JEE ADVANCED - 2017 (Paper 1)

SECTION-1 : (Maximum Marks : 28)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is (are) correct.
- For each question, darken the bubble(s) corresponding to the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks	:	+4	If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
Partial Marks	:	+1	For darkening a bubble corresponding **to each correct option,** Provided NO incorrect option is darkened.
Zero Marks	:	0	If non of the bubbles is darkened.
Negative Marks	:	–2	In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

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37. Which of the following is(are) NOT the squares of a 3×3 matrix with real entries ?

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Sol. : If given matrix B is square of matrix A.

Then, $A^2 = B$

$\therefore$ $|A^2| = |B| = -1$ in (A) and (B).

So, it is not possible.

$I^2 = I$ in (C).

If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$ then, $A^2 = B$ in (D)

So, (A) and (B) are not possible. **Ans. (A), (B)**

38. If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$ and midpoint (h, k), then which of the following is(are) possible value(s) of p, h and k ?

(A) $p = 5,\ h = 4,\ k = -3$ (B) $p = -1,\ h = 1,\ k = -3$

(C) $p = -2,\ h = 2,\ k = -4$ (D) $p = 2,\ h = 3,\ k = -4$

Sol. : The equation of a chord with midpoint (h, k) is

$$k \cdot y - 16\left(\frac{x+h}{2}\right) = k^2 - 16h$$

$\therefore \quad 8x - ky + k^2 - 8h = 0$

Comparing with $2x + y - p = 0$, we get

$\left(\frac{8}{2} = \frac{-k}{1} = \frac{k^2 - 8h}{-p}\right)$. So, $k = -4$.

$-4p = 16 - 8h$

i.e. $2h - p = 4$

$k = -4$ in (C) and (D)

$p = 2 \Rightarrow h = 3$ and $p = -2 \Rightarrow h = 1$

So, only (D) is true. **Ans. (D)**

39. Let a, b, x and y be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\operatorname{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is(are) possible value(s) of x ?

(A) $-1 - \sqrt{1-y^2}$ (B) $1 + \sqrt{1+y^2}$ (C) $1 - \sqrt{1+y^2}$ (D) $-1 + \sqrt{1-y^2}$

Sol. : $\operatorname{Im}\left(\frac{az+b}{z+1}\right) = y$ and $z = x + iy$

$\operatorname{Im}\left(\frac{a(x+iy)+b}{x+iy+1}\right) = y$

$\therefore \quad \operatorname{Im}\left(\frac{(ax+b+iay)(x+1-iy)}{(x+1)^2+y^2}\right) = y$

$\therefore \quad -y(ax + b) + ay(x + 1) = y((x + 1)^2 + y^2)$

$\therefore \quad (a - b)y = y((x + 1)^2 + y^2)$

$\because \quad y \neq 0$ and $a - b = 1$

$\therefore \quad (x + 1)^2 + y^2 = 1$

$\therefore \quad x = -1 \pm \sqrt{1-y^2}$ **Ans. (A), (D)**

40. Let X and Y be two events such that $P(X) = \frac{1}{3}$, $P(X \mid Y) = \frac{1}{2}$ and $P(Y \mid X) = \frac{2}{5}$. Then,

(A) $P(\bar{X} \mid Y) = \frac{1}{2}$ (B) $P(X \cap Y) = \frac{1}{5}$ (C) $P(X \cup Y) = \frac{2}{5}$ (D) $P(Y) = \frac{4}{15}$

Sol. : $P(X) = \frac{1}{3}$; $\frac{P(X \cap Y)}{P(Y)} = \frac{1}{2}$; $\frac{P(Y \cap X)}{P(X)} = \frac{2}{5}$

From this information, we get

$P(X \cap Y) = \frac{2}{15}$; (B) is not true.

$P(Y) = \frac{4}{15}$; (D) is true.

$\therefore \quad P(X \cup Y) = \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{7}{15}$; (C) is not true.

$$P(\bar{X} \mid Y) = \frac{P(\bar{X} \cap Y)}{P(Y)} = \frac{P(Y) - P(X \cap Y)}{P(Y)}$$

$\therefore$ $P(\bar{X} \mid Y) = 1 - \dfrac{\frac{2}{15}}{\frac{4}{15}} = \frac{1}{2}$; (A) is true. **Ans. (A), (D)**

41. Let $[x]$ be the greatest integer less than or equal to x. Then, at which of the following point(s) is the function $f(x) = x\, cos(\pi(x + [x]))$ discontinuous ?

(A) $x = -1$ (B) $x = 0$ (C) $x = 2$ (D) $x = 1$

Sol. : $f(x) = x\, cos(\pi x + [x]\pi)$

$\therefore$ $f(x) = (-1)^{[x]}\, x\, cos\pi x$

$\therefore$ Discontinuous at all integers except zero. **Ans. (A), (C), (D)**

42. If $2x - y + 1 = 0$ is a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$, then which of the following CANNOT be the sides of a right angled trianged ?

(A) $2a$, 4, 1 (B) $2a$, 8, 1 (C) a, 4, 1 (D) a, 4, 2

Sol. : If the line $y = mx + c$ is a tangent to hyperbola $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$, then $c^2 = a^2m^2 - b^2$

$\therefore$ $(1)^2 = 4a^2 - 16$.

So, $a^2 = \frac{17}{4}$

$\therefore$ $a = \frac{\sqrt{17}}{2}$

For option (A), sides are $\sqrt{17}$, 4, 1 (So, a Right angled triangle) as $(\sqrt{17})^2 = 4^2 + 1^2$

For option (B), sides are $\sqrt{17}$, 8, 1 (So, a Triangle is not possible) $\sqrt{17} + 1 \not> 8$

For option (C), sides are $\frac{\sqrt{17}}{2}$, 4, 1 (So, a Triangle is not possible) $\frac{\sqrt{17}}{2} + 1 \not> 4$

For option (D), sides are $\frac{\sqrt{17}}{2}$, 4, 2 (So, a Triangle exists but not right angled) $\left(\frac{\sqrt{17}}{2}\right)^2 + 2^2 \neq 4^2$

Ans. (B), (C), (D)

43. Let $f : R \to (0, 1)$ be a continuous function. Then, which of the following function(s) has (have) the value zero at some point in the interval (0, 1) ?

(A) $e^x - \int_0^x f(t)\, sin\, t\, dt$ (B) $x^9 - f(x)$

(C) $f(x) + \int_0^x f(t)\, sin\, t\, dt$ (D) $x - \int_0^{\frac{\pi}{2} - x} f(t)\, cos\, t\, dt$

Sol. : For option (A),

Let $g(x) = e^x - \int_0^x f(t)\, sin\, t\, dt$

$\therefore \quad g'(x) = e^x - (f(x) \sin x) > 0, \quad \forall x \in (0, 1) \qquad$ as $f(x) \in (0, 1)$

$\therefore \quad g(x)$ is strictly increasing function.

Also, $g(0) = 1$

$\therefore \quad g(x) > 1, \forall x \in (0, 1)$

$\therefore$ option (A) is not possible.

For option (B),

Let $k(x) = x^9 - f(x)$

Now, $k(0) = -f(0) < 0 \qquad$ (As $f \in (0, 1)$)

Also, $k(1) = 1 - f(1) > 0 \qquad$ (As $f \in (0, 1)$)

$\therefore \quad k(0) \cdot k(1)$ have opposite signs.

So, option (B) is correct.

For option (C), let

$$h(x) = f(x) + \int_0^{\frac{\pi}{2}} f(t) \cdot \sin t \, dt$$

$\therefore \quad h(x) > 0, \quad \forall x \in (0, 1) \qquad$ (As $f \in (0, 1)$)

So, option (C) is not possible.

For option (D),

$$\text{Let } q(x) = x - \int_0^{\frac{\pi}{2}-x} f(t) \cos t \, dt$$

$$\therefore \quad q(0) = 0 - \int_0^{\frac{\pi}{2}} f(t) \cdot \cos t \, dt < 0 \qquad \text{as } f(t) \in (0, 1), \cos t \in (0, 1)$$

$$\text{Also, } q(1) = 1 - \int_0^{\frac{\pi}{2}-1} f(t) \cdot \cos t \, dt > 0 \qquad \text{as } f(t) \in (0, 1), \cos t \in (0, 1)$$

$\therefore \quad q(0) \cdot q(1)$ have opposite signs.

$\therefore$ option (D) is correct. **Ans. (B), (D)**

SECTION-2 : (Maximum Marks : 15)

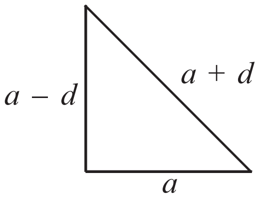
- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

  Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

  Zero Marks : 0 If all other cases.

44. The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side ?

Sol. :



Let the sides be $a - d$, a, $a + d$, where $d > 0$, $a > 0$, $a > d$

$\therefore$ the length of the smallest side = $a - d$

$\therefore$ Now, $(a + d)^2 = a^2 + (a - d)^2$

$\therefore$ $a(a - 4d) = 0$

$\therefore$ $a = 4d$ (As $a = 0$ is not possible.) (1)

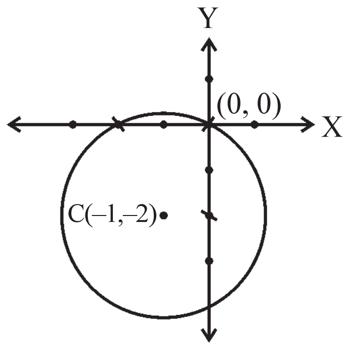
Also, $\frac{1}{2}a \cdot (a - d) = 24$ (Area)

$\therefore$ $a(a - d) = 48$ (2)

$\therefore$ From (1) and (2), we get $a = 8$, $d = 2$

Hence, the length of the smallest side is $(a - d) = (8 - 2) = 6$ **Ans. 6**

45. For how many values of p, the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points ?



Sol. : We shall consider 3 cases.

Case I : When $p = 0$.

(i.e. circle passes through origin)

Now, the equation of circle becomes

$x^2 + y^2 + 2x + 4y = 0$

$y = 0 \Rightarrow x^2 + 2x = 0 \Rightarrow (-2, 0), (0, 0)$ are points of intersection.

$x = 0 \Rightarrow y^2 + 4y = 0 \Rightarrow (0, -4), (0, 0)$ are points of intersection.

$\therefore$ Three points $(-2, 0), (0, -4), (0, 0)$ are on the axes and the circle.

Case II : When the circle intersects X-axis at 2 distinct points and touches Y-axis.

Now, $(g^2 - c) > 0$ and $f^2 - c = 0$

$\therefore$ $1 - (-p) > 0$ and $4 - (-p) = 0$, giving $p = -4$

$\therefore$ $p > -1$ and $p = -4$

$\therefore$ Not possible.

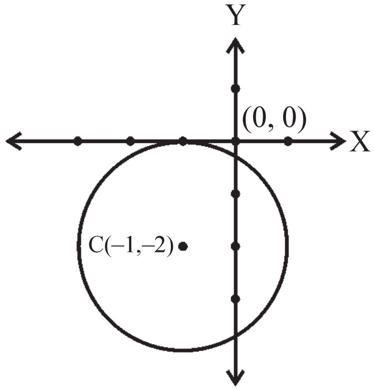
Case III : When the circle intersects Y-axis at 2 distinct points and touches X-axis.

Now, $g^2 - c = 0$ and $f^2 - c > 0$

$\therefore$ $1 - (-p) = 0$ and $4 - (-p) > 0$

$\therefore$ $p = -1$ and $p > -4$

$\therefore$ $p = -1$ is possible.



$\therefore$ Finally we conclude that $p = 0$ or -1

$\therefore$ Two possible values of p exist. **Ans. 2**

46. For a real number α, if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

of linear equations, has infinitely many solutions, then $1 + \alpha + \alpha^2 =$ ....

Sol. : Equations are

$x + \alpha y + \alpha^2 z = 1$

$\alpha x + y + \alpha z = -1$

$\alpha^2 x + \alpha y + z = 1$

The solution is unique if $D \neq 0$.

The solution is not unique, so $D = 0$.

$D = 0 \Rightarrow 1(1 - \alpha^2) - \alpha(\alpha - \alpha^3) + \alpha^2(\alpha^2 - \alpha^2) = 0$

$\therefore$ $(1 - \alpha^2) - \alpha^2 + \alpha^4 = 0$

$\therefore$ $(\alpha^2 - 1) = 0$

$\therefore$ $\alpha = \pm 1$

but at $\alpha = 1$

$x + y + z = 1$

$x + y + z = -1$. So there is no solution.

at $\alpha = -1$ all three equation become

$x - y + z = 1$ (coincident planes)

$\therefore$ $1 + \alpha + \alpha^2 = 1$ **Ans. 1**

47. Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then, $\frac{y}{9x}$ = ...

Sol. : $x = 10!$

Choose one letter from A, B, C, D, E, F, G, H, I, J and 8 from remaining n letters. These ten letters are arranged in 10 places, with one repeated.

$y = {}^{10}C_1 \, {}^{9}C_8 \, \frac{10!}{2!} = 45(10!)$

$\frac{y}{9x} = \frac{45(10!)}{9(10)!} = 5$ **Ans. 5**

48. Let $f : R \rightarrow R$ be a differentiable function such that $f(0) = 0, f\left(\frac{\pi}{2}\right) = 3$ and $f'(0) = 1$.

If $g(x) = \int_x^{\frac{\pi}{2}} [f'(t) \, cosec \, t - cot \, t \, cosec \, t \, f(t)] \, dt$

for $x \in \left(0, \frac{\pi}{2}\right]$, then $\lim_{x \to 0} g(x)$ = ...

Sol. : $g(x) = \int_x^{\frac{\pi}{2}} [f'(t) \, cosec \, t - cot \, t \, cosec \, t \, f(t)] \, dt$

$= \int_x^{\frac{\pi}{2}} (f(t) \, cosec \, t)' dt$

$= f\left(\frac{\pi}{2}\right) cosec\left(\frac{\pi}{2}\right) - \frac{f(x)}{sin \, x} = 3 - \frac{f(x)}{sin \, x}$

$\therefore \lim_{x \to 0} g(x) = 3 - \lim_{x \to 0} \frac{f(x)}{sin \, x} = 3 - \lim_{x \to 0} \frac{f'(x)}{cos \, x}$ (L-hospital' rule)

$= 3 - 1$

$= 2$ **Ans. 2**

SECTION-1 : (Maximum Marks : 21)

- This section contains **SIX** questions of matching type.
- This section contains **TWO** tables (each having 3 columns and 4 rows).
- Based on each table, there are **THREE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories :

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

Zero Marks : 0 If non of the bubble is darkened.

Negative Marks : –1 In all other cases.

Column 1, 2 and 3 contain conics, equations of tangents to the conics and points of contact, respectively.

Column 1	Column 2	Column 3
(I) $x^2 + y^2 = a^2$	(i) $my = m^2x + a$	(P) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
(II) $x^2 + a^2y^2 = a^2$	(ii) $y = mx + a\sqrt{m^2+1}$	(Q) $\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$
(III) $y^2 = 4ax$	(iii) $y = mx + \sqrt{a^2m^2-1}$	(R) $\left(\frac{-a^2m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$
(IV) $x^2 - a^2y^2 = a^2$	(iv) $y = mx + \sqrt{a^2m^2+1}$	(S) $\left(\frac{-a^2m}{\sqrt{a^2m^2-1}}, \frac{-1}{\sqrt{a^2m^2-1}}\right)$

49. The tangent to a suitable conic (column 1) at $\left(\sqrt{3}, \frac{1}{2}\right)$ is found to be $\sqrt{3}x + 2y = 4$, then which of the following options is the only CORRECT combination ?

(A) (II) (iii) (R) (B) (IV) (iv) (S) (C) (IV) (iii) (S) (D) (II) (iv) (R)

Sol. : $\sqrt{3}x + 2y = 4$ is a tangent at $\left(\sqrt{3}, \frac{1}{2}\right)$.

(C) and (D) are only possible.

From above $m = -\frac{\sqrt{3}}{2}$.

For S, $y < 0$, but $\frac{1}{2} > 0$

II (iv) R must be proper match.

$\frac{1}{\sqrt{a^2m^2+1}} = \frac{1}{2} \Rightarrow a^2m^2 + 1 = 4 \Rightarrow \frac{3}{4}a^2 = 3 \Rightarrow a = 2$

The curve is $x^2 + 4y^2 = 4$. Tangent is $y = -\frac{\sqrt{3}}{2}x + 2$.

So, $\sqrt{3}x + 2y = 4$

The point of contact is $\left(\sqrt{3}, \frac{1}{2}\right)$ (R).

Answer : (D) : II (iv) R **Ans. (D)**

50. If a tangent to a suitable conic (column 1) is found to be $y = x + 8$ and its point of contact is (8, 16), then which of the following options is the only CORRECT combination ?

(A) (III) (i) (P) (B) (III) (ii) (Q) (C) (II) (iv) (R) (D) (I) (ii) (Q)

Sol. : (III) (i) (P), II (iv) R, I (ii) Q may be proper matches.

$y = x + 8$ touches at (8, 16)

So, $m = 1$. Since $y = x + 8$, $a = 8$

For III $y^2 = 32x$, parabola, tangent is $y = x + 8$

Point of contact $\left(\frac{8}{1}, \frac{16}{1}\right) = (8, 16)$

Ans. : A : III (i) P is proper match. **Ans. (A)**

51. For $a = \sqrt{2}$, if a tangent is drawn to a suitable conic (column 1) at the point of contact (–1, 1), then which of the following options is the only **CORRECT** combination for obtaining its equation ?

(A) (II) (ii) (Q) (B) (III) (i) (P) (C) (I) (i) (P) (D) (I) (ii) (Q)

Sol. : (III) (i) (P) and I (ii) Q may be proper matches.

$a = \sqrt{2}$, point of contact (–1, 1)

$x = -1$ not possible for P as $\frac{a}{m^2} = \frac{\sqrt{2}}{m^2} > 0$

I (ii) Q is proper.

$x^2 + y^2 = 2$ is circle. $\frac{a}{\sqrt{1+m^2}} = 1 \Rightarrow \sqrt{2} = \sqrt{1+m^2} \Rightarrow m = 1$ **Ans. (D)**

52. Let $f(x) = x + \log x - x \log x,\ x \in (0, \infty)$.

* Column 1 contains information about zeroes of $f(x)$, $f'(x)$ and $f''(x)$.
* Column 2 contains information about the limiting behaviour of $f(x)$, $f'(x)$ and $f''(x)$ at infinity.
* Column 3 contains information about increasing / decreasing nature of $f(x)$ and $f'(x)$.

Column 1	Column 2	Column 3
(I) $f(x) = 0$ for some $x \in (1, e^2)$	(i) $\lim_{x\to\infty} f(x) = 0$	(P) f is increasing in (0, 1)
(II) $f'(x) = 0$ for some $x \in (1, e)$	(ii) $\lim_{x\to\infty} f(x) = -\infty$	(Q) f is decreasing in (e, e^2)
(III) $f'(x) = 0$ for some $x \in (0, 1)$	(iii) $\lim_{x\to\infty} f'(x) = -\infty$	(R) f' is increasing in (0, 1)
(IV) $f''(x) = 0$ for some $x \in (1, e)$	(iv) $\lim_{x\to\infty} f''(x) = 0$	(S) f' is decreasing in (e, e^2)

52. Which of the following options is the only **CORRECT** combination ?

(A) (IV) (i) (S) (B) (I) (ii) (R) (C) (III) (iv) (P) (D) (II) (iii) (S)

Ans. (D)

53. Which of the following options is the only **CORRECT** combination ?

(A) (III) (iii) (R) (B) (I) (i) (P) (C) (IV) (iv) (S) (D) (II) (ii) (Q)

Ans. (D)

54. Which of the following options is the only **INCORRECT** combination ?

(A) (II) (iii) (P) (B) (II) (iv) (Q) (C) (I) (iii) (P) (D) (III) (i) (R)

Ans. (D)

Sol. : 52 to 54

$f(x) = x + \log x - x \log x,\ x > 0$

$f'(x) = 1 + \frac{1}{x} - \log x - 1 = \frac{1}{x} - \log x$

$f''(x) = -\frac{1}{x^2} - \frac{1}{x} = \frac{-(x+1)}{x^2}$

(I) $f(1) = 1 + 0 - 0 = 1, f(e^2) = e^2 + 2 - 2e^2 = 2 - e^2 < 0$

$\therefore$ (I) is true.

(II) $f'(1) = 1 - 0 = 1, f'(e) = \frac{1}{e} - 1 < 0$

$\therefore$ (II) is true.

(III) Obviously III is false.

(IV) Since $x > 0$, IV is false.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x\left(1+\frac{\log x}{x} - \log x\right) = -\infty$$

as $\lim_{x\to\infty}\left(1+\frac{\log x}{x}\right) = 1$ and $\log x \to \infty$

(i) is false, (ii) is true.

$$\lim_{x\to\infty} f'(x) = \lim_{x\to\infty}\left(\frac{1}{x} - \log x\right) = -\infty$$

(iii) is true.

$$\lim_{x\to\infty} f''(x) = \lim_{x\to\infty}\frac{-(x+1)}{x^2} = 0$$

(iv) is true.

(P) $f'(x) = \frac{1}{x} - \log x > 0$ in $(0, 1)$ as $\log x < 0$

f is $\uparrow$ in $(0, 1)$. True.

(Q) $\log e < \log x < \log e^2$

$1 < \log x < 2$

So, $-2 < -\log x < -1$

$\frac{1}{e^2} < \frac{1}{x} < \frac{1}{e}$

$\frac{1}{e^2} - 2 < \frac{1}{x} - \log x < \frac{1}{e} - 1 < 0$

f is $\downarrow$ in (e, e^2) Q is true.

(R),(S) $f''(x) = \frac{-(x+1)}{x^2} < 0, \quad \forall x > 0$

$\therefore$ (R) is false and (S) is true.

(I) T	(i) F	(P) T
(II) T	(ii) T	(Q) T
(III) F	(iii) T	(R) F
(IV) F	(iv) T	(S) T

52. II (iii) S is TTT. **Ans. (D)**

53. II (ii) Q is TTT. **Ans. (D)**

54. III (i) R is FFF. **Ans. (D)**

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