ICSE X Mathematics
2022-23

Maximum Marks: 80
Time allowed: Two and half hours
Answers to this Paper must be written on the paper provided separately.
You will not be allowed to write during first 15 minutes.
This time is to be spent in reading the question paper.
The time given at the head of this Paper is the time allowed for writing the answers.

Attempt all questions from Section **A** and any four questions from Section **B**.
All working, including rough work, must be clearly shown, and must be done on the same sheet as the rest of the answer.
Omission of essential working will result in loss of marks.
The intended marks for questions or parts of questions are given in brackets [ ]
Mathematical tables and graph papers are provided.

SECTION A (40 Marks)
(Attempt **all** questions from this **Section**.)

Question 1
Choose the correct answers to the questions from the given options. [15]
(Do not copy the questions, write the correct answers only.)

(i) If $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$, the value of x and y respectively are:
(a) $1, -2$
(b) $-2, 1$
(c) 1, 2
(d) $-2, -1$
Answer: (a)
Explanation:

$2x + 0y = 2$
$x = 1$
$0x + 4y = -8$
$y = -2$

(ii) If x – 2 is a factor of $x^3 - kx - 12$, then the value of k is:
(a) 3
(b) 2
(c) -2
(d) -3
Answer: (c)
Explanation:

$x - 2 = 0$

$x = 2$

Substituting x in the equation,

$2^3 - 2k - 12 = 0$

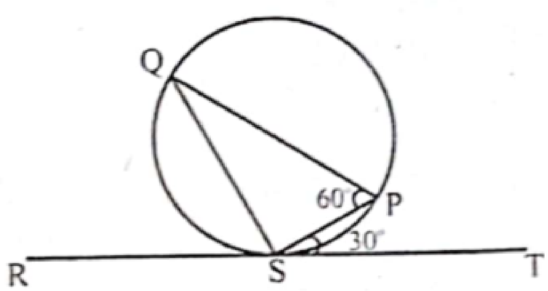
$8 - 2k - 12 = 0$

$-2k - 4 = 0$

$-2k = 4$

$k = -2$

(iii) In the given diagram RT is a tangent touching the circle at S. If $\angle PST = 30^\circ$ and $\angle SPQ = 60^\circ$ then $\angle PSQ$ is equal to:



(a) 40°
(b) 30°
(c) 60°
(d) 90°
Answer: (d)
Explanation:

Given, $\angle PST = 30^\circ$ and $\angle SPQ = 60^\circ$

From alternate segment theorem:

$\angle PQS = \angle PST = 30^{\circ}$ ...(i)

In ΔPQS,

∠PQS + ∠PSQ + ∠SPQ = 180°

$30^{\circ} + \angle PSQ + 60^{\circ} = 180^{\circ}$ (from (i))

∠PSQ = $180^{\circ} - 30^{\circ} - 60^{\circ}$

∠PSQ = 90°

(iv) A letter is chosen at random from all the letters of the English alphabets. The probability that the letter chosen is a vowel is:

(a) $\frac{4}{26}$

(b) $\frac{5}{26}$

(c) $\frac{21}{26}$

(d) $\frac{5}{24}$

Answer: (b)

Explanation:

The number of English alphabets = 26

The number of vowels in English alphabets = 5

The probability that the letter chosen is a vowel = The number of vowels in English alphabets / The number of English alphabets

The probability that the letter chosen is a vowel = 5/26

(v) If 3 is a root of the quadratic equation $x^2 - px + 3 = 0$ then p is equal to:

(a) 4

(b) 3

(c) 5

(d) 2

Answer: (a)

Explanation:

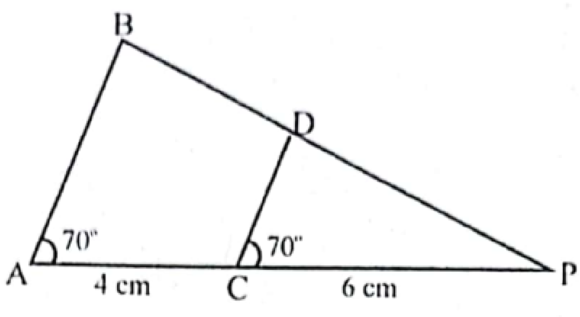
Substitute 3 in the equation,

$$3^2 - p(3) + 3 = 0$$

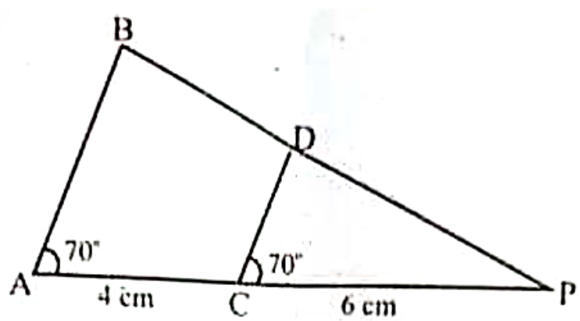
$$9 - 3p + 3 = 0$$

$12 - 3p = 0$
$- 3p = - 12$
$p = 4$

(vi) In the given figure $\angle BAP = \angle DCP = 70^{\circ}$, $PC = 6\ cm$ and $CA = 4\ cm$, then $PD : DB$ is:



(a) $5 : 3$
(b) $3 : 5$
(c) $3 : 2$
(d) $2 : 3$
Answer: (c)
Explanation:



In the given figure,
$\angle BAP = \angle DCP$
$\therefore AB \| CD$ {Corresponding angles are equal}
$\Rightarrow \frac{PC}{CA} = \frac{PD}{DB}$ {Applying Basic Proportionality Theorem}
$\Rightarrow \frac{PD}{DB} = \frac{6}{4}$
$\Rightarrow PD : DB = 3 : 2$

(vii) The printed price of an article is ₹3080. If the rate of GST is 10% then the GST charged is:
(a) ₹154
(b) ₹308
(c) ₹ 30.80
(d) ₹ ₹15.40
Answer: (b)
Explanation:
Printed price $= 3080$, rate of GST $= 10\%$
GST charged will be = $\frac{3080\times10}{100} = 308$

(viii) $(1 + sinA)(1 - sinA)$ is equal to:
(a) $cosec^2 A$
(b) $sin^2 A$
(c) $sec^2 A$
(d) $cos^2 A$
Answer: (d)
Explanation:
Given, $(1 + sinA)(1 - sinA)$ which equals to $1 - sin^2 A$
As we have $sin^2\theta + cos^2\theta = 1 \Rightarrow 1 - sin^2\theta = cos^2\theta$
So, from the above identity, $(1 + sinA)(1 - sinA) = cos^2 A$

(ix) The coordinates of the vertices of $\triangle ABC$ are respectively $(-4, -2)$, $(6, 2)$ and $(4, 6)$. The centroid G of $\triangle ABC$ is:
(a) $(2, 2)$
(b) $(2, 3)$
(c) $(3, 3)$
(d) $(0, -1)$
Answer: (a)
Explanation:
Given parameters are,
$(x_1, y_1) = (-4, -2)$

$(x_2, y_2) = (6,2)$
$(x_3, y_3) = (4, 6)$
The centroid formula of a given triangle can be expressed as,

$$C = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$C = \left(\frac{-4+6+4}{3}, \frac{-2+2+6}{3}\right) = \left(\frac{6}{3}, \frac{6}{3}\right)$$

$$C = (2,\ 2)$$

(x) The nth term of an AP is 2n + 5. The 10th term is:
(a) 7
(b) 15
(c) 25
(d) 45
Answer: (c)
Explanation:

$$t_n = 2n + 5$$

$$t_{10} = 2(10) + 5$$

$$t_n = 25$$

(xi) The mean proportional between 4 and 9 is:
(a) 4
(b) 6
(c) 9
(d) 36
Answer: (b)
Explanation:
Mean proportion between two numbers is defined as the square root of the product of the numbers.
i.e. $M = \sqrt{x \times y}$
Let the number $x = 4$ and $y = 9$.
So, Mean proportion between them is

$$M = \sqrt{4 \times 9} = \sqrt{36} = 6$$

Therefore, The mean proportional between 4 and 9 is 6.

(xii) Which of the following cannot be determined graphically for a grouped frequency distribution?
(a) Median
(b) Mode
(c) Quartiles
(d) Mean
Answer: (d)
Explanation:
Mean is the sum of the values of a subject divided by number of values.
As it is a single value and cannot be compared and represented in different values, therefore, the determination of mean by the graphical method is not possible.
Mean is a perfect method for the determination of the central tendency of a value.
Hence, mean can't be determined graphically.

(xiii) Volume of a cylinder of height $3\ cm$ is 48π. Radius of the cylinder is:
(a) $48\ cm$
(b) $16\ cm$
(c) $4\ cm$
(d) $24\ cm$
Answer: (c)
Explanation:

Let the radius of the base and height of the cylinder be $r\ cm$ and $h\ cm$ respectively. Then, $h = 3\ cm$

Now, Volume $= 48\pi cm^3$

$\pi r^2 h = 48\pi r^2 = \frac{48}{3} = 16r = 4\ cm$

(xiv) Naveen deposits Rs. 800 every month in a recurring deposit account for 6 months. If he receives Rs 4884 at the time of maturity, then the interest he earns is:
(a) ₹ 84
(b) ₹ 42
(c) ₹ 24
(d) ₹ 284
Answer: (a)
Explanation:

Total money deposited in 6 months = 800 x 6 = Rs. 4800
Money received at the time of maturity = Rs 4884
Therefore, the interest he earns = Rs. 4884 – Rs. 4800 = Rs. 84

(xv) The solution set for the inequation $2x + 4 \leq 14,\ x \in W$ is:
(a) $\{1, 2, 3, 4, 5\}$
(b) $\{0, 1, 2, 3, 4, 5\}$
(c) $\{1, 2, 3, 4\}$
(d) $\{0, 1, 2, 3, 4\}$
Answer: (b)
Explanation:

$2x + 4 \leq 14$
$2x \leq 14 - 4$
$2x \leq 10$
$x \leq 5,\ x \in W$

Therefore, the solution set is {0, 1, 2, 3, 4, 5}

Question 2
(i) Find the value of 'a ' if $x - a$ is a factor of the polynomial $3x^3 + x^2 - ax - 81$. [4]

Answer: a = 3
Explanation:
Since $(x - a)$ is a factor of the polynomial $p(x) = 3x^3 + x^2 - ax - 81 = 0$, by factor theorem, we have $p(a) = 0$

$\Rightarrow 3(a)^3 - (a)^2 + a(a) - 81 = 0$
$\Rightarrow 3a^3 = 81$
$\Rightarrow a^3 = 27$
$\Rightarrow a = 3$

(ii) Salman deposits ₹ 1000 every month in a recurring deposit account for 2 year If he receives ₹26000 on maturity, find:
(a) the total interest Salman earns.
(b) the rate of interest.

[4]

Answer: Rs 2000, 8%

Explanation:

Salman deposits in 2 years $(P) = 1000\times24$

$(P) = Rs\ 24000$

He receives (A) $= Rs\ 26000.$

(a) Total interest (I) $= A - P = 26000 - 24000 = Rs.\ 2000$

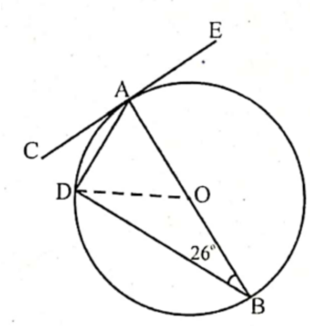
(b) $Interest\ (I) = p \times \frac{(n)(n+1)}{2\times12} \times \frac{r}{100}$

$2000 = 1000\times\frac{24\times25}{2\times12} \times \frac{r}{100}$

$2000 = 250r$

$r = 8\%$

(iii) In the given figure O, is the centre of the circle. CE is a tangent to the circle If ∠ABD=26° , then find:



(a) ∠BDA

(b) ∠BAD

(c) ∠CAD

(d) ∠ODB

[4]

Answer: $\angle BDA = 90^{\circ}, \angle BAD = 64^{\circ}, \angle CAD = 26^{\circ}, \angle ODB = 26^{\circ}$

Explanation:

a) The angle formed by semi-circle is 90°

$\therefore \angle BDA = 90^{\circ}$

b) In $\triangle ABD$,

$\angle A + \angle B + \angle D = 180^{\circ}$

$\angle A + 26^{\circ} + 90^{\circ} = 180^{\circ}$

$\angle A = 180^{\circ} - 116^{\circ}$

$\angle A = 64^{\circ}$

$\therefore \angle BAD = 64^{\circ}$

c) The tangent formed by a circle is perpendicular to is diameter.

$\Rightarrow CE \perp AB$

$\therefore \angle CAB = 90^{\circ}$

$\angle CAD + \angle DAB = 90^{\circ}$

$\angle CAD + 64^{\circ} = 90^{\circ}$

$\angle CAD = 26^{\circ}$

d) In $\triangle ODB$,
line $OD =$ line OB [$\because OD$ & OB are radius]
So $\triangle ODB$ is Isosceles $triangle$

$\therefore \angle OBD = \angle ODB$

$\therefore \angle ODB = 26^{\circ}$

Question 3
(i) Solve the following quadratic equation:

$x^2 + 4x - 8 = 0$

Give your answer correct to one decimal place. (Use mathematical tables if necessary.)

[4]

Answer: x = 1.4 or -5.4
Explanation:
Given quadratic equation is $x^2 + 4x - 8 = 0$
We have a formula for solving quadratic equation $ax^2 + bx + c = 0$ is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

By using quadratic formula, we have

$$x = \frac{-(4)\pm\sqrt{(4)^2-4(1)(-8)}}{2(1)} = \frac{-4\pm\sqrt{16+32}}{2}$$

$$= \frac{-4\pm\sqrt{48}}{2} = \frac{-4\pm4\sqrt{3}}{2} =-\ 2\pm2\sqrt{3}$$

$$= 2(-\ 1\pm\sqrt{3}) = 2(-\ 1\pm1.73205) = 2(0.73205)\ or\ 2(-\ 2.73205)$$

$$= 1.4641\ or\ -\ 5.4641$$

$$= 1.4\ or\ -\ 5.4$$

(ii) Prove the following identity: [4]

$$\left(sin^2\theta - 1\right)\left(tan^2\theta + 1\right) + 1 = 0$$

Explanation:

Identities: $sec^2\theta - tan^2\theta = 1$ and $sin^2\theta + cos^2\theta = 1$

$$LHS = \left(sin^2\theta - 1\right)\left(tan^2\theta + 1\right) + 1$$

$=-\ cos^2\theta \times sec^2\theta + 1$ (From the identity)

$$= (-\ cos^2\theta \times \frac{1}{cos^2\theta}) + 1$$

$$=-\ 1 + 1$$

$$= 0$$

$$= RHS$$

Hence, it is proved.

(iii) Use graph sheet to answer this question. Take $2\ cm = 1$ unit along both the axes.

(a) Plot A, B, C where $A(0, 4), B(1, 1)$ and $C(4, 0)$

(b) Reflect A and B on the x-axis and name them as E and D respectively.

(c) Reflect B through the origin and name it F. Write down the coordinates of F.

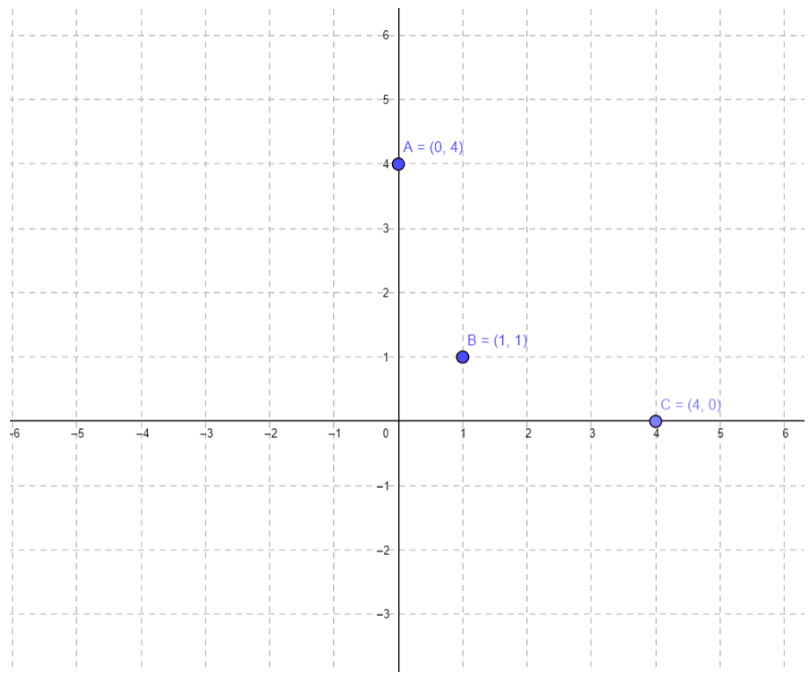
(d) Reflect B and C on the y-axis and name them as H and G respectively.

(e) Join points A, B, C, D, E, F, G, H and A in order and name the closed figure formed.

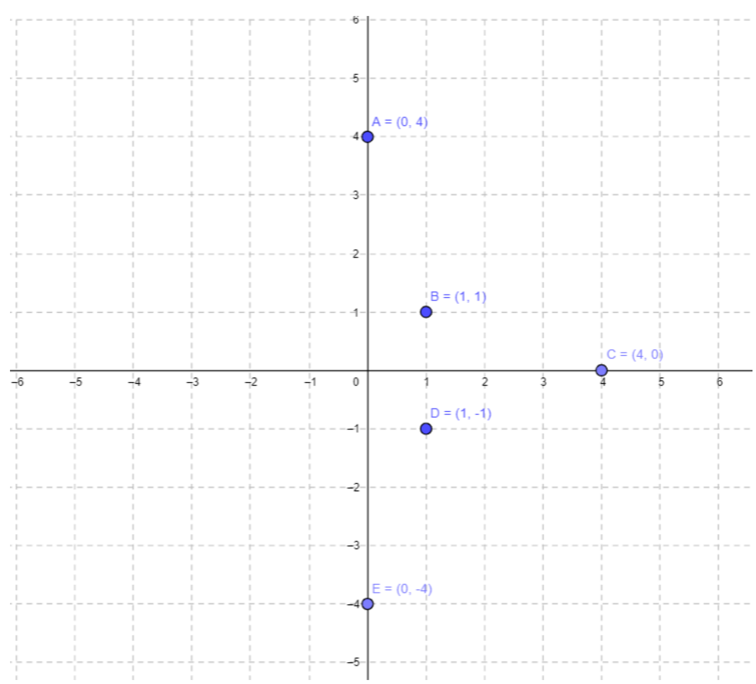
[5]

Explanation:

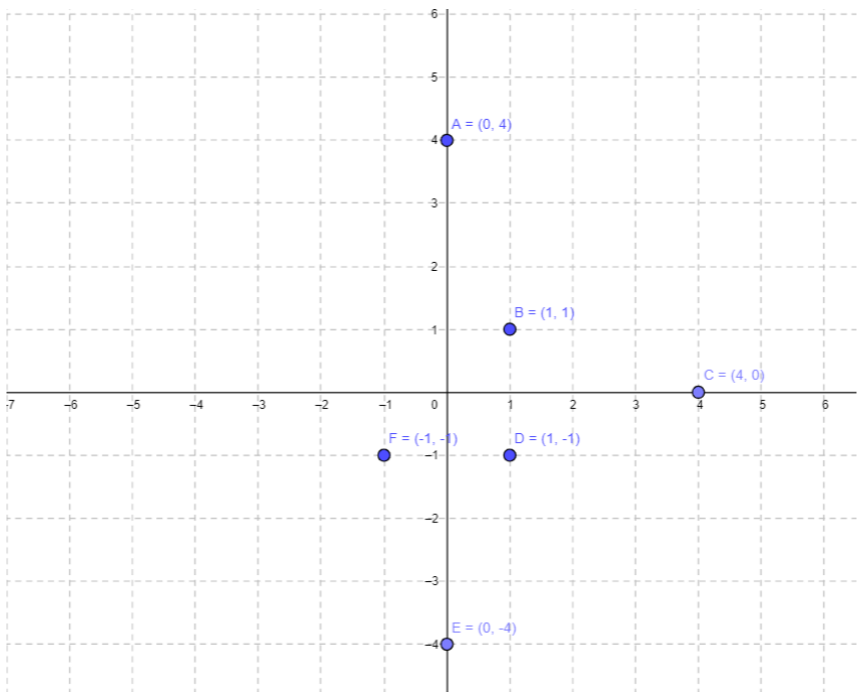
(a) Plot the points A, B, and C using their coordinates. A(0,4) will be located 4 units up the y-axis, B(1,1) will be located 1 unit up the y-axis and 1 unit to the right of the origin, and C(4,0) will be located 4 units to the right of the origin.



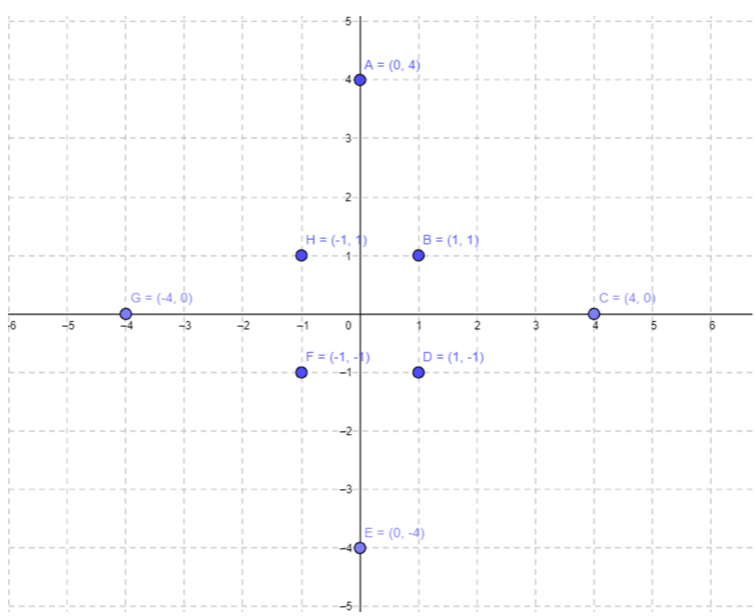
(b) To reflect points A and B on the x-axis, draw a horizontal line passing through A and B. The reflections of A and B on the x-axis will be E and D, respectively. The coordinates of E will be (0,-4), and the coordinates of D will be (1,-1).



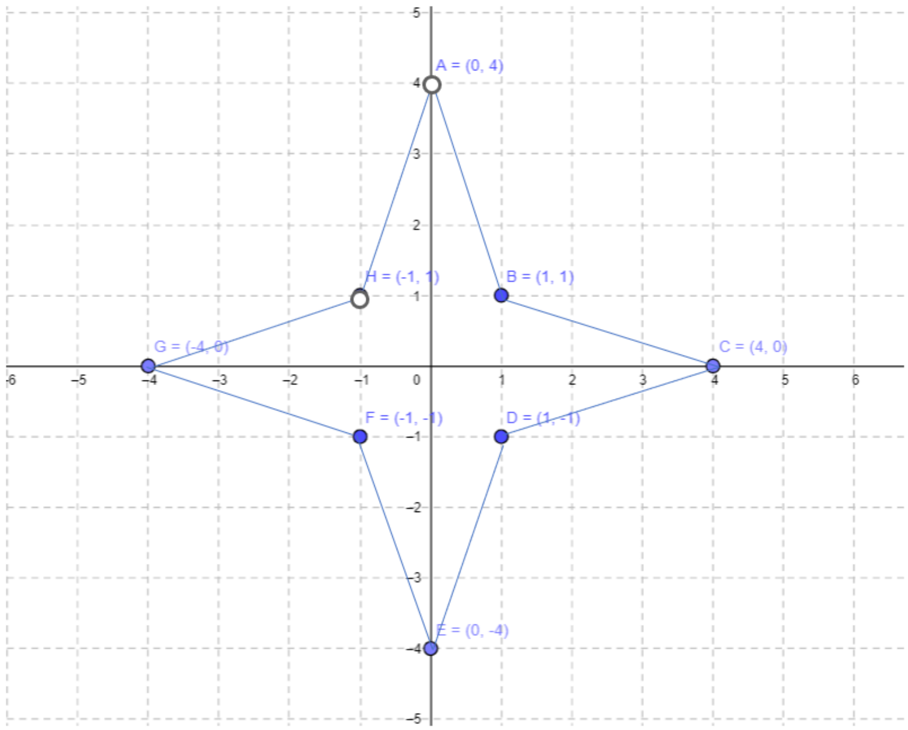
(c) To reflect point B through the origin, draw a line passing through the origin and point B. The reflection of B through the origin will be F. The coordinates of F will be (-1,-1).



(d) To reflect points B and C on the y-axis, draw a vertical line passing through points B and C. The reflections of B and C on the y-axis will be H and G, respectively. The coordinates of H will be (-1,1), and the coordinates of G will be (-4,0).



(e) Join points A, B, C, D, E, F, G, H, and A in order to form a closed figure. The closed figure formed is a trapezoid.



SECTION B (40 Marks)

(Attempt **any four** questions from this Section.)

Question 4

(i) If A = $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}, C = \begin{bmatrix} 4 & 1 \\ 1 & 5 \end{bmatrix} and\ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. $Find\ A(B + C) - 14I$ [3]

Answer: $A(B + C) - 14I$ = $\begin{bmatrix} 0 & 30 \\ 22 & 28 \end{bmatrix}$

Explanation:

$B + C = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 1 & 5 \end{bmatrix}$

$= \begin{bmatrix} 1+4 & 2+1 \\ 2+1 & 4+5 \end{bmatrix}$

$= \begin{bmatrix} 5 & 3 \\ 3 & 9 \end{bmatrix}$

$A(B + C) = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 3 & 9 \end{bmatrix}$

$= \begin{bmatrix} (1 \times 5) + (3 \times 3) & (1 \times 3) + (3 \times 9) \\ (2 \times 5) + (4 \times 3) & (2 \times 3) + (4 \times 9) \end{bmatrix}$

$= \begin{bmatrix} 5+9 & 3+27 \\ 10+12 & 6+36 \end{bmatrix}$

$= \begin{bmatrix} 14 & 30 \\ 22 & 42 \end{bmatrix}$

$14I = 14 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$

Therefore, $A(B + C) - 14I = \begin{bmatrix} 14 & 30 \\ 22 & 42 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$

$= \begin{bmatrix} 14-14 & 30-0 \\ 22-0 & 42-14 \end{bmatrix}$

$= \begin{bmatrix} 0 & 30 \\ 22 & 28 \end{bmatrix}$

(ii) ABC is a triangle whose vertices are A(1, -1), B(0, 4) and C(-6, 4). D is the midpoint of BC. Find the

a) Coordinates of D and

b) Equation of the median AD.

[3]

Answer: a) D is (-3, 4) **b)** The equation of line AD is $5x + 4y = 1$

Explanation:

a) Midpoint of BC = $\left(\frac{0-6}{2}, \frac{4+4}{2}\right)$

= (-3, 4)

Therefore, the point D is (-3, 4).

b) Equation of the median AD:

Equation of the line joining two points

$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

A (1, -1) and D (-3, 4)

$\frac{y-(-1)}{4-(-1)} = \frac{x-1}{-3-1}$

$\frac{y+1}{5} = \frac{x-1}{-4}$

$(-4)(y + 1) = 5(x - 1)$

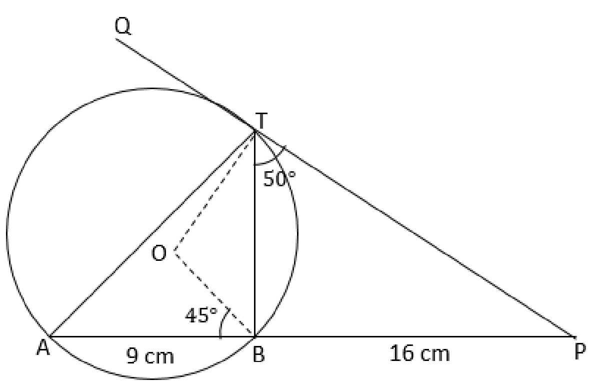
$-4y - 4 = 5x - 5$

$-4y - 5x = -1$

$5x + 4y = 1$

Therefore, the equation of line AD is $5x + 4y = 1$

(iii) In the given figure, O is the centre of the circle. PQ is a tangent to the circle at T. Chord AB produced meets the tangent at P.



$AB = 9\ cm, BP = 16\ cm, \angle PTB = 50^{\circ}$

$\angle OBA = 45^{\circ}$

Find:

(a) length of PT

(b) $\angle BAT$

(c) $\angle BOT$

(d) $\angle ABT$

[4]

Answer: $PT = 15cm$, $\angle BAT = 50°$, $\angle BOT = 100^{\circ}$, $\angle ABT = 85^{\circ}$

Explanation:

(a) Length of PT
We know that

$AB \times AP = PT^2$

$9 \times 25 = PT^2$

$PT = 15cm$

(b) Radius makes 90° at Tangent we have $\angle BTP = 50^{\circ}$

So, $\angle BTO = 90^{\circ} - 50^{\circ} \Rightarrow 40^{\circ}$

$\because \triangle BOT$ is isosceles $\triangle$

$\therefore \angle TBO = 40^{\circ}$

AID, $\triangle AOB$ is isosceles $\triangle$

$\therefore \angle OAB = 45^{\circ} \therefore \angle AOB = 90^{\circ}$

then $\angle AOT = 360^{\circ} - \left(90^{\circ} + 100^{\circ}\right) = 170^{\circ}$

$OA = OT$ (Radius)

$\therefore \triangle AOT$ is isosceles $\triangle$

$\therefore \angle OAT = \angle OTA = 5^{\circ}$

Now, $\angle BAT = \angle DAB + \angle OAT$

$= 45^{\circ} + 5^{\circ}$

$\angle BAT = 50°$

(c) $\angle BOT = 180^{\circ} - (\angle TBO + OTB)$

$= 180^{\circ} - 80^{\circ}$

$\angle BOT = 100^{\circ}$

(d) $\angle ABT = \angle ABO + \angle OBT$

$= 45^{\circ} + 40^{\circ}$

$\angle ABT = 85^{\circ}$

Question 5

(i) Mrs. Arora bought the following articles from a departmental store:

S.No.	Item	Price	Rate of GST	Discount
1.	Hair oil	Rs. 1200	18%	Rs. 100
2.	Cashew nuts	Rs. 600	12%	-

Find the:

(a) Total GST paid.

(b) Total bill amount including GST.

[3]

Explanation:

S.No.	Item	Price	Rate of GST	Discount	Discounted value
1.	Hair oil	Rs. 1200	18%	Rs. 100	Rs. 1100
2.	Cashew nuts	Rs. 600	12%	-	Rs. 600

Hair Oil:

CGST = 9% of 1100 = Rs. 99

SGST = 9% of 1100 = Rs. 99

Total GST = Rs. 198

Total amount = 1100 + 198 = Rs. 1298

Cashew Nuts:

CGST = 6% of 600 = Rs. 36

SGST = 6% of 600 = Rs. 36

Total GST = Rs. 72
Total amount = 600 + 72 = Rs. 672

Total GST = GST of Hair oil + GST of Cashew Nuts
= Rs. 198 + Rs. 72
= Rs. 270
Total amount = Rs 1298 + Rs. 672
= Rs. 1970

(ii) Solve the following inequation. Write down the solution set and represent it on the real number line.

$-5(x-9) \geq 17 - 9x > x + 2, x \in R$

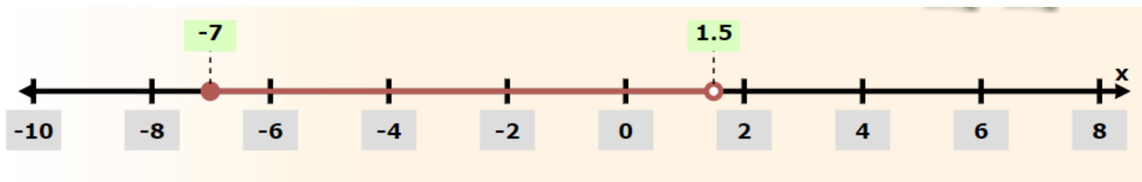
[3]

Answer: $x \in$ [-7, 1.5)
Explanation:
To solve this inequality, we'll first simplify it:

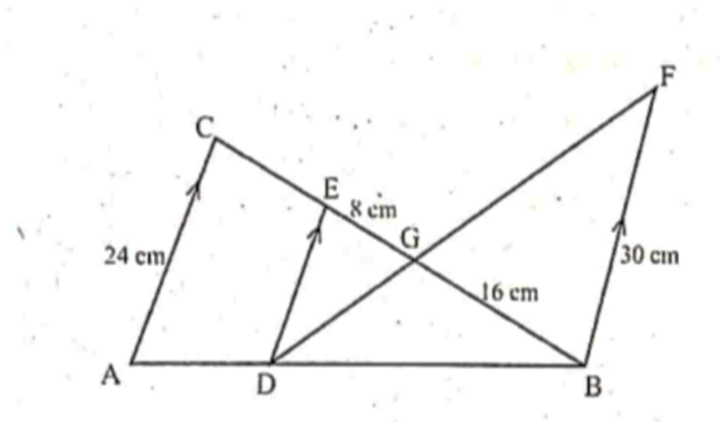
$$-5(x-9) \geq 17 - 9x > x + 2$$

-5 x + 45 ≥ 17 - 9 x > x + 2 (distribute -5 on the left and simplify)
-5 x + 45 ≥ 17 - 9 x and 17 - 9 x > x + 2 (split into two inequalities)
4 x ≥ - 28 and 15 > 10 x (add 9 x and subtract 17 from both sides of each inequality)
x ≥ - 7 and x < 1.5 (divide both sides of each inequality by 4 and 10, respectively)

To find the solution set for the given inequality x ≥ -7 and x < 1.5, we need to find the values of x that satisfy both conditions simultaneously.
x ≥ -7 means that x can take any value greater than or equal to -7.
x < 1.5 means that x can take any value less than 1.5.
To satisfy both conditions simultaneously, x must be a value that is greater than or equal to -7 AND less than 1.5.
Therefore, the solution set for x is:
$x \in$ [-7, 1.5)
We can represent this on number line as



(iii) In the given figure, $AC \parallel DE \parallel BF$.
If $AC = 24\ cm, EG = 8\ cm,\ GB = 16\ cm, BF = 30\ cm$. [4]



(a) Prove $\triangle GED \sim \triangle GBF$
(b) Find DE
(c) $DB : AB$

[4]

Explanation:

Given: $AC \parallel DE \parallel BF$

$AC = 24\ cm,\ EG = 8\ cm,\ GB = 16\ cm,\ BF = 30\ cm$

(i) Prove: $\Delta GED \sim \Delta GBF$

Explanation: From fig, we have

$\angle EGD = \angle BGF$ (Vertically Opposite Angles)

$\angle DEG = \angle FBG$ (Alternate Interior Angles)

$\therefore \Delta GED \sim \Delta GBF$ (AA similarity criterion)

(ii) Find DE

Explanation:

$\because \Delta GED \sim \Delta GBF$

$\Rightarrow \frac{GE}{GB} = \frac{DE}{FB}$

$\Rightarrow \frac{8}{16} = \frac{DE}{30}$

$\Rightarrow DE = \frac{1}{2} \times 30 = 15\ cm$

(iii) In figure, we have $ED \parallel AC$

∴ We have,

$\Delta BED \sim \Delta BCA$ (AA similarity)

$\Rightarrow \frac{DB}{DE} = \frac{AB}{AC}$

$\Rightarrow \frac{DB}{AB} = \frac{15}{24}$

$\Rightarrow DB : AB = 5 : 8$

Question 6

(i) The following distribution gives the daily wages of 60 workers of a factory.

Daily income in ₹	Number of workers (f)
$200 - 300$	6
$300 - 400$	10
$400 - 500$	14
$500 - 600$	16
$600 - 700$	10
$700 - 800$	4

Use graph paper to answer this question.
Take $2\ cm = ₹100$ along one axis and $2\ cm = 2$ workers along the other axis.
Draw a histogram and hence find the mode of the given distribution.

[3]

Explanation:

To draw a histogram for the given distribution, we will use the following steps:

Step 1: Draw the x and y-axis with suitable scales on the graph paper. We will take 2 cm = ₹100 along the x-axis and 2 cm = 2 workers along the y-axis.

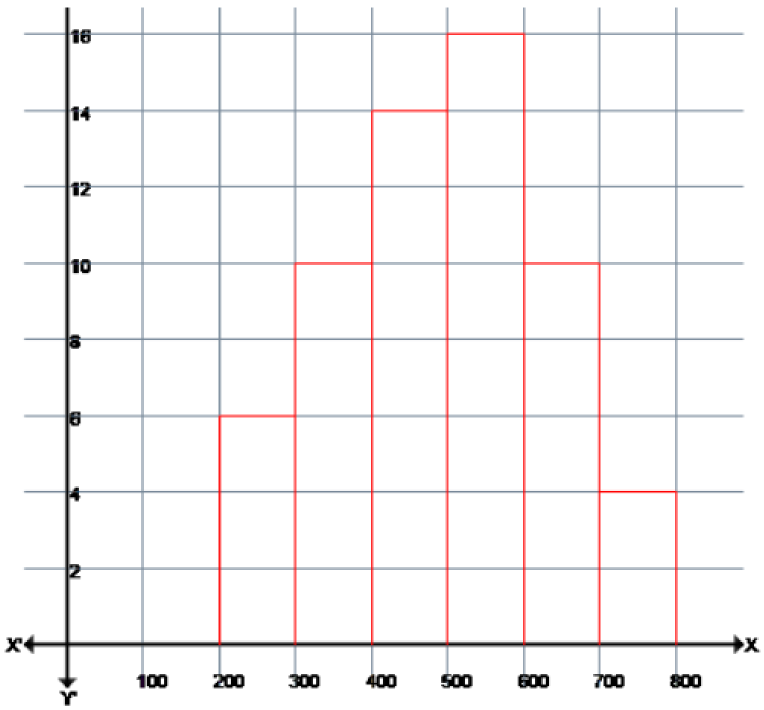
Step 2: Mark the class intervals on the x-axis and the corresponding frequencies on the y-axis.

Step 3: Draw rectangles for each class interval with the base of the rectangle on the x-axis and the height equal to the corresponding frequency on the y-axis.

Step 4: The histogram is obtained by placing the rectangles adjacent to each other.

Class	Lower	Upper	Frequency f
200 - 300	200	300	6
300 - 400	300	400	10
400 - 500	400	500	14
500 - 600	500	600	16
600 - 700	600	700	10
700 - 800	700	800	4

Using the above steps, the histogram for the given distribution is as follows:



Histogram for the given distribution

To find the mode of the given distribution, we look for the class interval with the highest frequency. From the histogram, we can see that the class interval 500-600 has the highest frequency of 16. Therefore, the mode of the given distribution is ₹500-₹600.

(ii) The 5^{th} term and the 9^{th} term of an Arithmetic Progression are 4 and -12 respectively. Find:
(a) the first term
(b) common difference
(c) sum of 16 terms of the AP.

[3]

Explanation:

Given 5^{th} term $= 4$

$9^{th}\ term\ =-\ 12$

Let a is the first term and d is the common difference of the A.P.

$t_5 \Rightarrow a + 4d = 4 - \ (i)$

$t_9 \Rightarrow a + 8d =- 12 - \ (ii)$

By subtracting (ii) - (i) we get -

$4d = \ - 16$

$d = \ - 4$

Now substitute the value of d in (1) eq we get -

$a + 4(- 4) = 4$

$- 16 + \ a = 4$

$a = 4 + 16 = 20$

Common difference is -4

Sum of 16 terms can be given by,

$S_{16} = = \frac{16}{2}[2\times20 + 15(- 4)]$

$S_{16} \ = 8[40 - 60]$

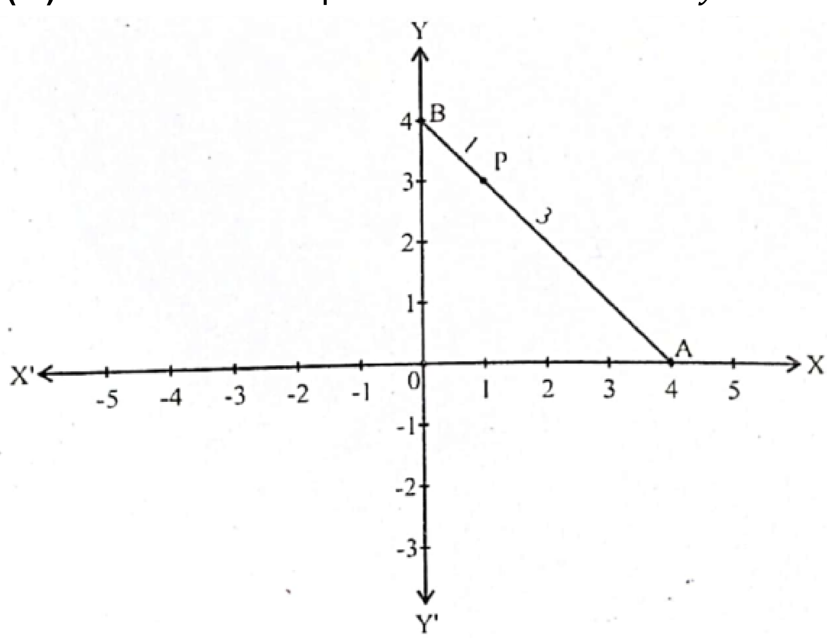
$S_{16} \ = \ - 160$

(a) First term = 20

(b) Common difference = - 4

(c) Sum of 16 terms = - 160

(iii) A and B are two points on the x-axis and y-axis respectively.



(a) Write down the coordinates of A and B.
(b) P is a point on AB such that $AP: PB = 3: 1$. Using section formula find the coordinates of point P.
(c) Find the equation of a line passing through P and perpendicular to AB.

[4]

Explanation:
(a) Co-ordinates of $A = (4, 0)$
co-ordinates of $B = (0, 4)$
(b) The coordinates of the point $P(x, y)$ which divides the line segment joining points $A\left(x_1, y_1\right)$ and $B\left(x_2, y_2\right)$ internally in the ratio $m: n$ are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

Here -

$$m = 3 \ n = 1 \ x_1 = 4 \ x_2 = 0 \ y_1 = 0 \ y_2 = 4$$

$$x = \frac{mx_2+nx_1}{m+n}, y = \frac{my_2+ny_1}{m+n}$$

$$x = \frac{3\times0+1\times4}{3+1}, \ y = \frac{3\times4+1\times0}{3+1}$$

$$x = \frac{4}{4}, \ y = \frac{12}{4}$$

x = 1, y = 3
Co-ordinates of P is $(1, 3)$

(c) Equation of line connecting AB formula:- $\left(y - y_1\right) = \frac{\left(y_2-y_1\right)}{\left(x_2-x_1\right)}\left(x - x_1\right)$

$$x_1 = 4 \, x_2 = 0 \ y_1 = 0 \, y_2 = 4$$

Equation of line segment will be -

$$\Rightarrow (y - 0) = \frac{(4-0)}{(0-4)}(x - 4)$$

$$\Rightarrow y = \frac{4}{-4}(x - 4) \ \Rightarrow \ -y = x - 4 \Rightarrow y = 4 - x$$

Comparing with $y = mx + 6$ we get the gradient is -1.
So the gradient of the line perpendicular to this line will be 1.
Now equation of the line parsing throng $P(1, 3)$ and $\perp$ to AB will be -

$$\Rightarrow y - y_1 = m\left(x - x_1\right) \ \Rightarrow (y - 3) = 1(x - 1)$$

$\Rightarrow y - 3 = x - 1 \quad \Rightarrow y = x - 1 + 3 \quad \Rightarrow y = x + 2$

Question 7

(i) A bag contains 25 cards, numbered through 1 to 25 . A card is drawn at random. What is the probability that the number on the card drawn is:

(a) multiple of 5

(b) a perfect square

(c) a prime number?

[3]

Explanation:

(a) Cards which are multiple of 5 will be - 5, 10, 15, 20, 25

Probability of drawn card to be multiple of 5 will be given by -

$\Rightarrow P = \frac{5}{25} = \frac{1}{5}$

$P = \frac{No\ of\ cards\ multiple\ of\ 5}{total\ No\ of\ cards}$

$P = \frac{5}{25}$ $P = \frac{1}{5}$

(b) Perfect numbers in 1 to 25 are $1, 4, 9, 16, 25$

Probability will be given by

$P = \frac{No\ of\ perfect\ Squares}{total\ Numbers}$

$p = \frac{5}{25} = \frac{1}{5}$

(c) Prime Numbers in 1 to 25 are $2, 3, 5, 7, 11, 13, 17, 19, 23$

Probability will be given by -

$P = \frac{Number\ of\ Primes}{Total\ Numbers}$ $P = \frac{9}{25}$

(ii) A man covers a distance of 100 km, travelling with a uniform speed of x km/hr. Had the speed been 5 km/hr more it would have taken 1 hour less. Find x the original speed.

[3]

Explanation:

Let the usual speed of train be km/hr.

The increased speed of the train $= (x + 5) km/hr$

Time taken by the train under usual speed to cover $100\ km = \frac{100}{x} hr$

Time taken by the train under increased speed to cover $100\ km = \frac{100}{x+5} hr$

Therefore,

$$\frac{100}{x} - \frac{100}{(x+5)} = \frac{60}{60}$$

$$\frac{100(x+5)-100x}{x(x+5)} = 1$$

$$\frac{100x+500-100x}{x^2+5x} = 1$$

$$500 = x^2 + 5x$$

$$x^2 + 5x - 500 = 0$$

$$x^2 + 25x - 20x - 500 = 0$$

$$x(x + 25) - 20(x + 25) = 0$$

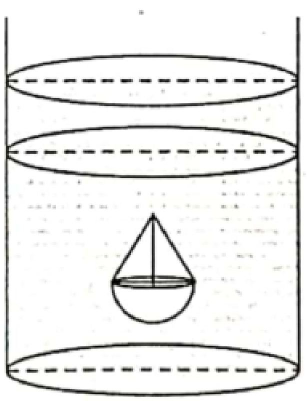
$$(x + 25)(x - 20) = 0$$

$$x = -25 \text{ or } x = 20$$

Since, the speed of the train can never be negative

Therefore, the original speed of the train is $20\ km/hr$.

(iii) A solid is in the shape of a hemisphere of radius $7\ cm$, surmounted by a cone of height $4\ cm$. The solid is immersed completely in a cylindrical container filled with water to a certain height. If the radius of the cylinder is $14\ cm$, find the rise in the water level.



[4]

Answer: 1.5 cm

Explanation:

First, we need to find the volume of the hemisphere and the cone, then we can add them together to find the total volume of the solid. Let's begin by finding the volume of the hemisphere.
The volume of a hemisphere of radius r is given by:

$$V = \frac{2}{3}\pi r^3$$

Substituting r = 7, we get:

$$V_1 = \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 = 718.66\ cm^3$$

Next, let's find the volume of the cone. The formula for the volume of a cone is

$$V = \frac{1}{3}\pi r^2 h$$

where r is the radius of the base and h is the height. Substituting r = 7 and h = 4, we get

$$V_2 = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 4 = 205.33\ cm^3$$

The total volume of the solid is:

V = V_1 + V_2 = 718.66 + 205.33 = 923.99 cm^3

Now, let's find the rise in the water level. We know that the volume of water displaced is equal to the volume of the solid. Let h be the height by which the water level rises when the solid is immersed in the water. The formula for the volume of a cylinder is:

$$V = \pi r^2 h$$

Substituting r = 14 and V = 923.99, we get

$$923.99 = \frac{22}{7} \times 14 \times 14 \times h$$

Solving for h, we get:
h = 1.49 ≈ 1.5 cm
Therefore, the water level rises by approximately 1.5 cm when the solid is completely immersed in the cylindrical container.

Question 8

(i) The following table gives the marks scored by a set of students in an examination. Calculate the mean of the distribution by using the short cut method.

Marks	Number of Students (f)
0 – 10	3
10 – 20	8
20 – 30	14
30 – 40	9
40 – 50	4
50 – 60	2

[3]

Answer: 27.25

Explanation:

Class (1)	**Frequency** (f) (2)	**Mid value** (x) (3)	$d = \frac{x - A}{h} = \frac{x - 35}{10}$ $A = 35, h = 10$ (4)	$f \cdot d$ (5) = (2) × (4)
0 - 10	3	5	-3	-9
10 - 20	8	15	-2	-16
20 - 30	14	25	-1	-14
30 - 40	9	35=A	0	0
40 - 50	4	45	1	4
50 - 60	2	55	2	4
---	---	---	---	---
	$n = 40$	-----	-----	$\sum f \cdot d = -31$

$$Mean\ \bar{x} = A + \frac{\sum fd}{n} \cdot h$$

$$= 35 + \frac{-31}{40} \cdot 10$$

$$= 35 + (-0.775) \cdot 10$$

$$= 35 - 7.75$$

$$= 27.25$$

(ii) What number must be added to each of the numbers $4, 6, 8, 11$ in order to get the four numbers in proportion?

[3]

Explanation:

Let we need to add x to make these numbers in proportion then -

According to the question -

$$(4 + x):(6 + x):\because(8 + x):(1 + x)$$

$$\Rightarrow (4 + x)(11 + x) = (8 + x)(6 + x) \Rightarrow 44 + 4x + 11x + x^2 = 48 + 8x + 6x + x^2 \Rightarrow 44$$

So we need to add 4 in all the numbers to get them in proportion.

(iii) Using ruler and compass construct a triangle ABC in which $AB = 6\ cm, \angle BAC = 120^\circ$ and $= 5\ cm$. Construct a circle passing through A, B and C. Measure and write down the radius of the circle.

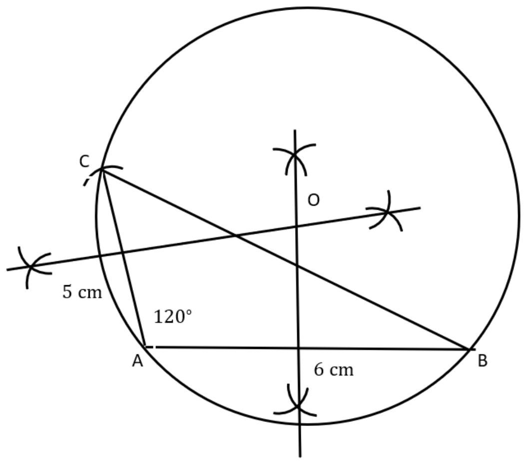
[4]

Answer: 5.5 cm

Explanation:

Steps of construction:

1. Draw a line segment AB of length 6 cm using a ruler.
2. At point A, construct an angle of 120 degrees using a compass.
3. From point A, draw a line segment AC of length 5 cm that intersects the angle at point C.
4. Connect points B and C with a line segment to form triangle ABC.
5. To construct the circumcircle of the triangle, place the compass on any point on the triangle and draw a circle that passes through all three vertices (A, B, and C). This circle is the circumcircle of the triangle.



By measuring the length of the radius it is 5.5 cm.

Question 9

(i) Using Componendo and Dividendo solve for x;

$$\frac{\sqrt{2x+2}+\sqrt{2x-1}}{\sqrt{2x+2}-\sqrt{2x-1}} = 3$$

[3]

Explanation:

We begin by simplifying the given equation using componendo and dividendo as follows:

$$\frac{\sqrt{2x+2}+\sqrt{2x-1}}{\sqrt{2x+2}-\sqrt{2x-1}} = 3$$

Adding 1 to both the numerator and denominator of the left-hand side, we get:

$$\frac{(\sqrt{2x+2}+\sqrt{2x-1})+(\sqrt{2x+2}-\sqrt{2x-1})}{\sqrt{2x+2}-\sqrt{2x-1}+1} = \frac{4}{2}$$

Simplifying the expression further, we get:

$$\frac{2\sqrt{2x+2}}{\sqrt{2x+2}-\sqrt{2x-1}+1} = 2$$

Multiplying both sides by $\sqrt{2x+2} - \sqrt{2x-1} + 1$, we get:

$2\sqrt{2x+2} = 2(\sqrt{2x+2} - \sqrt{2x-1} + 1)$

Simplifying, we get:

$\sqrt{2x+2} = (\sqrt{2x+2} - \sqrt{2x-1} + 1)$

Subtracting $\sqrt{2x+2}$ from both sides, we get:

$-\sqrt{2x-1} + 1 = 0$

Adding $\sqrt{2x-1}$ to both sides, we get:

$1 = \sqrt{2x-1}$

Squaring both sides, we get:

$1^2 = (\sqrt{2x-1})^2$

$1 = 2x - 1$

$2 = 2x$

$x = 1$

Therefore, the solution to the equation $\frac{\sqrt{2x+2}+\sqrt{2x-1}}{\sqrt{2x+2}-\sqrt{2x-1}} = 3$ is $x = 1$.

(ii) Which term of the Arithmetic Progression (A.P.) 15, 30, 45, 60... is 300 ? Hence find the sum of all the terms of the Arithmetic Progression (A.P.)

[3]

Answer: 20^{th} term, 3150

Explanation:

To find the term of an Arithmetic Progression (A.P.), we use the formula:

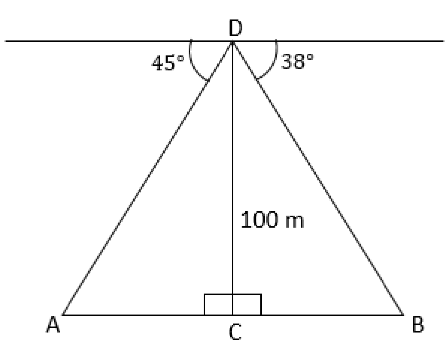
$a_n = a_1 + (n-1)d$

where a_n is the nth term of the A.P., a_1 is the first term of the A.P., n is the number of terms, and d is the common difference between consecutive terms.

Here, we have the first term a_1 = 15, and the common difference d = 30 - 15 = 15.

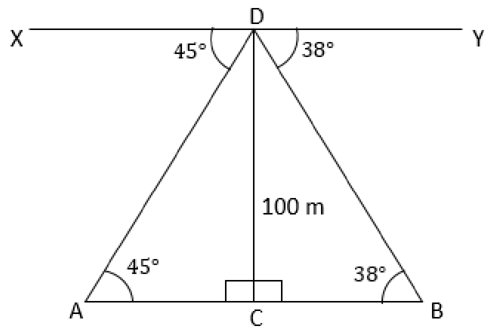
We need to find the term of the A.P. which is equal to 300. So, we substitute these values into the formula:
300 = 15 + (n-1)15
300 - 15 = 15(n-1)
285 = 15(n-1)
n-1 = 19
n = 20
Therefore, the 20th term of the A.P. is 300.
To find the sum of all the terms of an A.P., we use the formula:
S_n = (n/2)(a_1 + a_n)
where S_n is the sum of the first n terms of the A.P.
Substituting the values, we get:
S_n= (20/2)(15 + 300)
S_n = 10(315)
S_n = 315
Therefore, the sum of all the terms of the A.P. is 3150.

(iii) From the top of a tower $100\ m$ high a man observes the angles of depression of two ships A and B, on opposite sides of the tower as 45° and 38° respectively. If the foot of the tower and the ships are in the same horizontal line find the distance between the two ships A and B to the nearest metre.
(Use Mathematical Tables for this question.)



[4]

Answer: 228.04 m
Explanation:



In the figure,
A and B are the positions of the ships A and B.
CD = 100 m.
The angles of depression are ∠DAC and ∠DBC.
Since the line XY is parallel to AB, we have
∠YDB = ∠DBC = 38° and ∠XDA = ∠DAC = 45°
To find: Distance between the two ships
From right △ACD, tan 45° = CD/AC
1 = 100/AC
or AC = 100
From right △BCD:
tan 38° = CD/CB
0.781 = 100/BC
or BC = 128.04
Now, AB = AC + CB
= 100 + 128.04
= 228.04
Hence distance between the ships is 228.04 meters.

Question 10

(i) Factorize completely using factor theorem: $2x^3 - x^2 - 13x - 6$

[4]

Explanation:

For factorizing this cubic expression, no identity is useful.
Thus, we need to use factorization by regrouping terms.
We can write, $-x^2 = x^2 - 2x^2$ and $-13x = -x - 12x$

$$\Rightarrow 2x^3 - x^2 - 13x - 6 = 2x^3 + x^2 - 2x^2 - x - 12x - 6$$

We have, $-2x^2 = (-x)\times 2x$ and $-12x = (-6)\times 2x$

$$\Rightarrow 2x^3 - x^2 - 13x - 6 = 2x^3 + x^2 + (-x)\times 2x - x + (-6)\times 2x - 6$$

Observe that x^2 is common for the first two terms, $-x$ is common for the next two terms and -6 is common for the last two terms.

$$\Rightarrow 2x^3 - x^2 - 13x - 6 = x^2(2x+1) + (-x)(2x+1) + (-6)(2x+1)$$

Now, $(2x+1)$ is the common term.

$$\therefore 2x^3 - x^2 - 13x - 6 = (2x+1)\left(x^2 - x - 6\right)$$

So, one factor of the given expression is $(2x+1)$. Now, we need to factorize $\left(x^2 - x - 6\right)$.

For factorizing, this expression, split the middle term in such a way that the product of the coefficients of the new terms is equal to the product of the coefficients of the first and last terms in the expression.

Here, product of co-effs of first and last terms = $1\times(-6) = -6$

So, if the middle term $-x$ is split into two terms say ax, bx, then $a + b = -1$ and $ab = -6$.

Observe that values -3 and 2 satisfy these equations.

$$\Rightarrow x^2 - x - 6 = x^2 - 3x + 2x - 6$$

Observe that x is common for the first two terms and 2 is common for the next two terms.

$$\Rightarrow x^2 - 3x + 2x - 6 = x(x-3) + 2(x-3)$$

Now, $(x-3)$ is the common term.

$$\Rightarrow x^2 - x - 6 = (x-3)(x+2)$$

Thus, the factors of $x^2 - x - 6$ are $(x-3)$ and $(x+2)$.

$$\therefore 2x^3 - x^2 - 13x - 6 = (2x+1)(x-3)(x+2)$$

Hence, the factors of $2x^3 - x^2 - 13x - 6$ are $(2x+1), (x-3)$ and $(x+2)$.

(ii) Use **graph paper** to answer this question.

During a medical checkup of 60 students in a school, weights were recorded as follows:

Weight (in kg)	Number of Students
$28 - 30$	2
$30 - 32$	4
$32 - 34$	10
$34 - 36$	13
$36 - 38$	15
$38 - 40$	9
$40 - 42$	5
$42 - 44$	2

Taking $2\ cm = 2\ kg$ along one axis and $2\ cm = 10$ students along the other axis draw an ogive. Use your graph to find the:

(a) median

(b) upper Quartile

(c) number of students whose weight is above $37\ kg$.

[6]

Explanation:

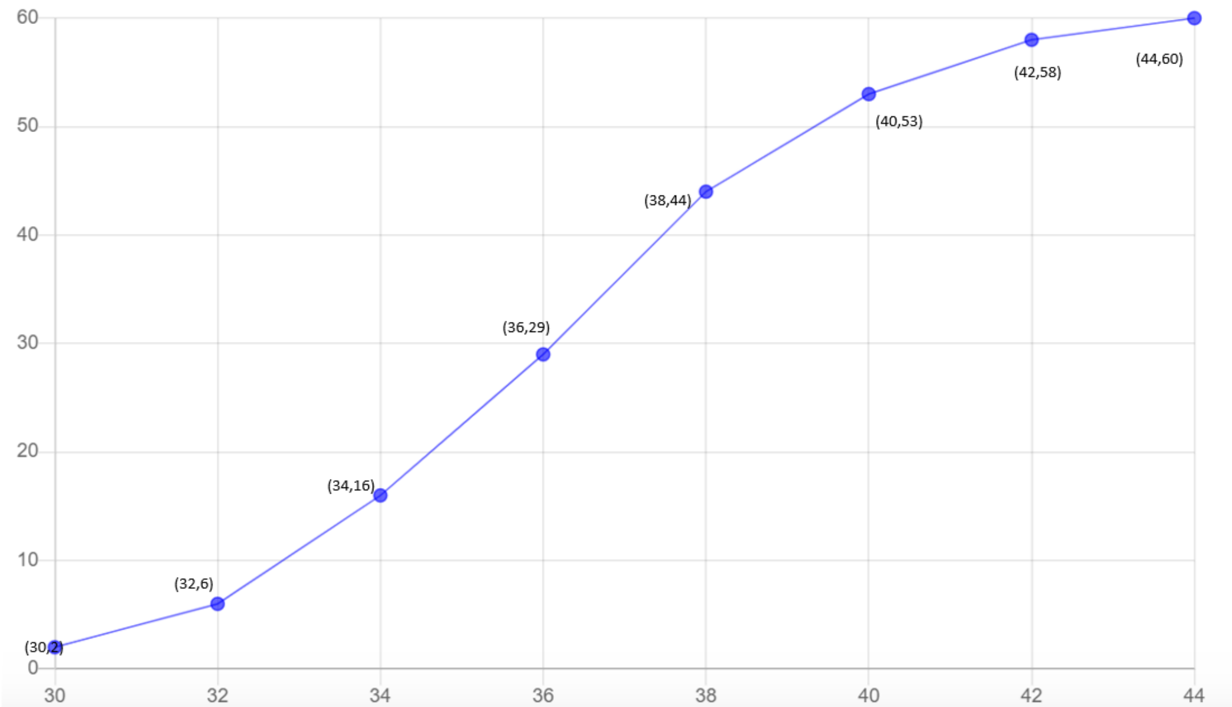
To draw the ogive, we first need to calculate the cumulative frequencies. The cumulative frequency is the total number of students up to and including a certain weight.

Weight (in kg)	Number of Students (f)	Cumulative Frequency (cf)
28 – 30	2	2
30 – 32	4	6
32 – 34	10	16
34 – 36	13	29
36 – 38	15	44
38 – 40	9	53
40 - 42	5	58
42 - 44	2	60

We can now plot the cumulative frequencies on the graph paper. The horizontal axis represents the weight (in kg), and the vertical axis represents the cumulative frequency (in number of students).

We use 2cm=2kg along the horizontal axis and 2cm=10 students along the vertical axis.

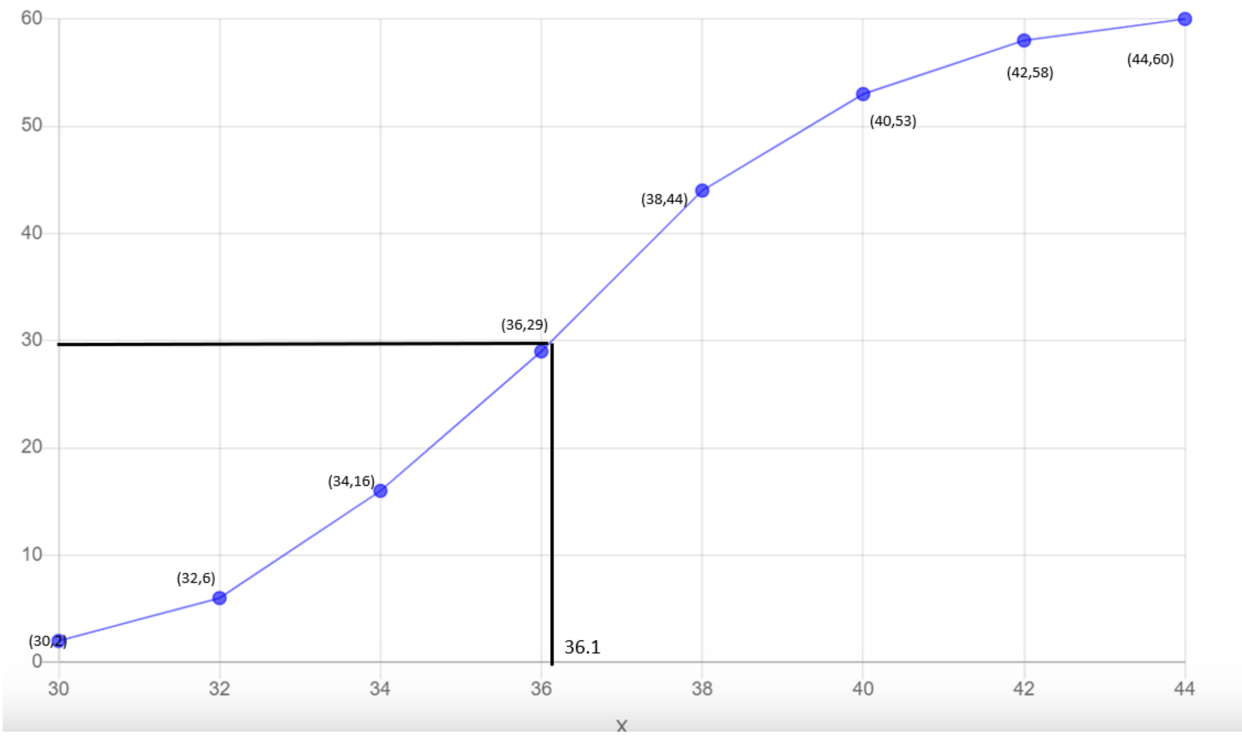
Ogive graph for weight distribution



(a) Using the graph drawn, we get

Median = $\left(\frac{n}{2}\right)^{th}$ term, where $n = 60$

= 30^{th} term = 36.15 kg (Approximately)



(b) Upper quartile (Q_3) :

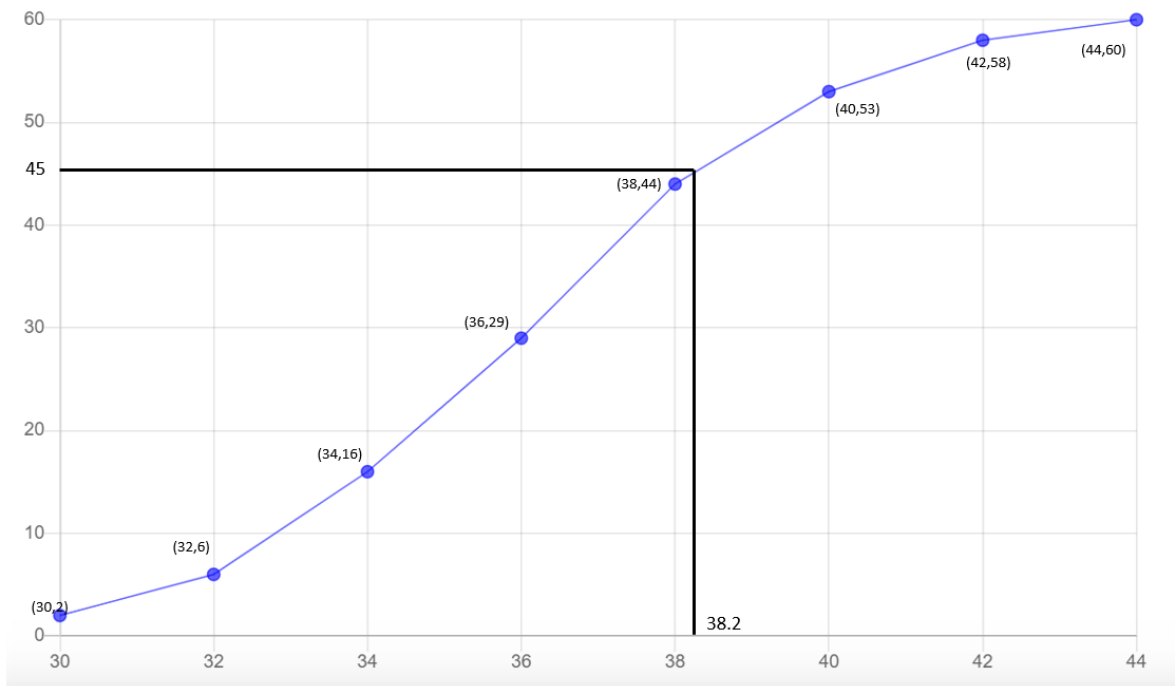
Here, $n = 60$

Q_3 class : $\left(\frac{3n}{4}\right)^{th}$

$= \left(\frac{3 \cdot 60}{4}\right)^{th}$ term

$= (45)^{th}$ term

= 38.2



(c) The number of students whose weight is above 37 kg is the total number of students (60) minus the cumulative frequency at the weight of 37 kg (which is 29). Therefore, the number of students whose weight is above 37 kg is:

60 - 29 = 31 students.