Long Answer Type Questions

[4 MARKS]

Que 1. If a transversal intersects two parallel lines, prove that the bisectors of any pair of corresponding angles so formed are parallel.

Sol. **Given:** A transversal EF cuts two parallel lines AB and CD at point G and H respectively. GL and HM are respectively the bisectors of a pair of corresponding angles $\angle EGB$ and $\angle GHD$ respectively [Fig. 6.26].

To prove: $GL \parallel HM$

Proof: Since $AB \parallel CD$ and EF is a transversal

$\therefore \quad \angle EGB = \angle GHD$ (Corresponding angles)

$\Rightarrow \quad \frac{1}{2}\angle EGB = \frac{1}{2}\angle GHD$

$\Rightarrow \quad \angle EGL = \angle GHM$

But these are corresponding angles formed by the lines GL and HM

$\therefore \; GL \parallel HM$

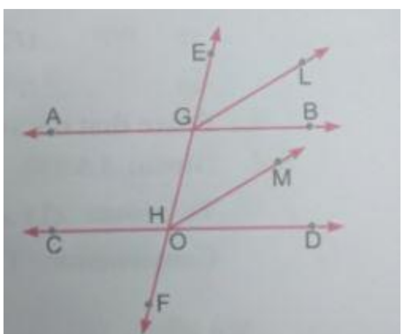

Fig. 6.26

Que 2. In Fig. 6.27, prove that $\angle ADC = \angle A + \angle B + \angle C$.

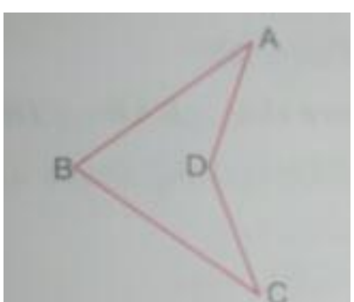

Fig. 6.27

Sol. $Join$ B and D and produce BD to E (Fig. 6.28). Since the exterior angle of a triangle is equal to sum of the two interior opposite angles.

Therefore, in ΔABD

$$x = w + \angle A \qquad \ldots(i)$$

In $\Delta CBD,$ $\quad y = z + \angle C \qquad \ldots(ii)$

Adding (i) and (ii), we get

$$x + y = w + \angle A + z + \angle C$$

$$x + y = w + z + \angle A + \angle C$$

$$x + y = \angle B + \angle A + \angle C$$

Hence, $\angle ADC = \angle A + \angle B + \angle C$

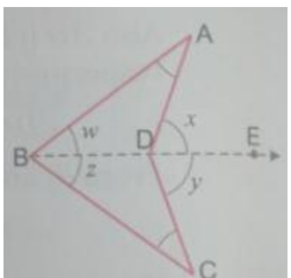

Fig. 6.28

Que 3. In Fig. 6.29, $DE \parallel QR$ and AP and BP are bisectors of $\angle EAB$ and $\angle RBA$ respectively. Find $\angle APB$.

Sol. Since interior angles on the same side of transversal are supplementary

$\therefore \qquad \angle EAB + \angle RBA = 180^0$

$\Rightarrow \qquad \frac{1}{2}\angle EAB + \frac{1}{2}\angle RBA = \frac{1}{2} \times 180^0 \qquad \ldots(i)$

As AP and BP are bisectors of $\angle EAB$ and $\angle RBA,$ respectively

$\therefore \qquad \angle 1 = \frac{1}{2}\angle EAB$ and $\angle 2 = \frac{1}{2}\angle RBA \qquad \ldots(ii)$

From (i) and (ii), we get

$$\angle 1 + \angle 2 = 90^0 \qquad \ldots(iii)$$

In Δ APB, we have

$$\angle 1 + \angle 2 + \angle APB = 180^0$$

$\Rightarrow \qquad 90^0 + \angle APB = 180^0 \qquad$ [Using (iii)]

$\Rightarrow \quad \angle APB = 180^0 - 90^0 \quad \therefore \angle APB = 90^0$

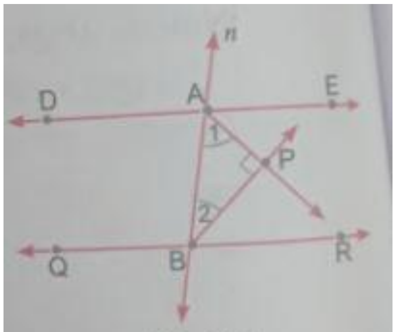

Fig. 6.29

Que 4. Prove that the sum of the angles of a triangle is 180^0.

Sol. **Given:** $\angle 1 + \angle 2 + \angle 3 = 180^0$

To prove: $\angle 1 + \angle 2 + \angle 3 = 180^0$

Construction: Through A, draw a line $XY \parallel BC$ (Fig. 6.30).

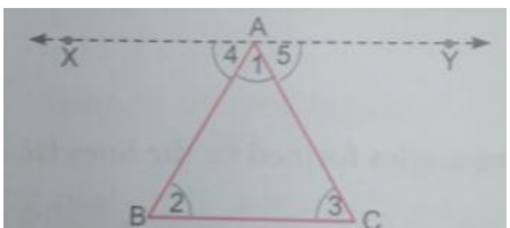

Fig. 6.30

Proof: Since $XY \parallel BC$ and AB is the transversal

$\therefore \quad \angle 4 = \angle 2$ (Alternate interior angles) $\ldots(i)$

Similarly, $XY \parallel BC$ and AC is the transversal $\ldots(ii)$

$\therefore \quad \angle 5 = \angle 3$

Adding (i) and(ii), we get

$\angle 4 + \angle 5 = \angle 2 + \angle 3$

Adding $\angle 1$ on both the sides

$\angle 4 + \angle 1 + \angle 5 = \angle 1 + \angle 2 + \angle 3$

But $\angle 4 + \angle 1 + \angle 5 = 180^0$ ($\therefore$ XAY is a straight line)

$\therefore \quad \angle 1 + \angle 2 + \angle 3 = 180^0$

Hence, the sum of the angles of a triangle is 180^0

Que 5. If the bisector of angles $\angle B$ and $\angle C$ of a triangle ABC meet at a point O, then prove that $\angle BOC = 90^0 + \frac{1}{2}\angle A$.

Sol. In $\Delta\ ABC$ (Fig. 6.31), we have

$$\angle A + \angle B + \angle C = 180^0$$

($\because$ Sum of the angles of a Δ is 180^0)

$\Rightarrow$ $\frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C = \frac{180^0}{2}$

$\Rightarrow$ $\frac{1}{2}\angle A + \angle 1 + \angle 2 = 90^0$

$\therefore$ $\angle 1 + \angle 2 = 90^0 - \frac{1}{2}$ $\quad \ldots(i)$

Now, in ΔOBC, we have:

$\angle 1 + \angle 2 + \angle BOC = 180^0$ ($\because$ Sum of the angles of Δ is 180^0)

$\Rightarrow$ $\angle BOC = 180^0 - (\angle 1 + \angle 2)$

$\Rightarrow$ $\angle BOC = 180^0 - (90^0 - \frac{1}{2}\angle A)$ [Using (i)]

$\Rightarrow$ $\angle BOC = 180^0 - 90^0 + \frac{1}{2}\angle A$

$\therefore$ $\angle BOC = 90^0 + \frac{1}{2}\angle A$

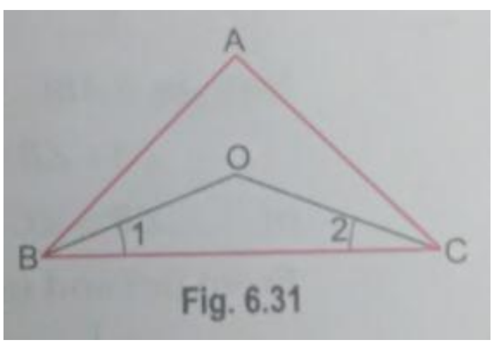

Fig. 6.31

Que 6. In ΔABC (Fig. 6.32), the sides AB and ΔABC are produced to points E and D respectively. If bisectors BO and CO of $\angle CBE$ and $\angle BCD$ respectively meet at point O, then prove that $\angle BOC = 90^0 - \frac{1}{2}\angle A$.

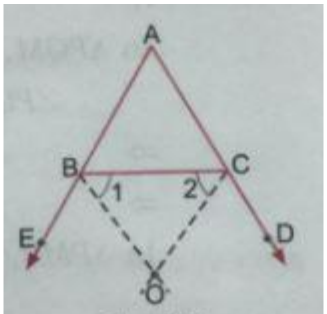

Fig. 6.32

Sol. As $\angle ABC$ and $\angle CBE$ form a linear pair

$\therefore$ $\angle ABC + \angle CBE = 180^0$

As BO is the bisector of $\angle CBE$

$\therefore$ $\angle CBE = 2\angle 1$

Therefore, $\angle ABC + 2\angle 1 = 180^0$

$\Rightarrow$ $2\angle 1 = 180^0$

$\Rightarrow$ $\angle 1 = 90^0 - \frac{1}{2}\angle ABC$ ...(i)

Again, $\angle ACB$ and $\angle BCD$ form a linear pair

$\therefore$ $\angle ACB + \angle BCD = 180^0$

As, CO is the bisector of $\angle BCD,$ therefore, $\angle BCD = 2\angle 2$

So, $\angle ACB + 2\angle 2 = 180^0$

$\Longrightarrow$ $2\angle 2 = 180^0 - 180^0 - \angle ACB$

$\Longrightarrow$ $\angle 2 = 90^0 - \frac{1}{2}\angle ACB$

In ΔOBC, we have

$\angle 1 + \angle 2 + \angle BOC = 180^0$ (Angle sum property of triangle)

From, $(i), (ii)$ and (iii), We have

$$90^0 - \frac{1}{2}\angle ABC + 90^0 - \frac{1}{2}\angle ACB + \angle BOC = 180^0$$

Now, in $\Delta\ ABC,$ we have

$\angle A + \angle B + \angle C = 180^0$

Or $\angle B + \angle C = 180^0 - \angle A$

From (iv) and (V), we have

$$180^0 - \frac{1}{2}(180^0 - \angle A) + \angle BOC = 180^0$$

$\Rightarrow \quad \angle BOC = 180^0 - 180^0 + \frac{1}{2}(180^0 - \angle A$

$\Rightarrow \quad BOC = \frac{1}{2}(180^0 - \angle A$

Hence, $\quad \angle BOC = 90^0 - \frac{1}{2}\angle A$