making intercepts PQ and QR.

Let the length of the intercept PQ be *a* and that of intercept QR be *b*.

$$\therefore \frac{PQ}{QR} = \frac{a}{b}$$

Now, let us draw another transversal *y* with respect to lines *l*, *m* and *n* making intercept ST and TU.



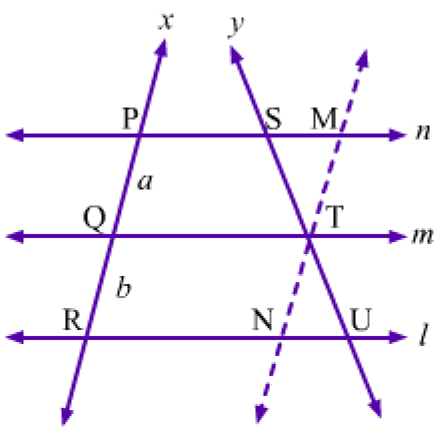
There is a property about intercepts made by three parallel lines on two different transversals. It is stated as follows:

The intercepts made by three parallel lines on one transversal are in the same ratio as the corresponding intercepts made by the same lines on any other transversal.

$$\Rightarrow \frac{PQ}{QR} = \frac{ST}{TU} = \frac{a}{b}$$

Let us prove this property.

For this, let us consider the same figure and draw a line parallel to *x* through T, which intersects *n* and *l* at points M and N respectively.



We have, PM || QT(As *n* || *m*)

PQ|| MT(By construction)

Thus, PQTM is a parallelogram.

⇒PQ = MT = *a*…(1) (Opposite sides of a parallelogram are equal.)

Similarly, QRNT is a parallelogram.

⇒QR = TN = *b*…(2) (Opposite sides of a parallelogram are equal.)

Now, in ΔTSM and ΔTUN:

∠STM= ∠UTN(Vertically opposite angles)

∠TSM= ∠TUN(Alternate angles)

∠TMS= ∠TNU(Alternate angles)

∴ ΔTSM~ΔTUN (By AAA criterion of similarity)

$$\Rightarrow \frac{TS}{TU} = \frac{MT}{NT} \qquad \text{(Property of similar triangles)}$$

$$\Rightarrow \frac{TS}{TU} = \frac{a}{b} \qquad \text{[Using (1) and (2)]}$$

$$\Rightarrow \frac{TS}{TU} = \frac{PQ}{QR}$$

Hence proved.

There may be a case when the intercepts made by three parallel lines on first transversal, i.e. *x*, are congruent.

∴PQ = QR

$\Rightarrow \frac{PQ}{QR} = 1$

Since, $\frac{PQ}{QR} = \frac{TS}{TU}$,

$\frac{TS}{TU} = 1$

⇒ST = TU

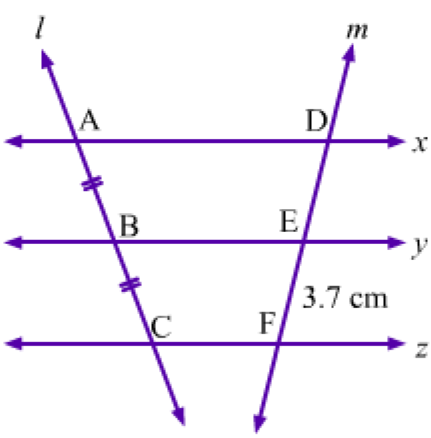
This property can be stated as follows:

If three parallel lines form congruent intercepts on one transversal,then the intercepts formed by them on other transversals are also congruent.

Let us go through a few examples to understand the concept better.

Example1:

In the given figure, *x* || *y* || *z* and AB = BC. Find DF, if EF = 3.7 cm.



Solution:

It is given that *x* || *y* || *z*, AB = BC and EF = 3.7 cm.

In the given figure, *l* and *m* are two transversals with respect to parallel lines *x*, *y* and *z*. Also, AB and BC are intercepts formed by three parallel lines *x*, *y* and *z* on transversal *l*, such that AB = BC.

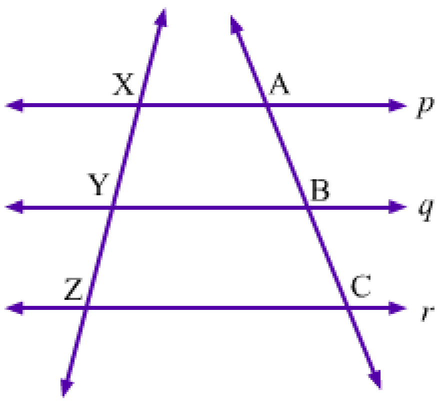
We know that if three parallel lines form congruent intercepts on one transversal, then intercepts formed by them on other transversals are also congruent.

∴DE = EF = 3.7cm (Since AB = BC)

Thus, DF = DE + EF = 3.7 cm + 3.7 cm = 7.4 cm.

Example 2:

In the given figure, $p \parallel q \parallel r$. If XY = 2.2 cm and BC = 5 cm, then what is the numerical value of AB× YZ?



Solution:

It is given that $p \parallel q \parallel r$, XY = 2.2 cm and BC = 5 cm.

Now, we know that the intercepts made by three parallel lines on one transversal are in the same ratio as the corresponding intercepts made by the same lines on any other transversal.

$$\therefore \frac{XY}{YZ} = \frac{AB}{BC}$$

⇒ AB × YZ = XY × BC

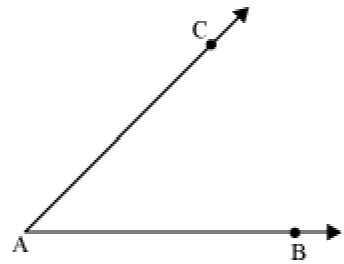
⇒ AB × YZ = 2.2 × 5

⇒ AB × YZ = 11

Hence, the required value is 11.

Construction of Copy of Angles Using Ruler and Compass

If we draw two rays from the same starting point as shown in the figure, then what figure will we obtain?



Yes, you are right. It is an angle.

Can you draw the copy of a given angle?

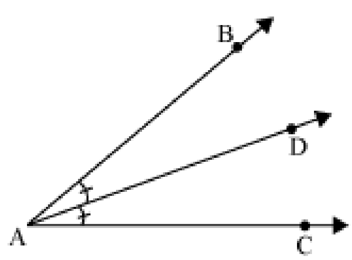
Yes, you can do so. You will first measure the angle using protractor and then draw an angle of the same measure.

What will you do if you do not have a protractor? Can you copy an angle using only ruler and compasses?

Construction of Bisector of Angles Using Ruler and Compass

Let us begin with the definition of the bisector of an angle.

"***The ray that divides an angle into two equal parts is called the bisector of that angle".***



In the given figure, $\overrightarrow{AD}$ will be the bisector of ∠BAC, if AD divides ∠BAC into two equal parts i.e., ∠BAD = ∠CAD.

Now, let us know the method of construction of bisector of an angle using ruler and compasses.

Construction of Angles of Special Measures and Its Verification

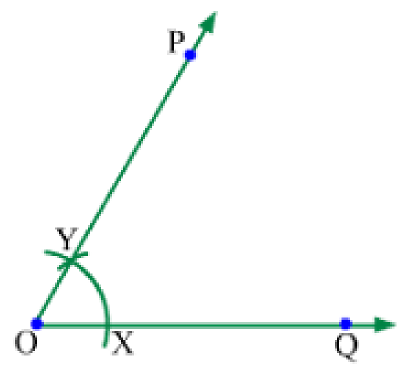
Constructing Angles of Special Measures

You can easily draw different angles using a **protractor**. Another way of constructing angles involves the use of a ruler and a compass. Angles that are multiples of 15° (e.g., 30°, 45°, 60°, 90°, 120°, 135°, etc.) can be constructed in this way. For example, we can construct an angle measuring 45° by bisecting an angle measuring 90°. Similarly, we can construct an angle measuring 120° with the help of an angle measuring 60°. In this lesson, we will learn how to construct such angles with the help of a ruler and a compass.

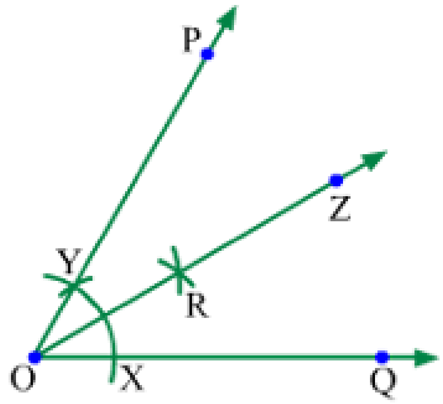
Constructing Angle of 60° and Its Verification

Constructing Angle of 30°

i) Construct an angle measuring 60° using a ruler and a compass, as is shown in the figure.



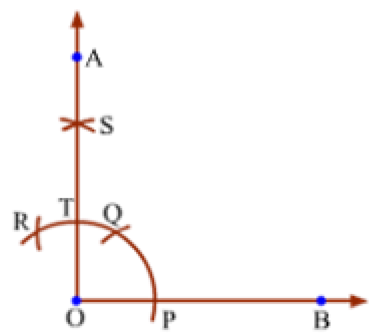
ii) Construct the bisector of ∠POQ in the following manner. Draw two intersecting arcs taking X and Y as the centres and the radius as more than half of XY. Let the arcs intersect at point R. Join O and R. Extend OR to form a ray OZ. OZ is the bisector of ∠POQ.



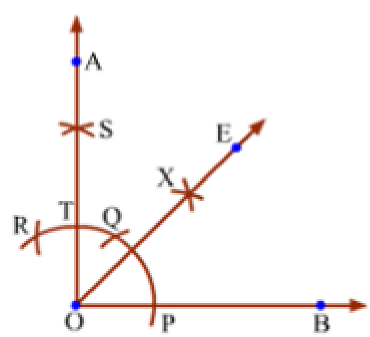
$$\therefore \angle POZ = \angle ZOQ = \frac{1}{2}\angle POQ = \frac{1}{2}\times 60^\circ = 30^\circ$$

Constructing Angle of 45°

i) Construct an angle measuring 90° using a ruler and a compass, as is shown in the figure.



ii) Construct the bisector of ∠AOB in the following manner. Draw two intersecting arcs taking T and P as the centres and the radius as more than half of TP. Let the arcs intersect at point X. Join O and X. Extend OX to form a ray OE. OE is the bisector of ∠AOB.

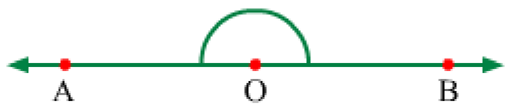


$$\therefore \angle AOE = \angle EOB = \frac{1}{2}\angle AOB = \frac{1}{2}\times 90^\circ = 45^\circ$$

Constructing Angle of 180°

i) Draw a line AB.

ii) Take a point O on AB, in between points A and B.



∠AOB is the required angle of measure 180°.

Solved Examples

Medium

Example 1:

Construct an angle measuring 135°.

Solution:

We know that 135° = 45° + 90°. So, we will first construct an angle measuring 90° and then an angle measuring 45°.

i) Draw a line AB and take a point O on it.

ii) Taking O as the centre and with a suitable radius, draw an arc such that it cuts AB at points C and F.

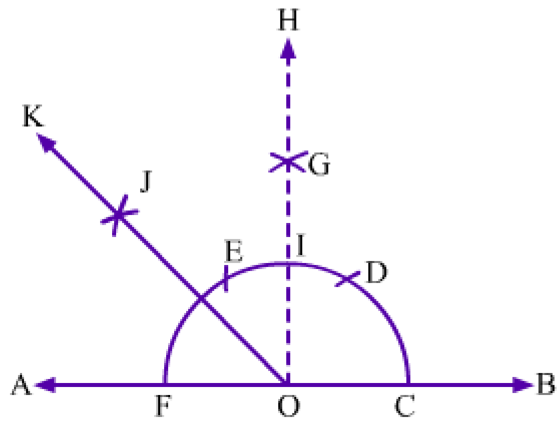
iii) Next, taking C as the centre and keeping the same radius as before, draw an arc intersecting the previous arc, say at point D. Now, with D as the centre and the same radius, draw an arc intersecting the first drawn arc, say at point E.

iv) Then, draw intersecting arcs taking D and E as the centres and the radius as more than of DE. Let the arcs intersect at point G. Join O and G. Extend OG to form a ray OH. Let OH intersect the first drawn arc at point I.

We will have ∠HOA = ∠HOB = 90°.

v) Construct the bisector of ∠HOA in the following manner. Draw intersecting arcs taking I and F as the centres and the radius as more than half of IF. Let the arcs intersect at point J. Join O and J. Extend OJ to form a ray OK. OK is the bisector of ∠HOA.

$$\therefore \angle HOK = \angle KOA = \frac{1}{2}\angle HOA = \frac{1}{2}\times 90° = 45°$$



Now, ∠KOB = ∠HOK + ∠HOB = 90° + 45° = 135°

Thus, ∠KOB is the required angle of measure 135°.

Hard

Example 1:

Construct an angle measuring 22.5°.

Solution:

We know that $22.5° = \frac{1}{2}\times 45°$ and $45° = \frac{1}{2}\times 90°$. So, we will first construct an angle measuring 90°; then, we will bisect it to get an angle measuring 45°; and, finally, we will bisect the 45° angle to get an angle measuring 22.5°.

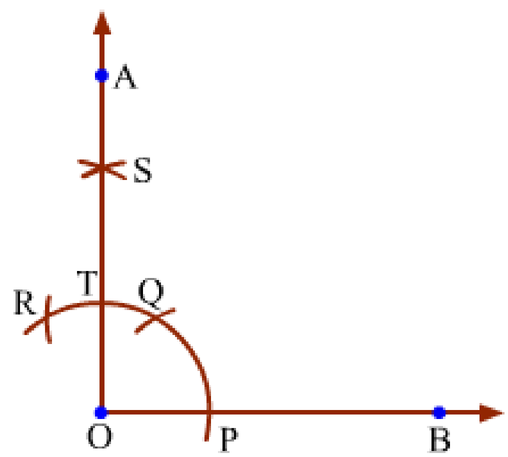
i) Draw a ray OB with the initial point O.

ii) Draw an arc taking O as the centre and with any radius, such that it cuts OB at point P.

iii) Taking P as the centre and with the same radius as before, draw an arc intersecting the previous arc, say at point Q. Now, with Q as the centre and the same radius, draw an arc intersecting the first drawn arc, say at point R.

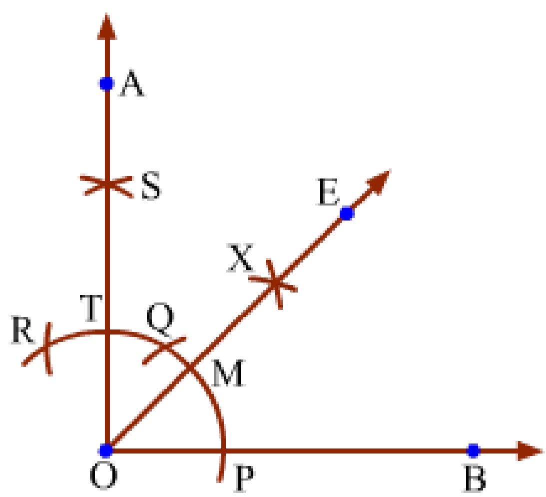
iv) Then, draw intersecting arcs taking Q and R as the centres and the radius as more than half of QR. Let the arcs intersect at point S. Join O and S. Extend OS to form a ray OA. Let OA intersect the first drawn arc at point T.

We will have ∠AOB = 90°.



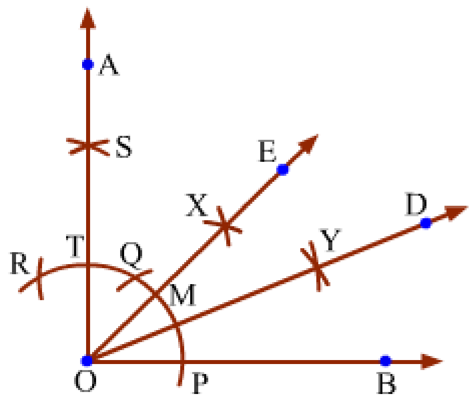
i) Construct the bisector of ∠AOB in the following manner. Draw intersecting arcs taking T and P as the centres and the radius as more than half of TP. Let the arcs intersect at point X. Join O and X. Extend OX to form a ray OE. Let OE intersect the first drawn arc at point M. OE is the bisector of ∠AOB.

$$\therefore \angle AOE = \angle EOB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 90^\circ = 45^\circ$$



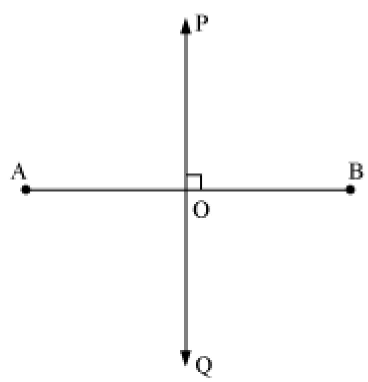
ii) Construct the bisector of ∠EOB in the following manner. Draw intersecting arcs taking M and P as the centres and the radius as more than half of MP. Let the arcs intersect at point Y. Join O and Y. Extend OY to form a ray OD. OD is the bisector of ∠EOB.

$$\therefore \angle EOD = \angle DOB = \frac{1}{2}\angle EOB = \frac{1}{2}\times 45° = 22.5°$$



Construction of Perpendicular Bisectors of Line Segments Using Ruler and Compass

If a line perpendicular to another line segment divides it into two equal parts, then it is called perpendicular bisector of the line segment.



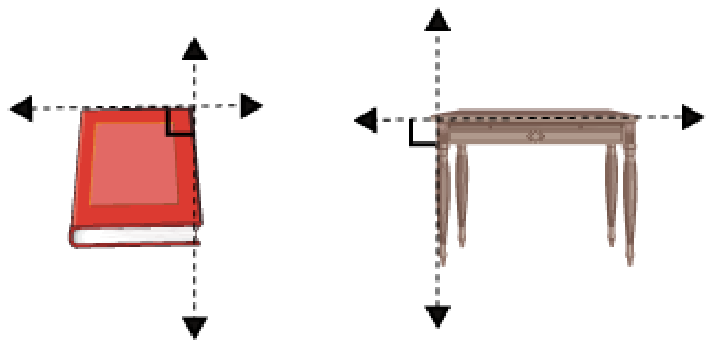
In this figure, AB is a line segment and the line PQ divides it into two equal parts and also, $\overline{PQ} \perp \overline{AB}$. Thus, we can say that PQ is the perpendicular bisector of $\overline{AB}$.

Now, let us learn how to construct the perpendicular bisector of a line segment using ruler and compasses.

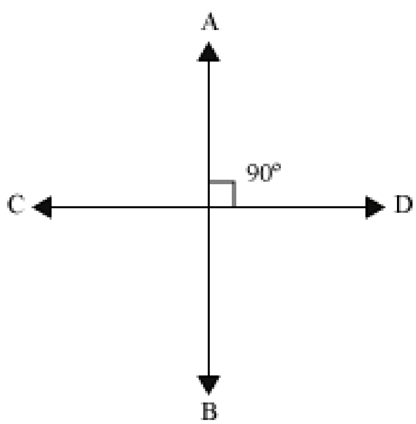
In this way, we can draw a perpendicular bisector of a given line segment.

Construction of Perpendiculars to Lines

As shown in the following figures, the line passing through the adjacent edges of a book and the top surface and legs of a table are the examples of perpendicular lines.



Perpendicular lines are two intersecting lines such that the angles formed by them are right angles.



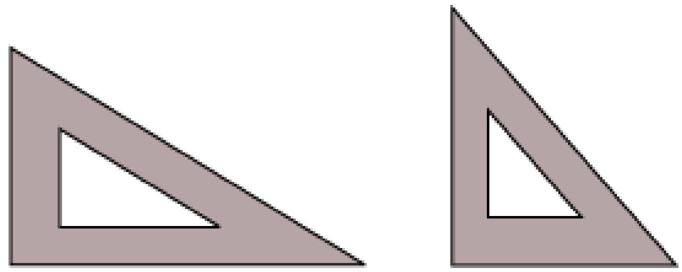
Now, we will learn to construct a perpendicular line to a given line. We can draw a perpendicular line to a given line at any point of the given line or from a point outside the given line.

Firstly, we will learn to draw the perpendicular line at any point of the given line. We can draw the perpendicular line by two methods:

1. Using ruler and compasses
2. Using ruler and set-square

Using ruler and set-square:

We can also draw a perpendicular line by using ruler and set-square. Therefore, first let us see what a set-square is.



The set-squares are two right triangular instruments. One of the instruments contains angles 60°, 90°, and 30° while the other contains angles 45°, 90°, and 45°.

The set-squares are used to draw perpendicular and parallel lines.

Now, let us know the method to draw perpendicular line using ruler and set-square.