GATE 2023
Mechanical Engineering
Exam Held on : 04-02-2023
Afternoon Session

SECTION - A GENERAL APTITUDE

Q.1 He did not manage of fix the car himself, so he ____ in the garage.

(a) got it fixed (b) getting it fixed
(c) gets fixed (d) got fixed

Ans. (a)

End of Solution

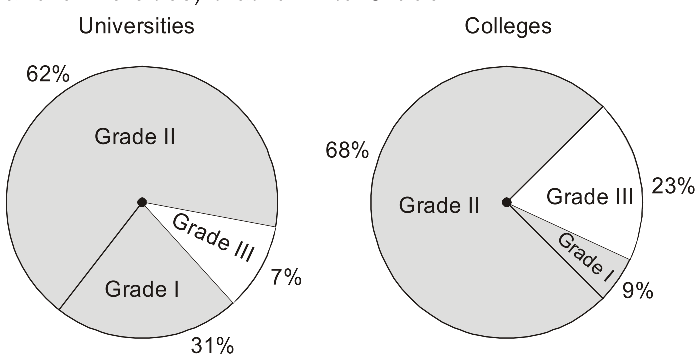
Q.2 Planting : Seed : : Raising : ______. [By word meaning]

(a) Child (b) Temperature
(c) Height (d) Lift

Ans. (a)

End of Solution

Q.3 A certain country has 504 universities and 25951 colleges. These are categorised into Grades I, II, and III as shown in the given pie charts.
What is the percentage, correct to one decimal place, of higher education institutions (colleges and universities) that fall into Grade III?



(a) 22.7 (b) 23.7
(c) 15.0 (d) 66.8

Ans. (a)

Given,

Number of colleges, $C = 25951$

Number of universities, $U = 504$

Number of students with grade 3 in colleges

$$G_3(C) = 0.23 \text{ of } 25951 = 0.23 \times 25951$$
$$G_3(C) = 5968.73$$

Number of students with grade 3 in universities

$$G_3(U) = 0.07 \text{ of } 504 = 0.07 \times 504$$
$$G_3(U) = 35.28$$

Thus, required percentage = $\dfrac{\text{Total number of students with Grade 3}}{\text{Total number of students}}$

$$= \frac{G_3(c)+G_3(u)}{c+u} = \frac{35.28+5968.73}{25951+504}$$

$= 0.2269$ or 22.7%

End of Solution

Q.4 The minute hand and second-hand of a clock cross each other ______ times between 09:15:00 AM and 09:45:00 AM on a day.

(a) 30
(b) 15
(c) 29
(d) 31

Ans. (a)

The minute hand and second hand will cross each other 1 time every minute after 9:15 i.e. 1 time each in 9:16, 9:17, 9.18 and so on upto 9:45. Thus, they will cross each other 30 times.

End of Solution

Q.5 The symbols ○, *, △, and □ are to be filled, one in each box, as shown below. The rules for filling in the four symbols are as follows:

1. Every row and every column must contain each of the four symbols.
2. Every 2 × 2 square delineated by bold lines must contain each of the four symbols.

Which symbol will occupy the box marked with '?' in the partially fileld figure?

	?		△
	○	*	
		□	*
*		△	○

(a) ○
(b) *
(c) △
(d) □

Ans. (b)

□	*	○	△
△	○	*	□
○	△	□	*
*	□	△	○

End of Solution

Q.6 In a recently held parent-teacher meeting, the teachers had very few complaints about Ravi. After all, Ravi was a hardworking and kind student. Incidentally, almost all of Ravi's friends at school were hardworking and kind too. But the teachers drew attention to Ravi's complete lack of interest in sports. The teachers believed that, along with some of his friends who showed similar disinterest in sports, Ravi needed to angage in some sports for his overall development.

Based only on the information provided above, which one of the following statements canbe logically inferred with certainity?

(a) All of Ravi's friends are hardworking and kind.

(b) No one who is not a friend of Ravi is hardworking and kind.

(c) None of Ravi's friends are interested in sports.

(d) Some of Ravi's friends are hardworking and kind.

Ans. **(d)**

End of Solution

Q.7 Consider the following inequalities

$p^2 - 4q < 4$

$3p + 2q < 6$

where p and q are positive integers.

The value of $(p + q)$ is

(a) 2 (b) 1

(c) 3 (d) 4

Ans. **(a)**

Given inequalities,

$$p^2 - 4q < 4 \quad \text{...(i)}$$

$$3p + 2q < 6 \quad \text{...(ii)}$$

Since p and q are positive integers and satisfy the given inequalities, only possibility is

$$p = q = 1$$

This can be easily verified by putting values of p and q in inequality (ii),

For $p = q = 1$; $3(1) + 2(1) < 6$

For $p = 1$ and $q = 2$; $3(1) + 2(2) > 6$

And, similarly any other value of p and q will not satisfy the inequality.

Hence, $p + q = 1 + 1 = 2$

Alternatively,

Multiplying inequality (ii) with 2,

$$6p + 4q < 12$$

or, $$4q < 12 - 6p \quad \text{...(iii)}$$

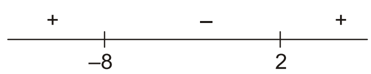
From (i) and (iii),

$$p^2 - 4 < 12 - 6p$$

$$p^2 + 6p - 16 < 0$$

$$(p + 8)(p - 2) < 0$$

Using wavy curve method,



$\therefore \quad p \in (-8, 2)$

But it is given that p is positive integer,

$\therefore \quad p = 1$

From (iii), $\quad 4q < 12 - 6(1)$

$4q < 6$

$\Rightarrow \quad q < \frac{3}{2}$

But again q is positive integer,

$\therefore \quad q = 1$

Hence, $\quad p + q = 1 + 1 = 2$

End of Solution

Q.8 Which one of the sentence sequences in the given options creates a coherent narrative?

(i) I could not bring myself to knock.

(ii) There was a murmur of unfamiliar voices coming from the big drawing room and the door was firmly shut.

(iii) The passage was dark for a bit, but then it suddenly opened into a bright kitchen.

(iv) I decided I would rather wander down the passage.

(a) (iv), (i), (iii), (ii)
(b) (iii), (i), (ii), (iv)
(c) (ii), (i), (iv), (iii)
(d) (i), (iii), (ii), (iv)

Ans. (c)

End of Solution

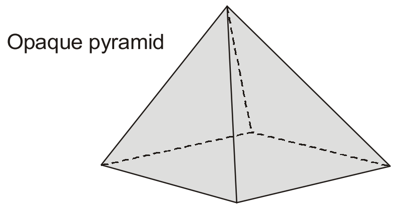
Q.9 How many pairs of sets (S,T) are possible among the subsets of {1, 2, 3, 4, 5, 6} that satisfy the the condition that S is a subset of T?

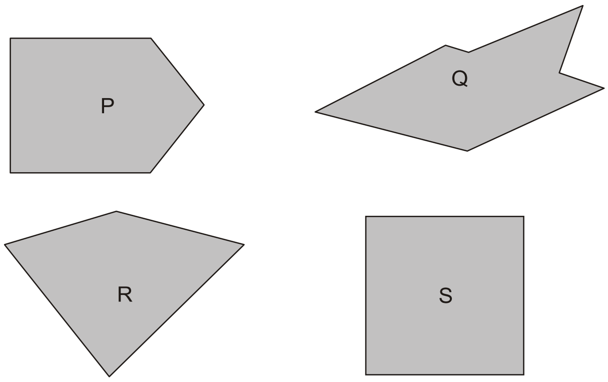
(a) 729
(b) 728
(c) 665
(d) 664

Ans. (a)

End of Solution

Q.10 An opaque pyramid (shown below), with a square base and isosceles faces, is suspended in the path of a parallel beam of light, such that its shadow is cast on a screen oriented perpendicular to the direction of the light beam. The pyramid can be reoriented in any direction within the light beam. Under these conditions, which one of the shadows P, Q, R and S is NOT possible?





(a) P
(b) Q
(c) R
(d) S

Ans. (b)

End of Solution

■■■■

SECTION - B TECHNICAL

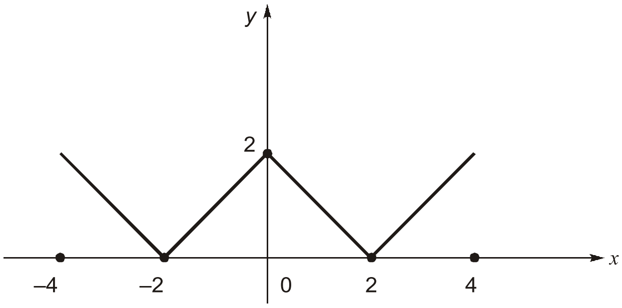
Q.11 A machine produces a defective component with a probability of 0.015. The number of defective components in a packed box containing 200 components produced by the machine follows a Poisson distribution. The mean and the variance of the distribution are

(a) 3 and 3, respectively
(b) $\sqrt{3}$ and $\sqrt{3}$ respectively
(c) 0.015 and 0.015, respectively
(d) 3 and 9, respectively

Ans. (a)

End of Solution

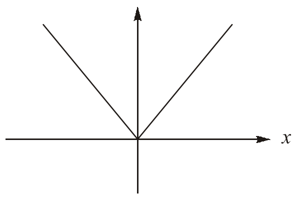
Q.12 The figure shows the plot of a function over the interval [–4, 4]. Which one of the options given CORRECTLY identifies the function?



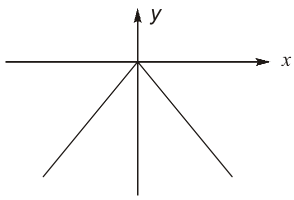
(a) $|2-x|$
(b) $|2-|x||$
(c) $|2+|x||$
(d) $2-|x|$

Ans. (b)

We know that the graph of $y = |x|$ is



and the graph of $y = -|x|$ is



It can be observed that the given graph can be obtained by first shifting the graph of $y = -|x|$ up by 2 units and then taking the modulus of resultant function. Shifting up by 2 units transforms the equation to,

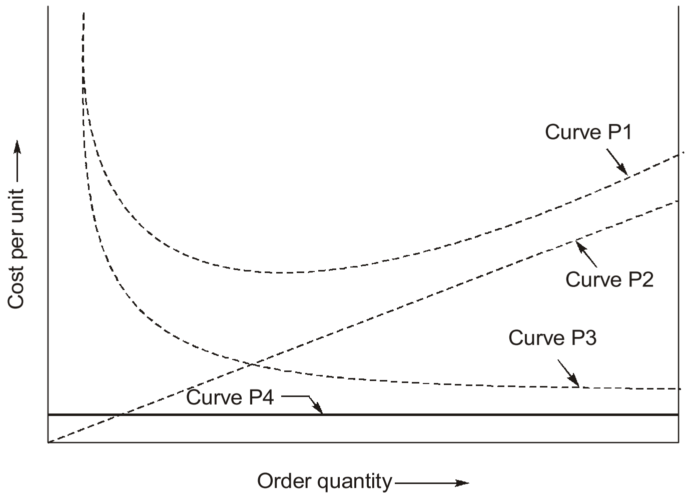
$$y = 2 - |x|$$

and, taking modulus gives the resultant equation as

$$y = |2 - |x||$$

End of Solution

Q.13 With reference to the Economic Order Quantity (EOQ) model, which one of the options given is correct?



(a) Curve P1: Total cost, Curve P2: Holding cost, Curve P3: Setup cost, and Curve P4: Production cost.

(b) Curve P1: Holding cost, Curve P2: Setup cost, Curve P3: Production cost, and Curve P4: Total cost.

(c) Curve P1: Production cost, Curve P2: Holding cost, Curve P3: Total cost, and Curve P4: Setup cost.

(d) Curve P1: Total cost, Curve P2: Production cost, Curve P3: Holding cost, and Curve P4: Setup cost.

Ans. (a)

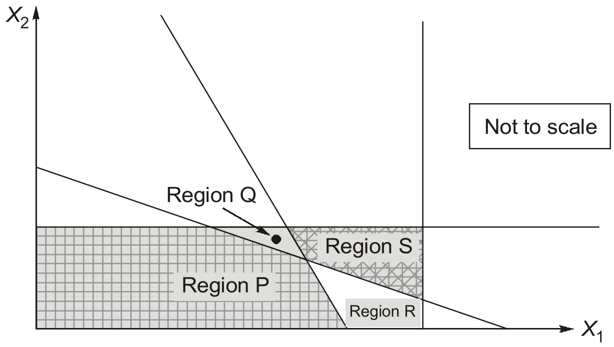
Holding cost is directly proportional to order quantity. so, P2 is holding cost curve. At EOQ holding and setup cost are same, also setup cost is inversely proportional to order quantity. So P3 is setup cost curve.

End of Solution

Q.14 Which one of the options given represents the feasible region of the linear programming model:

Maximize $45X_1 + 60X_2$

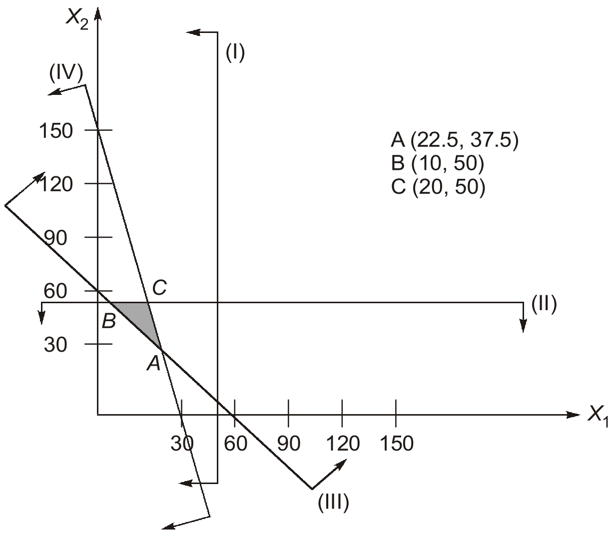
$$X_1 \leq 45$$
$$X_2 \leq 50$$
$$10X_1 + 10X_2 \geq 600$$
$$25X_1 + 5X_2 \leq 750$$



(a) Region P (b) Region Q

(c) Region R (d) Region S

Ans. (b)



$$Z = 45X_1 + 60X_2$$

$$X_1 \leq 45 \Rightarrow \frac{X_1}{45} = 1 \quad \text{... (i)}$$

$$X_2 \leq 50 \Rightarrow \frac{X_2}{50} = 1 \quad \text{... (ii)}$$

$$10X_1 + 10X_2 \geq 600 \Rightarrow \frac{X_1}{60} + \frac{X_2}{60} = 1 \quad \text{... (iii)}$$

$$25X_1 + 5X_2 \leq 750 \Rightarrow \frac{X_1}{30} + \frac{X_2}{150} = 1 \quad \text{... (iv)}$$

Point A

$$X_1 = 60 - X_2 \quad \text{(From (iii))}$$

$$25(60 - X_2) + 5X_2 = 750 \quad \text{(From (iv))}$$

$$X_2 = 37.5$$

$$X_1 = 60 - 37.5 = 22.5$$

Point B

$$X_2 = 50 \quad \text{(From (ii))}$$

$$10X_1 + 10(50) = 600 \quad \text{(From (iii))}$$

$$X_1 = 10$$

Point C

$$X_2 = 50 \quad \text{(From (ii))}$$

$$25X1 + 5 \times 50 = 750 \quad \text{(From (iv))}$$

$$X_1 = 20$$

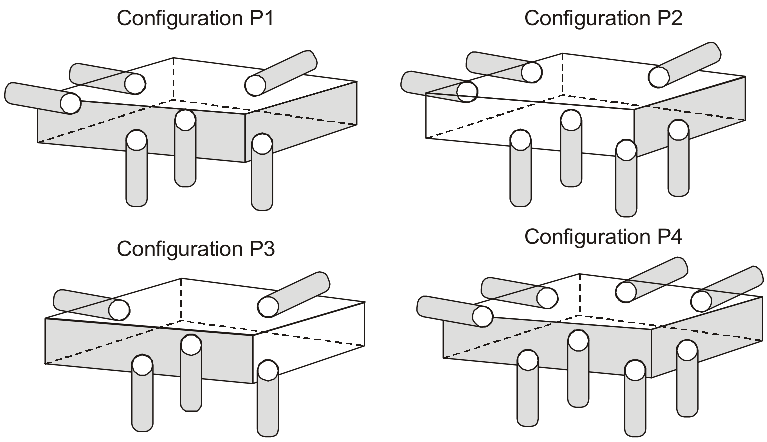
$$Z(A) = 45 \times 22.5 + 60 \times 37.5 = 3262.5$$

$$Z(B) = 45 \times 10 + 60 \times 50 = 3450$$

$$Z(C) = 45 \times 20 + 60 \times 50 = 3900$$

End of Solution

Q.15 A cuboidal part has to be accurately positioned first, arresting six degrees of freedom and then clamped in a fixture, to be used for machining. Locating pins in the form of cylinders with hemi-spherical tips are to be placed on the fixture for positioning. Four different configurations of locating pins are proposed as shown. Which one of the options given is correct?



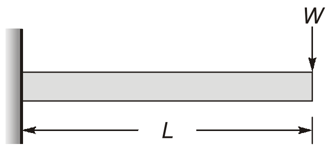
(a) Configuration P1 arrests 6 degrees of freedom, while Configurations P2 and P4 are over-constrained and Configuration P3 is under-constrained.
(b) Configuration P2 arrests 6 degrees of freedom, while Configurations P1 and P3 are over-constrained and Configuration P4 is under-constrained.
(c) Configuration P3 arrests 6 degrees of freedom, while Configurations P2 and P4 are over-constrained and Configuration P1 is under-constrained.
(d) Configuration P4 arrests 6 degrees of freedom, while Configurations P1 and P3 are over-constrained and Configuration P2 is under-constrained.

Ans. **(a)**

As per 3 - 2 - 1 principle by providing six pins, six degree of freedom are arrested. So, configuration P1 arrest six degree of freedom. Configuration P2 and P4 are over constrained while configuration 3 is under constrained.

End of Solution

Q.16 The effective stiffness of a cantilever beam of length L and flexural rigidity EI subjected to a transverse tip load W is



(a) $\frac{3EI}{L^3}$

(b) $\frac{2EI}{L^3}$

(c) $\frac{L^3}{2EI}$

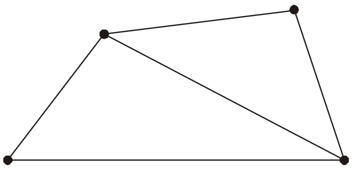
(d) $\frac{L^3}{3EI}$

Ans. **(a)**

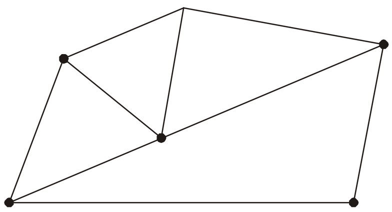
End of Solution

Q.17 The options show frames consisting of rigid bars connected by pin joints. Which one of the frames is non-rigid?

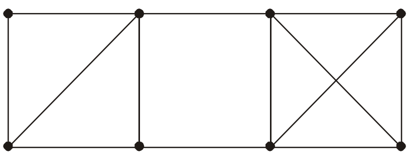
(a)



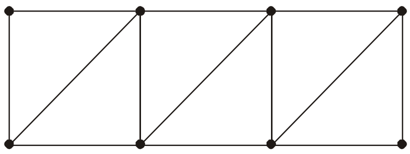
(b)



(c)



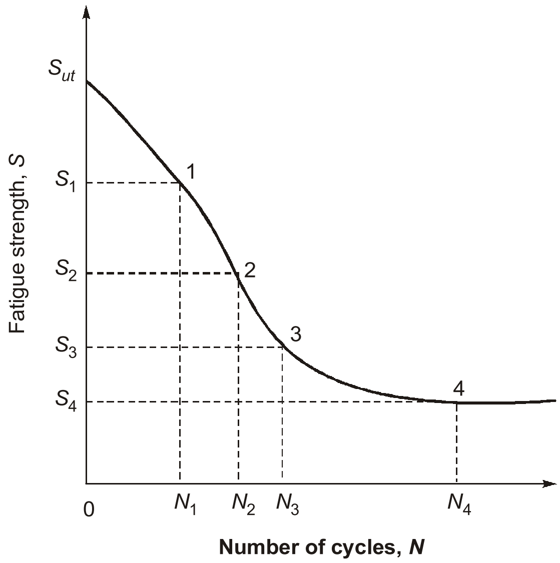
(d)



Ans. **(c)**

End of Solution

Q.18 The S-N curve from a fatigue test for steel is shown. Which one of the options gives the endurance limit?



(a) S_{ut} (b) S_2

(c) S_3 (d) S_4

Ans. **(d)**

End of Solution

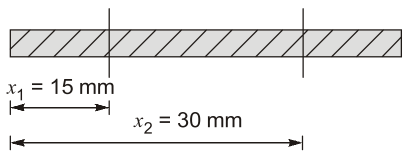
Q.19 Air (density = 1.2 kg/m^3, kinematic viscosity = 1.5 × 10^{-5} m^2/s) flows over a flat plate with a free-stream velocity of 2 m/s. The wall shear stress at a location 15 mm from the leading edge is τ_w. What is the wall shear stress at a location 30 mm from the leading edge?

(a) $\dfrac{\tau_w}{2}$ (b) $\sqrt{2}\tau_w$

(c) $2\tau_w$ (d) $\dfrac{\tau_w}{\sqrt{2}}$

Ans. **(d)**

Air, ρ = 1.2 kg/m^3, ν = 1.5 × 10^{-5} m^2/s, U_∞ = 2 m/s



At x_1 = 15 mm $\tau_1 = \tau_w$

At x_2 = 30 mm; τ_2 = ?

$$\text{Reynolds number} = \frac{U_\infty x}{\nu} = \frac{2 \times 30 \times 10^{-3}}{1.5 \times 10^{-5}} = 4000$$

As Re < (Re)$_{critical}$

∴ Flow is laminar

For laminar flow over flat plate

We know, $$\tau_w \propto \frac{1}{\sqrt{x}}$$

$$\frac{\tau_1}{\tau_2} = \sqrt{\frac{x_2}{x_4}}$$

$$\frac{\tau_w}{\tau_2} = \sqrt{\frac{30}{15}}$$

$$\tau_2 = \frac{\tau_w}{\sqrt{2}}$$

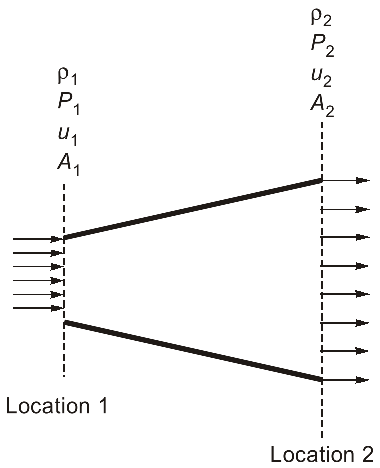
End of Solution

Q.20 Consider an isentropic flow of air (ratio of specific heats = 1.4) through a duct as shown in the figure.

The variations in the flow across the cross-section are negligible. The flow conditions at Location 1 are given as follows:

$$P_1 = 100 \text{ kPa},\ \rho_1 = 1.2 \text{ kg/m}^3,\ u_1 = 400 \text{ m/s}$$

The duct cross-sectional area at Location 2 is given by $A_2 = 2A_1$, where A_1 denotes the duct cross-sectional area at Location 1. Which one of the given statements about the velocity u_2 and pressure P_2 at Location 2 is TRUE?



(a) $u_2 < u_1,\ P_2 < P_1$ (b) $u_2 < u_1,\ P_2 > P_1$

(c) $u_2 > u_1,\ P_2 < P_1$ (d) $u_2 > u_1,\ P_2 > P_1$

Ans. (c)

First, we need to find the Mach number to know the nature of fluid at entry and exit.

$$M = \frac{V}{\sqrt{\gamma R T_1}} \quad \text{...(1)}$$

Now, from equation

$$P_1 = \rho R T_1$$

$$\Rightarrow \qquad T_1 = \frac{P_1}{\rho R} = 290.6\,\text{K}$$

Putting value of T_1 in equation (1)

$$M = 1.17 \text{ (flow is supersonic)}$$

Therefore, it will work as a nozzle, therefore velocity will increase and pressure will decrease.

End of Solution

Q.21 Consider incompressible laminar flow of a constant property Newtonian fluid in an isothermal circular tube. The flow is steady with fully-developed temperature and velocity profiles. The Nusselt number for this flow depends on

(a) neither the Reynolds number nor the Prandtl number

(b) both the Reynolds and Prandtl numbers

(c) the Reynolds number but not the Prandtl number

(d) the Prandtl number but not the Reynolds number

Ans. **(a)**

End of Solution

Q.22 A heat engine extracts heat (Q_H) from a thermal reservoir at a temperature of 1000 K and rejects heat (Q_L) to a thermal reservoir at a temperature of 100 K, while producing work (W). Which one of the combinations of [Q_H, Q_L and W] given is allowed?

(a) Q_H = 2000 J, Q_L = 500 J, W = 1000 J

(b) Q_H = 2000 J, Q_L = 750 J, W = 1250 J

(c) Q_H = 6000 J, Q_L = 500 J, W = 5500 J

(d) Q_H = 6000 J, Q_L = 600 J, W = 5500 J

Ans. **(b)**

From energy conservation, $W = Q_H - Q_L$

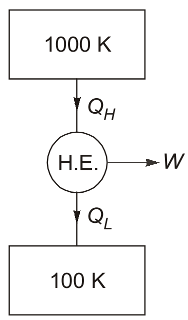
For option (a)

$$W = Q_H - Q_L = 2000 - 500$$
$$= 1500\text{J} \neq 1000 \text{ J}$$

So, this option is incorrect.

For option (b)

$$W = Q_H - Q_L = 2000 - 750$$
$$= 1250 \text{ J}$$



$$\oint \frac{\delta Q}{T} = \frac{2000}{1000} - \frac{750}{100} = 2 - 7.5$$
$$= -5.5 < 0$$

So, this option is correct.

For option (c)

$$W = Q_H - Q_L = 6000 - 500 = 5500 \text{ J}$$

$$\oint \frac{\delta Q}{T} = \frac{6000}{1000} - \frac{500}{100} = 6 - 5 = 1 > 0$$

So, this option is incorrect.

for option (d)

$$W = Q_H - Q_L = 6000 - 600 = 5400 \neq 5500$$

So, this option is incorrect.

End of Solution

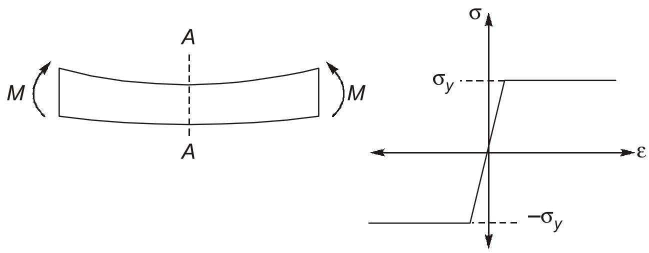
Q.23 Two surfaces *P* and *Q* are to be joined together. In which of the given joining operation(s), there is no melting of the two surfaces *P* and *Q* for creating the joint?

(a) Arc welding
(b) Brazing
(c) Adhesive bonding
(d) Spot welding

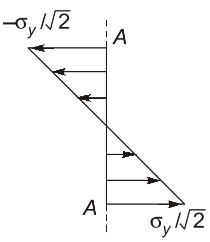
Ans. (b, c)

End of Solution

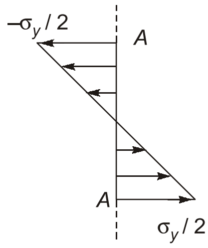
Q.24 A beam is undergoing pure bending as shown in the figure. The stress (σ)-strain (ε) curve for the material is also given. The yield stress of the material is σ_Y. Which of the option(s) given represent(s) the bending stress distribution at cross-section AA after plastic yielding?



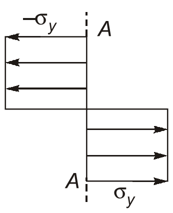
(a)



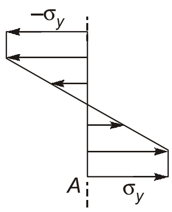
(b)



(c)



(d)



Ans. (c, d)

If bending stress at every fibre is equal to yield strength of the material, then entire cross-section undergoes plastic bending. Then, option (c) is correct.

If bending stress at inner fibres is less than yield strength and bending stress at extreme fibres is equal to yield strength, then, option (d) is correct.

End of Solution

Q.25 In a metal casting process to manufacture parts, both patterns and moulds provide shape by dictating where the material should or should not go. Which of the option(s) given correctly describe(s) the mould and the pattern?

(a) Mould walls indicate boundaries within which the molten part material is allowed, while pattern walls indicate boundaries of regions where mould material is not allowed.

(b) Moulds can be used to make patterns.

(c) Pattern walls indicate boundaries within which the molten part material is allowed, while mould walls indicate boundaries of regions where mould material is not allowed.

(d) Patterns can be used to make moulds.

Ans. **(a, b, d)**

In some moulding processes like investment and full moulding, permanent moulds can be used for preparing the pattern. (Language of the question is ambiguous)

End of Solution

Q.26 The principal stresses at a point P in a solid are 70 MPa, –70 MPa and 0. The yield stress of the material is 100 MPa. Which prediction(s) about material failure at P is/are CORRECT?

(a) Maximum normal stress theory predicts that the material fails

(b) Maximum shear stress theory predicts that the material fails

(c) Maximum normal stress theory predicts that the material does not fail

(d) Maximum shear stress theory predicts that the material does not fail

Ans. **(b, c)**

Given, $\sigma_1 = 70$ MPa. $\sigma_2 = -70$ MPa

If, $S_{yt} = 100$ MPa

Maximum shear stress,

$$\tau_{max} = 100 \text{ MPa}$$

As per MPST, $(\sigma_1 = 70 \text{ MPa}) < (S_{yt} = 100)$

It is safe.

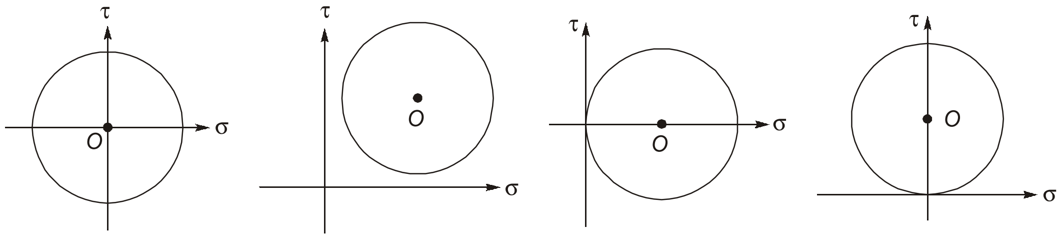
As per MSST, $(\tau_{max} = 70 \text{ MPa}) > \left(S_{ys} = \frac{S_{yt}}{2} = 50\right)$

It is unsafe.

Answer will be (a) and (d)

End of Solution

Q.27 Which of the plot(s) shown is/are valid Mohr's circle representations of a plane stress state in a material? (The center of each circle is indicated by O.)

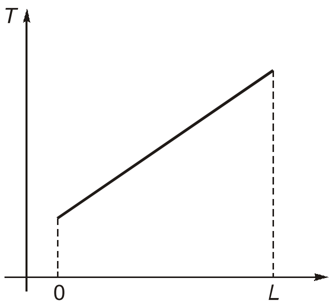


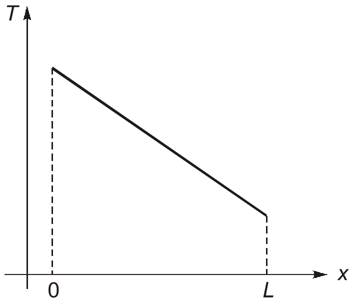
(a) M1
(b) M2
(c) M3
(d) M4

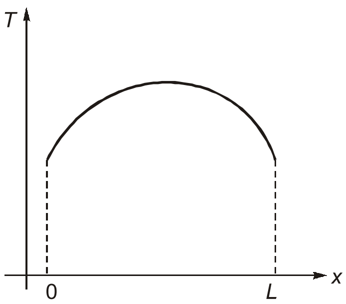
Ans. (a, c)

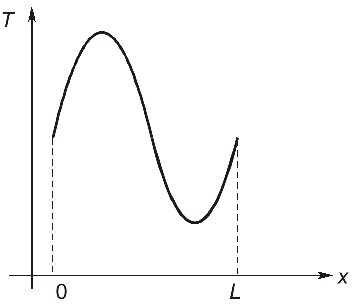
End of Solution

Q.28 Consider a laterally insulated rod of length L and constant thermal conductivity. Assuming one-dimensional heat conduction in the rod, which of the following steady-state temperature profile(s) can occur without internal heat generation?

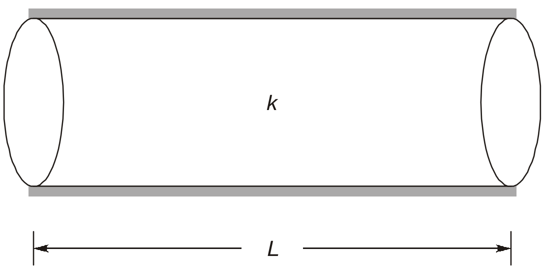
(a)


(b)


(c)


(d)


Ans. (a, b)



For 1-d steady state without internal heat generation heat conduction equation

$$\frac{d^2T}{dx^2} = 0$$

Taking integration

$$\frac{dT}{dx} = C_1$$

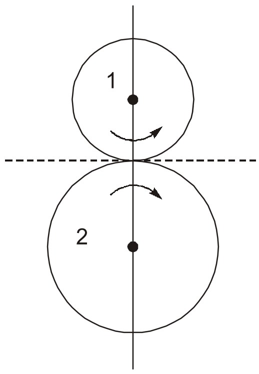
Again taking integration,

$$T(x) = C_1x + C_2$$

Temperature distribution is linear.

End of Solution

Q.29 Two meshing spur gears 1 and 2 with diametral pitch of 8 teeth per mm and an angular velocity ratio $\frac{|\omega_2|}{|\omega_1|} = \frac{1}{4}$, have their centers 30 mm apart. The number of teeth on the driver (gear 1) is _______ . (Answer in integer)



Ans. **(96) (95.999 to 96.001)**

Given:

$$\text{Diametrical pitch, } P_d = \frac{8 \text{ teeth}}{\text{mm}} = \frac{T}{D}$$

$$\Rightarrow \quad \text{module } (m) = \frac{D(\text{mm})}{T} = \frac{1}{P_d}$$

$$\therefore \quad m = \frac{1}{8}\text{mm}$$

$$\frac{\omega_2}{\omega_1} = \frac{1}{4} = \frac{T_1}{T_2}$$

$$\Rightarrow \quad T_2 = 4T_1$$

$$\because \quad R + r = 30$$

$$\left\{ \begin{aligned} m &= \frac{D}{T} = \frac{2R}{T} \\ \therefore \quad R &= \frac{mT}{2} \end{aligned} \right\}$$

$$\Rightarrow \quad \frac{mT_2}{2} + \frac{mT_1}{2} = 30$$

$$\Rightarrow \quad \frac{m}{2}(T_2 + T_1) = 30$$

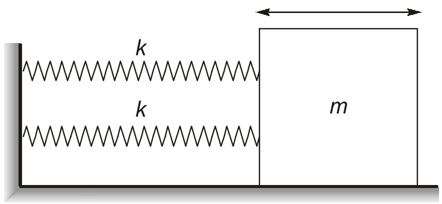
$$\Rightarrow \quad \frac{1}{8 \times 2}(4T_1 + T_1) = 30$$

$$\Rightarrow \quad 5T_1 = 30 \times 16$$

$$\therefore \quad T_1 = \frac{30 \times 16}{5} = 96$$

End of Solution

Q.30 The figure shows a block of mass $m = 20$ kg attached to a pair of identical linear springs, each having a spring constant $k = 1000$ N/m. The block oscillates on a frictionless horizontal surface. Assuming free vibrations, the time taken by the block to complete ten oscillations is ________ seconds. (Rounded off to two decimal places) Take $\pi = 3.14$.



Ans. **(6.28) (6.27 to 6.29)**

Equivalent stiffness: $k_{eq} = k + k = 2k = 2 \times 1000$

$\therefore \quad k_{eq} = 2000$ N/m

$m = 20$ kg

Natural frequency: $\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{2000}{20}} = 10$ rad/s

Time period of oscillation:

$$T_n = \frac{2\pi}{\omega_n}$$

Time taken to complete 10 oscillations $= 10T_n$

$$= 10 \times \frac{2\pi}{\omega_n} = 10 \times \frac{2\pi}{10} = 2\pi = 2 \times 3.14 = 6.28 \text{ sec}$$

End of Solution

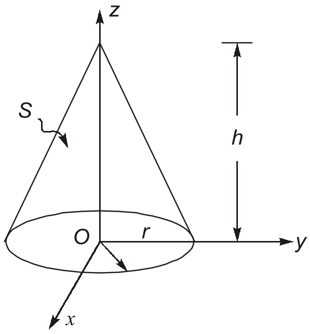
Q.31 A vector field

$$B(x, y, z) = x\hat{i} + y\hat{j} - 2z\hat{k}$$

is defined over a conical region having height $h = 2$, base radius $r = 3$ and axis along z, as shown in the figure. The base of the cone lies in the x-y plane and is centered at the origin. If n denotes the unit outward normal to the curved surface S of the cone, the value of the integral

$$\int_S B \cdot n dS$$

equal ______. [answer in integer]



Ans. **(0) (–0.001 to 0.001)**

Given, $\vec{B}(x,y,z) = x\hat{i} + y\hat{j} - 2z\hat{k}$

Since surface s is a closed surface, Gauss-divergence theorem can be applied

$$\therefore \quad \int_s \vec{B}\cdot\hat{n}ds = \iiint \vec{\nabla}\cdot\vec{B}dv$$

where, $$\vec{\nabla}\cdot\vec{B} = \frac{\partial}{\partial x}x + \frac{\partial}{\partial y}(y) - \frac{\partial}{\partial z}(2z) = 1+1-2=0$$

Thus, $$\int_s \vec{B}\cdot\hat{n}ds = 0$$

End of Solution

Q.32 A linear transformation maps a point (x, y) in the plane to the point $(\hat{x},\hat{y})$ according to the rule

$$\hat{x} = 3y, \;\; \hat{y} = 2x$$

Then, the disc $x^2 + y^2 \leq 1$ gets transformed to a region with an area equal to ______.
(Rounded off to two decimals)
Use $\pi = 3.14$.

Ans. **(18.84) (18.80 to 18.90)**

Given that,

$$\hat{x} = 3y \text{ and } \hat{y} = 2x$$

$$\therefore \quad y = \frac{\hat{x}}{3} \text{ and } x = \frac{\hat{y}}{2}$$

The given region, $$x^2 + y^2 \leq 1$$

or $$\left(\frac{\hat{y}}{2}\right)^2 + \left(\frac{\hat{x}}{3}\right)^2 \leq 1$$

$$\frac{(\hat{y})^2}{4} + \frac{(\hat{x})^2}{9} \leq 1$$

This is equation of an ellipse with semi major axis, $a = 3$ and semi-minor axis, $b = 2$, thus area of transformed region is

$$A = \pi ab = 6\pi \simeq 18.84 \text{ units}$$

End of Solution

Q.33 The value of k that makes the complex-valued function
$f(z) = e^{-kx}(\cos 2y - i \sin 2y)$
analytic, where $z = x + iy$, is______. [Answer in integer]

Ans. **(2) (1.999 to 2.001)**

Given,
$$f(z) = u(x, y) + iv(x, y)$$
$$f(z) = e^{-kx}\cos 2y - ie^{-kx}\sin 2y$$

Here,
$$\frac{\partial u}{\partial x} = -ke^{-kx}\cos 2y$$
$$\frac{\partial u}{\partial y} = -2e^{-kx}\sin 2y$$

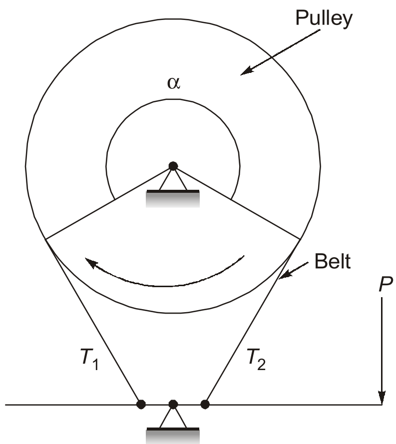
and,
$$\frac{\partial v}{\partial x} = -ke^{-kx}\sin 2y$$
$$\frac{\partial v}{\partial y} = -2e^{-kx}\cos 2y$$

Since $f(z)$ is analytic, it will satisfy Cauchy–Riemann equation,
$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$
$$\Rightarrow \quad -ke^{kx}\cos 2y = -2e^{-kx}\cos 2y$$
$$\Rightarrow \quad k = 2$$

End of Solution

Q.34 The braking system shown in the figure uses a belt to slow down a pulley rotating in the clockwise direction by the application of a force P. The belt wraps around the pulley over an angle α = 270 degrees. The coefficient of friction between the belt and the pulley is 0.3. The influence of centrifugal forces on the belt is negligible.
During braking, the ratio of the tensions T_1 to T_2 in the belt is equal to ________. (Rounded off to two decimal places)

Take π = 3.14.



Ans. **(4.11) (4.05 to 4.15)**

Using the equation,

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.3 \times \frac{3\pi}{2}}$$

$$\Rightarrow \quad \frac{T_1}{T_2} = 4.11$$

End of Solution

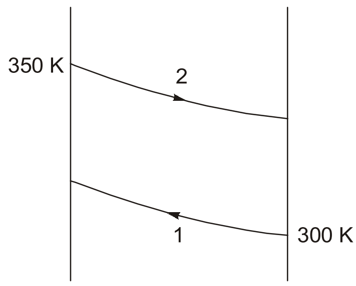
Q.35 Consider a counter-flow heat exchanger with the inlet temperatures of two fluids (1 and 2) being $T_{1,\,in}$ = 300 K and $T_{2,\,in}$ = 350 K. The heat capacity rates of the two fluids are C_1 = 1000 W/K and C_2 = 400 W/K, and the effectiveness of the heat exchanger is 0.5. The actual heat transfer rate is _____ kW. (Answer in integer)

Ans. **(10) (9.999 to 10.001)**

Given:

$$C_1 = 1000 \text{ W/K}$$

$$C_2 = 400 \text{ W/K} = C_{min}$$



$$\text{Effectiveness, } \in = \frac{\dot{q}_{actual}}{\dot{q}_{max}}$$

$$\in = \frac{\dot{q}_{actual}}{C_{min}\left(T_{h_i} - T_{c_i}\right)} = 0.5$$

$$q_{actual} = 0.5 \times 400\ (350 - 300)$$

$$= 10000 \text{ Watt} = 10 \text{ kW}$$

End of Solution

Q.36 Which one of the options given is the inverse Laplace transform of $\frac{1}{S^3 - S}$?

$u(t)$ denotes the unit-step function.

(a) $\left(-1+\frac{1}{2}e^{-t}+\frac{1}{2}e^{t}\right)u(t)$

(b) $\left(\frac{1}{3}e^{-t}-e^{t}\right)u(t)$

(c) $\left(-1+\frac{1}{2}e^{-(t-1)}+\frac{1}{2}e^{(t-1)}\right)u(t-1)$

(d) $\left(-1-\frac{1}{2}e^{-(t-1)}-\frac{1}{2}e^{(t-1)}\right)u(t-1)$

Ans. **(a)**

Given,

$$F(s) = \frac{1}{s^3 - s} = \frac{1}{s(s^2-1)} = \frac{1}{s(s-1)(s+1)}$$

On partial fraction decomposition,

$$F(s) = -\frac{1}{s} + \frac{\frac{1}{2}}{s-1} + \frac{\frac{1}{2}}{s+1}$$

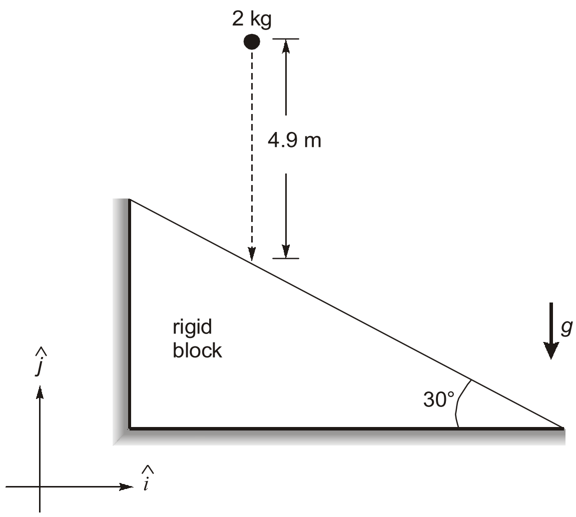
Thus, $$L^{-1}(F(s)) = L^{-1}\left(-\frac{1}{s}\right)+L^{-1}\left(\frac{\frac{1}{2}}{s-1}\right)+L^{-1}\left(\frac{\frac{1}{2}}{s+1}\right)$$

$$= -L^{-1}\left(\frac{1}{s}\right)+\frac{1}{2}L^{-1}\left(\frac{1}{s-1}\right)+\frac{1}{2}L^{-1}\left(\frac{1}{s+1}\right)$$

$$= -1+\frac{1}{2}e^{t}+\frac{1}{2}e^{-t}$$

End of Solution

Q.37 A spherical ball weighing 2 kg is dropped from a height of 4.9 m onto an immovable rigid block as shown in the figure. If the collision is perfectly elastic, what is the momentum vector of the ball (in kg m/s) just after impact?
Take the acceleration due to gravity to be g = 9.8 m/s^2. Options have been rounded off to one decimal place.



(a) $19.6\hat{i}$ (b) $19.6\hat{j}$

(c) $17.0\hat{i}+9.8\hat{j}$ (d) $9.8\hat{i}+17.0\hat{j}$

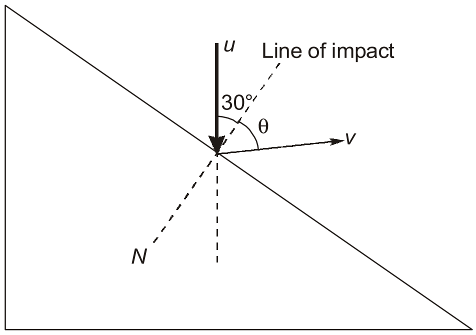
Ans. **(c)**

Velocity of the ball, just before making an impact with the incline

$$u = \sqrt{2gh} = \sqrt{2\times 9.8\times 4.9} = 9.8 \text{m/s}$$

For perfectly elastic collision, along the line of impact,

Velocity of approach = Velocity of separation



$$9.8\cos 30^\circ = v\cos\theta \quad \text{...(i)}$$

Now, momentum along the inclined plane will remain conserved,

$$P_i = P_f$$

$$\Rightarrow \quad 2u\sin 30^\circ = 2v\sin\theta$$

$$\Rightarrow \quad v\sin\theta = 9.8\sin 30^\circ = 4.9 \quad \text{...(ii)}$$

Using equation (i) and (ii), we get

$$v = 9.8\sqrt{\sin^2 30 + \cos^2 30^\circ} = 9.8 \text{ m/s}$$

Therefore, $\theta = 30^\circ$

Now, momentum is given by,

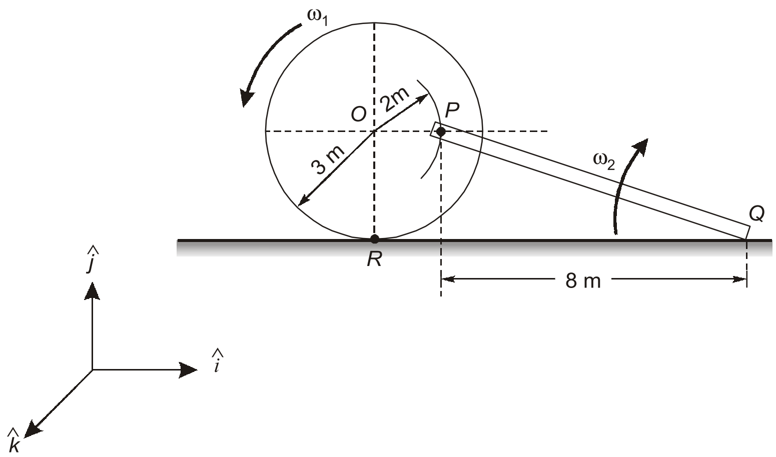
$$\vec{P}_f = 2 \times 9.8\sin 60^\circ \hat{i} + 2 \times 9.8 \cos 60^\circ \hat{j}$$

$$= 17\hat{i} + 9.8\hat{j}$$

End of Solution

Q.38 The figure shows a wheel rolling without slipping on a horizontal plane with angular velocity ω_1. A rigid bar PQ is pinned to the wheel at P while the end Q slides on the floor.

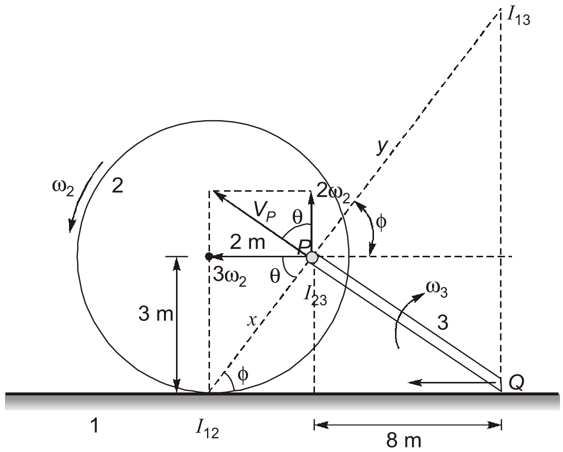
What is the angular velocity ω_2 of the bar PQ?



(a) $\omega_2 = 2\omega_1$ (b) $\omega_2 = \omega_1$

(c) $\omega_2 = 0.5\omega_1$ (d) $\omega_2 = 0.25\omega_1$

Ans. (d)



Let us assume fixed link as 1, disc as link 2 and rod as link 3.

Applying Kennedy theorem:

$$\omega_2(I_{12}I_{23}) = \omega_3(I_{13}I_{23}) \qquad ...(1)$$

$$\tan\theta = \frac{3}{2}$$

$$\tan\phi = \frac{3}{2}$$

$\Rightarrow$ $\theta = \phi$

$\therefore$ $x\cos\phi = 2$

$y\cos\phi = 8$

From equation (1),

$$\omega_1\left(\frac{2}{\cos\phi}\right) = \omega_2\left(\frac{8}{\cos\phi}\right)$$

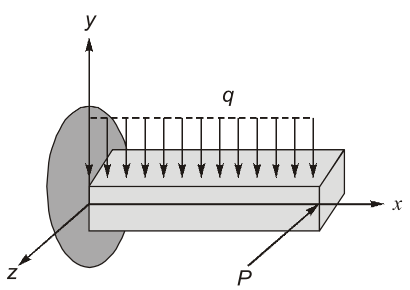
($\because$ According to the question, ω_1 is the angular velocity of disc and ω_2 is the angular velocity of rod.)

$\Rightarrow$ $\omega_1 = 4\omega_2$

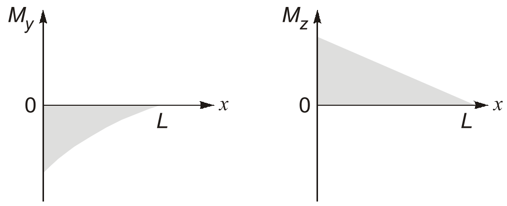
$\Rightarrow$ $\omega_2 = 0.25\omega_1$

End of Solution

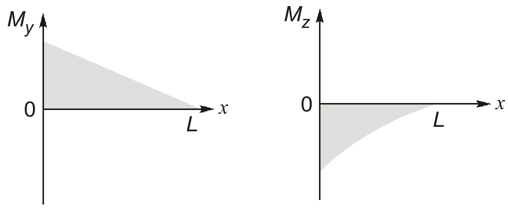
Q.39 A beam of length L is loaded in the xy-plane by a uniformly distributed load, and by a concentrated tip load parallel to the z-axis, as shown in the figure. The resulting bending moment distributions about the y and the z axes are denoted by M_y and M_z, respectively. Which one of the options given depicts qualitatively CORRECT variations of M_y and M_z along the length of the beam?



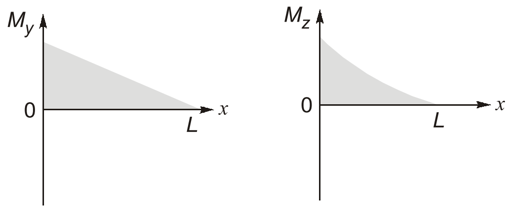
(a)



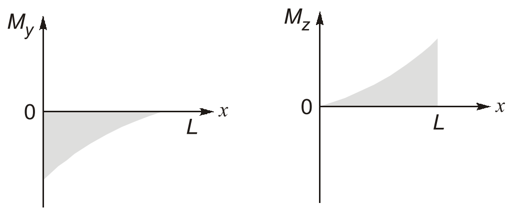
(b)



(c)



(d)



Ans. **(b)**

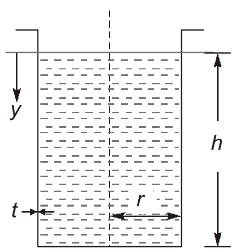
Due to UDL, BM is negative and varies parabolically in vertical plane i.e bending moment is acting about Z-axis (M_z).

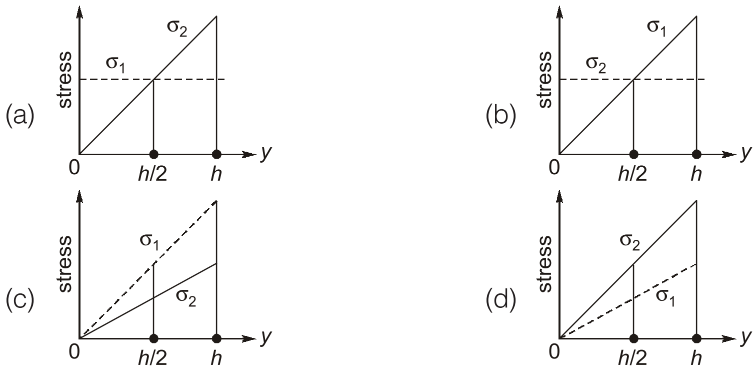
Due to Horizontal concentrated point load, bending moment is positive and varies linearly in horizontal plane i.e. bending moment is acting about Y-axis(M_y).

End of Solution

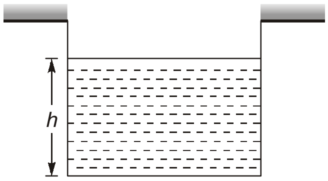
Q.40 The figure shows a thin-walled open-top cylindrical vessel of radius r and wall thickness t. The vessel is held along the brim and contains a constant-density liquid to height h from the base. Neglect atmospheric pressure, the weight of the vessel and bending stresses in the vessel walls.

Which one of the plots depicts qualitatively correct dependence of the magnitude of axial wall stress (σ_1) and circumferential wall stress (σ_2) on y?





Ans. **(a)**



As we move from the free surface towards bottom, the pressure variation can be written as:

$$P = \rho g h$$

Now, $$\sigma_h = \frac{PD}{2t} = \rho g h\left(\frac{D}{2t}\right)$$

$\therefore$ $$\sigma_h \propto h$$

Longitudinal stress will be developed because of the total amount of fluid and since this amount is constant, the longitudinal stress will be constant.

End of Solution

Q.41 Which one of the following statements is FALSE?

(a) For an ideal gas, the enthalpy is independent of pressure.

(b) For a real gas going through an adiabatic reversible process, the process equation is given by PV^{γ} = constant, where P is the pressure, V is the volume and γ is the ratio of the specific heats of the gas at constant pressure and constant volume.

(c) For an ideal gas undergoing a reversible polytropic process $PV^{1.5}$ = constant, the equation connecting the pressure, volume and temperature of the gas at any point along the process is $\frac{P}{R} = \frac{mT}{V}$, where R is the gas constant and m is the mass of the gas.

(d) Any real gas behaves as an ideal gas at sufficiently low pressure or sufficiently high temperature.

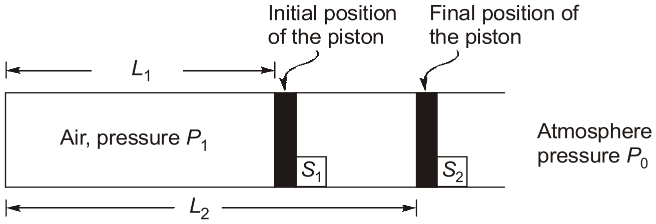
Ans. **(b)**

The process $PV^{\gamma} = C$ is applicable for an ideal gas going through an adiabatic reversible process.

End of Solution

Q.42 Consider a fully adiabatic piston-cylinder arrangement as shown in the figure. The piston is massless and cross-sectional area of the cylinder is A. The fluid inside the cylinder is air (considered as a perfect gas), with γ being the ratio of the specific heat at constant pressure to the specific heat at constant volume for air. The piston is initially located at a position L_1. The initial pressure of the air inside the cylinder is $P_1 >> P_0$, where P_0 is the atmospheric pressure. The stop S_1 is instantaneously removed and the piston moves to the position L_2, where the equilibrium pressure of air inside the cylinder is $P_2 >> P_0$.

What is the work done by the piston on the atmosphere during this process?



(a) 0

(b) $P_0A(L_2 - L_1)$

(c) $P_1AL_1 \ln \dfrac{L_1}{L_2}$

(d) $\dfrac{(P_2L_2 - P_1L_1)A}{(1-\gamma)}$

Ans. **(b)**

From the given data, $V_1 = L_1 \times A$

$$V_2 = L_2 \times A$$

$$W_{atm} = P_0(V_2 - V_1)$$

$$= P_0(L_2A - L_1A)$$

$$= P_0A(L_2 - L_1)$$

End of Solution

Q.43 A cylindrical rod of length h and diameter d is placed inside a cubic enclosure of side length L, S denotes the inner surface of the cube. The view-factor $F_{S\text{-}S}$ is

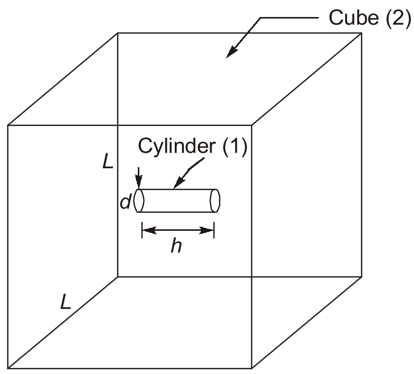
(a) 0

(b) 1

(c) $\dfrac{\left(\pi dh + \dfrac{\pi d^2}{2}\right)}{6L^2}$

(d) $1 - \dfrac{\left(\pi dh + \dfrac{\pi d^2}{2}\right)}{6L^2}$

Ans. **(d)**



$$A_1 = \pi dh + \frac{\pi}{4}d^2 + \frac{\pi}{4}d^2$$

$$A_1 = \left(\pi dh + \frac{\pi d^2}{2}\right); \; A_2 = 6L^2$$

Summation rule (for surface 1)

$$F_{11} + F_{12} = 1$$

$$F_{12} = 1 \qquad \{F_{11} = 0\}$$

Reciprocity rule (1, 2):

$$A_1F_{12} = A_2F_{21}$$

$$A_1 = A_2F_{21} \qquad \{F_{12} = 1\}$$

$$F_{21} = \frac{A_1}{A_2}$$

Summation rule (for surface 2)

$$F_{21} + F_{22} = 1$$

$$F_{22} \text{ or } F_{ss} = 1 - F_{21} = 1 - \frac{A_1}{A_2}$$

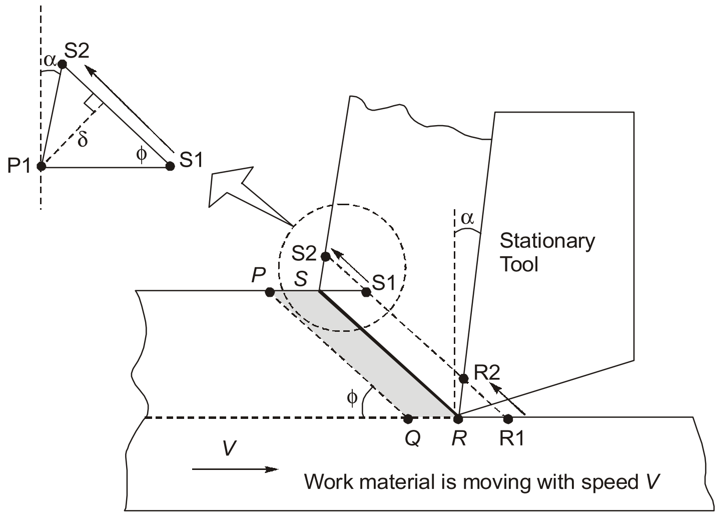
$$F_{22} = 1 - \frac{\left(\pi dh + \frac{\pi d^2}{2}\right)}{6L^2}$$

End of Solution

Q.44 In an ideal orthogonal cutting experiment (see figure), the cutting speed V is 1 m/s, the rake angle of the tool α = 5°, and the shear angle, ϕ, is known to be 45°.

Applying the ideal orthogonal cutting model, consider two shear planes PQ and RS close to each other. As they approach the thin shear zone (shown as a thick line in the figure), plane RS gets sheared with respect to PQ (point R1 shears to R2, and S1 shears to S2).

Assuming that the perpendicular distance between PQ and RS is δ = 25 μm, what is the value of shear strain rate (in s^{-1}) that the material undergoes at the shear zone?



(a) 1.84×10^4 (b) 5.20×10^4
(c) 0.71×10^4 (d) 1.30×10^4

Ans. **(b)**

Given:

Cutting velocity, $V = 1$ m/s,

Rake angle, $\alpha = 5°$,

Shear angle, $\phi = 45°$,

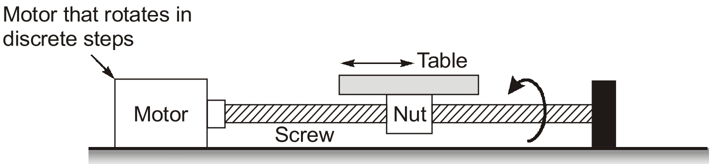
Thickness of shear plane, $\delta = 25\ \mu m$

Shear velocity, $$V_s = \frac{V\cos\alpha}{\cos(\phi-\alpha)} = \frac{1\times\cos 5}{\cos(45-5)} = 1.3 \text{ m/s}$$

Shear strain rate $$= \frac{V_s}{\delta} = \frac{1.3}{25\times10^{-6}} = 5.2\times10^4\, s^{-1}$$

End of Solution

Q.45 A CNC machine has one of its linear positioning axes as shown in the figure, consisting of a motor rotating a lead screw, which in turn moves a nut horizontally on which a table is mounted. The motor moves in discrete rotational steps of 50 steps per revolution. The pitch of the screw is 5 mm and the total horizontal traverse length of the table is 100 mm. What is the total number of controllable locations at which the table can be positioned on this axis?



(a) 5000 (b) 2
(c) 1000 (d) 200

Ans. **(c)**

Given,

Steps per revolution = 50

Total table movement = 100 mm

Pitch of lead screw = 5 mm

So, In 50 steps lead screw moves 5 mm

50 steps = 5 mm

hence 10 steps are required to move 1 mm.

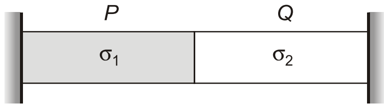
So, number of steps required are = $100 \times 10 = 1000$

End of Solution

Q.46 Cylindrical bars P and Q have identical lengths and radii, but are composed of different linear elastic materials. The Young's modulus and coefficient of thermal expansion of Q are twice the corresponding values of P. Assume the bars to be perfectly bonded at the interface, and their weights to be negligible.

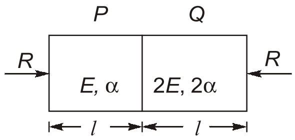
The bars are held between rigid supports as shown in the figure and the temperature is raised by ΔT. Assume that the stress in each bar is homogeneous and uniaxial. Denote the magnitudes of stress in P and Q by σ_1 and σ_2, respectively.

Which of the statement(s) given is/are CORRECT?



(a) The interface between P and Q moves to the left after heating.
(b) The interface between P and Q moves to the right after heating.
(c) $\sigma_1 < \sigma_2$
(d) $\sigma_1 = \sigma_2$

Ans. **(a, d)**



As the areas of both the bars are same, therefore, the stress developed in both the bars will remain same.

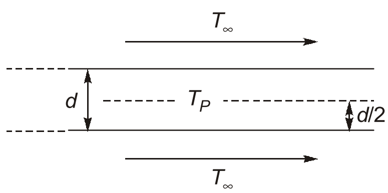
Deflection in rod P, $\Delta_P = l\alpha T - \frac{RL}{AE}$

Deflection in rod Q, $\Delta_Q = l(2\alpha)T - \frac{RL}{A(2E)}$

We can say that $\Delta_Q > \Delta_P$. Therefore, interface will move towards left side.

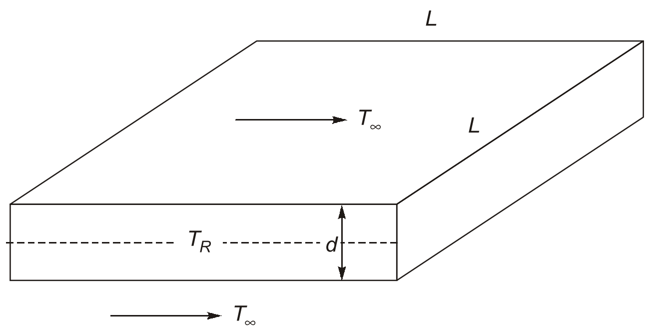
End of Solution

Q.47 A very large metal plate of thickness d and thermal conductivity k is cooled by a stream of air at temperature $T_\infty = 300$ K with a heat transfer coefficient h, as shown in the figure. The centerline temperature of the plate is T_P. In which of the following case(s) can the lumped parameter model be used to study the heat transfer in the metal plate?



(a) $h = 10$ Wm^{-2}K^{-1}, $k = 100$ Wm^{-1}K^{-1}, $d = 1$ mm, $T_P = 350$ K
(b) $h = 100$ Wm^{-2}K^{-1}, $k = 100$ Wm^{-1}K^{-1}, $d = 1$ m, $T_P = 325$ K
(c) $h = 100$ Wm^{-2}K^{-1}, $k = 1000$ Wm^{-1}K^{-1}, $d = 1$ mm, $T_P = 325$ K
(d) $h = 1000$ Wm^{-2}K^{-1}, $k = 1$ Wm^{-1}K^{-1}, $d = 1$ m, $T_P = 350$ K

Ans. **(a, c)**



Characteristic length, $L_C = \dfrac{V}{A_s} = \dfrac{L \times L \times d}{(L \times L) + (L \times L)} = \dfrac{d}{2}$

As edge area is negligible

Lumped analysis is used when Biot number is less than 0.1.

i.e. $\qquad \text{Bi} = \dfrac{hL_C}{k} < 0.1$

(A) $\qquad \text{Bi} = \dfrac{10 \times 0.5}{100 \times 1000} = 5 \times 10^{-5} < 0.1$

(B) $\qquad \text{Bi} = \dfrac{100 \times 0.5}{100} = 0.5 > 0.1$

(C) $\qquad \text{Bi} = \dfrac{100 \times 0.5}{1000 \times 1000} = 5 \times 10^{-5} < 0.1$

(D) $\qquad \text{Bi} = \dfrac{1000 \times 0.5}{1} = 500 > 0.1$

End of Solution

Q.48 The smallest perimeter that a rectangle with area of 4 square units can have is ________ units. (Answer in integer)

Ans. **(8) (7.999 to 8.001)**

Let x and y be the dimensions of the rectangle.

Given, $\qquad xy = 4 \qquad$ ...(i)

Perimeter, P = 2(x + y)

By equation (i),

$$P = 2\left(x + \frac{4}{x}\right) = 2x + \frac{8}{x}$$

For smallest perimeter,

$$\frac{dP}{dx} = 0$$

$$\frac{d}{dx}\left(2x + \frac{8}{x}\right) = 0$$

$$2-\frac{8}{x^2} = 0$$

$\Rightarrow$ $$x^2 - 4 = 0$$

$$x = 2 \quad \text{(ignoring netagive value)}$$

$\therefore$ $$y = 2$$

Thus, the smallest perimeter is

$$P = 2(2 + 2) = 8 \text{ units}$$

End of Solution

Q.49 Consider the second-order linear ordinary differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0,\ x\geq 1$$

with the intial conditions

$$y(x=1)=6,\quad \left.\frac{dy}{dx}\right|_{x=1}=2$$

the value of y at $x = 2$ equals _______. [Answer in integer]

Ans. (9) (8.999 to 9.001)

It is Euler differential equation

$\therefore$ $$D(D-1)y + Dy - y = 0$$

$$[D(D-1) + (D-1)]y = 0$$

Corresponding auxiliary equation,

$$m^2 - m + m - 1 = 0$$

On solving, $$m^2 = 1$$

$$m = \pm 1$$

$\therefore$ $$y = C_1e^t + C_1e^{-t}$$

But, $$t = \ln x$$

$\therefore$ $$y = C_1x+\frac{C_2}{x}$$

Also, $$\frac{dy}{dx} = C_1-\frac{C_2}{x^2}$$

Now applying boundary conditions,

i.e. $$y(x=1) = 6;\quad C_1 + C_2 = 6 \qquad \text{...(i)}$$

and, $$\left.\frac{dy}{dx}\right|_{x=1} = 2;\quad C_1 - C_2 = 2 \qquad \text{...(ii)}$$

On solving (i) and (ii),

$$C_1 = 4;\ C_2 = 2$$

Thus, $$y = 4x+\frac{2}{x}$$

Substituting, x = 2,

$$y = 4(2)+\frac{2}{2}=9$$

End of Solution

Q.50 The initial value problem

$$\frac{dy}{dt} + 2y = 0, \quad y(0) = 1$$

is solved numerically using the forward Euler's method with a constant and positive time step of Δt.

Let y_n represent the numerical solution obtained after n steps. The condition $|y_{n+1}| \le |y_n|$ is satisfied if and only if Δt does not exceed _______. (Answer in integer)

Ans. **(1) (0.999 to 1.001)**

$$\frac{dy}{dt} + 2y = 0 \qquad y(0) = 1 \qquad h = \Delta t = \text{positive}$$

$$t_0 = 0 \qquad y_0 = 1$$

$$\frac{dy}{dt} = -2y$$

$$f(t, y) = -2y$$

$$y_{n+1} = y_n + hf(t_n, y_n)$$

$$y_{n+1} = y_n + \Delta t(-2y_n)$$

$$y_{n+1} = y_n - \Delta t 2y_n$$

$$y_{n+1} = y_n(1 - 2\Delta t)$$

$$\frac{y_{n+1}}{y_n} = 1 - 2\Delta t$$

$$\frac{|y_{n+1}|}{|y_n|} \le 1$$

$$\Rightarrow \quad |1 - 2\Delta t| \le 1$$

$$-1 \le 1 - 2\Delta t \le 1$$

$$-2 \le -2\Delta t \le 0$$

$$0 \le 2\Delta t \le 2$$

$$0 \le \Delta t \le 1$$

End of Solution

Q.51 The atomic radius of a hypothetical face-centered cubic (FCC) metal is $\left(\frac{\sqrt{2}}{10}\right)$nm. The atomic weight of the metal is 24.092 g/mol. Taking Avogadro's number to be 6.023×10^{23} atoms/mol, the density of the metal is kg/m^3. (Answer in integer)

Ans. **(2500) (2499.999 to 2500.001)**

Given:

$$\text{radius of atom, } r = \left(\frac{\sqrt{2}}{10}\right)\text{nm}$$

Atomic weight of metal, $m =$ 24.092 g/mol

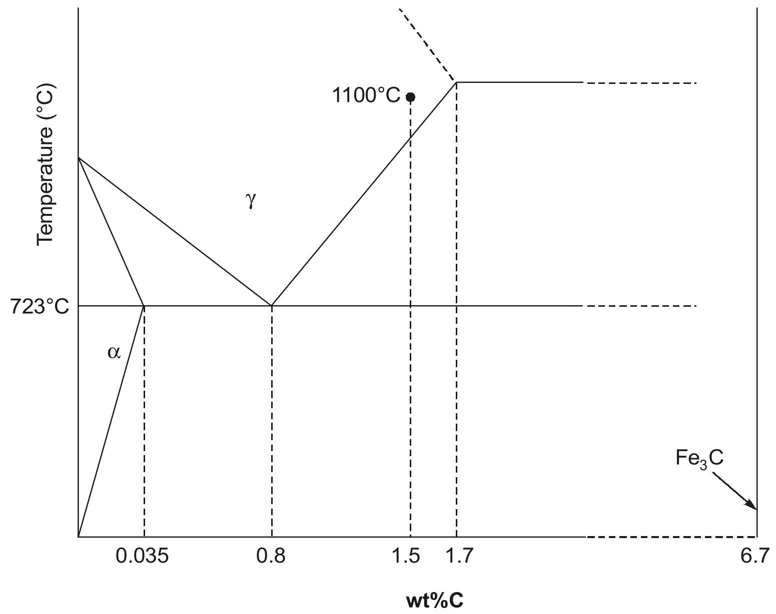
Avogadro's number, $A = 6.023 \times 10^{23}$ atoms/mol

$$\text{Volume of unit cell, } a^3 = \left(\frac{4r}{\sqrt{2}}\right)^3 = \left(\frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{10}\right)^3 = 0.064 \text{ nm}^3$$

$$\text{Density of metal } \rho = \frac{4 \times 24.092 \times (10^9)^3}{6.023 \times 10^{23} \times 1000 \times 0.064} = 2500 \text{ kg/m}^3$$

End of Solution

Q.52 A steel sample with 1.5 wt.% carbon (no other alloying elements present) is slowly cooled from 1100 °C to just below the eutectoid temperature (723 °C). A part of the iron-cementite phase diagram is shown in the figure. The ratio of the pro-eutectoid cementite content to the total cementite content in the microstructure that develops just below the eutectoid temperature is ______. (Rounded off to two decimal places)



Ans. **(0.54) (0.53 to 0.55)**

Mass fraction of preoeutectiod cementite,

$$m_1 = \frac{1.5 - 0.8}{6.7 - 0.8} = 0.1186$$

Mass fraction of total cementite,

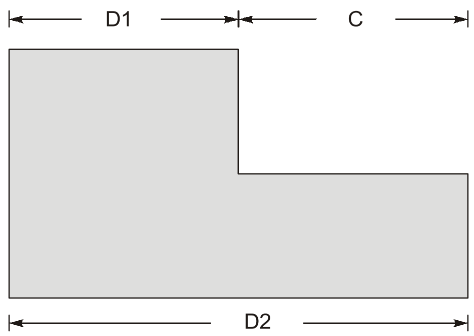
$$m_2 = \frac{1.5 - 0.035}{6.7 - 0.035} = 0.2198$$

$$\frac{m_1}{m_2} = \frac{0.1186}{0.2198} = 0.54$$

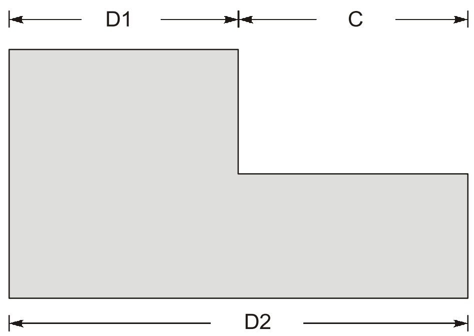
End of Solution

Q.53 A part, produced in high volumes, is dimensioned as shown. The machining process making this part is known to be statistically in control based on sampling data. The sampling data shows that D1 follows a normal distribution with a mean of 20 mm and a standard deviation of 0.3 mm, while D2 follows a normal distribution with a mean of 35 mm and a standard deviation of 0.4 mm. An inspection of dimension C is carried out in a sufficiently large number of parts.

To be considered under six-sigma process control, the upper limit of dimension C should be ________ mm. (Rounded off to one decimal place)



Ans. (16.587) (16.4 to 16.6)



D1 and D2 follow normal distribution.

So, Upper limit = $\mu + 3\sigma$

Lower limit = $\mu - 3\sigma$

$$\bar{D}1 = 20 \text{ mm}, \ \bar{D}2 = 35 \text{ mm}, \sigma_{D2} = 0.4 \text{ mm}, \sigma_{D1} = 0.3 \text{ mm}$$

$$\text{Mean value of } (\bar{C}) = \bar{D}2 - \bar{D}1 = 35 - 20 = 15 \text{ mm}$$

Standard deviation of C

$$\sigma_C^2 = \sigma_{D2}^2 - \sigma_{D1}^2$$

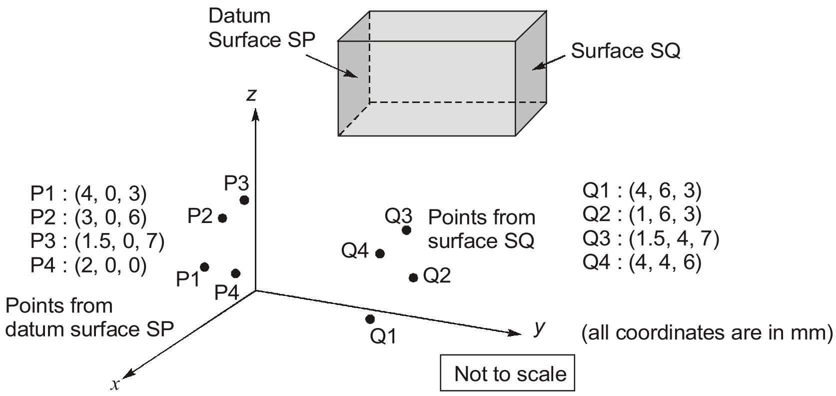
$$\sigma_C = \sqrt{0.4^2 - 0.3^2} = 0.26457$$

$$\text{Upper limit of } C = \bar{C} + 6 \times \sigma_C$$

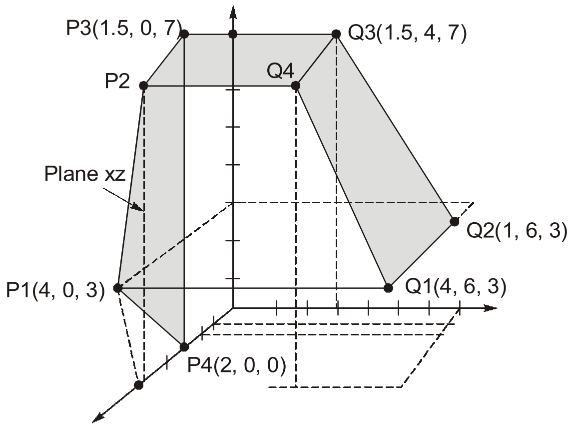
$$= 15 + 6 \times 0.26457 = 16.587 \text{ mm}$$

End of Solution

Q.54 A coordinate measuring machine (CMM) is used to determine the distance between Surface SP and Surface SQ of an approximately cuboidal shaped part. Surface SP is declared as the datum as per the engineering drawing used for manufacturing this part. The CMM is used to measure four points P1, P2, P3, P4 on Surface SP, and four points Q1, Q2, Q3, Q4 on Surface SQ as shown. A regression procedure is used to fit the necessary planes. The distance between the two fitted planes is__________ mm. (Answer in integer)



Ans. **(5) (4.999 to 5.001)**



Points P1, P2, P3, P4 lie on xz plane. Thus distances of Q1, Q2, Q3, Q4 from the SP surface are as described below.

Distance of Q1 from x-z plane = 6

Distance of Q2 from x-z plane = 6

Distance of Q3 from x-z plane = 4

Distance of Q4 from x-z plane = 4

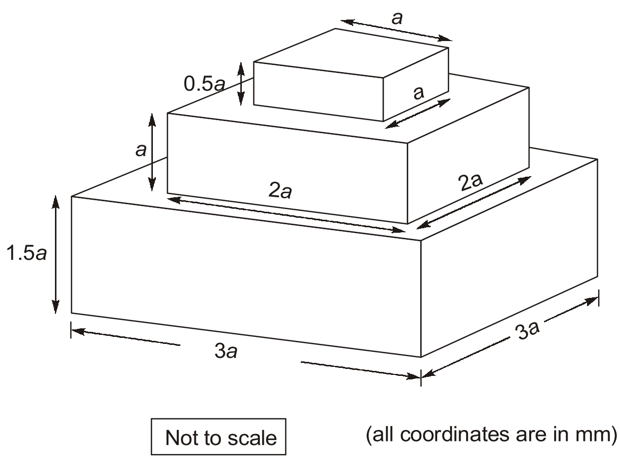
Thus,Avarage distance between the two fitted planes = 5 mm

End of Solution

Q.55 A solid part (see figure) of polymer material is to be fabricated by additive manufacturing (AM) in square-shaped layers starting from the bottom of the part working upwards. The nozzle diameter of the AM machine is $a/10$ mm and the nozzle follows a linear serpentine path parallel to the sides of the square layers with a feed rate of $a/5$ mm/min.

Ignore any tool path motions other than those involved in adding material, and any other delays between layers or the serpentine scan lines.

The time taken to fabricate this part is ________ minutes. (Answer in integer)



Ans. (9000) (8999.999 to 9000.001)

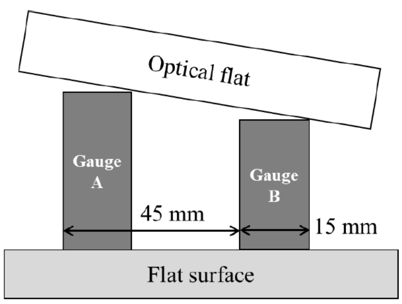
$$V = (3a)^2 \times 1.5a + (2a)^2 \times a + a^2 \times 0.5a$$
$$= 18a^3$$

$$T = \frac{V}{\left(\frac{a}{10}\right)^2 \times \left(\frac{a}{5}\right)}$$

$$= \frac{18a^3}{a^3} \times 100 \times 5 = 9000 \text{ min}$$

End of Solution

Q.56 An optical flat is used to measure the height difference between a reference slip gauge A and a slip gauge B. Upon viewing via the optical flat using a monochromatic light of wavelength 0.5 μm, 12 fringes were observed over a length of 15 mm of gauge B. If the gauges are placed 45 mm apart, the height difference of the gauges is ________ μm. (Answer in integer)



Ans. **(9) (8.999 to 9.001)**

Given

Number of fringes = 12 per 15 mm

Wave length = 0.5 μm

Distance between gauges = 45 mm

Height difference between gauges,

$$h = \frac{n\lambda}{2} = \frac{12}{15} \times \frac{0.5}{2} \times 45 = 9 \ \mu\text{m}$$

End of Solution

Q.57 Ignoring the small elastic region, the true stress (σ) – true strain (ε) variation of a material beyond yielding follows the equation $\sigma = 400\varepsilon^{0.3}$ MPa. The engineering ultimate tensile strength value of this material is ________ MPa. (Rounded off to one decimal place)

Ans. **(206.55) (206.4 to 206.6)**

Given: $\sigma_T = 400\varepsilon_T^{0.3}$

At ultimate tensile strength

$$n = \varepsilon_T = 0.3$$

So, true stress at ultimate tensile strength

$$\sigma_T = 400 \times 0.3^{0.3} = 278.74 \text{ MPa}$$

True strain, $\varepsilon_T = \ln(1 + \varepsilon)$

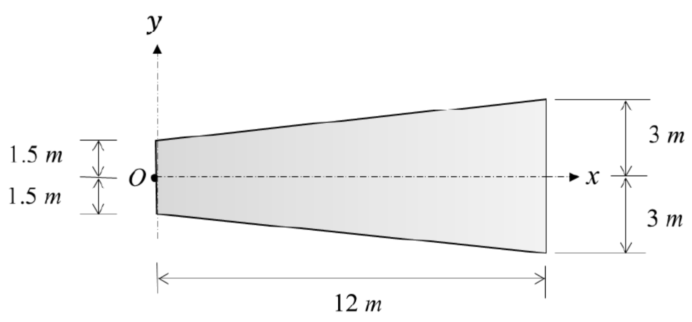
$\Rightarrow$ $1 + \varepsilon = e^{0.3} = 1.35$

Engineering ultimate tensile strength

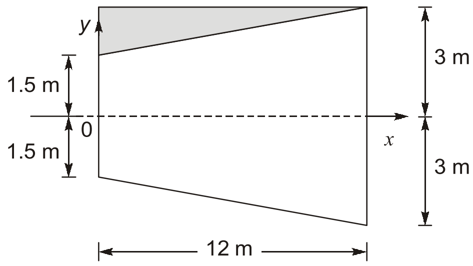
$$\sigma = \frac{\sigma_T}{1+\varepsilon} = \frac{278.74}{1.35} = 206.55 \text{MPa}$$

End of Solution

Q.58 The area moment of inertia about the y-axis of a linearly tapered section shown in the figure is ________ m^4. (Answer in integer)



Ans. (3024) (3023.999 to 3024.001)



Considering upper half of the section:

Moment of inertia of rectangular secton about y-axis:

$$I_{\text{rectangular sec.}} = \frac{1}{3} \times 3 \times (12)^3 = (12)^3$$

Moment of inertia of triangular section about y-axis:

$$I_{\text{triangular sec.}} = \frac{1}{12} \times 1.5 \times (12)^3 = \frac{1}{8} \times (12)^3$$

$$I = (12)^3 - \frac{(12)^3}{8} = \frac{7}{8} \times (12)^3$$

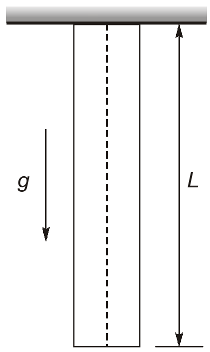
$\therefore$ Moment of inertia for the whole section

$$= 2 \times \frac{7}{8} \times (12)^3 = 3024 \text{ m}^3$$

End of Solution

Q.59 A cylindrical bar has a length L = 5 m and cross section area S = 10 m^2. The bar is made of a linear elastic material with a density ρ = 2700 kg/m^3 and Young's modulus E = 70 GPa. The bar is suspended as shown in the figure and is in a state of uniaxial tension due to its self-weight. The elastic strain energy stored in the bar equals ________ J. (Rounded off to two decimal places)

Take the acceleration due to gravity as g = 9.8 m/s^2.



Ans. **(2.09) (2.00 to 2.16)**

The strain energy is given by:

$$U = \frac{W^2L}{6AE} = \frac{\gamma^2 AL^3}{6E}$$

$$\Rightarrow \quad U = \frac{(2700 \times 9.81)^2 \times 10 \times 5^3}{6 \times 70 \times 10^9} = 2.09 \text{ N-m}$$

End of Solution

Q.60 A cylindrical transmission shaft of length 1.5 m and diameter 100 mm is made of a linear elastic material with a shear modulus of 80 GPa. While operating at 500 rpm, the angle of twist across its length is found to be 0.5 degrees.

The power transmitted by the shaft at this speed iskW. (Rounded off to two decimal places) Take π = 3.14.

Ans. **(239.246) (237 to 240)**

Using the equation $\theta = \dfrac{TL}{GJ}$

$$\therefore \quad T = \frac{GJ\theta}{L} = \frac{80 \times 10^3 \times \frac{\pi}{32} \times (100)^4 \times 0.5 \times \frac{\pi}{180}}{1500}$$

$$\Rightarrow \quad T = 4569261.297 \text{ N-mm}$$

$$\Rightarrow \quad T = 4569.261 \text{ N-m}$$

$$\therefore \quad P = \frac{2\pi NT}{60}$$

$$\Rightarrow \quad P = \frac{2\pi \times 500 \times 4569.261}{60}$$

$$\Rightarrow \quad P = 239245.96 \text{ W}$$

$$\Rightarrow \quad P = 239.246 \text{ kW}$$

End of Solution

Q.61 Consider a mixture of two ideal gases, X and Y, with molar masses $\overline{M}_X$ = 10 kg/kmol and $\overline{M}_Y$ = 20 kg/kmol, respectively, in a container. The total pressure in the container is 100 kPa, the total volume of the container is 10 m^3 and the temperature of the contents of the container is 300 K. If the mass of gas-X in the container is 2 kg, then the mass of gas-Y in the container is _________ kg. (Rounded off to one decimal place) Assume that the universal gas constant is 8314 J $kmol^{-1}K^{-1}$.

Ans. **(4) (3.9 to 4.1)**

We are given with two gases:

$$P_t = 100 \text{ kPa}$$
$$V = 10 \text{ m}^3$$
$$T = 300 \text{ K}$$

X	Y
M_x : 10 kg/k-mol	
M_y : 20 kg/k-mol	

Using the equation, $PV = n\bar{R}T$

$\Rightarrow$ $100 \times 10 = n \times 8.314 \times 300$

$\Rightarrow$ $n = 0.4$

Now, $n = n_x + n_y$

$$n = \frac{m_x}{M_x} + \frac{m_y}{M_y}$$

$\Rightarrow$ $0.4 = \dfrac{2}{10} + \dfrac{m_y}{20}$

$\Rightarrow$ $m_y = 4$ kg

End of Solution

Q.62 The velocity field of a certain two-dimensional flow is given by

$$V(x, y) = k(x\hat{i} - y\hat{j})$$

where $k = 2$ s^{-1}. The coordinates x and y are in meters. Assume gravitational effects to be negligible.

If the density of the fluid is 1000 kg/m^3 and the pressure at the origin is 100 kPa, the pressure at the location (2 m, 2 m) is ________ kPa.

(Answer in integer)

Ans. **(84) (83.999 to 84.001)**

Given $v(x, y) = k(x\hat{i} - y\hat{j})$

$$v_x = kx = u$$
$$v_y = -ky = v$$

Using N-S equation in x-direction

$$-\frac{\partial P}{\partial x} + \mu\frac{\partial^2 u}{\partial x^2} = \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)$$

$$-\frac{\partial P}{\partial x} = \rho(kx \times k + (-ky)0)$$

$$-\frac{\partial P}{\partial x} = k^2\rho x$$

$$P(x) = \frac{-k^2\rho x^2}{2} + C_1 \qquad \text{...(i)}$$

Using N-S equation in y-direction

$$-\frac{\partial P}{\partial y}+\mu\frac{\partial^2 u}{\partial y^2} = \rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)$$

$$-\frac{\partial P}{\partial y} = \rho\left(kx\times 0+(-ky)(-k)\right)$$

$$-\frac{\partial P}{\partial y} = k^2\rho y$$

$$P(y) = \frac{-k^2\rho y^2}{2}+C_2 \qquad \text{...(ii)}$$

From equation (i) and (ii), we get

$$\text{P(x, y)} = -\frac{k^2\rho x^2}{2}-\frac{k^2\rho y^2}{2}+c$$

Now, it is given that

$$\text{P(0, 0)} = 100 \text{ kPa}$$

$$100 = 0 + c$$

$$c = 100$$

then,

$$\text{P(x, y)} = \frac{-\rho k^2}{2}\left(x^2+y^2\right)+100$$

$$\text{P(2, 2)} = \frac{-4}{2}\left(2^2+2^2\right)+100$$

$$\text{P(2, 2)} = -16 + 100 = 84 \text{ kPa}$$

End of Solution

Q.63 Consider a unidirectional fluid flow with the velocity field given by

$$V(x,y,z,t) = u(x,t)\hat{i}$$

where $u(0, t) = 1$. If the spatially homogeneous density field varies with time t as

$\rho(t) = 1 + 0.2e^{-t}$

the value of $u(2,1)$ is. (Rounded off to two decimal places)

Assume all quantities to be dimensionless.

Ans. **(1.137) (1.13 to 1.15)**

Given: $u(0, t) = 1$

$\rho(t) = 1 + 0.2e^{-t}$

Using continuity equation

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\cancelto{0}{\frac{\partial}{\partial y}(\rho v)}+\cancelto{0}{\frac{\partial}{\partial z}(\rho\omega)}=0$$

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho u) = 0 \qquad \text{...(i)}$$

$$\frac{\partial \rho}{\partial t} = \frac{\partial\left(1+0.2e^{-t}\right)}{\partial t} = -0.2e^{-t}$$

$$\frac{\partial}{\partial x}(\rho u) = \frac{\partial}{\partial x}((1+0.2e^{-t})u(x,t))$$

$$= \left(1+0.2e^{-t}\right)\cdot\frac{\partial u}{\partial x}$$

Now, put in equation (i)

$$-0.2e^{-t} + \left(1+0.2e^{-t}\right)\cdot\frac{\partial u}{\partial x} = 0$$

$$\frac{\partial u}{\partial x} = \frac{0.2e^{-t}}{1+0.2e^{-t}}$$

$$u = \left(\frac{0.2e^{-t}}{1+0.2e^{-t}}\right)\cdot x + c \qquad \text{...(ii)}$$

Using boundary condition,

$$u(0,\ t) = 1$$

$$c = 1$$

From equation (ii),

$$u = \frac{0.2e^{-t}}{1+0.2e^{-t}}x + 1$$

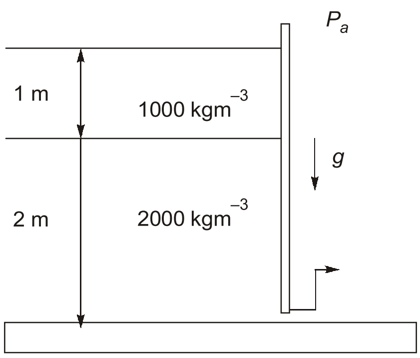
$$u(2,\ 1) = \frac{0.2e^{-1}}{1+0.2e^{-1}}\times 2 + 1$$

$$u(2,\ 1) = 1.137\ \text{m/s}$$

End of Solution

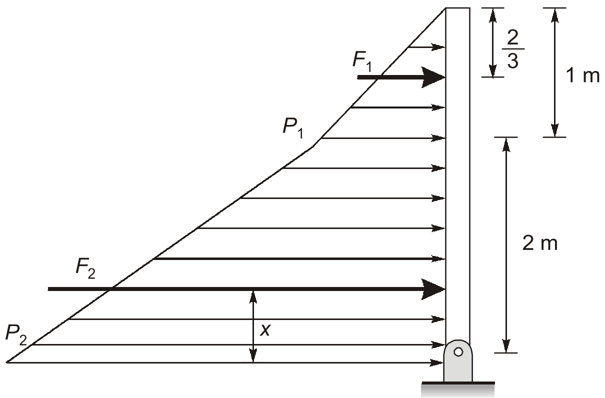
Q.64 The figure shows two fluids held by a hinged gate. The atmospheric pressure is Pa = 100 kPa. The moment per unit width about the base of the hinge is ________ kNm/m. (Rounded off to one decimal place)

Take the acceleration due to gravity to be g = 9.8 m/s^2.



Ans. **(57.225) (57.1 to 57.3)**

Making the pressure prism for the system.



$$P_1 = 10^3 \times 9.81 \times 1$$
$$P_1 = 9.81 \text{ kPa}$$
$$P_2 = P_1 + [2000 \times 9.81 \times 2]$$
$$P_2 = 49.05 \text{ kPa}$$

The resultant forces F_1 and F_2 due to pressure P_1 and P_2 respectively are given as:

F_1 = Volume of triangular pressure prism

$$= \frac{1}{2} \times P_1 \times 1 \times 1 = 4.9 \text{kN}$$

F_2 = Volume of trapezoidal pressure prism

$$= \frac{1}{2} \times [P_1 + P_2] \times 2 \times 1 = 58.86 \text{ kN}$$

Now, the moment due to forces is given as

$$M = F_1 \times \left[2 + \frac{1}{3}\right] + F_2 \times \left[\frac{2P_1 + P_2}{P_1 + P_2}\right] \times \frac{2}{3}$$

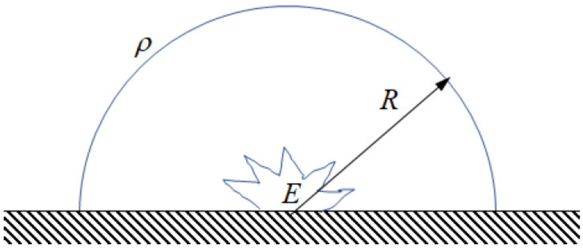
Putting the values, we get

$$M = 57.225 \text{ kNm}$$

End of Solution

Q.65 An explosion at time $t = 0$ releases energy E at the origin in a space filled with a gas of density ρ. Subsequently, a hemispherical blast wave propagates radially outwards as shown in the figure.

Let R denote the radius of the front of the hemispherical blast wave. The radius R follows the relationship $R = k t^a E^b \rho^c$, where k is a dimensionless constant. The value of exponent a is__________. (Rounded off to one decimal place)



Ans. (0.4) (0.39 to 0.41)

$$R = k\ t^a\ E^b\ \rho^c$$

$$M^0L^1T^0 = (M^0L^0T^0)\,(T)^a(ML^2T^{-2})^b(ML^{-3})^c$$

Comparing powers of M, L, T

$$b + c = 0$$

$$\Rightarrow \qquad b = -c$$

$$2b - 3c = 1$$

$$-2c - 3c = 1$$

$$c = -\frac{1}{5}$$

$$\Rightarrow \qquad b = \frac{1}{5}$$

$$a - 2b = 0$$

$$a = 2\times\frac{1}{5} = 0.4$$

End of Solution

■■■■