Chapter 8
Indices
Exercise 8

1.

(i) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

Solution:

$\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

$= \left[\left(\frac{3^4}{2^4}\right)\right]^{-\frac{3}{4}}$

$= \left[\left(\frac{3}{2}\right)^4\right]^{-\frac{3}{4}}$

$= \left(\frac{3}{2}\right)^{-\frac{3}{4}\times 4}$

$= \left(\frac{3}{2}\right)^{-3}$

$= \left(\frac{2}{3}\right)^{3}$

$= \frac{2^3}{3^3}$

$= \frac{2\times 2\times 2}{3\times 3\times 3}$

$= \frac{8}{27}$

(ii) $\left(1\frac{61}{64}\right)^{-\frac{2}{3}}$

Solution:

$\left(1\frac{61}{64}\right)^{-\frac{2}{3}}$

$= \left(\frac{125}{64}\right)^{-\frac{2}{3}} = \left(\frac{5^3}{4^3}\right)^{-\frac{2}{3}} = \left[\left(\frac{5}{4}\right)^3\right]^{-\frac{2}{3}}$

$= \left(\frac{5}{4}\right)^{3\times-\frac{2}{3}}$

$= \left(\frac{5}{4}\right)^{-2}$

$= \left(\frac{4}{5}\right)^{2}$

$= \frac{16}{25}$

2. (i) $(2a^{-3}b^2)^3$

Solution:

$(2a^{-3}b^2)^3$

$= 2^3\ a^{-3\times3}\ b^{2\times3}$

$= 8a^{-1}b^6$

(ii) $\dfrac{\mathbf{a^{-1} + b^{-1}}}{\mathbf{ab^{-1}}}$

Solution:

$$= \frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{ab}} = \frac{\frac{b+a}{ab}}{\frac{1}{ab}} = \frac{b+a}{ab} \times \frac{ab}{1} = a + b$$

3. (i) $\dfrac{\mathbf{x^{-1}\, y^{-1}}}{\mathbf{x^{-1} + y^{-1}}}$

Solution:

$$= \frac{\frac{xy^{-1}}{1}}{x} + \frac{1}{y} = \frac{\frac{1}{xy}}{\frac{y+x}{xy}}$$

$$= \frac{1}{x+y}$$

(ii) $\dfrac{\mathbf{(4 \times 10^{7})(6 \times 10^{-5})}}{\mathbf{8 \times 10^{10}}}$

Solution:

$$= \frac{4 \times 10^{7} \times 6 \times 10^{-5}}{8 \times 10^{10}}$$

$$= \frac{3 \times 10^{7-5}}{10^{10}}$$

$$= \frac{3 \times 10^{2}}{10^{10}}$$

$= \frac{3 \times 10^2}{10^{10}}$

$= \frac{3}{10^{10-2}}$

$= \frac{3}{10^8}$

4. (i) $\frac{\mathbf{3a}}{\mathbf{b^{-1}}} + \frac{\mathbf{2b}}{\mathbf{a^{-1}}}$

Solution:

$= \frac{3a}{b^{-1}} + \frac{2b}{a^{-1}}$

$= \frac{3a}{\frac{1}{b}} + \frac{2b}{\frac{1}{a}}$

$= \frac{3a \times b}{1} + \frac{2b \times a}{1}$

= 3ab + 2ab = 5ab

(ii) $\mathbf{5^0 \times 4^{-1} + 8^{\frac{1}{3}}}$

Solution:

$5^0 \times 4^{-1} + 8^{\frac{1}{3}}$

$= 1 \times \left(\frac{1}{4}\right) + (2)^{3 \times \frac{1}{3}}$

$= \frac{1}{4} + 2$

$= \frac{1+8}{4}$

$= \frac{9}{4} = 2\frac{1}{4}$

5. (i) $\left(\frac{8}{125}\right)^{-\frac{1}{3}}$

Solution:

$$\left(\frac{8}{125}\right)^{-\frac{1}{3}}$$

$$= \left[\frac{2 \times 2 \times 2}{5 \times 5 \times 5}\right]^{-\frac{1}{3}}$$

$$= \left(\frac{2^3}{5^3}\right)^{-\frac{1}{3}}$$

$$= \left(\frac{2}{5}\right)^{3 \times -\frac{1}{3}}$$

$$= \left(\frac{2}{5}\right)^{-1}$$

$$= \frac{5}{2} = 2\frac{1}{2}$$

(ii) $(0.027)^{-\frac{1}{3}}$

Solution:

$$(0.027)^{-\frac{1}{3}}$$

$$= \left(\frac{27}{1000}\right)^{-\frac{1}{3}}$$

$$= \left[\frac{3 \times 3 \times 3}{10 \times 10 \times 10}\right]^{-\frac{1}{3}}$$

$= \left(\frac{3^3}{10^3}\right)^{-\frac{1}{3}}$

$= \left(\frac{3}{10}\right)^{3\times -\frac{1}{3}}$

$= \left(\frac{3}{10}\right)^{-1}$

$= \frac{10}{3}$

6. (i) $\left(-\frac{1}{27}\right)^{-\frac{2}{3}}$

Solution:

$\left(-\frac{1}{27}\right)^{-\frac{2}{3}}$

$= \left(-\frac{1}{3^3}\right)^{-\frac{2}{3}}$

$= \left(-\frac{1}{3}\right)^{3\times -\frac{2}{3}}$

$= \left(-\frac{1}{3}\right)^{-2}$

$= (-3)^2$

$= 9$

(ii) $(64)^{-\frac{2}{3}} \div 9^{-\frac{3}{2}}$

Solution:

$(4^3)^{-\frac{2}{3}} \div (9^2)^{-\frac{3}{2}}$

$$= \quad 4^{3\times -\frac{2}{3}} \div 3^{7\times -\frac{3}{2}}$$

$$= 4^{-2} \div 3^{-3}$$

$$= \frac{4^{-2}}{3^{-3}}$$

$$= \frac{\frac{1}{4^2}}{\frac{1}{3^3}}$$

$$= \frac{3^3}{4^2}$$

$$= \frac{27}{16}$$

7.

Solution

(i) $\frac{(27)^{\frac{2n}{3}} \times 8^{-\frac{n}{6}}}{(18)^{-\frac{n}{2}}}$

$$= \frac{(3^3)^{\frac{2n}{9}} \times (2^3)^{-\frac{n}{6}}}{2\times 9^{-\frac{n}{2}}}$$

$$= \frac{3^{3\times\frac{2n}{3}} \times 2^{3\times -\frac{n}{6}}}{2^{-\frac{n}{2}} \times 3^{2\times -\frac{n}{2}}}$$

$$= \frac{3^{2n} \times \ 2^{-\frac{n}{2}}}{2^{-\frac{n}{2}} \times 3^{-n}}$$

$$= \frac{3^{\ 2n}}{2^{-\frac{n}{2}}} \times \frac{1}{2^{\frac{n}{2}}} \ \times \ 3^{n}$$

$$= \ \frac{3^{2n+n}}{2^{-\frac{n}{2}}.\ 2^{\frac{n}{2}}}$$

$$= \frac{3^{3n}}{2^{-\frac{n}{2}}.2^{\frac{n}{2}}}$$

$$= \ \frac{3^{3n} \times 2^{\frac{n}{2}}}{2^{\frac{n}{2}}}$$

$$= 3^{3n}$$

(ii) $\frac{5.(25)^{n+1} - 25.(5)^{2n}}{5.(5)^{2n+3} - (25)^{n+1}}$

$$= \ \frac{5^{1}.(5^{2})^{n+1} - 5^{2}.\ 5^{2n}}{5^{1}\ .5^{2n+3} - (5^{2})^{n+1}}$$

$$= \frac{5^{1+2n+2} - \ 5^{2+2n}}{5^{1+2n+3} \ - 5^{2n+2}}$$

$$= \ \frac{5^{2n+3} - 5^{2+2n}}{5^{2n+4} - 5^{2+2n}}$$

$= \frac{5^{2n+3} - 5^{2+2n}}{5^{2n+4} - 5^{2+2n}}$

$= \frac{5^{2n}.5^{3} - 5^{2}.5^{2n}}{5^{2n}\ .5^{4} - 5^{2}.5^{2n}}$

$= \frac{5^{2n}[5^{3} - 5^{2}]}{5^{2n}[5^{4} - 5^{2}]}$

$= \frac{125 - 25}{625 - 25}$

$= \frac{100}{600}$

$= \frac{1}{6}$

8.

Solution

(i) $(8^{-\frac{4}{3}} \ \div \ 2^{-2})^{\frac{1}{2}}$

$= ((2^{3})^{-\frac{4}{3}} \ \div \ 2^{-2})^{\frac{1}{2}}$

$= \left(\frac{2^{-4}}{2^{-2}}\right)^{\frac{1}{2}}$

$$= (2^{-4+2})^{\frac{1}{2}}$$

$$= (2^{-2})^{\frac{1}{2}}$$

$$= 2^{-1}$$

$$= \frac{1}{2}$$

(ii) $\left(\frac{27}{8}\right)^{\frac{2}{3}} - \left(\frac{1}{4}\right)^{-2} + 5^{0}$

$$= \left(\frac{3^3}{2^3}\right)^{\frac{2}{3}} - \left(\frac{1}{2^2}\right)^{-2} + 1$$

$$= \left(\frac{3}{2}\right)^{3 \times \frac{2}{3}} - \frac{1}{2^{2x-2}} + 1$$

$$= \left(\frac{3}{2}\right)^{2} - \frac{1}{2^{-4}} + 1$$

$$= \frac{9}{4} - 2^4 + 1$$

$$= \frac{9}{4} - 16 + 1$$

$= \frac{9}{4} - \ 15$

$= \frac{9-60}{4}$

$= -\frac{51}{4}$

9.

Solution

(i) $(3x^2)^{-3} \times (x^9)^{\frac{2}{3}}$

$= \frac{1}{(3x^2)^3} \times x^{9\times\frac{2}{3}}$

$= \frac{1}{3^3 . x^{2\times3}} \times x^{3\times2}$

$= \frac{1}{27. \ x^6} \times x^6$

$= \frac{1}{27}$

(ii) $(8x^4)^{\frac{1}{3}} \div x^{\frac{1}{3}}$

$= (2^3 . x^4)^{\frac{1}{3}} \div x^{\frac{1}{3}}$

$$= \frac{2^{3 \cdot \frac{1}{3}} . \ x^{4 \cdot \frac{1}{3}}}{x^{\frac{1}{3}}}$$

$$= \frac{2 . x^{\frac{4}{3}}}{x^{\frac{1}{3}}}$$

$$= \frac{2. \ x^{\frac{4}{3}}}{x^{\frac{1}{3}}}$$

$$= 2.x^{4 \cdot \frac{1}{3}} - \ x^{\frac{1}{3}}$$

$$= x^{\frac{1}{3}}[\ 2x^4 - \ 1]$$

10.

Solution

(i) $(3^2)^0 + \ 3^{-4} \times 3^6 + \left(\frac{1}{3}\right)^{-2}$

$= 3^0 + 3^{-4+6} \ + \frac{1}{3^{-2}}$

$=1 + 3^2 + 3^2$

$= 1 + 9 + 9$

$= 19$

(ii) $9^{\frac{5}{2}} - 3.(5)^0 - \left(\frac{1}{81}\right)^{-\frac{1}{2}}$

$= 9^{\frac{5}{2}}$ - 3(1) - $\left(\frac{1}{9^2}\right)^{-\frac{1}{2}}$

$= 9^{\frac{5}{2}}$ - 3 - $\frac{1}{9^{2\times\frac{1}{2}}}$

$= 3^{2\times\frac{5}{2}}$ - 3 - $\frac{1}{9^{-1}}$

$= 3^5 - 3 - 9$

$= 3^5 - 3 - 3^2$

$= 3(3^4 - 1 - 3)$

$= 3(81 - 1 - 3)$

$= 3(77)$

$= 231$

11.

Solution

(i) $16^{\frac{3}{4}} + 2\left(\frac{1}{2}\right)^{-1} .3^0$

$= (2^4)^{\frac{3}{4}} + 2\left(\frac{1}{2^{-1}}\right) .1$

$= 2^{4\times\frac{3}{4}} + 2.2$

$= 2^3 + 4$

$= 8 + 4$

$= 12$

(ii) $(81)^{\frac{3}{4}} - \left(\frac{1}{32}\right)^{-\frac{2}{5}} + (8)^{\frac{1}{3}}\left(\frac{1}{2}\right)^{-1} . (2)^0$

$= (3^4)^{\frac{3}{4}} - \left(\frac{1}{2^5}\right)^{-\frac{2}{5}} + 2^{2\times\frac{1}{3}}\left(\frac{1}{2^{-1}}\right)^1$

$= 3^3 - \frac{1}{2^{5\times-\frac{2}{5}}} + 2\,(2)^1$

$= 3^3 - \frac{1}{2^{-2}} + 2(2)$

$= 27 - 2^2 + 4$

$= 27 - 4 + 4$

$= 27$

12

Solution

(i) $\left(\frac{64}{125}\right)^{-\frac{2}{3}} \div \frac{1}{\left(\frac{256}{625}\right)^{\frac{1}{4}}} + \left(\frac{\sqrt{25}}{\sqrt[3]{64}}\right)^0$

$= \left(\frac{4^3}{5^3}\right)^{-\frac{2}{3}} \div \frac{1}{\left(\frac{4^4}{5^4}\right)^{\frac{1}{4}}} + 1$

$= \left(\frac{4}{5}\right)^{-2} \div \frac{1}{\left(\frac{4}{5}\right)} + 1$

$$= \frac{\left(\frac{5}{4}\right)^2}{\left(\frac{4}{5}\right)} + 1$$

$$= \left(\frac{5}{4}\right)^2 \times \left(\frac{4}{5}\right) + 1$$

$$= \frac{5}{4} + 1$$

$$= \frac{9}{4}$$

(ii) $\frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 2^2 \times 5^n}$

$$= \frac{5^n.5^3 - 6 \times 5^n.5}{9 \times 5^n - 2^2 \times 5^n}$$

$$= \frac{5^n[5^3 - 6 \times 5]}{5^n[9-4]}$$

$$= \frac{125-30}{5}$$

$$= \frac{95}{5}$$

$$=19$$

13.

Solution

(i) $[(64)^{\frac{2}{3}}\ .2^{-2}\ \div 8^{0}]^{-\frac{1}{2}}$

$= \left((4^{3})^{\frac{2}{3}} . \frac{1}{2^{2}} \div 1\right)^{-\frac{1}{2}}$

$= \left(\frac{4^{2}}{2^{2}}\right)^{-\frac{1}{2}}$

$= \left(\frac{4}{2}\right)^{2 \times -\frac{1}{2}}$

$= 2^{-1}$

$= \frac{1}{2}$

(ii) $3^{n} \times 9^{n+1} \div 3^{n-1} \times 9^{n-1}$

$= 3^{n} \times 3^{2(n+1)} \div 3^{n-1} \times 3^{2(n-1)}$

$= 3^{n} \times 3^{2n+2} \div 3^{n-1} \times 3^{2n-2}$

$= \frac{3^{n+2n+2}}{3^{n-1+2n-2}}$

$= \frac{3^{3n+2}}{3^{3n-3}}$

$$= \frac{3^{3n}.\ 3^{2}}{3^{3n}.\ 3^{-3}}$$

$= 3^2 \times 3^3$

$= 3^{2+3}$

$= 3^5$

$= 243$

14.

Solution

(i) $\frac{\sqrt{2^2} \times \sqrt[4]{256}}{\sqrt[3]{64}} - \left(\frac{1}{2}\right)^{-2}$

$$= \frac{(2^2)^{\frac{1}{2}} \times (4)^{4\times\frac{1}{4}}}{4^{3\times\frac{1}{3}}} - \frac{1}{2^{-2}}$$

$= \frac{2 \times 4}{4} - 2^2$

$= 2-4$

$= -2$

(ii) $\frac{3^{-\frac{6}{7}} \times 4^{-\frac{3}{7}} \times 9^{\frac{3}{7}} \times 2^{\frac{6}{7}}}{2^2 + 2^0 + 2^{-2}}$

$$= \frac{3^{-\frac{6}{7}} \times 3^{2 \times \frac{3}{7}} \times 2^{2 \times -\frac{3}{7}} \times 2^{\frac{6}{7}}}{4 + 1 + \frac{1}{2^2}}$$

$$= \frac{3^{-\frac{6}{7}} \times 3^{\frac{6}{7}} \times 2^{-\frac{6}{7}} \times 2^{\frac{6}{7}}}{4 + 1 + \frac{1}{4}}$$

$$= \frac{3^{-\frac{6}{7}+\frac{6}{7}} \times 2^{-\frac{6}{7}+\frac{6}{7}}}{\frac{16+4+1}{4}}$$

$$= \frac{3^0 \times 2^0}{\left(\frac{21}{4}\right)}$$

$$= \frac{1}{\frac{21}{4}}$$

$$= \frac{4}{21}$$

15.

Solution

(i) $\dfrac{(32)^{\frac{2}{5}} \times (4)^{-\frac{2}{2}} \times (8)^{\frac{1}{3}}}{2^{-2} \div (64)^{-\frac{1}{3}}}$

$$= \frac{(2^5)^{-\frac{2}{5}} \times (2^2)^{-\frac{1}{2}} \times \left(2^3\right)^{\frac{1}{3}}}{\frac{1}{2^2} \div 4^{3\times-\frac{1}{3}}}$$

$$= \frac{2^{-2} \times 2^{-1} \times 2^1}{\frac{1}{2^2} \div 4^{-1}}$$

$$= \frac{2^{-1-2+1}}{\frac{\frac{1}{2^2}}{\frac{1}{2^2}}}$$

$$= 2^{-2}$$

$$= \frac{1}{2^2}$$

$$= \frac{1}{4}$$

(ii) $\dfrac{5^{2(x+6)} \times 25^{-7+2x}}{125^{2x}}$

$$= \frac{5^{2x+12} \times 5^{2(-7+2x)}}{(5^3)^{2x}}$$

$$= \frac{5^{2x+12-14+14x}}{5^{6x}}$$

$$= \frac{5^{6x-2}}{5^{6x}}$$

$= 5^{-2}$

$$= \frac{1}{5^2}$$

$$= \frac{1}{25}$$

16.

Solution

(i) $\dfrac{7^{2n+3} - 49^{n+2}}{(343^{n+1})^{\frac{2}{3}}}$

$$= \frac{7^{2n+3} - 7^{2(n+2)}}{\left(7^{3(n+1)}\right)^{\frac{2}{3}}}$$

$$= \frac{7^{2n+3} - 7^{2n+4}}{7^{2(n+1)}}$$

$$= \frac{7^{2n+3} - 7^{2n+4}}{7^{2n} . 7^2}$$

$= \frac{7^{2n}[7^3-7^4]}{7^{2n}.7^2}$

$= \frac{343-2401}{49}$

$= -\frac{2058}{49}$

$=$ - 42

(ii) $(27)^{\frac{4}{3}} + (32)^{0.8} + (0.8)^{-1}$

$= 3^{3\times\frac{4}{3}} + 2^{5\times\frac{8}{10}} + \left(\frac{8}{10}\right)^{-1}$

$= 3^4 + 2^{5\times\frac{4}{5}} + \left(\frac{4}{5}\right)^{-1}$

$= 3^4 + 2^4 + \frac{5}{4}$

$= 81 + 16 + \frac{5}{4}$

$= 97 + \frac{5}{4}$

$= \frac{388+5}{4}$

$= \frac{393}{4}$

17.

Solution

(i) $\left(\sqrt{32}-\sqrt{5}\right)^{\frac{1}{3}}.\left(\sqrt{32}+\sqrt{5}\right)^{\frac{1}{3}}$

$= \left[\left(\sqrt{2^5}-\sqrt{5}\right).\left(\sqrt{25}+\sqrt{5}\right)\right]^{\frac{1}{3}}$

$= \left[\left(\sqrt{2^5}\right)^2 - \left(\sqrt{5}\right)^2\right]^{\frac{1}{3}}$

$= (2^5 - 5)^{\frac{1}{3}}$

$= (32-5)^{\frac{1}{3}}$

$= (27)^{\frac{1}{3}}$

$= (3^3)^{\frac{1}{3}}$

$= 3$

(ii) $\left(x^{\frac{1}{3}} - x^{-\frac{1}{3}}\right)\left(x^{\frac{2}{3}} + 1 + x^{-\frac{2}{3}}\right)$

$= \left(x^{\frac{1}{3}} - \frac{1}{x^{\frac{1}{3}}}\right)\left(x^{\frac{2}{3}} + 1 + x^{\frac{2}{3}}\right)$

∴ it is in the form of

$(a - b)(a^2 + ab + b^2) = a^3 - b^3$

Here $a = x^{\frac{1}{3}}$; $b = \frac{1}{x^{\frac{1}{3}}}$

∴ $(x^{\frac{1}{3}})^3 - \left(\frac{1}{x^{\frac{1}{3}}}\right)^3$

$= x^{\frac{3}{5}} - \frac{1}{x^{\frac{3}{5}}}$

$= x - \frac{1}{x}$

18.

Solution

(i) $\left(\frac{x^m}{x^n}\right)^l . \left(\frac{x^n}{x^l}\right)^m . \left(\frac{x^l}{x^m}\right)^n$

$= (x^{m-n})^l . (x^{n-l})^m . (x^{l-m})^n$

$= x^{ml-nl} . x^{mn-ml} . x^{nl-nm}$

$= x^{ml-nl+ mn-ml+nl-nm}$

$= x^{0}$

$= 1$

(ii) $\left(\frac{x^{a+b}}{x^{c}}\right)^{a-b} . \left(\frac{x^{b+c}}{x^{a}}\right)^{b-c} . \left(\frac{x^{c+a}}{x^{b}}\right)^{c-a}$

$= \frac{x^{(a+b)(a-b)}}{x^{c(a-b)}} . \frac{x^{(b+c)(b-c)}}{x^{a(b-c)}} . \frac{x^{(c+a)(c-a)}}{x^{b(c-a)}}$

$= \frac{x^{a^2-b^2}}{x^{ac-bc}} . \frac{x^{b^2-c^2}}{x^{ab-ac}} . \frac{x^{c^2-a^2}}{x^{bc-ab}}$

$= \frac{x^{a^2-b^2+ b^2-c^2+c^2-a^2}}{x^{ac-bc+ab-ac+bc-ab}}$

$= \frac{x^{0}}{x^{0}}$

$=1$

19

Solution

(i) $\sqrt[lm]{\frac{x^l}{x^m}} \cdot \sqrt[mn]{\frac{x^m}{x^n}} \cdot \sqrt[nl]{\frac{x^n}{x^l}}$

$= \sqrt[lm]{x^{l-m}} \cdot \sqrt[mn]{x^{m-n}} \cdot \sqrt[nl]{x^{n-l}}$

$= (x^{l-m})^{\frac{1}{lm}} \cdot (x^{m-n})^{\frac{1}{mn}} \cdot (x^{n-l})^{\frac{l}{nl}}$

$= x^{\frac{l-m}{lm}} \cdot x^{\frac{m-n}{nm}} \cdot x^{\frac{n-l}{nl}}$

$= x^{\frac{n(l-m)+l(m-n)+m(n-l)}{lm.n}}$

$= x^{\frac{nl-nm+lm-ln+mn-ln}{lmn}}$

$= x^0$

$=1$

(ii) $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \cdot \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \cdot \left(\frac{x^c}{x^a}\right)^{c^2+ac+a^2}$

$= x^{(a-b)(a^2+ab+b^2)} \cdot x^{(b-c)(b^2+bc+c^2)} \cdot x^{(c-a)(c^2+ac+a^2)}$

$= x^{a^3-b^3} \cdot x^{b^3-c^3} \cdot x^{c^3-a^3}$

$= x^{a^3-b^3+b^3-c^3+c^3-a^3}$

$= x^0$

$= 1.$

(iii) $\left(\frac{x^a}{x^{-b}}\right)^{a^2-ab+b^2} \cdot \left(\frac{x^b}{x^{-c}}\right)^{b^2-bc+c^2} \cdot \left(\frac{x^c}{x^{-a}}\right)^{c^2-ac+a^2}$

$= x^{(a-c-b)(a^2-ab+b^2)} \cdot x^{(b-c-c)(b^2-bc+c^2)} \cdot x^{(c-c-a)(c^2-ac+a^2)}$

$= x^{a^3+b^3} \cdot x^{b^3+c^3} \cdot x^{c^3+a^3}$

$= x^{a^3+b^3+b^3+c^3+c^3+a^3}$

$= x^{2a^3+2b^3+2c^3}$

$= x^{2(a^3+b^3+c^3)}$

20.

Solution

(i) $(a^{-1} + b^{-1}) \div (a^{-2} - b^{-2})$

$$= \left(\frac{1}{a} + \frac{1}{b}\right) \div \left(\frac{1}{a^2} - \frac{1}{b^2}\right)$$

$$= \left(\frac{b+a}{ab}\right) \div \left(\frac{b^2-a^2}{a^2b^2}\right)$$

$$= \frac{\frac{b+a}{ab}}{\frac{(b^2-a^2)}{a^2b^2}}$$

$$= \frac{b+a}{ab} \times \frac{(ab)^2}{(b^2-a^2)}$$

$$= \frac{b+a}{ab} \cdot \frac{ab^2}{(b+a)(b-a)}$$

$$= \frac{ab}{b-a}$$

(ii) $\frac{1}{1+a^{m-n}} + \frac{1}{a^{n-m}+1}$

$= \frac{1}{1+a^{m-n}} + \frac{1}{1+a^{-(m-n)}}$

$= \frac{1}{1+a^{m-n}} + \frac{1}{1+\frac{1}{a^{(m-n)}}}$

$= \frac{1}{1+a^{m-n}} + \frac{1}{\frac{a^{m-n}+1}{a^{m-n}}}$

$= \frac{1}{1+a^{m-n}} + \frac{a^{m-n}}{a^{m-n}+1}$

$= \frac{1+a^{m-n}}{1+a^{m-n}}$

$=1$

21.

Solution

(i) $(a+b)^{-1}\ (a^{-1}+\ b^{-1}) = \frac{1}{ab}$

LHS $= (a+b)^{-1}\ (a^{-1}+\ b^{-1})$

$= \frac{1}{a+b}\left(\frac{1}{a}+\frac{1}{b}\right)$

$= \frac{1}{a+b}\ \left(\frac{b+a}{ab}\right)$

$$= \frac{1}{a+b}\left(\frac{a+b}{ab}\right)$$

$$= \frac{1}{ab}$$

= RHS

(ii) $\frac{x+y+z}{x^{-1}y^{-1}+y^{-1}z^{-1}+z^{-1}x^{-1}}$ = xyz

LHS = $\frac{x+y+z}{x^{-1}y^{-1}+y^{-1}z^{-1}+z^{-1}x^{-1}}$

$$= \frac{x+y+z}{\frac{1}{xy}+\frac{1}{yz}+\frac{1}{xz}}$$

$$= \frac{x+y+z}{\frac{z+x+y}{xyz}}$$

$$= \frac{x+y+z}{\frac{x+y+z}{xyz}}$$

= xyz

= RHS

22

Solution

Given $a = c^z$; $b = a^x$; $c = b^y$

$= a = c^z$

$a = (b^y)^z \quad (\because c = b^y)$

$a = b^{yz}$

$a = (a^x)^{yz} \quad (\therefore b = a^x)$

$a^1 = a^{xyz}$

∴ bases are equal so exponents are also equal

∴ $xyz = 1$

Hence proved

23.

Solution

Given $a = xy^{p-1}$; $b = xy^{q-1}$; $c = xy^{r-1}$

LHS = $a^{q.r} . b^{r-p} . c^{p-q}$

$= (xy^{p-1})^{q-r} .(xy)^{q-1(r-p)} . (xy^{r-1})^{(p-q)}$

$= xy^{(p-1)(q-r)} \; .xy^{(q-1)(r-p)} \; . \; xy^{(r-1)(p-q)}$

$= xy^{pq-pr-q+r} \; .xy^{qr-qp-r+p} \; . \; xy^{rp-rq-p+q}$

$= xy^{pq-pr-q+r+qr-qp-r+p+rp-rq-p+q}$

$= xy^0$

=1

=RHS

Hence proved

24.

Solution

Given $2^x = 3^y = 6^{-z}$

Let $2^x = 3^y = 6^{-z} = k$

$= 2 = k^{\frac{1}{x}}$

$3 = k^{\frac{1}{y}}$

$6 = k^{-\frac{1}{z}}$

$\frac{1}{6} = k^{\frac{1}{z}}$

$= \frac{1}{2 \times 3} = k^{\frac{1}{z}}$

$= \frac{1}{k^{\frac{1}{x}} . k^{\frac{1}{y}}} = k^{\frac{1}{z}}$

$1 = k^{\frac{1}{z}} . k^{\frac{1}{x} . \frac{1}{y}}$

$k = k^{\frac{1}{x} + \frac{1}{y} + \frac{1}{z}}$

$\therefore \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

25.

Solution

Given $2^x = 3^y = 12^z$

Let $2^x = 3^y = 12^z = k$

$= \quad 2 = k^{\frac{1}{x}}$

$= \ 3 = k^{\frac{1}{y}}$

$= 12 = k^{\frac{1}{z}}$

$= 2^2 . 3 = k^{\frac{1}{z}}$

$= \left(k^{\frac{1}{x}}\right)^2 . k^{\frac{1}{y}} = k^{\frac{1}{z}}$

$=k^{\frac{2}{x}} . k^{\frac{1}{4}} = k^{\frac{1}{z}}$

$= \frac{2}{x} + \frac{1}{y} = \frac{1}{z}$

$= \frac{2y+x}{xy} = \frac{1}{z}$

Or $\frac{2}{x} = \frac{1}{z} - \frac{1}{y}$

$\frac{2}{x} = \frac{y-z}{yz}$

$x = \frac{2yz}{y-z}$

Hence proved

26.

Solution

(i) $(3x^2)^0$

$= 1$

(ii) $(xy)^{-2}$

$= \frac{1}{(xy)^2}$

$= \frac{1}{x^2 y^2}$

(iii) $(-27a^{a})^{\frac{2}{3}}$

$= (3^3a^a)^{\frac{2}{3}}$

$= (3a^3)^{3\times\frac{2}{3}}$

$= (3a^3)^2$

$= 3^2a^{3\times2}$

$= 9a^6$

27

Solution

Given a =3 ; b = -2

(i) $a^a + b^b$

$= 3^3 +(-2)^{-2}$

$= 3^3 + \frac{1}{(-2)^2}$

$= 27 + \frac{1}{4}$

$= \frac{108+1}{4}$

$= \frac{109}{4}$

(ii) $a^b + b^a$

$= 3^{-2} + (-2)^3$

$= \frac{1}{3^2} - 8$

$= \frac{1}{9} - 8$

$= \frac{1-72}{9}$

$= -\frac{71}{9}$

28.

Solution

Given $x = 10^3 \times 0.99$; $y = 10^{-2} \times 110$

$= \sqrt{\frac{x}{y}} = \sqrt{10^3 \times \frac{0.0099}{10^{-2} \times 110}}$

$= \sqrt{10^{3+2} \times \frac{0.0099}{110}}$

$= \sqrt{10^5 \times \frac{0.0099}{110}}$

$= \sqrt{\frac{990}{110}}$

$= \sqrt{9}$

= 3

29.

Solution

Given x =9 , y = 2 , z = 8

$x^{\frac{1}{2}}.y^{-1}.z^{\frac{2}{3}}$

$= 9^{\frac{1}{2}}.2^{-1}.8^{\frac{2}{3}}$

$= (3^2)^{\frac{1}{2}}.\left(\frac{1}{2}\right)\ (2^3)^{\frac{2}{3}}$

$= 3\,\frac{1}{2}\,.\,2^2$

$= 3\,.\frac{1}{2}\,.\,4^2$

= 6

30

Solution

Given $x^4y^2z^3 = 49392$

2	49392
2	24696
2	12348
2	6174
2	3087
3	1029
7	343
7	49
	7

$x^4y^2z^3 = 2^4 . 3^2 . 7^4$

$\therefore$ x, y, z are different primes

$\therefore x = 2$; $y = 3$; $z = 7$

31.

Solution

Given $\sqrt[3]{a^6b^{-4}} = a^x . b^2y$

$(a^6b^{-4})^{\frac{1}{3}} = a^x\ b^{2y}$

$a^{\frac{6}{3}} \cdot b^{-\frac{4}{3}} = a^x b^{2y}$

$\therefore x = \frac{6}{3}$ $\qquad$ $2y = -\frac{4}{3}$

$x = 2$ $\qquad$ $y = -\frac{4}{3\times 2}$

$y = -\frac{2}{3}$

32.

Solution

Given $(p+q)^{-1}\ (p^{-1}+q^{-1}) = p^a \cdot q^b$

$\frac{1}{p+q}\ \left(\frac{1}{p}+\frac{1}{q}\right) = p^q \cdot q^b$

$\frac{1}{p+q}\ \left(\frac{q+p}{qp}\right) = p^a . q^b$

$\frac{1}{qp} = p^a q^b$

$(qp)^{-1} = p^a \cdot q^b$

$p^{-1} . q^{-1} = p^a \cdot q^b$

$a = -1$

$b = -1$

LHS = a + b + 2

= -1 + 2 -1

= 0

= RHS

33.

Solution

Given $\left(\frac{p^{-1}q^{2}}{p^{2}q^{-4}}\right)^{7} \div \left(\frac{p^{3}q^{-5}}{p^{-2}q^{3}}\right)^{-5} = p^{x}q^{y}$

$= \left(\frac{p^{-7}.\ q^{2\times7}}{p^{2\times7}q^{-4\times7}}\right) \div \frac{p^{3\times5}.\ q^{-5\times-5}}{p^{-2\times-5}q^{3\times-5}} = p^{x}q^{y}$

$= \left(\frac{p^{-7}q^{14}}{p^{14}.q^{-28}}\right) \div \left(\frac{p^{15}.q^{25}}{p^{10}q^{-15}}\right) = p^{x}.q^{y}$

$= (p^{-7-14}.q^{14+28}) \div (p^{15-10}.q^{25+15}) = p^{x}.q^{y}$

$=(p^{-21}.q^{42}) \div (p^{5}.q^{40}) = p^{x}.q^{y}$

$= \frac{p^{-21}.\ q^{42}}{p^{5}.\ q^{40}} = p^{x}.q^{y}$

$= (p^{-21-5}.q^{42-20}) = p^{x}q^{y}$

$= (p^{-26}.q^{2}) = p^{x}.q^{y}$

$= \therefore x = -26 \quad ; \quad y = 2$

$\therefore x + y = -26 + 2$

$= -24$

34.

Solution

(i) $5^{2x+3} = 1$

$5^{2x+3} = 5^0 \quad (\therefore\ 5^0 = 10)$

$\therefore 2x + 3 = 0$

$2x = -3$

$X = -\frac{3}{2}$

(ii) $(13)^{\sqrt{x}} = 4^4 - 3^4 - 6$

$(13)^{\sqrt{x}} = 256 - 81 - 6$

$(13)^{\sqrt{x}} = 169$

$(13)^{\sqrt{x}} = (13)^2$

$\sqrt{x} = 2$

$x = 2^{\frac{1}{2}}$

(iii) $\left(\sqrt{\frac{3}{5}}\right)^{x+1} = \frac{125}{27}$

$\left(\frac{3}{5}\right)^{\frac{x+1}{2}} = \frac{5^3}{3^3}$

$\left(\frac{3}{5}\right)^{\frac{x+1}{2}} = \left(\frac{3}{5}\right)^{-3}$

$\frac{x+1}{2}$ = -3

x + 1 = -6

x = -6 – 1

x = -7

(iv) $\left(\sqrt[3]{4}\right)^{2x+\frac{1}{2}} = \frac{1}{32}$

$\left[(2^2)^{\frac{1}{3}}\right]^{4x+\frac{1}{2}} = \frac{1}{32}$

$\left(2^{\frac{2}{3}}\right)^{4x+\frac{1}{2}} = \frac{1}{2^5}$

$2\ \frac{4x+1}{3} = 2^{-5}$

$= \frac{4x+1}{3}$ = -5

4x + 1 = -15

4x = -15 -1

4x = -16

x = $-\frac{16}{4}$

x = - 4

35.

Solution

(i) $\sqrt{\frac{p}{q}} = \left(\frac{q}{p}\right)^{1-2x}$

$\left(\frac{p}{q}\right)^{\frac{1}{2}} = \frac{p}{q}^{-(1-2x)}$

$\left(\frac{p}{q}\right)^{\frac{1}{2}} = \left(\frac{p}{q}\right)^{-1+2x}$

$\frac{1}{2} = -1 + 2x$

-1 +2x = $\frac{1}{2}$

$2x = \frac{1}{2} + 1$

$2x = \frac{1+2}{2}$

$2x = \frac{3}{2}$

$x = \frac{3}{2 \times 2}$

$x = \frac{3}{4}$

(ii) $4^{x-1} \times (0.5)^{3-2x} = \left(\frac{1}{8}\right)^{x}$

$2^{2(x-1)} \times \left(\frac{1}{2}\right)^{3-2x} = \left(\frac{1}{2^3}\right)^{x}$

$2^{2x-2} \times \frac{1}{2^{2x-3}} = \frac{1}{2^{3x}}$

$2^{2x-2} \times 2^{2x-3} = 2^{-3x}$

2x-2 + 2x -3 = -3x

4x – 5 = -3x

4x + 3x = 5

7x = 5

$x = \frac{5}{7}$

36.

Solution

Given $5^{3x} = 125$

$10^{y} = 0.001$

= $5^{3x} = 125$

$5^{3x} = 5^{3}$

3x = 3

$$X = \frac{3}{3}$$

$$X = 1$$

$$10^{y} = 0.001$$

$$10^{y} = \left(\frac{10}{1000}\right)$$

$$10^{y} = \left(\frac{1}{10^{3}}\right)$$

$$10^{y} = 10^{-3}$$

$$Y = -3$$

$$\therefore x = 1\ ;\ y = -3$$

37.

Solution

Given

$$\frac{9^{n}3^{2}.3^{n} - 27^{n}}{3^{3m}.2^{3}} = \frac{1}{27}$$

$$\frac{3^{2n}3^{2}.3^{n} - 3^{3n}}{3^{3m}.2^{3}} = \frac{1}{3^{3}}$$

$$\frac{3^{2n+2+n} - 3^{3n}}{3^{3m}.8} = \frac{1}{3^{3}}$$

$$\frac{3^{3n+2} - 3^{3n}}{3^{3m}.8} = \frac{1}{3^{3}}$$

$$\frac{3^{3n}.(3^2-1)}{3^{3m}.8} = \frac{1}{3^3}$$

$$\frac{3^{3n}(9-1)}{3^{3m}.8} = \frac{1}{3^3}$$

$$\frac{3^{3n}.8}{3^{3m}.8} = \frac{1}{3^3}$$

$3^{3n}.3^3 = 3^{3m}$

$3^{3(n+1)} = 3^{3m}$

$\therefore$ m = n +1

38.

Solution

Given

$3^{4x} = (81)^{-1}$

$3^{4x} = (3^4)^{-1}$

4x = -4

$x = -\frac{4}{4}$

x = -1

and

$10^{\frac{1}{y}} = 0.0001$

$10^{\frac{1}{y}} = \frac{1}{10000}$

$10^{\frac{1}{y}} = \frac{1}{10^4}$

$10^{\frac{1}{y}} = 10^{-4}$

$\frac{1}{y} = -4$

$y = -\frac{1}{4}$

$2^{-x} .(16)^{y}$

$2^{-(-1)} (16)^{-\frac{1}{4}}$

$2^{1} . (2^4)^{-\frac{1}{4}}$

$2^{1}. 2^{-1}$

$2. \frac{1}{2}$

1

39

Solution

Given $3^{x+1} = 9^{x-2}$

$3^{x+1} = 3^{2(x-2)}$

$3^{x+1} = 3^{2x-4}$

$x + 1 = 2x - 4$

$2x - x = 1 + 4$

$x = 5$

$2^{1+x} = 2^{1+5}$

$= 2^6$

$= 64$

40.

Solution

(i) $3(2^x + 1) - 2^{x+2} + 5 = 0$

$3.2^x + 3 - 2^x . 2^2 + 5 = 0$

$3. 2^x + 3 - 2^x . 4 + 5 = 0$

$3.2^x - 4.2^x + 8 = 0$

$-2^x + 8 = 0$

$2^x = 8$

$2^x = 2^3$

$x = 3$

(ii) $3^x = 9.3^y$

$3^x = 3^2 . 3^y$

$3^x = 3^{2+y}$

$x = 2 + y$

and $8.2^y = 4^x$

$2^3.2^y = 2^{2x}$

$3 + y = 2x$

$X = \frac{3+y}{2}$

Chapter test

1. if $2^x. 3^y.5^z = 2160$ find the value of x,y and z hence compute the value of $3^x . 2^{-y} . 5^{-z}$

Solution

$2^x. 3^y.5^z = 2160$

$2^x. 3^y.5^z = 2\times2\times2\times2\times3\times3\times3\times5$

$2^x. 3^y.5^z = 2^4 .3^3 . 5^1$

Comparing powers of 2,3 and 5 on both sides of above equation

we get x = 4 , y =3 , z = 1

Also $2^x. 3^{-y}.5^{-z} = (3)^4 \times (2)^{-3} \times (5)^{-1}$

$= 3\times3\times3\times3\times\frac{1}{2\times2\times2} \times \frac{1}{5} = \frac{81}{40} = 2\frac{1}{40}$

2	2160
2	1080
2	540
2	270
3	130
3	45
3	15
5	5

2.

If x = 2 and y = -3 find the values of

(i) $x^x + y^y$

(ii) $x^y + y^x$

Solution

(i) $x^x + y^y$ given that x =2 and y = 3

Then $x^x + y^y = (2)^2 + (-3)^{-3} = 2 \times 2 + \frac{1}{(-3)^3}$

$= 4 + \frac{1}{-3\times(-3)\times(-3)} = 4 + \frac{1}{-27}$

$= 4 - \frac{1}{27} = \frac{108-1}{27} = \frac{107}{27} = 3\frac{26}{27}$

(ii) $x^y + y^x = (2)^{-3} + (-3)^2 = \frac{1}{(2)^3} + (-3) \times (-3)$

$= \frac{1}{2\times2\times2} + 9 = \frac{1}{8} + 9 = \frac{1+9\times8}{8} = \frac{1+72}{8}$

$= \frac{73}{8} = 9\frac{1}{8}$

3. if $p = x^{m+n} . y^{t}$, $q = x^{n+1} . y^{m}$ and $r = x^{l+m} . y^{n}$. prove that $p^{m-n} . q^{n-1} . r^{l-n} = 1$

solution

given that $p = x^{m+n} . y^{t}$ ....(1)

$q = x^{n+l} . y^{m}$ ....(2)

$r = x^{l+m} . y^{n}$ ....(3)

L.H.S = $p^{m-n} . q^{n-1} . r^{l-m}$ ......(4)

Putting the value of a, b, c from (1) (2) (3) respectively in (4) we get

L.H.S = $(x^{m+n}y^{t})^{m-n} , (x^{n+1} , y^{m})^{n-1} . (x^{l+m} y^{n})^{l-m}$

$= (x^{m+n})^{m-n} . y^{l(m-n)} . (x)^{(n+l)(n-1)} . y^{m(n-1)} . (x)^{(l+m)(l-m)} . y^{m(l-m)}$

$= (x^{(m+n)(m-n)} . y^{lm + ln} (x)^{(n+l)(n-1)} . y^{mn-1} . (x)^{(l+m)(l-m)} . y^{ln + nm}$

$= (x)^{m^2-n^2} . y^{lm-ln} . (x)^{n^2-l^2} . y^{mn-ml} . (x)^{l^2-m^2} . y^{nl-nm}$

$=(x)^{m^2-n^2+n^2-l^2+l^2= m^2} \; (y)^{lm-\ln +mn-ml+nl-nm}$

$= (x)^{o} (y)^{o} = 1\times1 = 1$

Hence proved L.H.S = R.H.S

4. if x $= a^{m+n}$, $y = a^{n+1}$ and $z = a^{l+m}$, prove that $x^m \cdot y^n z^l = x^n y^t z^m$

Answer

$x = a^{m+n}$, $y = a^{n+l}$, $z = a^{l+m}$

L.H.S $= x^m\, y^n\, z^p$

$= a^{m[m+n]} \cdot y^{x[n+1]} \cdot z^{l[l+m]}$

$= a^{m^2+mn} \cdot y^{n^2+nl} \cdot z^{l^2+lm}$

$= a^{m^2+mn+n^2+nl+l^2+lm} = a^{l^2+m^2+n^2+lm+mn+np}$

R.H.S $= x^n \cdot y^l \cdot z^m$

$= a^{n(m+n)} \cdot a^{l(n+p)} \cdot a^{m(l+m)}$

$= a^{mn+n^2} \cdot a^{l(n+p)} \cdot a^{m(l+m)}$

$= a^{mn+n^2+\ln+l^2+lm+m^2}$

$= a^{l^2+m^2+n^2+lm+mn+nl}$

∴ L.H.S = R.H.S

5.Show that

$$\frac{\left[p+\frac{1}{q}\right]^m \times \left[p-\frac{1}{q}\right]^n}{\left[q+\frac{1}{p}\right]^m \times \left[q-\frac{1}{p}\right]^n} = \left(\frac{p}{q}\right)^{m+n}$$

Solution

$$\text{L.H.S} = \frac{\left[p+\frac{1}{q}\right]^m \times \left[p-\frac{1}{q}\right]^n}{\left[q+\frac{1}{p}\right]^m \times \left[q-\frac{1}{p}\right]^n}$$

$$= \frac{\left(\frac{pq+1}{q}\right)^m \times \left(\frac{pq-1}{q}\right)^n}{\left(\frac{pq+1}{p}\right)^m \times \left(\frac{pq-1}{p}\right)^n}$$

$$= \frac{\frac{(pq+1)^m}{(q)^m} \times \frac{(pq-1)^n}{(q)^n}}{\frac{(pq+1)^m}{(p)^m} \times \frac{(pq-1)^n}{(p)^n}}$$

$$= \frac{(pq+1)^m}{(q)^m} \times \frac{(p)^m}{(pq+1)^m} \times \frac{(pq-1)^n}{(q)^n} \times \frac{(p)^n}{(pq-1)^n}$$

$$= \frac{1}{(q)^m} \times \frac{(p)^m}{1} \times \frac{1}{q^n} \times \frac{p^n}{1} = \frac{p^m \times p^n}{q^m \times q^n} = \frac{(p)^{m+n}}{(q)^{m+n}}$$

$$= \left(\frac{p}{q}\right)^{m+n} = R.H.S$$

Hence , L.H.S = R.H.S proved the result

6.

If x is a positive real number and exponents are rational numbers then simplify the following

(i) $\dfrac{(x^{(a+b)})^2\,(x^{(b+c)})^2\,(c^{(c+a)})^2}{\left(x^a x^b x^c\right)^4}$

(ii) $\left[\dfrac{x^{a^2}}{x^{b^2}}\right]^{\frac{1}{a+b}}\quad\left[\dfrac{x^{b^2}}{x^{c^2}}\right]^{\frac{1}{b+c}}\quad\left[\dfrac{x^{c^2}}{x^{a^2}}\right]^{\frac{1}{c+a}}$

(iii) $\left(\dfrac{x^b}{x^c}\right)^{b+c-a}\quad\left(\dfrac{x^c}{x^a}\right)^{c+a-b}\quad\left(\dfrac{x^a}{x^b}\right)^{a+b-c}$

Solution

(i) $\dfrac{(x^{(a+b)})^2\,(x^{(b+c)})^2\,(c^{(c+a)})^2}{\left(x^a x^b x^c\right)^4}$

$= \dfrac{x^{2a+2b}.x^{2b+2c}.x^{2c+2a}}{x^{4a}.x^{4b}.x^{4c}}\quad\left\{\begin{matrix}\therefore\ (a^n)^m = a^{mn}\\ x^m.x^n = x^{m+n}\end{matrix}\right\}$

$= \dfrac{x^{2a+2b+2b+2c+2c+2a}}{x^{4a+4b+\ 4c}}$

$$= \frac{x^{4a+4b+4c}}{x^{4a+4b+4c}} = x^{4a+4b+4c-4a-4b-4c}$$

$$= x^0 = 1$$

(ii) $\left[\frac{x^{a^2}}{x^{b^2}}\right]^{\frac{1}{a+b}} \quad \left[\frac{x^{b^2}}{x^{c^2}}\right]^{\frac{1}{b+c}} \quad \left[\frac{x^{c^2}}{x^{a^2}}\right]^{\frac{1}{c+a}}$

$$= (x^{a^2-b^2})^{\frac{1}{a+b}} \times (x^{b^2-c^2})^{\frac{1}{b+c}} \times (x^{c^2-a^2})^{\frac{1}{c+a}}$$

$\{\because (x^m)^n = x^{mn}\}$

$$= x^{\frac{a^2-b^2}{a+b}} \times x^{\frac{b^2-c^2}{b+c}} \times x^{\frac{c^2-a^2}{c+a}}$$

$$= x^{\frac{(a+b)(a-b)}{a+b}} \times x^{\frac{(b+c)(b-c)}{b+c}} \times x^{\frac{(c+a)(c-a)}{c+a}}$$

$\{\because a^2 - b^2 = (a+b)(a-b)\}$

$$= x^{a-b} . x^{b-c} . x^{c-a}$$

$$= x^{a-b+b-c+c-a} = x^0 = 1 \quad \{\because x^0 = 1\}$$

(iii) $\left(\frac{x^b}{x^c}\right)^{b+c-a} \left(\frac{x^c}{x^a}\right)^{c+a-b} \left(\frac{x^a}{x^b}\right)^{a+b-c}$

$$= (x^{b-c})^{b+c-a} . (x^{c-a})^{c+a-b} . (x^{a-b})^{a+b-c}$$

$$= (x^{b-c})^{(b+c-a)} . (x^{c-a})^{(c+a-b)} . (x^{a-b})^{(a+b-c)}$$

$$\left\{\begin{array}{l} \because \ x^m \div x^n = x^{m-n} \\ \quad (x^m)^n = x^{mn} \end{array}\right\}$$

$$= x^{b^2-c^2-ab+ca} . x^{c^2-a^2-bc+ab} . x^{a^2-b^2-ca+bc}$$

$$= x^{b^2-c^2-ab+ca+c^2-a^2-bc+ab+a^2-b^2-ca+bc}$$

$= x^0 = 1$ $\left\{\begin{array}{l} \because (x^m)^n = x^{mn} \\ \quad x^0 = 1 \end{array}\right\}$