Chapter 9

logarithms

Exercise 9.1

1. convert the following to logarithmic form :

(i) $5^2 = 25$

(ii) $a^5 = 64$

(iii) $7^x = 100$

(iv) $9^0 = 1$

(v) $6^1 = 6$

(vi) $3^{-2} = \frac{1}{9}$

(vii) $10^{-2} = 0.01$

(viii) $(81)^{\frac{3}{4}} = 27$

Solution

(i) $5^2 = 25 = \log_5 25 = 2$

(ii) $a^5 = 64 = \log_a 64 = 5$

(iii) $7^x = 100 = \log_7 100 = x$

(iv) $9^0 = 1 = \log_9 1 = 0$

(v) $6^1 = 6 = \log_6 6 = 1$

(vi) $3^{-2} = \frac{1}{9} = \log_3 \frac{1}{9} = -2$

(vii) $10^{-2} = 0.01 = \log_{10} 0.01 = -2$

(viii) $(81)^{\frac{3}{4}} = 27$ $= \log_{81} 27 = \frac{3}{4}$

2. convert the following into exponential form :

(i) $\log_2 32 = 5$

(ii) $\log_3 81 = 4$

(iii) $\log_3 \frac{1}{3} = -1$

(iv) $\log_3 4 = \frac{2}{3}$

(v) $\log_8 32 = \frac{5}{3}$

(vi) $\log_{10}(0.001) = -3$

(vii) $\log_2 0.25 = -2$

(viii) $\log_a \left(\frac{1}{a}\right) = -1$

Solution

(i) $\log_2 32 = 5$ $= 2^5 = 32$

(ii) $\log_3 81 = 4$ $= 3^4 = 81$

(iii) $\log_3 \frac{1}{3} = -1$ $= 3^{-1} = \frac{1}{3}$

(iv) $\log_3 4 = \frac{2}{3}$ $= (8)^{\frac{2}{3}}$ 4

(v) $\log_8 32 = \frac{5}{3}$ $= (8)^{\frac{5}{3}} = 32$

(vi) $\log_{10}(0.001) = -3$ $= 10^{-3} = 0.001$

(vii) $\log_2 0.25 = -2$ = 2^{-2} = 0.25

(viii) $\log_a\left(\frac{1}{a}\right) = -1$ = a^{-1} = $\frac{1}{a}$

3. by converting to exponential form , find the values of :

(i) $\log_2 16$

(ii) $\log_5 125$

(iii) $\log_4 8$

(iv) $\log_9 27$

(v) $\log_{10}(.01)$

(vi) $\log_7 \frac{1}{7}$

(vii) $\log_5 256$

(viii) $\log_2 0.25$

Solution

(i) let , $\log_2 16$ = x

= $2^x = 16$

= $2^x = 2 \times 2 \times 2 \times 2$

= $2^x = 2^4$

$\therefore x = 4$

(ii) let $\log_5 125 = x = 5^x = 125$

$= 5^x = 5\times5\times5 \ = 5^x = 5^3$

$x = 3$

(iii) let $\log_4 8 = x \ = 4^x = 8$

$= (2\times2)^x = 2\times2\times2 \ = (2)^{2x} \ = (2)^3$

$= 2x = 3$

$x= \frac{3}{2}$

(iv) $\log_9 27 \ = x$

$= 9^x \ = 27$

$= (3\times3)^x \ = 3\times3\times3 = (3)^{2x} = (3)^3$

$= 2x = 3$

$\therefore x = \frac{3}{2}$

(v) $\log_{10}(.01) = x = (10)^x = .01$

$= (10)^x = \frac{1}{100} \ = (10)^x = \frac{1}{10} \times \frac{1}{10}$

$= (10)^x = \frac{1}{(10)^2} \ = (10)^x = (10)^{-2}$

$\therefore x = -2$

(vi) $\log_7 \frac{1}{7} = x =$ $(7)^x = \frac{1}{7}$

$= (7)^x = (7)^{-1}$

$x = -1$

(vii) let $\log_5 256 = x$

$= (.5)^x = 256$ $= \left(\frac{5}{10}\right)^x = 256$

$= \left(\frac{1}{2}\right)^x = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= (2)^x = 2^8$

$= -x = 8$

$x = -8$

(viii) let , $\log_3 0.25 = x$

$= (2)^x = 0.25$

$= (2)^x = \frac{25}{100} = (2)^x = \frac{1}{4}$

$= (2)^x = (2)^{-2}$

$X = -2$

4. solve the following equations for x.

(i) $\log_3 x = 2$

(ii) $\log_x 25 = 2$

(iii) $\log_{10} x = -2$

(iv) $\log_4 x = \frac{1}{2}$

(v) $\log_x 11 = 2.5$

(vi) $\log_x \frac{1}{4} = -1$

(vii) $\log_{81} x = \frac{3}{2}$

(viii) $\log_9 x = 2.5$

(ix) $\log_4 x = -1.5$

Solution

(i) $\log_3 x = 2$

Let us simplify the expression

$(3)^2 = x$

X = 9

(ii) $\log_x 25 = 2$

Let us simplify the expression

$(x)^2 = 25$

$= 5\times 5$

$= x^2 = 5^2$

Since the powers are same

So,

$X = 5$

(iii) $\log_{10} x = -2$

Let us simplify the expression ,

$(10)^{-2} = x$

$X = \frac{1}{10^2}$

$= \frac{1}{100}$

$= 0.01$

(iv) $\log_4 x = \frac{1}{2}$

Let us simplify the expression,

$4^{\frac{1}{2}} = x$

$x = (2 \times 2)^{\frac{1}{2}}$

$= 2^{2\times\frac{1}{2}}$

$= 2$

(v) $\log_x 11 = 2.5$

Let us simplify the expression,

$(x)^1 = 11$

X = 11

(vi) $\log_x \frac{1}{4} = -1$

Let us simplify the expression

$(x)^{-1} = \frac{1}{4}$

$x^{-1} = 4^{-1}$

since the powers are same,

so,

x = 4

(vii) $\log_{81} x = \frac{3}{2}$

Let us simplify the expression

$81^{\frac{3}{2}} = x$

$x = 81^{\frac{3}{2}}$

$= (3^4)^{\frac{3}{2}}$

$= 3^{4\times\frac{3}{2}}$

$= 3^6$

$= 3 \times 3 \times 3 \times 3 \times 3 \times 3$

= 729

X= 729

(viii) $\log_9 x = 2.5$

$\log_9 x = \frac{5}{2}$

Let us simplify the expression ,

$9^{\frac{5}{2}} = x$

x = $(3^2)^{\frac{5}{2}}$

$= 3^{2\times\frac{5}{2}}$

= 3^5

= 3×3×3×3×3

= 234

X = 234

(ix) $\log_4 x = -1.5$

$\log_4 x = -\frac{3}{2}$

Let us simplify the expression

$(4)^{-\frac{3}{2}} = x$

$x = (2^2)^{-\frac{3}{2}}$

$= 2^{2\times-\frac{3}{2}}$

$= 2^{-3}$

$= \frac{1}{2^3}$

$= \frac{1}{2\times2\times2}$

$= \frac{1}{8}$

$= \frac{1}{8}$

5. given $\log_{10} a = b$, express 10^{2b-3} in terms of a .

Solution

Given :

$\log_{10} a = b$

$(10)^b = a$

Now ,

$10^{2b-3} = \frac{(10)^{2b}}{(10)^3}$

$\frac{(10^b)^2}{10\times10\times10}$

Substitute the value of $(10)^b = a$, we get

$= \frac{a^2}{1000}$

6. given $\log_{10} x = a$, $\log_{10} y = b$ and $\log_{10} z = c$,

(i) write down 10^{2a-3} in terms of x.

(ii) write down 10^{3b-1} in terms of y.

(iii) if $\log_{10} p = 2a + \frac{b}{2} - 3c$ express p in terms of x, y and z

Solution

Given :

$\log_{10} x = a$

- $(10)^a = x$

$\log_{10} y = b$

- $(10)^b = y$

$\log_{10} z = c$

- $(10)^c = z$

(i) write down 10^{2a-3} in terms of x.

$10^{2a-3} = \frac{(10)^{2a}}{(10)^3}$

$= \frac{(10)^{2a}}{(10\times10\times10)}$

Substitute the value of $(10)^a = x$, we get

$= \frac{x^2}{1000}$

(ii) write down 10^{3b-1} in terms of y.

$10^{3b-1} = \frac{(10)^{3b}}{(10)^1}$

$= \frac{(10)^{3b}}{10}$

Substitute the value of $(10)^b = y$ we get

$\frac{y^3}{10}$

(iii) if $\log_{10} p = 2a + \frac{b}{2} - 3c, express\ p\ in\ term\ of\ x, y\ and\ z$

We know that ,
$(10)^a = x$
$(10)^b = y$
$(10)^c = z$

By substituting the values
$\log_{10} p = 2a + \frac{b}{2} - 3c$
$= 2 \log_{10} x + \frac{1}{2} log_{10} y - 3 log_{10} z$
$= log_{10} x^2 + \log_{10} y^{\frac{1}{2}} - \log_{10} z^3$

$= \log_{10} \left(x^2 + y^{\frac{1}{2}}\right) - \log_{10} z^3$

$= \frac{\log_{10} x^2\sqrt{y}}{z^3}$

$P = \frac{(x^2\sqrt{y})}{z^3}$

7. if $\log_{10} x = a$ and $\log_{10} y = b$, find the value of xy .

solution

given
$\log_{10} x = a$

$(10)^a = x$
$\log_{10} y = b$
$(10)^b = y$
Then ,
$xy = (10)^a \times (10)^b$
$= (10)^{a+b}$

8. given $\log_{10} a = m$ and $\log_{10} b = n$, express $\frac{a^3}{b^2}$ In terms of m and n

Solution
Given:
$\log_{10} a = m$
$(10)^m = a$
$\log_{10} b = n$
$(10)^n = b$
So,

$$\frac{a^3}{b^2} = \frac{(10^m)^3}{(10^n)^2}$$

$$= \frac{10^{3m}}{10^{2n}}$$

$$= 10^{3m-2n}$$

9. given $\log_{10} a = 2a$ and $\log_{10} y = -\frac{b}{2}$

(i) write 10^a in terms of x.
(ii) write 10^{2b+1} in terms of y.
(iii) if $\log_{10} p = 3a - 2b$ express p in terms of x and y

solution

given :

$\log_{10} a$ =2a

$(10)^{2a} = a$

$\log_{10} y = -\frac{b}{2}$

$(10)^{-\frac{b}{2}} = y$

(i) write 10^a in terms of x.

$10^a = (10^{2a})^{\frac{1}{2}}$

$= x^{\frac{1}{2}}$

$= \sqrt{x}$

(ii) write 10^{2b+1} in terms of y .

$10^{2b+1} = 10^{2b} \times 10^1$

$= 10^{4\left(\frac{b}{2}\right)} \times 10^1$

$= y^4 \times 10^1$

$= 10y^4$

(iii) if $\log_{10} p$ = 3a – 2b , express p in terms of x and y.

$\log_{10} p$ = 3a – 2b

Substitute the values ,

$\log_{10} p = \frac{3}{2}\ (2a) - \ 4\left(\frac{b}{2}\right)$

$= \frac{3}{2}\log_{10} x - 4\log_{10} y$

$= \log_{10} \frac{x^{\frac{3}{2}}}{y^4}$

$P = \frac{x^{\frac{3}{2}}}{y^4}$

10 . if $\log_2 y = x$ and $\log_3 z = x$, find 72^x in terms of y and z

solution

Given
$\log_2 y = x$
$2^x = y$
$\log_3 z = x$
$3^x = z$
So,
$72^x = (2\times2\times2\times3\times3)^x$

$= (2^3 \times 3^2)^x$

$= 2^{3x} \times 3^{2x}$

$= (2^x)^3 \times (3^x)^2$

$= y^3 \times z^2$

$= y^3z^2$

11. if $\log_2 x = a$ and $\log_5 y = a$, write 100^{2a-1} in terms of x and y.

solution

given

$log_2 x = a$

$2^a = x$

$\log_5 y = a$

$5^a = y$

So,

$100^{2a-1} = (2\times2\times5\times5)^{2a-1}$

$= (2^2 \times 5^2)^{2a-1}$

$= 2^{4a-2} \times 5^{a-2}$

$= \frac{(2^{4a})}{2^2} \times \frac{(5^{4a})}{5^2}$

$= \frac{(2^a)^4 \times (5^a)^4}{4\times25}$

$= \frac{x^4 y^4}{100}$

Exercise 9.2

1. simplify the following

(i) $log a^3 - log a^2$

(ii) $log a^3 \div log a^2$

(iii) $\frac{log 4}{log 2}$

(iv) $\frac{log 8 \, log 9}{log 27}$

(v) $\frac{log 27}{log \sqrt{3}}$

(vi) $\frac{log 9 - log 3}{log 27}$

Solution

(i) $log a^3 - log a^2$

By using quotient law,

$log a^3 - log a^2 = \log \frac{a^3}{a^2}$

$= \log a$

(ii) $log a^3 \div log a^2$

By using power law,

$log a^3 \div log a^2 = 3\log a \div 2 \log a$

$= \frac{3\log a}{2\log a}$

$= \frac{3}{2}$

(iii) $\boldsymbol{\frac{log4}{log2}}$

Let us simplify the expression ,

$\frac{log4}{log2} = \frac{\log(2\times2)}{\log2}$

By using power law,

$= \frac{2log2}{log2}$

= 2

(iv) $\boldsymbol{\frac{log8\ log9}{log27}}$

Let us simplify the expression

$\frac{log8\ log9}{log27} = \frac{log2^3.log3^2}{log3^3}$

By using power law,

$= \frac{(3log2)\ .\ (2log3)}{3log3}$

$= \frac{log2.2}{1}$

= 2 log 2

= log 2^2

= log 4

(v) $\boldsymbol{\frac{log27}{log\sqrt{3}}}$

Let us simplify the expression,

$\frac{log27}{log\sqrt{3}} = \frac{\log(3\times3\times3)}{\log3^{\frac{1}{2}}}$

$= \frac{log3^3}{log3^{\frac{1}{2}}}$

= by using power law

$= \frac{3log3}{\frac{1}{2}log3}$

$= \frac{3\times2}{1\left(\frac{log3}{log3}\right)}$

= (6)(1)

= 6

(vi) $\boldsymbol{\frac{log9-log3}{log27}}$

Let us simplify the expression

$\frac{log9-log3}{log27} = \frac{\log(3\times3)-log3}{\log(3\times3\times3)}$

$= \frac{log3^2-log3}{log3^3}$

$= \frac{log3}{3log3}$

$= \frac{1}{3}$

2. evaluate the following

(i) $\log(10\div\sqrt[3]{10})$

(ii) $2 + \frac{1}{2}\log(10^{-3})$

(iii) $2\log 5 + \log 8 - \frac{1}{2}\log 4$

(iv) $2\log 10^3 + 3\log 10^{-2} - \frac{1}{3}\ log 5^{-3} + \frac{1}{2}\log 4$

(v) $2\log 2 + \log 5\ - \frac{1}{2}\log 36 - \log\frac{1}{30}$

(vi) $2\log 5 + \log 3 + 3\log 2 - \frac{1}{2}\log 36 - 2\log 10$

(vii) $\log 2 + 16\log\frac{16}{15} + 12\log\frac{25}{24} + 7\log\frac{81}{80}$

(viii) $2\log 10^5 + \log 10^8 - \frac{1}{2}\log 10^4$

Solution

(i) $\log(10 \div \sqrt[3]{10})$

Let us simplify the expression,

$\log(10 \div \sqrt[3]{10}) = \log(10 \div 10^{\frac{1}{3}})$

$= \log(10^{1-\frac{1}{3}})$

$= \log(10^{\frac{2}{3}})$

$= \frac{2}{3}\log 10$

$= \frac{2}{3}(1)$

$= \frac{2}{3}$

(ii) $2+\frac{1}{2}\log(10^{-3})$

Let us simplify the expression ,

$2+\frac{1}{2}\log(10^{-3}) = 2+\frac{1}{2}\times(-3)\log 10$

$= 2-\frac{3}{2}\log 10$

$= 2-\frac{3}{2}(1)$

$= 2-\frac{3}{2}$

$= \frac{4-3}{2}$

$= \frac{1}{2}$

(iii) $2\log 5+\log 8-\frac{1}{2}\log 4$

Let us simplify the expression

$2\log 5+\log 8-\frac{1}{2}\log 4 = \log 5^2+\log 8-\frac{1}{2}\log 2^2$

$= \log 25+\log 8-\frac{1}{2}2\log 2$

$= \log 25+\log 8-\log 2$

$= \log \frac{(25\times 8)}{2}$

$= \text{Log}(25\times 4)$

$= \log 100$
$= \log 10^2$
$= 2 \log 10$
$= 2(1)$
$= 2$

(iv) $\mathbf{2 \log 10^3 + 3 \log 10^{-2} - \frac{1}{3} \log 5^{-3} + \frac{1}{2} \log 4}$

Let us simplify the expression,
$2 \log 10^3 + 3 \log 10^{-2} - \frac{1}{3} \log 5^{-3} + \frac{1}{2} \log 4 = 2 \times 3 \log 10 + 3 (-2) \log 10 - \frac{1}{3}(-3) \log 5 + \frac{1}{2} \log 2^2$

$= 6 \log 10 - 6 \log 10 + \log 5 + \frac{1}{2} 2 \log 2$

$= 6 \log 10 - 6 \log 10 + \log 5 + \log 2$

$= 0 + \log 5 + \log 2$

$= \log(5 \times 2)$

$= \log 10$

$= 1$

(v) 2 log 2 + log 5 $-\frac{1}{2}$ log 36 – log$\frac{1}{30}$

let us simplify the expression ,

2 log 2 + log 5 $-\frac{1}{2}$ log 36 – log$\frac{1}{30}$ = log 2^2 + log 5 – $\frac{1}{2}$ log 6^2 – log $\frac{1}{30}$

= log 4+ log 5 – log 6 – log$\frac{1}{30}$

= log 4 + log 5 - log 6 –(log 1 – log 30)

= log4 + log 5 – log 6 – log1 + log 30

= log 4 + log 5 + log 30 – (log6 + log1)

= log($4\times 5 \times 30$) – log (6×1)

= log$\frac{4\times5\times30}{6\times1}$

= log($4 \times 5 \times 5$)

= log 100

= log 10^2

= 2 log 10

= 2 log 10

= 2(1)

= 2

(vi) $2\log 5 + \log 3 + 3\log 2 - \frac{1}{2}\log 36 - 2\log 10$

Let us simplify the expression ,

$2\log 5 + \log 3 + 3\log 2 - \frac{1}{2}\log 36 - 2\log 10 = \log 5^2 + \log 3 + \log 2^3 - \frac{1}{2}\log 6^2 - \log 10^2$

$= \log 25 + \log 3 + \log 8 - \log 6 - \log 100$

$= \log(25 \times 3 \times 8) - \log(6 \times 100)$

$= \log \frac{25\times3\times8}{6\times100}$

$= \log \frac{1\times3\times8}{6\times100}$

$= \log \frac{1\times3\times8}{6\times4}$

$= \log \frac{24}{24}$

$= \log 1$

$= 0$

(vii) $\log 2 + 16\log\frac{16}{15} + 12\log\frac{25}{24} + 7\log\frac{81}{80}$

Let us simplify the expression,

$\log 2 + 16\log\frac{16}{15} + 12\log\frac{25}{24} + 7\log\frac{81}{80} = \log 2 + 16(\log 16 - \log 15) + 12(\log 25 - \log 24) + 7(\log 81 - \log 80)$

$= \log 2 + 16(\log 2^4 – \log(3 \times 5)) + 12(\log 5^2 – \log(3\times 2 \times 2 \times 2)) + 7 (\log(3 \times 3 \times 3 \times 3) - \log(2^4 \times 5))$

$= \log 2 + 16(4 \log 2 –(\log 3 + \log 5)) + 12(2\log 5 – \log(3\times 2^3)) + 7(\log 3^4 –(\log 2^4 + \log 5))$

$= \log 2 + 16(4\log 2- \log 3 – \log 5) + 12(2\log 5 –(\log 3 + 3 \log 2)) + 7 (4 \log 3 – 4 \log 2 – \log 5)$

$= \log 2 + 64 \log 2 – 16 \log 3 – 16 \log 5 + 24 \log 5 – 12 \log 3 – 36 \log 2 + 28 \log 3 – 28 \log 2 – 7 \log 5$

$= (\log 2 + 64 \log 2 – 36 \log 2 – 28\log 2) + (-16 \log 3 – 12 \log 3 + 28 \log 3)+ (-16 \log 5 + 24 \log 5 – 7 \log 5)$

$= (65 \log 2 – 64 \log 2) +(-28 \log 3 + 28 \log 3) +(-23 \log 5 + 24 \log 5)$

$= \log 2 + 0 + \log 5$

$= \log(2\times 5)$

$= \log 10$

$= 1$

(viii) $2\log 10^5 + \log 10^8 - \frac{1}{2}\log 10^4$

Let us simplify the expression

$2\log 10^5 + \log 10^8 - \frac{1}{2}\log 10^4 = \log_{10}5^2 + \log 10^8 - \log 10^{4^{\frac{1}{2}}}$

$= \log 10^{25} + \log 10^8 - \log 10^{2^{2\times\frac{1}{2}}}$

$= \log 10^{25} + \log 10^8 - \log 10^2$

$= \log 10^{\frac{25\times 8}{2}}$

$= \log 10^{25\times 4}$

$= \log 10^{100}$

$= \log 10^{10^2}$

$= 2\log 10^{10}$

$= 2(1)$

$= 2$

3. express each of the following as a single logarithm :

(i) $2\log 3 - \frac{1}{2}\log 16 + \log 12$

(ii) $2\log 10^5 - \log 10^2 + 3\log 10^{4+1}$

(iii) $\frac{1}{2}\log 36 + 2\log 8 - \log 1.5$

(iv) $\frac{1}{2}$ log 25 – 2 log 3 + 1

(v) $\frac{1}{2}$log 9 + 2 log 3 – log 6 + log 2 -2

Solution

(i) 2 log 3 – $\frac{1}{2}$ log 16 + log 12
Let us simplify the expression into single logarithm ,
2 log 3 – $\frac{1}{2}$ log 16 + log 12 = 2 log 3 – $\frac{1}{2}$ log 4^2 + log 12

= 2 log 3 – log 4 + log 12

= log 3^2 – log 4 + log 12

= Log 9 – log 4 + log 12

= log$\frac{9\times 12}{4}$

= log (9× 3)

= log 27

(ii) 2 $\log 10^5 - \log 10^2 + 3\ log 10^{4+1}$

Let us simplify the expression into single logarithm ,
2 $\log 10^5 - \log 10^2 + 3\ log 10^{4+1}$ = $\log 10^{5^2}$- $\log 10^2$ +$\log 10^{4^3}$ + $\log 10^{10}$

= $\log_{10} 25$- $\log_{10} 2$ + $\log_{10} 64$ + $\log_{10} 10$

$= \log_{10}(25\times 64 \times 10) - \log_{10} 2$

$= \log_{10} (16000) - \log_{10} 2$

$= \log_{10}\left(\frac{16000}{2}\right)$

$= \log_{10} 8000$

(iii) $\frac{1}{2} \log 36 + 2 \log 8 - \log 1.5$

Let us simplify the expression into single logarithm,

$\frac{1}{2} \log 36 + 2 \log 8 - \log 1.5 = \log 36^{\frac{1}{2}} + \log 8^2 - \log 1.5$

$= \log 6^{2\times\frac{1}{2}} + \log 64 - \log 1.5$

$= \log 6 + \log 64 - \log\left(\frac{15}{10}\right)$

$= \log 6 + \log 64 - (\log 15 - \log 10)$

$= \log (6\times 64) - \log 15 + \log 10$

$= \log(6 \times 64 \times 10)- \log 15$

$= \log\frac{6\times 64\times 10}{15}$

$= \log (4\times 64)$

$= \log 256$

(iv) $\frac{1}{2}$ log 25 – 2 log 3 + 1

Let us simplify the expression into single logarithm,

$\frac{1}{2}$ log 25 – 2 log 3 +1 = log $25^{\frac{1}{2}} - \ log 3^2 + \log 10$

= log$5^{2\times\frac{1}{2}}$ - log 9 + log 10

= log (5× 10) – log 9

= log $\left(\frac{5\times10}{9}\right)$

= log $\frac{50}{9}$

(v) $\frac{1}{2}$log 9 + 2 log 3 – log 6 + log 2 -2

Let us simplify the expression into single logarithm,

$\frac{1}{2}$log 9 + 2 log 3 – log 6 + log 2 -2 = log$9^{\frac{1}{2}}$ + log3^2- log 6 + log 2 – log 100

= log$3^{2\times\frac{1}{2}}$ + log 9 – log 6 + log 2 – log 100

= log 3 + log 9 – log 6 + log 2 – log 100

= log $\frac{3\times9\times2}{6\times100}$

= log $\frac{9}{100}$

4. prove the following :

(i) $\mathbf{\log_{10} 4 \div \log_{10} 2 = \log_3 9}$

(ii) $\mathbf{\log_{10} 25 + \log_{10} 4 = \log_5 25}$

Solution

(i) $\log_{10} 4 \div \log_{10} 2 = \log_3 9$

Let us consider LHS, $\log_{10} 4 \div \log_{10} 2$

$\log_{10} 4 \div \log_{10} 2 = \log_{10} 2^2 \div log_{10} 2$

$= 2 \log_{10} 2 \div log_{10} 2$

$= 2 \frac{\log_{10} 2}{log_{10} 2}$

= 2(1)

= 2

Now let us consider RHS,

$\log_3 9 = log_3 3^2$

$= 2\ log_3 3$

=2(1)

=2

∴ LHS = RHS

Hence proved.

(ii) $\log_{10} 25 + \log_{10} 4 = \log_5 25$

Let us consider LHS, $\log_{10} 25 + \log_{10} 4$

$\log_{10} 25 + \log_{10} 4 = \log_{10}(25 \times 4)$

$= \log_{10} 100$

$= \log_{10} 10^2$

$= 2 \log_{10} 10$

=2(1)
= 2

Now let us consider RHS,
$\log_5 25 = log_5 5^2$
= $2\log_5 5$
= 2(1)
=2
LHS = RHS
Hence proved .

5. if x =(100)^a , y = (10000)^b and z = (10)^c , express log $\left[\frac{10\sqrt{y}}{x^2z^3}\right]$ in terms of a,b, c.

Solution

Given :
x = $(100)^a = (10^2)^a = 10^{2a}$
y = $(10000)^b = (10^4)^b = 10^{4b}$
z = $(10)^c$

it is given that , log $\left[\frac{10\sqrt{y}}{x^2z^3}\right]$
$\log\left[\frac{10\sqrt{y}}{x^2z^3}\right]$ = (log 10 + $\log\sqrt{y}$) - (log x^2 + log z^3)
= $(1 + \log y^{\frac{1}{2}}) - (log x^2 + log z^3)$ [we know that , log 10 =1]

= $(1 + \frac{1}{2}\log y) - (2\log x + 3\log z)$
Now substitute the values of x,y, z we get

= $\left(1 + \frac{1}{2}\, log 10^{4b}\right) - (2\, log 10^{2a} + 3 log 10^c)$

$= (1 + \frac{1}{2}\ 4b \log 10\ ) - (\ 2 \times 2a \log 10 + 3 \times c\ log10)$

$= \left(1 + \frac{1}{2} 4b\right) - (\ 2 \times 2a + 3c)[\ since\ , \log 10\ = 1]$

= (1 + 2b) – (4a + 3c)

= 1 + 2b – 4a – 3c

6. if a = $\log_{10} x$, find the following in terms of a :

(i) x
(ii) $\log_{10} \sqrt[5]{x^2}$
(iii) $\log_{10} 5x$

Solution
Given :
a = $\log_{10} x$

(i) x
$10^a = x$
$x = 10^a$

(ii) $\log_{10} \sqrt[5]{x^2}$
$\log_{10} \sqrt[5]{x^2} = \log_{10}(x^2)^{\frac{1}{5}}$
$= \log_{10} x^{\frac{2}{5}}$
$= \frac{2}{5} \log_{10} x$

$= \frac{2}{5}\ (a)$

$= \frac{2a}{5}$

(iii) $\log_{10} 5x$

x = $(10)^a$

$= \log_{10} 5x$

$=\log_{10} 5(10)^a$

$= \log_{10} 5 + log_{10} 10$

$= \log_{10} 5 + a(1)$

$= a + \log_{10} 5$

7. if a = $\log\frac{2}{3}$, b = $\log\frac{3}{5}$ and c = $2\log\sqrt{\frac{5}{2}}$. find the values of

(i) a+ b + c

(ii) 5^{a+b+c}

Solution

Given:

$a = \log\frac{2}{3}$

$b = \log\frac{3}{5}$

$c = 2\log\sqrt{\frac{5}{2}}$

(i) a + b + c

Let us substitute the given values we get

a + b + c = $\log \frac{2}{3} + \log \frac{3}{5} + 2 \log \sqrt{\frac{5}{2}}$

= (log 2 – log 3) + (log 3 – log 5) + 2 $\log \frac{5}{2}^{\frac{1}{2}}$

= log 2 – log 3 + log3 – log 5 + 2 $\times \frac{1}{2}$ $(log 5 - \log 2)$

= log 2 – log 3 + log 3 – log 5 + log 5 – log 2

= 0

(ii) 5^{a+b+c}

5^{a+b+c} = 5^0

= 1

8. if x = $\log \frac{3}{5}$, $y = \log \frac{5}{4}$ $and\ z = 2 \log \frac{\sqrt{3}}{2}$, find the value of

(i) x + y – z

(ii) 3^{x+y-z}

Solution

Given

X = $\log \frac{3}{5} = \log 3 - \log 5$

Y = $\log \frac{5}{4} = \log 5 - \log 4$

Z = 2 $\log \frac{\sqrt{3}}{2} = \log \left(\frac{\sqrt{3}}{2}\right)^2 = \log \frac{3}{4} = \log 3 - \log 4$

(i) x + y – z

Let us substitute the given values we get
x + y – z = log 3 – log 5 + log 5 – log 4 – (log 3 – log 4)
= log 3 – log 5 + log 5 – log 4 – log 3 + log 4
= 0

(ii) 3^{x+y-z}
$3^{x+y-z} = 3^0$
= 1

9. if x = $\log_{10} 12$, $y = \log_4 2 \times \log_{10} 9$ *and* $z = \log_{10} 0.4$, find the values of

(i) x- y – z
(ii) 7^{x-y-z}

Solution
Given
X = $\log_{10} 12$
Y = $\log_4 2 \times log_{10} 9$
Z = $\log_{10} 0.4$

(i) x – y – z
Let us substitute the given values we get
x- y –z $= \log_{10} 12 - log_4 2 \times log_{10} 9 - log_{10} 0.4$
$= \log_{10}(3 \times 4) - log_4 4^{\frac{1}{2}} \times log_{10} 3^2 - log_{10} \frac{4}{10}$

$= \log_{10} 3 + \log_{10} 4 - \frac{1}{2} log_4 4 \times 2\, log_{10} 3 - (log_{10} 4 - log_{10} 10)$

$= log_{10} 3 + log_{10} 4 - \frac{1}{2} \times 1 \times 2\, log_{10} 3 - (log_{10} 4 - log_{10} 10)$

$= \log_{10} 3 + \ log_{10} 4 - \frac{1}{2} \times 1 \times 2\ log_{10} 3 - \ log_{10} 4 + 1$

$= \log_{10} 3 + \ log_{10} 4 - \ log_{10} 3 - \ log_{10} 4 + 1$

= 1

(ii) 7^{x-y-z}

$7^{x-y-z} = 7^1$

= 7

10. if log V + log 3 = log π + log 4 + 3 log r , find v in terms of other quantities .

Solution

Given

Log v + log 3 = log π + log 4 + 3 log r

Let us simplify the given expression to find V,

Log(v × 3) = log $(\pi \times 4 \times r^3)$

Log 3v = log$4\pi r^3$

3V = $4\pi r^3$

V = $\frac{4\pi r^3}{3}$

11. given 3(log 5 – log 3) – (log 5 – 2 log 6) = 2 – log n , find n.

Solution

Given :

3(log 5 – log 3) – (log 5 – 2 log6) = 2 – log n

Let us simplify the given expression to find n,

3 log 5 – 3 log 3 – log 5 + 2 log 6 = 2 – log n

2 log 5 – 3 log 3 + 2 log 6 = 2(1) – log n

Log 5^2 – log3^3 + log 6^2 = 2 log 10 – log n [since , 1 = log 10]
Log 25 – log 27 +log36 – log 10^2 = - log n
Log n = - log 25 + log 27 – log 36 + log 100
= (log 100 + log 27) – (log 25 + log 36)
= log (100 × 27) – log (25 × 36)
= log $\frac{(100 \times 27)}{25\times 36}$
= log n = log 3
n = 3

12. given that $\log_{10} y + 2 \log_{10} x = 2$, express y in terms of x .

Solution
Given :
$\log_{10} y + 2\log_{10} x = 2$
Let us simplify the given expression,
$\log_{10} y + \log_{10} x^2 = 2(1)$

$\log_{10} y + \log_{10} x^2 = 2\log_{10} 10$

$\log_{10}(y \times x^2) = \log_{10} 10^2$

$yx^2 = 100$
y = $\frac{100}{x^2}$

13. express $\log_{10} 2 + 1$ in the form $\log_{10} x$.

Solution
Given
$\log_{10} 2 + 1$
Let us simplify the given expression
$\log_{10} 2 + 1 = \log_{10} 2 + \log_{10} 10$ [since 1 = $\log_{10}$ 10]

$= \log_{10}(2 \times 10)$
$= \log_{10} 20$

14. if $a^2 = \log_{10} x$, $b^2 = \log_{10} y$ and $\frac{a^2}{2} - \frac{b^2}{3} = \log_{10} z$. express z in terms of x and y.

Solution
Given
$a^2 = \log_{10} x$
$b^2 = \log_{10} y$
$\frac{a^2}{2} - \frac{b^2}{3} = \log_{10} z$

Let us substitute the given values in the expression we get
$\log_{10} \frac{x}{2} - \log_{10} \frac{y}{3} = \log_{10} z$
$\log_{10} x^{\frac{1}{2}} - \log_{10} y^{\frac{1}{3}} = \log_{10} z$

$\log_{10} \sqrt{x} - \log_{10} \sqrt[3]{y} = \log_{10} z$
$\log_{10} \frac{\sqrt{x}}{\sqrt[3]{y}} = \log_{10} z$

$\frac{\sqrt{x}}{\sqrt[3]{y}} = z$

$Z = \frac{\sqrt{x}}{\sqrt[3]{y}}$

15. given that log m = x + y and log n = x – y , express the value of log m^2n in terms of x and y.

Solution

Given :

Log m = x + y

Log n = x –y

Log m^2n

Let us simplify the given expression,

Log m^2n = log m^2 + log n

= 2 log m + log n

By substituting the given values, we get

= 2(x+ y) +(x- y)

= 2x + 2y + x – y

= 3x + y

16. given that log x = m + n and log y = m – n , express the value of log$\left(\frac{10x}{y^2}\right)$ in terms of m and n .

Solution

Given

Log x = m + n

Log y = m –n

log$\left(\frac{10x}{y^2}\right)$

let us simplify the given expression,

log$\left(\frac{10x}{y^2}\right)$ = log 10x – log y^2

= log 10 + log x – 2 log y

= 1 + log x – 2 log y

= 1 +(m +n) – 2(m –n)

= 1 + m + n – 2m + 2n
= 1 – m + 3n

17. if $\log \frac{x}{2} = \log \frac{y}{3}$ find the value of $\frac{y^4}{x^6}$

Solution
Given :
Log $\frac{x}{2} = \log \frac{y}{3}$
Let us simplify the given expression ,
By cross multiplying , we get

3 log x = 2 log y
Log x^3 = log y^2
So , $x^3 = y^2$

Now square on both sides we get
$(x^3)^2 = (y^2)^2$

$x^6 = y^4$

$\frac{y^4}{x^6} = 1$

18. solve for x :
(i) log x + log 5 = 2 log 3

(ii) $\log_3 x - \log_3 2 = 1$

(iii) $x = \frac{\log 125}{\log 25}$
(iv) $\left(\frac{log8}{log2}\right) \times \left(\frac{log3}{log\sqrt{3}}\right) = 2 \log x$

Solution

(i) log x + log 5 = 2 log 3

Let us solve for x,
Log x = 2 log 3 – log 5
= log 3^2 – log 5
= log 9 – log 5
= log $\left(\frac{9}{5}\right)$
x = $\frac{9}{5}$

(ii) $\log_3$ x – $\log_3$ 2 = 1
Let us solve for x ,
Log_3 x = 1 + $\log_3$ 2
= $\log_3$ 3 + $\log_3$ 2 [since , 1 can be written as $\log_3$ 3 = 1]
= $\log_3$ (3× 2)
= $\log_3$ 6
x = 6

(iii) x = $\frac{\log 125}{\log 25}$
x = $\frac{\log 5^3}{\log 5^2}$
= $\frac{3 \log 5}{2\log 5}$
= $\frac{3}{2}$ $[\ since\ , \frac{log 5}{log\ 5} = 1$
$\therefore x = \frac{3}{2}$

(iv) $\left(\frac{log 8}{log 2}\right) \times \left(\frac{log 3}{log\sqrt{3}}\right) = 2 \log x$
$\left(\frac{\log 2^3}{\log 2}\right) \times \left(\frac{\log 3}{\log 3^{\frac{1}{2}}}\right) = 2 \log x$

$$\left(\frac{3\log 2}{\log 2}\right) \times \left(\frac{\log 3}{\frac{1}{2}\log 3}\right) = 2\log x$$

$$\frac{3\times 1}{\frac{1}{2}} = 2\log x$$

$3 \times 2 = 2 \log x$

$6 = 2 \log x$

$\text{Log } x = \frac{6}{2}$

$\text{Log } x = 3$

$x = (10)^3$

$= 1000$

$x = 1000$

19. given $2 \log_{10} x + 1 = \log_{10} 250$, find

(i) x

(ii) $\log_{10} 2x$

Solution

Given :

$2 \log_{10} x+1 = \log_{10} 250$

(i) let us simplify the above expression ,

$\text{Log}_{10} x^2 + \log_{10} 10 = \log_{10} 250$ [since 1 can be written as $\log_{10}$ 10]

$\text{Log}_{10} (x^2 \times 10) = \log_{10} 250$

$(x^2 \times 10) = 250$

$x^2 = \frac{250}{10}$

$x^2 = 25$

$x = \sqrt{25}$

$= 5$

$\therefore x = 5$

(ii) $\log_{10} 2x$

We know that , x = 5
So $\log_{10} 2x = \log_{10} 2\times5$
$= \log_{10}10$
$= 1$

20 . if $\frac{\log x}{\log 5} = \frac{\log y^2}{\log 2} = \frac{log 9}{log\left(\frac{1}{3}\right)}$ find x and y

Solution

Given :

$$\frac{\log x}{\log 5} = \frac{\log y^2}{\log 2} = \frac{log 9}{log\left(\frac{1}{3}\right)}$$

Let us consider,

$$\frac{\log x}{\log 5} = \frac{log 9}{log\left(\frac{1}{3}\right)}$$

$$\text{Log x} = \frac{(\log 9 \times \log 5)}{\log\left(\frac{1}{3}\right)}$$

$$= \frac{\log 3^2 \times \log 5}{\log 1 - \log 3}$$

$$= \frac{(2 \log 3 \times \log 5)}{-\log 3} \text{ [log 1 = 0]}$$

$= - 2\times \log 5$
$= \log 5^{-2}$
$x = 5^{-2}$
$= \frac{1}{5^2}$

$= \frac{1}{25}$

Now,

$\frac{\log y^2}{\log 2} = \frac{\log 9}{\log\left(\frac{1}{3}\right)}$

Log y^2 = $\frac{\log 9 \times \log 2}{\log\left(\frac{1}{3}\right)}$

$= \frac{log 3^2 \times log 2}{log 1 - log 3}$

$= \frac{2 log 3 \times log 2}{-log 3}$ [log1 = 0]

= -2 × log 2

= log 2^{-2}

$y^2 = 2^{-2}$

$= \frac{1}{2^2}$

$= \frac{1}{4}$

$= \sqrt{\frac{1}{4}}$

$= \frac{1}{2}$

21. prove the following

(i) $3^{\log 4} = 4^{\log 3}$

(ii) $27^{\log 2} = 8^{\log 3}$

Solution

(i) $3^{\log 4} = 4^{\log 3}$

Let us take log on both sides,

If log $3^{\log 4}$ = log $4^{\log 3}$

Log4 . log 3 = log 3. Log 4

Log 2^2 . log 3 = log 3. Log 2^2

2log 2 . log 3 = log 3 . 2log 2

Which is true

Hence proved.

(ii) $27^{\log 2} = 8^{\log 3}$

Let us take log on both sides,

If $\log 27^{\log 2} = \log 8^{\log 3}$

$\text{Log } 2 . \log 27 = \log 3 . \log 8$

$\text{Log } 2 . \log 3^3 = \log 3 . \log 2^3$

$\text{Log } 2 . 3 \log 3 = \log 3 . 3 \log 2$

$3 \log 2 . \log 3 = 3 \log 2 . \log 3$

Which is true.

Hence proved.

22. solve the following equations:

(i) $\log(2x+3) = \log 7$

(ii) $\log(x+1) + \log(x-1) = \log 24$

(iii) $\log (10x+5) - \log(x - 4) = 2$

(iv) $\log_{10} 5 + \log_{10}(5x + 1) = \log_{10}(x + 5) + 1$

(v) $\log(4y - 3) = \log(2y + 1) - \log 3$

(vi) $\log_{10} (x+2) + \log_{10} (x - 2) = \log_{10} 3 + 3 \log_{10} 4$

(vii) $\log (3x+2) + \log(3x - 2) = 5 \log 2$

Solution

(i) $\log(2x+3) = \log 7$

Let us simplify the expression ,

$2x + 3 = 7$

$2x = 7 - 3$

$2x = 4$

$X = \frac{4}{2}$

$= 2$

(ii) $\log(x+1) + \log(x-1) = \log 24$

Let us simplify the expression,

$\text{Log } [(x + 1)(x-1)] = \log 24$

$\text{Log}(x^2 - 1) = \log 24$

$(x^2 - 1) = 24$

$x^2 = 24 + 1$
$= 25$
$X = \sqrt{25}$
$= 5$
(iii) log (10x+5) –log(x – 4) =2
Let us simplify the expression ,
Log $\frac{10x + 5}{x-4}$ = 2 log 10
log$\frac{10x + 5}{x-4}$ = log 10^2
$\frac{10x+5}{x-4} = 100$
10x + 5 = 100(x -4)
10x + 5 = 100x – 400
5 + 400 = 100x – 10x
90x = 405
X = $\frac{405}{90}$
= $\frac{81}{18}$
= $\frac{9}{2}$
= 4.5

(iv) $\log_{10} 5 + \log_{10}(5x +1) = \log_{10}(x + 5) + 1$
Let us simplify the expression
$\text{Log}_{10}[\ 5\times(5x+1)] = \log_{10} (x + 5) + \log_{10} 10$
$\text{Log}_{10}[\ 5\times(5x+1)] = \log_{10} [(x + 5) \times 10\ ]$ [5×(5x + 1)] = [(x + 5)×10]
25x + 5 = 10x + 50
25x – 10x = 50 -5
15x = 45
X = $\frac{45}{15}$
= 3

(v) $\log(4y - 3) = \log(2y + 1) - \log 3$
Let us simplify the expression ,
$\text{Log}(4y - 3) = \log \frac{2y+1}{3}$
$(4y - 3) = \frac{2y+1}{3}$
By cross multiplying we get
$3(4y - 3) = 2y + 1$
$12y - 9 = 2y + 1$
$12y - 2y = 9 + 1$
$10y = 10$
$Y = \frac{10}{10}$
$= 1$

(vi) $\log_{10}(x+2) + \log_{10}(x - 2) = \log_{10}3 + 3\log_{10}4$
Let us simplify the expression,
$\text{Log}_{10}[(x+2) \times (x-2)] = \log_{10}3 + \log 10^{4^3}$
$\text{Log}_{10}[(x + 2) \times (x - 2)] = \log_{10}(3 \times 4^3)$
$[(x + 2) \times (x - 2)] = (3 \times 4^3)$
$(x^2 - 4) = (3 \times 4 \times 4 \times 4)$
$(x^2 - 4) = 192$
$x^2 = 192 + 4$
$= 196$
$x = \sqrt{196}$
$= 14$

(vii) $\log(3x+2) + \log(3x - 2) = 5\log 2$
Let us simplify the expression
$\text{Log}(3x + 2) + \log(3x - 2) = \log 2^5$
$\text{Log}[(3x + 2) \times (3x - 2)] = \log 32$
$\text{Log}(9x^2 - 4) = \log 32$
$(9x^2 - 4) = 32$
$9x^2 = 32 + 4$

$9x^2 = 36$

$x^2 = \frac{36}{9}$

$x^2 = 4$

$x = \sqrt{4}$

$= 2$

23. solve for x :

$\text{Log}_3(x+1) - 1 = 3 + \log_3(x-1)$

Solution :

Given :

$\text{Log}_3(x + 1) - 1 = 3 + \log_3(x - 1)$

Let us simplify the expression,

$\text{Log}_3(x+1) - \log_3(x-1) = 3 + 1$

$\text{Log}_3 \frac{x+1}{x-1} = 4 \log_3 3$ [since $\log_3 3 = 1$]

$\text{Log}_3 \frac{x+1}{x-1} = \log_3 3^4$

$\frac{x+1}{x-1} = 3^4$

By cross multiplying , we get

$(x + 1) = 81(x - 1)$

$X + 1 = 81x - 81$

$81x - x = 1 + 81$

$80x = 82$

$X = \frac{82}{80}$

$= \frac{41}{40}$

$= 1\frac{1}{40}$

24. solve for x :

$$\mathbf{5^{\log x} + 3^{\log x} = 3^{\log x + 1} - 5^{\log x - 1}}$$

Solution

Given

$$5^{\log x} + 3^{\log x} = 3^{\log x + 1} - 5^{\log x - 1}$$

Let us simplify the expression,

$$5^{\log x} + 3^{\log x} = 3^{\log x} . 3^{1} - 5^{\log x} . 5^{-1}$$

$$5^{\log x} + 3^{\log x} = 3.3^{\log x} - \frac{1}{5} . 5^{\log x}$$

$$5^{\log x} + \frac{1}{5} . 5^{\log x} = 3.3^{\log x} - 3^{\log x}$$

$$\left(1 + \frac{1}{5}\right) 5^{\log x} = (3-1)\, 3^{\log x}$$

$$\left(\frac{6}{5}\right) 5^{\log x} = 2(3^{\log x})$$

$$\frac{5^{log\, x}}{3^{log\, x}} = \frac{2\times 5}{6}$$

$$\left(\frac{5}{3}\right)^{log\, x} = \frac{10}{6}$$

$$\left(\frac{5}{3}\right)^{log\, x} = \frac{5}{3}$$

$$\left(\frac{5}{3}\right)^{log\, x} = \left(\frac{5}{3}\right)^{1}$$

So , by comparing the powers

Log x = 1

Log x = log 10

x = 10

25. if log $\frac{x-y}{2} = \frac{1}{2}$ (log x + log y) , prove that $x^2 + y^2 = 6xy$

Solution

Given

Log $\frac{x-y}{2} = \frac{1}{2}$ (log x + log y)

Let us simplify

Log $\frac{x-y}{2} = \frac{1}{2}$ (log x × y)

Log $\frac{x-y}{2} = \frac{1}{2}$ log xy

Log $\frac{x-y}{2} = \log\,(xy)^{\frac{1}{2}}$

$\frac{x-y}{2} = xy^{\frac{1}{2}}$

By squaring on both sides we get

$\left[\frac{x-y}{2}\right]^2 = \left[xy^{\frac{1}{2}}\right]^2$

$\frac{(x-y)^2}{4} = xy$

By cross multiplying , we get

$(x-y)^2 = 4xy$

$x^2 + y^2 - 2xy = 4xy$

$x^2 + y^2 = 4xy + 2xy$

$x^2 + y^2 = 6xy$

hence proved

26. if $x^2 + y^2 = 23xy$, prove that log $\frac{x+y}{5} = \frac{1}{2}(\log x + \log y)$

Solution

Given

$x^2 + y^2 = 23xy$

so the above equation can be written as

$x^2 + y^2 = 25xy - 2xy$

$x^2 + y^2 + 2xy = 25xy$

$(x+y)^2 = 25xy$

$\frac{(x+y)^2}{25} = xy$

Now by taking log on both sides we get

$log \frac{(x+y)^2}{25} = \log xy$

$\log \left[\frac{x+y}{5}\right]^2 = \log xy$

$2 \log \frac{x+y}{5} = \log x + \log y$

$\text{Log}\frac{x+y}{5} = \frac{1}{2} \log x + \log y$

Hence proved .

27. if p = $\log_{10} 20$ and q = $\log_{10} 25$, find the value of x if 2 $\log_{10}(x+1) = 2p-q$

Solution

Given

$P = \log_{10} 20$

$Q = \log_{10} 25$

Then ,

$2 \log_{10} (x+1) = 2p - q$

Now substitute the values of p and q we get

$2 \log_{10} (x+ 1) = 2 \log_{10} 20 - \log_{10} 25$

$= 2 \log_{10}20 - \log_{10} 5^2$

$= 2 \log_{10} 20 - 2 \log_{10} 5$

$2 \text{Log}_{10} (x+1) = 2(\log_{10}20 - \log_{10}5)$

$\text{Log}_{10} (x+1) = (\log_{10}20 - \log_{10}5)$

$= \log_{10}\left(\frac{20}{5}\right)$

$= \log_{10}(x+1) = \log_{10}4$

$(x+1) = 4$

$x = 4 - 1$

$= 3$

28. show that :

(i) $\frac{1}{\log_2 42} + \frac{1}{\log_3 42} + \frac{1}{\log_7 42} = 1$

(ii) $\frac{1}{\log_8 36} + \frac{1}{\log_9 36} + \frac{1}{\log_{18} 36} = 2$

Solution

(i) $\frac{1}{\log_2 42} + \frac{1}{\log_3 42} + \frac{1}{\log_7 42} = 1$

Let us consider LHS:

$\frac{1}{\log_2 42} + \frac{1}{\log_3 42} + \frac{1}{\log_7 42}$

By using the formula , $\log_n m = \frac{log_m}{log_n}$

$\frac{1}{\log_2 42} + \frac{1}{\log_3 42} + \frac{1}{\log_7 42} = \frac{1}{\left(\frac{\log 42}{\log 2}\right)} + \frac{1}{\left(\frac{log 42}{log\, 3}\right)} + \frac{1}{\left(\frac{log 42}{log\, 7}\right)}$

$= \frac{log_2}{log 42} + \frac{log_3}{log 42} + \frac{log_7}{log 42}$

$= \frac{log_2 + log_3 + log_7}{log 42}$

$= \frac{log\ 2 \times 3 \times 7}{log 42}$

$= \frac{log 42}{log 42}$

$= 1$

$=$ RHS

(ii) $\frac{1}{\log_8 36} + \frac{1}{\log_9 36} + \frac{1}{\log_{18} 36} = 2$

Let us consider LHS:

$\frac{1}{\log_8 36} + \frac{1}{\log_9 36} + \frac{1}{\log_{18} 36}$

By using the formula , $\log_n m = \frac{logm}{logn}$

$\frac{1}{\log_8 36} + \frac{1}{\log_9 36} + \frac{1}{\log_{18} 36} = \frac{1}{\frac{log 36}{log_8}} + \frac{1}{\frac{log 36}{log_9}} + \frac{1}{\frac{log 36}{log_{18}}}$

$= \frac{log_8}{log36} + \frac{log_9}{log36} + \frac{log_{18}}{log36}$

$= \frac{log_8+log_9+log_{18}}{log36}$

$= \frac{log8\times9\times18}{log36}$

$= \frac{log36^2}{log36}$

$= \frac{2log36}{log36}$

$= 2$

$=$ RHS

29. prove the following identities :

(i) $\mathbf{\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc} = 1}$

(ii) $\mathbf{\log_b a \,.\, \log_c b \,.\, \log_d c = \log_d a}$

Solution

(i) $\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc} = 1$

Let us consider LHS:

$\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$

By using the formula , $\log_n m = \frac{log_m}{log_n}$

$\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc} = \frac{1}{\frac{\log abc}{log_a}} + \frac{1}{\frac{log\, abc}{log_b}} + \frac{1}{\frac{log\, abc}{log_c}}$

$= \frac{\log a}{\log abc} + \frac{\log b}{\log abc} + \frac{log\ c}{log\ abc}$

$= \frac{\log a+\log b+\log c}{\log abc}$

$= \frac{(\log a \times b \times c)}{\log abc}$

$= \frac{\log abc}{\log abc}$

=1

RHS

(ii) $\log_b a \,.\log_c b \,.\log_d c = \log_d a$

Let us consider LHS

$\log_b a \,.\log_c b \,.\log_d c = \frac{\log a}{\log b} \times \frac{log\ b}{log\ c} \times \frac{log\ c}{log\ d}$

$= \frac{log\ a}{log\ d}$

$= \log_d a$

= RHS

30. given that $\log_a x = \frac{1}{\alpha}$, $\log_b x = \frac{1}{\beta}$, $\log_c x = \frac{1}{\gamma}$, find $\log_{abc} x$.

Solution

It is given that :

$\log_a x = \frac{1}{\alpha}$, $\log_b x = \frac{1}{\beta}$, $\log_c x = \frac{1}{\gamma}$

so,

$\log_a x = \frac{1}{\alpha} = \frac{\log x}{\log a} = \frac{1}{a} = \log a = \alpha \log x$

$\log_b x = \frac{1}{\beta} = \frac{\log x}{\log b} = \frac{1}{\beta}\ log\ b = \beta\ log\ x$

$\log_c x = \frac{1}{\gamma} = \frac{\log x}{\log c} = \frac{1}{\gamma}\ log\ b = \gamma\ log\ x$

now ,

$\log_{abc} x = \frac{\log x}{\log abc}$

$= \frac{\log x}{\log a + \log b + \log c}$

$$= \frac{\log x}{a \ \log x + b \log x + y \log x}$$

$$= \frac{\log x}{log_x(\alpha+\beta+\gamma)}$$

$$= \frac{1}{(\alpha+\beta+\gamma)}$$

31. solve for x :

(i) $\log_3 x + \log_9 x + \log_{81} x = \frac{7}{4}$

(ii) $\log_2 x + \log_8 x + \log_{32} x = \frac{23}{15}$

Solution

(i) $\log_3 x + \log_9 x + \log_{81} x = \frac{7}{4}$

Let us simplify the expression ,

$$\frac{1}{\log_x 3} + \frac{1}{\log_x 9} + \frac{1}{\log_x 81} = \frac{7}{4}$$

$$\frac{1}{\log_x 3^1} + \frac{1}{\log_x 3^2} + \frac{1}{\log_x 3^4} = \frac{7}{4}$$

$$\frac{1}{\log_x 3} + \frac{1}{2} \log_x 3 + \frac{1}{4} \log_x 3 = \frac{7}{4}$$

$$\frac{1}{\log_x 3\left[1+\frac{1}{2}+\frac{1}{4}\right]} = \frac{7}{4}$$

$$\frac{1}{\log_x 3\left[\frac{4+2+1}{4}\right]} = \frac{7}{4}$$

$$\log_3 x \frac{7}{4} = \frac{7}{4}$$

$\log_3 x = 1$

$\log_3 x = \log_3 3$ [since $1 = \log_a a$]

On comparing we get

x = 3

(ii) $\log_2 x + \log_8 x + \log_{32} x = \frac{23}{15}$

Let us simplify the expression ,

$$\frac{1}{\log_x 2} + \frac{1}{\log_x 8} + \frac{1}{\log_x 32} = \frac{23}{15}$$

$$\frac{1}{\log_x 2^1} + \frac{1}{\log_x 2^3} + \frac{1}{\log_x 2^5} = \frac{23}{15}$$

$$\frac{1}{\log_x 2} + \frac{1}{3}\log_x 2 + \frac{1}{5}\log_x 2 = \frac{23}{15}$$

$$\frac{1}{\log_x 2}\left[1 + \frac{1}{3} + \frac{1}{5}\right] = \frac{23}{15}$$

$$\text{Log}_2\, x\, \frac{15+5+3}{15} = \frac{23}{15}$$

$$\text{Log}_2\, x\, \frac{23}{15} = \frac{23}{15}$$

$$\text{Log}_2\, x = \frac{23}{15} \times \frac{15}{23}$$

$\text{Log}_2\, x = 1$

$\text{Log}_2\, x = \text{Log}_2\, 2$ [since , $1 = \log_a a$]

On comparing , we get

X = 2