LINEAR EQUATION IN ONE VARIABLE

LINEAR EQUATION :

Equation : A statement of equality which contains one or more unknown quantity or variable (literals) is called an equation. For example -

Ex.1 $3x + 7 = 12$, $\frac{5}{2}x - 9 = 1$, $x^2 + 1 = 5$ and $\frac{x}{3} + 5 = \frac{x}{2} - 3$ are equations in one variable x.

Ex.2 $2x + 3y = 15$, $7x - \frac{y}{3} = 3$ are equations in two variables x and y.

Linear Equation : An equation involving only linear polynomials is called a linear equation.

Ex.3 $3x - 2 = 7$, $\frac{3}{2}x + 9 = \frac{1}{2}$, $\frac{y}{3} + \frac{y-2}{4} = 5$ are linear equations in one variable, because the highest power of the variable in each equation is one whereas the equations $3x^2 - 2x + 1 = 0$, $y^2 - 1 = 8$ are not linear equations, because the highest power of the variable in each equation is not one.

Solution of a Linear Equation :

Solution : A value of the variable which when substituted for the variable in an equation, makes L.H.S. = R.H.S. is said to satisfy the equation and is called a solution or a root of the equation.

◈ **Rules for Solving Linear Equations in One Variable :**

Rule-1 Same quantity (number) can be added to both sides of an equation without changing the equality.

Rule-2 Same quantity can be subtracted from both sides of an equation without changing the equality.

Rule-3 Both sides of an equation may be multiplied by the same non-zero number without changing the equality.

Rule-4 Both sides of an equation may be divided by the same non-zero number without changing the equality.

Solving Equations having Variable Terms on One Side and Number(s) on the Other Side :

❖ EXAMPLES ❖

Ex.1 Solve the equation : $\frac{x}{5} + 11 = \frac{1}{15}$ and check the result.

Sol. We have,

$$\frac{x}{5} + 11 = \frac{1}{15}$$

$$\Rightarrow \frac{x}{5} + 11 - 11 = \frac{1}{15} - 11 \quad \text{[Subtracting 11 from both sides]}$$

$$\Rightarrow \frac{x}{5} = \frac{1}{15} - 11$$

$\Rightarrow \frac{x}{5} = \frac{1-165}{15}$

$\Rightarrow \frac{x}{5} = -\frac{164}{15}$

$\Rightarrow 5 \times \frac{x}{5} = 5 \times -\frac{164}{15}$

$\Rightarrow x = -\frac{164}{3}$

Thus, $x = -\frac{164}{3}$ is the solution of the given equation.

Check Substituting $x = \frac{-164}{3}$ in the given equation, we get

L.H.S. $= \frac{x}{5} + 11$

$= \frac{-164}{3} \times \frac{1}{5} + 11 = \frac{-164}{15} + 11 = \frac{-164+165}{15} = \frac{1}{15}$ and,

R.H.S. $= \frac{1}{15}$

$\therefore$ L.H.S. = R.H.S. for $x = \frac{-164}{3}$

Hence, $x = \frac{-164}{3}$ is the solution of the given equation.

Ex.2 Solve : $\frac{1}{3}x - \frac{5}{2} = 6$

Sol. We have,

$\frac{1}{3}x - \frac{5}{2} = 6$

$\Rightarrow \frac{1}{3}x - \frac{5}{2} + \frac{5}{2} = 6 + \frac{5}{2}$ [Adding $\frac{5}{2}$ on both sides]

$\Rightarrow \frac{1}{3}x = 6 + \frac{5}{2}$

$\Rightarrow \frac{1}{3}x = \frac{12+5}{2}$

$\Rightarrow \frac{1}{3}x = \frac{17}{2}$

$\Rightarrow 3 \times \frac{1}{3}x = 3 \times \frac{17}{2}$ [Multiplying both sides by 3]

$\Rightarrow x = \frac{51}{2}$

Thus, $x = \frac{51}{2}$ is the solution of the given equation.

Check Substituting $x = \frac{51}{2}$ in the given equation, we get

$$\text{L.H.S.} = \frac{1}{3}x - \frac{5}{2} = \frac{1}{3} \times \frac{51}{2} - \frac{5}{2} = \frac{17}{2} - \frac{5}{2}$$

$$= \frac{17-5}{2} = \frac{12}{2} = 6$$

and,

R.H.S. = 6

$\therefore$ L.H.S. = R.H.S. for $x = \frac{51}{2}$

Hence, $x = \frac{51}{2}$ is the solution of the given equation.

Ex.3 Solve : $\frac{x}{2} - \frac{x}{3} = 8$

Sol. We have, $\frac{x}{2} - \frac{x}{3} = 8$

LCM of denominators 2 and 3 on L.H.S. is 6.

Multiplying both sides by 6, we get

$\Rightarrow 3x - 2x = 6 \times 8$

$\Rightarrow x = 48$

Check Substituting $x = 48$ in the given equation, we get

$$\text{L.H.S.} = \frac{x}{2} - \frac{x}{3} = \frac{48}{2} - \frac{48}{3} = 24 - 16 = 8 \text{ and,}$$

R.H.S. = 8

$\therefore$ L.H.S. = R.H.S. for $x = 48$

Hence, $x = 48$ is the solution of the given equation.

Ex.4 Solve : $\frac{x}{2} + \frac{x}{3} - \frac{x}{4} = 7$

Sol. We have, $\frac{x}{2} + \frac{x}{3} - \frac{x}{4} = 7$

LCM of denominators 2, 3, 4 on L.H.S. is 12. Multiplying both sides by 12, we get

$6x + 4x - 3x = 7 \times 12$

$\Rightarrow 7x = 7 \times 12$

$\Rightarrow 7x = 84$

$\Rightarrow \frac{7x}{7} = \frac{84}{7}$ [Dividing both sides by 7]

$\Rightarrow x = 12$

Check Substituting $x = 12$ in the given equation, we get

$$\text{L.H.S.} = \frac{12}{2} + \frac{12}{3} - \frac{12}{4} = 6 + 4 - 3 = 7 \text{ and, R.H.S.} = 7$$

$\therefore$ L.H.S. = R.H.S. for $x = 12$.

Hence, $x = 12$ is the solution of the given equation.

Ex.5 Solve : $\frac{y-1}{3} - \frac{y-2}{4} = 1$

Sol. We have, $\frac{y-1}{3} - \frac{y-2}{4} = 1$

LCM of denominators 3 and 4 on L.H.S. is 12.

Multiplying both sides by 12, we get

$$12 \times \left(\frac{y-1}{3}\right) - 12 \times \left(\frac{y-2}{4}\right) = 12 \times 1$$

$\Rightarrow 4(y - 1) - 3(y - 2) = 12$

$\Rightarrow 4y - 4 - 3y + 6 = 12 \Rightarrow 4y - 3y - 4 + 6 = 12$

$\Rightarrow y + 2 = 12$

$\Rightarrow y + 2 - 2 = 12 - 2$ [Subtracting 2 from both sides]

$\Rightarrow y = 10$

Thus, $y = 10$ is the solution of the given equation.

Check Substituting $y = 10$ in the given equation, we get

$$\text{L.H.S.} = \frac{10-1}{3} - \frac{10-2}{4} = \frac{9}{3} - \frac{8}{4} = 3 - 2 = 1$$

and, R.H.S. = 1

$\therefore$ L.H.S. = R.H.S. for $y = 10$.

Hence, $y = 10$ is the solution of the given equation.

Transposition Method for Solving Linear Equations in One Variable

The transposition method involves the following steps:

Step-I Obtain the linear equation.

Step-II Identify the variable (unknown quantity) and constants(numerals).

Step-III Simplify the L.H.S. and R.H.S. to their simplest forms by removing brackets.

Step-IV Transpose all terms containing variable on L.H.S. and constant terms on R.H.S.
Note that the sign of the terms will change in shifting them from L.H.S. to R.H.S. and vice-versa.

Step-V Simplify L.H.S. and R.H.S. in the simplest form so that each side contains just one term.

Step-VI Solve the equation obtained in step V by dividing both sides by the coefficient of the variable on L.H.S.

❖ EXAMPLES ❖

Ex.1 Solve : $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$

Sol. We have, $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$

The denominators on two sides are 2, 5, 3 and 4. Their LCM is 60. Multiplying both sides of the given equation by 60, we get

$$60 \times \left(\frac{x}{2} - \frac{1}{5}\right) = 60\left(\frac{x}{3} + \frac{1}{4}\right)$$

$\Rightarrow 60 \times \frac{x}{2} - 60 \times \frac{1}{5} = 60 \times \frac{x}{3} + 60 \times \frac{1}{4}$

$\Rightarrow 30x - 12 = 20x + 15$

$\Rightarrow 30x - 20x = 15 + 12$ [On transposing 20x to

LHS and –12 to RHS]

$\Rightarrow 10x = 27$

$\Rightarrow x = \frac{27}{10}$

Hence, $x = \frac{27}{10}$ is the solution of the given equation.

Check Substituting $x = \frac{27}{10}$ in the given equation, we get

$$\text{L.H.S.} = \frac{x}{2} - \frac{1}{5} = \frac{27}{10} \times \frac{1}{2} - \frac{1}{5} = \frac{27}{10} - \frac{1}{5}$$

$$= \frac{27 - 1 \times 4}{20} = \frac{27 - 4}{20} = \frac{23}{20}$$

and,

$$\text{R.H.S.} = \frac{x}{3} + \frac{1}{4} = \frac{27}{10} \times \frac{1}{3} + \frac{1}{4}$$

$$= \frac{9}{10} + \frac{1}{4} = \frac{9 \times 2 + 1 \times 5}{20} = \frac{18 + 5}{20} = \frac{23}{20}$$

Thus, for $x = \frac{27}{10}$, we have L.H.S. = R.H.S.

Ex.2 Solve : $x + 7 - \frac{8x}{3} = \frac{17}{6} - \frac{5x}{8}$

Sol. We have, $x + 7 - \frac{8x}{3} = \frac{17}{6} - \frac{5x}{8}$

The denominators on two sides are 3, 6 and 8. Their LCM is 24.
Multiplying both sides of the given equation 24, we get

$$24\left(x + 7 - \frac{8x}{3}\right) = 24\left(\frac{17}{6} - \frac{5x}{8}\right)$$

$\Rightarrow 24x + 24 \times 7 - 24 \times \frac{8x}{3} = 24 \times \frac{17}{6} - 24 \times \frac{5x}{8}$

$\Rightarrow 24x + 168 - 64x = 68 - 15x$

$\Rightarrow 168 - 40x = 68 - 15x$

$\Rightarrow -40x + 15x = 68 - 168$ [Transposing –15x to LHS and 168 to RHS]

$\Rightarrow -25x = -100$

$\Rightarrow 25x = 100$

$\Rightarrow x = \frac{100}{25}$ [Dividing both sides by 25]

$\Rightarrow x = 4$

Thus, x = 4 is the solution of the given equation.

Check Substituting x = 4 in the given equation, we get

$$\text{L.H.S.} = x + 7 - \frac{8x}{3} = 4 + 7 - \frac{8 \times 4}{3}$$

$$= 11 - \frac{32}{3} = \frac{33 - 32}{3} = \frac{1}{3}$$

and, $\text{R.H.S.} = \frac{17}{6} - \frac{5x}{8} = \frac{17}{6} - \frac{5 \times 4}{8} = \frac{17}{6} - \frac{5}{2}$

$$= \frac{17 - 15}{6} = \frac{2}{6} = \frac{1}{3}$$

Thus, for x = 4, we have L.H.S. = R.H.S.

Ex.3 Solve : $\frac{3t-2}{4}-\frac{2t+3}{3}=\frac{2}{3}-t$

Sol. We have, $\frac{3t-2}{4}-\frac{2t+3}{3}=\frac{2}{3}-t$

The denominators on two sides are 4, 3 and 3. Their LCM is 12.

Multiplying both sides of the given equation by 12, we get

$$12\left(\frac{3t-2}{4}\right)-12\left(\frac{2t+3}{3}\right)=12\left(\frac{2}{3}-t\right)$$

$\Rightarrow 3(3t-2)-4(2t+3)=12\left(\frac{2}{3}-t\right)$

$\Rightarrow 9t-6-8t-12=12\times\frac{2}{3}-12t$

$\Rightarrow 9t-6-8t-12=8-12t$

$\Rightarrow t-18=8-12t$

$\Rightarrow t+12t=8+18$ [Transposing $-12t$ to LHS and -18 to RHS]

$\Rightarrow 13t=26$

$\Rightarrow t=\frac{26}{13}$ [Dividing both sides by 13]

$\Rightarrow t=2$

Check Substituting $t=2$ on both sides of the given equation, we get

$$\text{L.H.S.}=\frac{3t-2}{4}-\frac{2t+3}{3}$$

$$=\frac{3\times2-2}{4}-\frac{2\times2+3}{3}=\frac{6-2}{4}-\frac{4+3}{3}$$

$$\frac{4}{4}-\frac{7}{3}=1-\frac{7}{3}=\frac{3-7}{3}=\frac{-4}{3}$$

and,

$$\text{R.H.S.}=\frac{2}{3}-t=\frac{2}{3}-2=\frac{2-6}{4}=\frac{-4}{3}$$

Thus, for $t=2$, we have L.H.S. = R.H.S.

Ex.4 Solve : $\frac{x+2}{6}-\left(\frac{11-x}{3}-\frac{1}{4}\right)=\frac{3x-4}{12}$

Sol. We have, $\frac{x+2}{6}-\left(\frac{11-x}{3}-\frac{1}{4}\right)=\frac{3x-4}{12}$

The denominators on two sides of the given equation are 6, 3, 4 and 12. Their LCM is 24.

Multiplying both sides of the given equation by 24, we get

$$24\left(\frac{x+2}{6}\right)-24\left(\frac{11-x}{3}-\frac{1}{4}\right)=24\left(\frac{3x-4}{12}\right) \Rightarrow 4(x+2)-24\left(\frac{11-x}{3}\right)+24\times\frac{1}{4}=2(3x-4)$$

$\Rightarrow 4(x+2)-8(11-x)+6=2(3x-4)$

$\Rightarrow 4x+8-88+8x+6=6x-8$

$\Rightarrow 12x-74=6x-8$

$\Rightarrow 12x-6x=74-8$ [Transposing $6x$ to LHS and -74 to RHS]

$\Rightarrow 6x=66$

$\Rightarrow x=\frac{66}{6}$ [Dividing both sides by 6]

$\Rightarrow x=11$

Check Substituting x = 11 on both sides of the given equation, we get

$$\text{L.H.S.} = \frac{x+2}{6} - \left(\frac{11-x}{3} - \frac{1}{4}\right)$$

$$= \frac{11+2}{6} - \left(\frac{11-11}{3} - \frac{1}{4}\right) = \frac{13}{6} - \left(0 - \frac{1}{4}\right)$$

$$= \frac{13}{6} + \frac{1}{4} = \frac{26+3}{12} = \frac{29}{12}$$

and, $\text{R.H.S.} = \frac{3x-4}{12} = \frac{3\times 11-4}{12} = \frac{33-4}{12} = \frac{29}{12}$

Thus, for x = 11, we have L.H.S. = R.H.S.

Ex.5 Solve : $x - \frac{2x+8}{3} = \frac{1}{4}\left(x - \frac{2-x}{6}\right) - 3$

Sol. We have,

$$x - \frac{2x+8}{3} = \frac{1}{4}\left(x - \frac{2-x}{6}\right) - 3$$

$$\Rightarrow x - \frac{2x+8}{3} = \frac{x}{4} - \frac{2-x}{24} - 3$$

The denominators on the two sides of this equation are 3, 4 and 24. Their LCM is 24. Multiplying both sides of this equation by 24, we get

$$24x - 24\left(\frac{2x+8}{3}\right) = 24 \times \frac{x}{4} - 24\left(\frac{2-x}{24}\right) - 3 \times 24$$

$\Rightarrow 24x - 8(2x + 8) = 6x - (2 - x) - 72$

$\Rightarrow 24x - 16x - 64 = 6x - 2 + x - 72$

$\Rightarrow 8x - 64 = 7x - 74$

$\Rightarrow 8x - 7x = 64 - 74$ [Transposing 7x to LHS and – 64 to RHS]

$\Rightarrow x = -10$

Thus, x = – 10 is the solution of the given equation.

Check Putting $x = -\frac{2x+8}{3} = -10 - \frac{2\times -10+8}{3}$

$$= -10 - \frac{-20+8}{3} = -10 - \left(\frac{-12}{3}\right) = -10 + 4 = -6$$

and,

$$\text{R.H.S.} = \frac{1}{4}\left(x - \frac{2-x}{6}\right) - 3 = \frac{1}{4}\left(-10 - \frac{2+10}{6}\right) - 3$$

$$= \frac{1}{4}(-10 - 2) - 3 = -3 - 3 = -6$$

Thus, L.H.S. = R.H.S. for x = – 10.

Ex.6 Solve : $0.16(5x - 2) = 0.4x + 7$

Sol. We have,

$0.16(5x - 2) = 0.4x + 7$

$\Rightarrow 0.8x - 0.32 = 0.4x + 7$ [Expanding the bracket on LHS]

$\Rightarrow 0.8x - 0.4x = 0.32 + 7$ [Transposing 0.4x to LHS and –0.32 to RHS]

$\Rightarrow 0.4x = 7.32$

$$\Rightarrow \frac{0.4x}{0.4} = \frac{7.32}{0.4}$$

$$\Rightarrow x = \frac{732}{40} \quad \Rightarrow x = \frac{183}{10} = 18.3$$

Hence, x = 18.3 is the solution of the given equation.

Ex.7 Solve : $\frac{2}{5x} - \frac{5}{3x} = \frac{1}{15}$

Sol. We have, $\frac{2}{5x}-\frac{5}{3x}=\frac{1}{15}$

Multiplying both sides by 15x, the LCM of 5x and 3x, we get

$15x \times \frac{2}{5x}-15x \times \frac{5}{3x}=15x \times \frac{1}{15}$

$\Rightarrow 6 - 25 = x \Rightarrow -19 = x \Rightarrow x = -19$

Hence, $x = -19$ is the solution of the given equation.

Ex.8 Solve : $\frac{17-3x}{5}-\frac{4x+2}{3}=5-6x+\frac{7x+14}{3}$

Sol. Multiplying both sides by 15 i.e. the LCM of 5 and 3, we get

$3(17 - 3x) - 5(4x + 2) = 15(5 - 6x) + 5(7x + 14)$

$\Rightarrow 51 - 9x - 20x - 10 = 75 - 90x + 35x + 70$

$\Rightarrow 41 - 29x = 145 - 55x$

$\Rightarrow -29x + 55x = 145 - 41$

$\Rightarrow 26x = 104$

$\Rightarrow \frac{26x}{26}=\frac{104}{26}$

$\Rightarrow x = 4$

Thus, $x = 4$ is the solution of the given equation.

Ex.9 Solve : $\frac{x+2}{3}-\frac{x+1}{5}=\frac{x-3}{4}-1$

Sol. Multiplying both sides by 60 i.e. the LCM of 3, 5, and 4, we get

$20(x + 2) - 12(x + 1) = 15(x - 3) - 1 \times 60$

$\Rightarrow 20x + 40 - 12x + 12 = 15x - 45 - 60$

$\Rightarrow 8x + 28 = 15x - 105$

$\Rightarrow 8x - 15x = 105 - 28$

$\Rightarrow -7x = -133$

$\Rightarrow \frac{-7x}{-7}=\frac{-133}{-7}$ [Dividing both sides by –7]

$\Rightarrow x = \frac{133}{7} = 19$

Thus, $x = 19$ is the solution of the given equation.

Ex.10 Solve : $(2x + 3)^2 + (2x - 3)^2 = (8x + 6)(x - 1) + 22$

Sol. We have,

$(2x + 3)^2 + (2x - 3)^2 = (8x + 6)(x - 1) + 22$

$\Rightarrow 2\{(2x)^2 + 3^2\}$

$= x(8x + 6) - (8x + 6) + 22$ [Using:$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$ on LHS]

$\Rightarrow 2(4x^2 + 9) = 8x^2 + 6x - 8x - 6 + 22$

$\Rightarrow 8x^2 + 18 = 8x^2 - 2x - + 16$

$\Rightarrow 8x^2 - 8 + 2x = 16 = - 18$

$\Rightarrow 2x = - 2$

$\Rightarrow x = - 1$

Hence, $x = - 1$ is the solution of the given equation.

Cross-Multiplication Method for Solving Equations of the form :

$$\frac{ax+b}{cx+d}=\frac{m}{n}$$

$\Rightarrow n(ax + b) = m(cx + d)$

❖ EXAMPLES ❖

Ex.1 Solve : $\frac{2x+1}{3x-2}=\frac{9}{10}$

Sol. We have, $\frac{2x+1}{3x-2}=\frac{9}{10}$

$\Rightarrow 10 \times (2x + 1) = 9 \times (3x - 2)$
[By cross-multiplication]

$\Rightarrow 20x + 10 = 27x - 18$

$\Rightarrow 20x - 27x = -18 - 10$ [Using transposition]

$\Rightarrow -7x = -28$

$\Rightarrow \frac{-7x}{-7}=\frac{-28}{-7}$ [Dividing both sides by –7]

$\Rightarrow x = 4$

Hence, $x = 4$ is the solution of the given equation.

Ex.2 Solve : $\frac{3x+5}{2x+7}=4$

Sol. We have, $\frac{3x+5}{2x+7}=4$

$\Rightarrow \frac{3x+5}{2x+7}=\frac{1}{4}$

$\Rightarrow 1 \times (3x + 5) = 4 \times (2x + 7)$
[By cross-multiplication]

$\Rightarrow 3x + 5 = 8x + 28$

$\Rightarrow 3x - 8x = 28 - 5$ [Using transposition]

$\Rightarrow -5x = 23$

$\Rightarrow \frac{-5x}{-5}=\frac{23}{-5}$

$\Rightarrow x = -\frac{23}{5}$

Hence, $x = -\frac{23}{5}$ is the solution of the given equation.

Ex.3 Solve : $\frac{17(2-x)-5(x+12)}{1-7x}=8$

Sol. We have, $\frac{17(2-x)-5(x+12)}{1-7x}=8$

$\Rightarrow \frac{34-17x-5x+60}{1-7x}=\frac{8}{1}$

$\Rightarrow \frac{-22x-26}{1-7x}=\frac{8}{1}$

$\Rightarrow 1 \times (-22x - 26) = 8 \times (1 - 7x)$
[By cross-multiplication]

$\Rightarrow -22x - 26 = 8 - 56x$

$\Rightarrow -22x + 56x = 8 + 26$

$\Rightarrow 34x = 34$

$\Rightarrow \frac{34x}{34}=\frac{34}{34}$

Hence, $x = 1$ is the solution of the given equation.

Ex.4 Solve : $\frac{x+b}{a-b}=\frac{x-b}{a+b}$

Sol. We have, $\frac{x+b}{a-b}=\frac{x-b}{a+b}$

$\Rightarrow (x + b) \times (a + b) = (x - b) \times (a - b)$

[By cross-multiplication]

$\Rightarrow x (a + b) + b(a + b) = x(a - b) - b(a - b)$

$\Rightarrow ax + bx + ba + b^2 = ax - bx - ba - b^2$

$\Rightarrow ax + bx - ax + bx = - bx + b^2 - ba - b^2$

$\Rightarrow 2bx = - 2ba$

$\Rightarrow \frac{2bx}{2b}=-\frac{2ab}{2b}$

$\Rightarrow x = - a$

Hence, $x = - a$ is the solution of the given equation.

Ex.5 Solve : $\frac{(4+x)(5-x)}{(2+x)(7-x)}=1$

Sol. We have, $\frac{(4+x)(5-x)}{(2+x)(7-x)}=1$

$\Rightarrow \frac{20-4x+5x-x^2}{14-2x+7x-x^2}=1$

$\Rightarrow \frac{20+x-x^2}{14+5x-x^2}=1$

$\Rightarrow 20 + x - x^2 = 14 + 5x - x^2$

[By cross-multiplication]

$\Rightarrow x - x^2 = - 5x + x^2 = 14 - 20$

$\Rightarrow - 4x = - 6$

$\Rightarrow \frac{-4x}{-4}=\frac{-6}{-4}$

$\Rightarrow x=\frac{3}{2}$

Hence, $x=\frac{3}{2}$ is the solution of the given equation.

Ex.6 Solve : $\frac{1}{x+1}+\frac{1}{x+2}=\frac{2}{x+10}$

Sol. We have, $\frac{1}{x+1}+\frac{1}{x+2}=\frac{2}{x+10}$

Multiplying both sides by $(x + 1)(x + 2)(x + 10)$

i.e., the LCM of $x + 1$, $x + 2$ and $x + 10$, we get

$\frac{(x+1)(x+2)(x+10)}{x+1}+\frac{(x+1)(x+2)(x+10)}{x+2}$

$=\frac{2(x+1)(x+2)(x+10)}{x+10}$

$\Rightarrow (x + 2)(x + 10) = (x + 1)(x + 10)$

$= 2(x + 1)(x + 2)$

$\Rightarrow x^2 + 2x + 10x + 20 + x^2 + 10x + x + 10$

$= 2(x^2 + x + 2x + 2)$

$\Rightarrow 2x^2 + 23x + 30 = 2(x^2 + 3x + 2)$

$\Rightarrow 2x^2 + 23x + 30 = 2x^2 + 6x + 4$

$\Rightarrow 2x^2 + 23x - 2x^2 + 6x = 4 - 30$

$\Rightarrow 17x = - 26$

$\Rightarrow x = -\frac{26}{17}$

Hence, $x = -\frac{26}{17}$ is the solution of the given equation.

Ex.7 Solve : $\frac{6x^2+13x-4}{2x+5} = \frac{12x^2+5x-2}{4x+3}$

Sol. We have, $\frac{6x^2+13x-4}{2x+5} = \frac{12x^2+5x-2}{4x+3}$

$\Rightarrow (6x^2 + 13x - 4)(4x + 3)$
$= (12x^2 + 5x - 2)(2x + 5)$
[By cross-multiplication]

$\Rightarrow (6x^2 + 13x - 4) \times 4x + (6x^2 + 13x - 4) \times 3$
$= (12x^2 + 5x - 2) \times 2x + (12x^2 + 5x - 2) \times 5$

$\Rightarrow 24x^3 + 52x^2 - 16x + 18x^2 + 39x^2 - 12$
$= 24x^3 + 10x^2 - 4x + 60x^2 + 25x - 10$

$\Rightarrow 24x^3 + 70x^2 + 23x - 12$
$= 24x^3 + 70x^2 + 12x - 10$

$\Rightarrow 24x^3 + 70x^2 + 23x - 24x^3 - 70x^2 - 21x$
$= -10 + 12$

$\Rightarrow 2x = 2$

$\Rightarrow x = 1$

Hence, $x = 1$ is the solution of the given equation.

Ex.8 Solve : $\frac{4x+17}{18} - \frac{13x-2}{17x-32} + \frac{x}{3} = \frac{7x}{12} - \frac{x+16}{36}$

Sol. We have,

$\frac{4x+17}{18} - \frac{13x-2}{17x-32} + \frac{x}{3} = \frac{7x}{12} - \frac{x+16}{36}$

$\Rightarrow \frac{4x+17}{18} - \frac{7x}{12} + \frac{x+16}{36} + \frac{x}{3} = \frac{13x-2}{17x-32}$

Multiplying both sides by 36 i.e., the LCM of 18, 12, 36 and 3, we get

$36 \times \frac{4x+17}{18} - 36 \times \frac{7x}{12} + 36 \times \frac{x+16}{36} + 36 \times \frac{x}{3}$
$= 36 \times \left(\frac{13x-2}{17x-32}\right)$

$\Rightarrow 2(4x + 17) - 3 \times 7x + x + 16 + 12x$
$= 36 \times \left(\frac{13x-2}{17x-32}\right)$

$\Rightarrow 8x + 34 - 21x + x + 16 + 12x = 36 \times \left(\frac{13x-2}{17x-32}\right)$

$\Rightarrow 50 = 36 \times \left(\frac{13x-2}{17x-32}\right)$ [By cross-multiplication]

$\Rightarrow 50 \times (17x - 32) = 36(13x - 2)$

$\Rightarrow 850x - 1600 = 468x - 72$

$\Rightarrow 850x - 468x = 1600 - 72$

$\Rightarrow 382x - 1528$

$\Rightarrow x = \frac{1528}{382} = 4$

Hence, $x = 4$ is the solution of the given equation.

APPLICATIONS OF LINEAR EQUATIONS TO PRACTICAL PROBLEMS

The following steps should be followed to solve a word problem:

Step-I Read the problem carefully and note what is given and what is required.
Step-II Denote the unknown quantity by some letters, say x, y, z, etc.
Step-III Translate the statements of the problem into mathematical statements.
Step-IV Using the condition(s) given in the problem, form the equation.
Step-V Solve the equation for the unknown.
Step-VI Check whether the solution satisfies the equation.

❖ EXAMPLES ❖

Ex.1 A number is such that it is as much greater than 84 as it is less than 108. Find it.

Sol. Let the number be x. Then, the number is greater than 84 by $x - 84$ and it is les than 108 by $108 - x$. [Given]

$\therefore\ x - 84 = 108 - x$

$\Rightarrow x + x = 108 + 84$

$\Rightarrow 2x = 192$

$\Rightarrow \frac{2x}{2} = \frac{192}{2}$

$\Rightarrow x = 92$

Hence, the number is 96.

Ex.2 A number is 56 greater than the average of its third, quarter and one-twelfth. Find it.

Sol. Let the number be x. Then,

One third of x is $= \frac{1}{3}x$, Quarter of x is $= \frac{x}{4}$,

One-twelfth of x is $= \frac{x}{12}$

Average of third, quarter and one-twelfth of x is $= \frac{\left(\frac{x}{3}+\frac{x}{4}+\frac{x}{12}\right)}{3} = \frac{1}{3}\left(\frac{x}{3}+\frac{x}{4}+\frac{x}{12}\right)$

It is given that the number x is 56 greater than the average of the third, quarter and one-twelfth of x.

$\therefore\ x = \frac{1}{3}\left(\frac{x}{3}+\frac{x}{4}+\frac{x}{12}\right) + 56$

$\Rightarrow x = \frac{x}{9}+\frac{x}{12}+\frac{x}{36} + 56$

$\Rightarrow x - \frac{x}{9} - \frac{x}{12} - \frac{x}{36} + 56$

$\Rightarrow 36x - 4x - 3x - x = 36 \times 56$

[Multiplying both sides by 36
i.e., the L.C.M. of 9, 12 and 36]

$\Rightarrow 36x - 8x = 36 \times 56$

$\Rightarrow 28x = 36 \times 56$

$\Rightarrow \frac{28x}{28} = \frac{36 \times 56}{28}$ [Dividing both sides by 28]

$\Rightarrow x = 36 \times 2$

$\Rightarrow x = 72$

Hence, the number is 72.

Ex.3 A number consists of two digits whose sum is 8. If 18 is added to the number.

Sol. Let one's digit be x.

Since the sum of the digits is 8. Therefore, ten's digit = 8 – x.

$\therefore$ Number = $10 \times (8 - x) + x = 80 - 10x + x$

$= 80 - 9x$ ... (i)

Now,

Number obtained by reversing the digit

$= 10 \times x + (8 - x) = 10x + x - x = 9x + 8.$

It is given that if 18 is added to the number its digits are reversed.

$\therefore$ Number + 18 = Number obtained by reversing the digits

$\Rightarrow 80 - 9x + 18 = 9x + 8$

$\Rightarrow 98 - 9x = 9x + 8$

$\Rightarrow 98 - 8 = 9x + 9x$

$\Rightarrow 90 = 18x$

$\Rightarrow \frac{18x}{18} = \frac{90}{18}$

$\Rightarrow x = 5$

Putting the value of x in (i), we get

Number = $80 - 9 \times 5 = 80 - 45 = 35$

Ex.4 Divide 34 into two parts in such a way that $\left(\frac{4}{7}\right)^{th}$ of one part is equal to $\left(\frac{2}{5}\right)^{th}$ of the other.

Sol. Let one part be x. Then, other part is (34 – x). It is given that

$\left(\frac{4}{7}\right)^{th}$ of one part = $\left(\frac{2}{5}\right)^{th}$ of the other part

$\Rightarrow \frac{4}{7}x = \frac{2}{5}(34 - x)$

$\Rightarrow 20x = 14(34 - x)$ [Multiplying both sides by 35, the LCM of 7 and 5]

$\Rightarrow 20x = 14 \times 34 - 14x$

$\Rightarrow 20x + 14x \times 14 - 34$

$\Rightarrow 34x = 14 \times 34$

$\Rightarrow \frac{34x}{34} = \frac{14 \times 34}{34}$ [Dividing both sides by 34]

$\Rightarrow x = 14$

Hence, the two parts are 14 and 34 – 14 = 20

Ex.5 The numerator of a fraction is 4 less that the denominator. If 1 is added to both its numerator and denominator, it becomes 1/2. Find the fraction.

Sol. Let the denominator of the fraction be x. Then,

Numerator of the fraction = x – 4

$\therefore$ Fraction = $\frac{x-4}{x}$ ...(i)

If 1 is added to both its numerator and denominator, the fraction becomes $\frac{1}{2}$.

$\therefore \quad \frac{x-4+1}{x+1}=\frac{1}{2}$

$\Rightarrow \quad \frac{x-3}{x+1}=\frac{1}{2}$

$\Rightarrow 2(x-3)=x+1$ [Using cross-multiplication]

$\Rightarrow 2x-6=x+1$

$\Rightarrow 2x-x=6+1$

$\Rightarrow x=7$

Putting $x = 7$ in (i), we get

Fraction $=\frac{7-4}{7}=\frac{3}{7}$.

Hence, the given fraction is $\frac{3}{7}$.

Ex.6 Saurabh has Rs 34 paise and twenty-five paise coins. If the number of 25-paise coins be twice the number of 50-paise coins, how mahy coins of each kind does he have ?

Sol. Let the number of 50-paise coins be x. Then,

Number of 25-paise coins = 2x

$\therefore$ Value of x fifty-paise coins = 50 × x paise

$= \text{Rs}\frac{50\times x}{100}=\text{Rs}\frac{x}{2}$

Value of 2x twenty-five paise coins = 25 × 2x paise

$= \text{Rs}\frac{50x}{100}=\text{Rs}\frac{x}{2}$

$\therefore$ Total value of all coins $= \text{Rs}\left(\frac{x}{2}+\frac{x}{2}\right)=\text{Rs } x$

But, the total value of the money is Rs 34

$\therefore \quad x = 34$

Thus, number of 50-paise coins = 34

Number of twenty-five paise coins

$= 2x = 2 \times 34 = 68$

Ex.7 Arvind has Piggy bank. It is full of one-rupee and fifty-paise coins. It contains 3 times as many fifty paise coins as one rupee coins. The total amount of the money in the bank is Rs 35. How many coins of each kind are there in the bank ?

Sol. Let there be x one rupee coins in the bank. Then,

Number of 50-paise coins = 3x

$\therefore$ Value of x one rupee coins = Rs x

Value of 3x fifty-paise coins = 50 × 3x paise

$= 150\ x = \text{paise} = \text{Rs}\frac{150}{100}x = \text{Rs}\frac{3x}{2}$

$\therefore$ Total value of all the coins $= \text{Rs}\left(x+\frac{3x}{2}\right)$

But, the total amount of the money in the ban is given as Rs 35.

$\therefore \quad x+\frac{3x}{2}=35$

$\Rightarrow 2x + 3x = 70$ [Multiplying both sides by 2]

$\Rightarrow 5x = 70 \quad \Rightarrow \quad \frac{5x}{5}=\frac{70}{5} \quad \Rightarrow x = 14$

$\therefore$ Number of one rupee coins = 14, Number of 50 paise coins = $3x = 3 \times 14 = 42$.

Ex.8 Kanwar is three years older than Anima. Six years ago, Kanwar's age was four times Anima's age. Find the ages of Knawar and Anima.

Sol. Let Anima's age be x years. Then, Kanwar's age is (x + 3) years.

Six years ago, Anima's age was (x – 6) years

It is given that six years ago Kanwar's age was four times Anima's age.

$\therefore$ $x - 3 = 4(x - 6)$

$\Rightarrow$ $x - 3 = 4x - 24$

$\Rightarrow$ $x - 4x = -24 + 3$

$\Rightarrow$ $-3x = -21$

$\Rightarrow$ $\frac{-3x}{-3} = \frac{-21}{-3}$

$\Rightarrow$ $x = 7$

Hence, Anima's age = 7 years

Kanwar's age = (x + 3) yers

= (7 + 3) years = 10 years.

Ex.9 Hamid has three boxes of differen fruits. Box A weighs $2\frac{1}{2}$ kg more than Box B and Box C weighs $10\frac{1}{4}$ kg more than Box B. The total weight of the boxes is $48\frac{3}{4}$ Hox many kg does Box A weigh ?

Sol. Suppose the box B weights x kg.

Sinc box A weighs $2\frac{1}{2}$ kg more than box B and C weighs $10\frac{1}{4}$ kg more than box B.

$\therefore$ Weight of box A $= \left(x + 2\frac{1}{2}\right)$ kg

$= \left(x + \frac{5}{2}\right)$ kg ... (i)

Weight of box C $= \left(x + 10\frac{1}{4}\right)$ kg

$= \left(x + \frac{41}{4}\right)$ kg

$\therefore$ Total weight of all the boxes

$= \left(x + \frac{5}{2} + x + x + \frac{41}{4}\right)$ kg

But, the total weight of the boxes is given as $48\frac{3}{4}$ kg $= \frac{195}{4}$ kg

$\therefore$ $x + \frac{5}{2} + x + x + \frac{41}{4} = \frac{195}{4}$

$\Rightarrow$ $4x + 10 + 4x + 4x + 41 = 195$

[Multiplying both sides by 4]

$\Rightarrow$ $12x + 51 = 195$

$\Rightarrow$ $12x + 195 - 51$

$\Rightarrow$ $12x = 144$

$\Rightarrow$ $\frac{12x}{12} = \frac{144}{12}$

$\Rightarrow$ $x = 12$

Putting x = 12 in (i), we get

Weight of box A $= \left(12 + \frac{5}{2}\right)$ kg = $14\frac{1}{2}$ kg.

Ex.10 A man sold an artical for Rs 495 and gained 10% on it. Find the cost price of the article.

Sol. Let the cost price of teh article be Rs s

It is given that the man gained 10%. This means that :

On Rs 100 gain = Rs 10

$\therefore$ On Re 1 gain = Rs $\frac{10}{100}$

On Rs x gain = Rs $\frac{10}{100} \times x$ = Rs $\frac{x}{10}$

We know that

S.P. = C.P. + Gain

$\therefore \quad 495 = x + \frac{x}{10}$

$\Rightarrow \quad 495 = x + \frac{x}{10}$ [Multiplying both sides by 10]

$\Rightarrow \quad 4950 = 11x$

$\Rightarrow \quad \frac{11x}{11} = \frac{4950}{11}$

$\Rightarrow \quad x = 450$

Hence, cost price of the article = Rs 450.

Ex.11 50 kg of an alloy of lead and tin contains 50% lead. How much lead must be melted into it to make an alloy containing 75% lead ?

Sol. Lean contents in 100 kg alloy = 60 kg

$\therefore$ Lead contents in 50 kg alloy

$= \left(\frac{60}{100} \times 50\right)$ kg = 30 kg

Let x kg of lead be melted into the alloy to make it an alloy containing 75% lead. Then,

Weight of the new alloy = (50 + x) kg

Weight of lead in the new alloy = (30 + x) kg

Now,

Lead contents in (50 + x)kg alloy = (30 + x)kg

$\therefore$ Lead contents in one kg alloy $= \left(\frac{30+x}{50+x}\right)$ kg

Lead contents in 100 kg alloy

$= \left(\frac{30+x}{50+x}\right) \times 100$ kg

$\Rightarrow$ Percentage of lead in new alloy

$= \left(\frac{30+x}{50+x} \times 100\right)\%$

But, the percentage of lead in the new alloy is given as 75%.

$\therefore \quad \frac{30+x}{50+x} \times 100 = 75$

$\Rightarrow \quad (30 + x) \times 100 = 75 \times (50 + x)$

[Using cross-multiplication]

$\Rightarrow \quad 3000 + 100x = 3750 + 75x$

$\Rightarrow \quad 100x - 75x = 3750 - 3000$

$\Rightarrow 25x = 750$

$\Rightarrow \frac{25x}{25} = \frac{750}{25}$

$\Rightarrow x = 30$

Hence, required lead to be added = 30 kg.

Ex.12 The sum of two numbers is 2490. If 6.5% of one number is equal to 8.5% of the other, find the numbers.

Sol. Let the first number be x. Then,

Second number = 2490 – x.

[$\therefore$ Sum of the numbers is given to be 2490]

Now,

6.5% of the first number $= \frac{6.5}{100} \times x = \frac{65x}{1000}$

and,

8.5% of the second number $= \frac{8.5}{100} \times (2490 - x)$

$= \frac{85}{1000}(2490 - x)$

It is given that 6.5% of the first number is equal to 8.5% of the other.

$\therefore \frac{65x}{1000} = \frac{85}{1000}(2490 - x)$

$\Rightarrow 65x = 85\ (2490 - x)$

[Multiplying both sides by 1000]

$\Rightarrow 65x = 2490 \times 85 - 85x$

$\Rightarrow 65x + 85x = 2490 \times 85$

$\Rightarrow 150x = 2490 \times 85$

$\Rightarrow x = \frac{2490 \times 85}{150}$

$\Rightarrow x = 1411$

$\therefore$ First number $= 1411$, Second number

$= 2490 - 1411 = 1079$

Check We have,

6.5% of first number $= \frac{6.5}{100} \times 1411 = \frac{91715}{1000}$

8.5% of the second number $= \frac{8.5}{100} \times 1079$

$= \frac{91715}{1000}$

Clearly, 6.5% of the first number is equal to 8.5% of the second number, which is the same as given in the problem.

Ex.13 The sum of two numbers is 45 and their ratio is 7 : 8. Find the numbers.

Sol. Let one of the numbers be x. Since the sum of the two numbes is 45. Therefore, the other number will be 45 – x.

It is given that the ratio of the numbers is 7 : 8.

$\therefore \frac{x}{45 - x} = \frac{7}{8}$

$\Rightarrow 8 \times x = 7 \times (45 - x)$ [By cross-ultiplication]

$\Rightarrow 8x = 315 - 7x$

$\Rightarrow 8x + 7x = 315$

$\Rightarrow 15x = 315$

$\Rightarrow x = \frac{315}{15} = 21$

Thus, one number is 21 and, Other number

$= 45 - x = 45 - 21 = 24$

Check Clearly, sum of the numbers = 21 + 24 = 45, which is same as given in the problem.

Ratio of the numbers $= \frac{21}{24} = \frac{7}{8}$, which is same as given in the problem.

Thus, our solution is correct.

Ex.14 Divide Rs 1380 among Ahmed, John and Babita so that the amount Ahmed receives is 5 times a smuch as Babita's share and is 3 times as much as John's share.

Sol. Let Babita's share be Rs x. Then,

Ahmed's share = Rs 5x

$\therefore$ John's share = Total amount – (Babita's share + Ahmed's share)

$= \text{Rs}[1380 - (x + 5x)] = \text{Rs}\ (1380 - 6x)$

It is given that Ahmed's share is three times John's share.

$\therefore 5x = 3(1380 - 6x)$

$\Rightarrow 5x = 4140 - 18x$

$\Rightarrow 5x + 18x = 4140$

$\Rightarrow 23x = 4140$

$\Rightarrow x = \frac{4140}{23} = 180$

$\therefore$ Babita's share = Rs 180, Ahmed's share

$= \text{Rs}(5 \times 180) = \text{Rs}\ 900$

John's share = Rs $(1380 - 6 \times 180)$ = Rs 300

Ex.15 The length of a rectangle exceeds its breadth by 4 cm. If length and breadth are each increased by 3 cm, the area of the new rectangle will be 81 cm^2 more than that of the given rectangle. Find the length and breadth of the given rectangle.

Sol. Let the breadth of the given rectangle be x cm. Then, Length = (x + 4) cm

$\therefore$ Area = Length × Breadth = $(x + 4)x = x^2 + 4x$.

When length and breadth are each increased by 3 cm.

New length = (x + 4 + 3) cm = (x + 7) cm,

New breadth = (x + 3) cm

$\therefore$ Area of new rectangle = Length × Breadth

$= (x + 7)(x + 3)$

$= x(x + 3) + 7(x + 3)$

$= x^2 + 3x + 7x + 21 = x^2 + 10x + 21$

It is given that the are of new rectangle is 81 cm^2 more than the given rectangle.

$\therefore x^2 + 10x + 21 = x^2 + 4x + 81$

$\Rightarrow x^2 + 10x - x^2 - 4x = 81 - 21$

$\Rightarrow 6x = 60$

$\Rightarrow x = \frac{60}{6} = 10$

Thus,

Length of the given rectangle

$= (x + 4)\text{cm} = (10 + 4)\ \text{cm} = 14\ \text{cm}$

Breadth of the given rectangle = 10 cm

Check Area of the given rectangle $= (x^2 + 4x)\text{cm}^2$

$= (10^2 + 4 \times 10)\text{cm}^2 = 140\ \text{cm}^2$

Area of the new rectangle $= (x^2 + 10x + 21)\text{cm}^2$

$= (10^2 + 10 \times 10 + 21)\text{cm}^2 = 221\ \text{cm}^2$

Clearly, area of the new rectangle is 81 cm^2 more than that of the given rectangle, which is the same as given in the problem. Hence, our answer is correct.

Ex.16 An altitutde of a triangle is five-thirds the length of its corresponding base. If the altitude were increased by 4 cm and the base be decreased by 2 cm, the area of the triangle would remain the same. Find the base and the altitude of the triangle.

Sol. Let the length of the base of the triangle be x cm. Then,

$$\text{Altitude} = \left(\frac{5}{3} \times x\right)\text{cm} = \frac{5x}{3}\text{cm}$$

$$\therefore \text{ Area} = \frac{1}{2}(\text{Base} \times \text{Altitude})\ \text{cm}^2$$

$$= \frac{1}{2}\left(x \times \frac{5x}{3}\right)\text{cm}^2 = \frac{5x^2}{6}\text{cm}^2$$

When the altitude is increased by 4 cm and the base is decreased by 2 cm, we have

$$\text{New base} = (x - 2)\ \text{cm, New altitude} = \left(\frac{5x}{3} + 4\right)\text{cm}$$

$\therefore$ Area of the new triangle

$$= \frac{1}{2}(\text{Base} \times \text{Altitude})$$

$$= \frac{1}{2}\left\{\left(\frac{5x}{3} + 4\right) \times (x - 2)\right\}\text{cm}^2$$

$$= \frac{1}{2}\left\{(x - 2) \times \left(\frac{5x}{3} + 4\right)\right\}\text{cm}^2$$

$$= \frac{1}{2}\left\{\frac{5x}{3}(x - 2) + 4(x - 2)\right\}\text{cm}^2$$

$$= \frac{1}{2}\left\{\frac{5x^2}{3} - \frac{10x}{3} + 4x - 8\right\}\text{cm}^2$$

$$= \frac{1}{2}\left(\frac{5x^2}{6} - \frac{5x}{3} + 2x - 4\right)\text{cm}^2$$

It is given that the area of the given triangle is same as the area of the new triangle.

$$\therefore \quad \frac{5x^2}{6} = \frac{5x^2}{6} - \frac{5x}{3} + 2x - 4$$

$$\Rightarrow \quad \frac{5x^2}{6} - \frac{5x^2}{6} + \frac{5x}{3} - 2x = -4$$

$$\Rightarrow \quad \frac{5x}{3} - 2x = -4 \quad \text{[Multiplying both sides by 3]}$$

$$\Rightarrow -x = -12$$

$$\Rightarrow x = 12\text{cm}$$

Hence, base of the triangle = 12 cm.

Altitude of the triangle $= \left(\frac{5}{3} \times 12\right)$ cm $= 20$ cm

Check We have,

Area of the given triangle $= \left(\frac{5}{6} \times 12^2\right)$ cm $= 120\text{cm}^2$

Area of the given triangle

$$= \left(\frac{5}{6} \times 12^2 - \frac{15}{3} \times 12 + 2 \times 12 - 4\right) \text{cm} = 120\text{cm}^2$$

Therefore, area of the given triangle is the same as that of the new triangle, which is the same as given in the problem. Thus, our answer is correct.

EXERCISE - 1

Q.1 A number is as much greater than 36 as is less than 86. Find the number.

Q.2 Find a number such that when 15 is subtracted from 7 times the number, the result is 10 more than twixe the number.

Q.3 The sum of a rational number and its reciprocal is $\frac{13}{6}$. Find the number.

Q.4 The sum of two numbers is 184. If one-third of the one exceeds one-seventh of the other by 8, find the smaller number.

Q.5 The difference of two numbers is 11 and one-fifth of their sum is 9. Find the numbers.

Q.6 If the sum of two numbers is 42 and their product is 437, then find the absolute difference between the numbers.

Q.7 The sum of two numbers is 15 and the sum of their squares is 113. Find the numbers.

Q.8 The average of four consecutive even numbers is 27. Find the largest of these numbers.

Q.9 The sum of the squares of three consecutive od numbers is 2531. Find the numbers.

Q.10 Of two numbers, 4 times the smaller one is less than 3 times the larger one by 5. If the sum of the numbers is larger than 5 times their difference by 6, find the two numbers.

Q.11 The ratio between a two-digit number and the sum of the digits of that number is 4 : 1. If the digit in the unit's place is 3 more than the digit in the ten's place, what is the number ?

Q.12 A number consists of two digits. The sum of the digits is 9. if 63 is subtracted from the number, its digits are interchanged. Find the number.

Q.13 A fraction becomes $\frac{2}{3}$ when 1 is added to both, its numerator and denominator. And, it becomes $\frac{1}{2}$ when 1 is subtracted from both the numerator and denominator. Find the fraction.

Q.14 50 is divided into two parts such that the sum of their reciprocals is $\frac{1}{12}$. Find the two parts.

Q.15 If three nyumbers are added in pairs, the sums equal 10, 19 and 21. Find the numbers.

Q.16 Rajeev's age after 15 years will be 5 times his age 5 years back. What is te present age of Rajeev?

Q.17 The ages of two persons differe by 16 years. If 6 years ago, the elder one be 3 times as old as the younger one, find their present agaes.

Q.18 The product of the ages of Anket and Nikita is 240 . If twice the age or Nikita is more than Ankit's age by 4 years, what is Nikita's age?

Q.19 The present age of a father is 3 years more than three times the age of his son. Three years hence, father's age will be 10 years more than twice the age of the son. Find the present age of the father.

Q.20 Rohit was 4 times as old as his son 8 years ago. After 8 years, Rohit will be wtice as old as his son. What are their present ages ?

Q.21 One year ago, the ratio of Gaurav's and Sachin's age was 6 : 7 respectively. Four years hence, this ratio would become 7 : 8. How old is Sachin ?

Q.22 Abhay's age after six years will be three-seventh of his father's age at present ?

ANSWERS

1. 61 **2.** 5 **3.** 2/3 or 3/2 **4.** 72 **5.** 28 & 17 **6.** 4
7. 7 & 8 **8.** 30 **9.** 27, 29, 31 **10.** 59 & 43 **11.** 36 **12.** 81
13. 3/5 **14.** 30 & 20 **15.** 6, 4, 15 **16.** 10 years **17.** 14 & 30 years **18.** 12 years
19. 33 years **20.** 16 & 40 years **21.** 36 years **22.** 50 years

EXERCISE - 2

Q.1 The differene between a number and its three-fifth is 50. What is the number ?
(A) 75 (B) 100
(C) 125 (D) None of these

Q.2 If a number is decreased by 4 and divided by 6, the result is 8. What would be the result if 2 is subtracted from the number and then it is divided by 5?
(A) $9\frac{2}{3}$ (B) 10
(C) $11\frac{1}{5}$ (D) None of these

Q.3 If one-third of one-fourth of a number is 15, then three-tenth of that number is :
(A) 35 (B) 36 (C) 45 (D) 54

Q.4 A number is doubled and 9 is added. If the resultant is trebled, it becomes 75. What is that number ?
(A) 3.5 (B) 6
(C) 8 (D) None of these

Q.5 Three-fourth of a number is 60 more than its one-third. The number is :
(A) 84 (B) 108
(C) 144 (D) None of these

Q.6 When 24 is subtracted froma number, it reduces to its four-seventh. What is the sum of the digits of that number?
(A) 1 (B) 9
(C) 11 (D) None of these

Q.7 Find the number which when multiplied by 15 is increased by 196.
(A) 14 (B) 20 (C) 26 (D) 28

Q.8 If a number, when divided by 4, is reduced by 21, the number is :
(A) 18 (B) 20 (C) 28 (D) 38

Q.9 A number whose fifth part increased by 4 is equal to its fourth part diminished by 10, is
(A) 240 (B) 260
(C) 270 (D) 280

Q.10 The difference of two numbers is 20% of the larger number. If the smaller number is 12, the larger one is :
(A) 15 (B) 16
(C) 18 (D) 20

Q.11 If one-seventh of a number exceeds its eleventh pary by 100, then the number is :
(A) 770 (B) 1100
(C) 1825 (D) 1925

Q.12 If the sum of one-half and one-fifth of a number exceeds one-third of that number by $7\frac{1}{3}$, the number is
(A) 15 (B) 18 (C) 20 (D) 30

Q.13 If doubling a number and adding 20 to the result gives the same answer as multiplying the number by 8 and taking away 4 from the product, the number is:
(A) 2 (B) 3 (C) 4 (D) 6

Q.14 If 50 is subtracted from two-third of a number, the result is equal to sum of 40 and one-fourth of that number. What is the number ?
(A) 174 (B) 216 (C) 246 (D) 336

Q.15 If the sum of a number and its square is 182, what is the number ?
(A) 15 (B) 26
(C) 28 (D) None of these

Q.16 Twenty times a positive integer is less than its square by 96. What is the integer ?
(A) 20
(B) 24
(C) Cannot be determined
(D) None of these

Q.17 Thrice the square of a natural number decreased by 4 times the number is equal to 50 more than the number. The number is :
(A) 4 (B) 5 (C) 6 (D) 10

Q.18 The sum of a number and its reciprocal is one-eighth of 34. What is the product of the number and its square root ?
(A) 8 (B) 27
(C) 32 (D) None of these

Q.19 Two-third of a positive number and $\frac{25}{216}$ of its reciprocal are equal. The number is :
(A) $\frac{5}{12}$ (B) $\frac{12}{5}$
(C) $\frac{25}{144}$ (D) $\frac{144}{25}$

Q.20 Find a positive number when increased by 17 is equal to 60 times the reciprocal of the number.
(A) 3 (B) 10 (C) 17 (D) 20

Q.21 A positive number when decreased by 4 is equal to 21 times the reciprocal of the number. The number is:
(A) 3 (B) 5 (C) 7 (D) 9

Q.22 The sum of a positve number and its reciprocal is thrice the difference of the number and its reciprocal. The number is:
(A) $\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$
(C) $\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$

Q.23 The product of two natural numbers is 17. Then, the sum of the reciprocals of their square is :

(A) $\frac{1}{289}$ (B) $\frac{289}{290}$

(C) $\frac{290}{289}$ (D) 289

Q.24 If $2\frac{1}{2}$ is added to a number and the sum multiplied by $4\frac{1}{2}$ and 3 is added to the product and then dividing the sum by $1\frac{1}{5}$, the quotient become 25. What is the number?

(A) $2\frac{1}{2}$ (B) $3\frac{1}{2}$

(C) $4\frac{1}{2}$ (D) $5\frac{1}{2}$

Q.25 Three numbers are in the ratio 4 : 5 : 6 and their average is 25. The largest number is :

(A) 30 (B) 32 (C) 36 (D) 42

Q.26 Three numbers are in the ratio 3 : 4 : 6 and their product is 1944. The largest of these number is :

(A) 6 (B) 12

(C) 18 (D) None of these

Q.27 Two numbers are such that the square of one is 244 less than 8 times the square of the other. If the number be in ratio 3 : 4 the number are:

(A) 6, 8 (B) 9, 12

(C) 12, 16 (D) None of these

Q.28 Two numbers are such that the ratio between them is 4 : 7. If each is increased by 4, the ratio becomes 3 : 5. The larger number is :

(A) 36 (B) 48

(C) 56 (D) 64

Q.29 The sum of three numbers is 264. If the first number be twice the second and third number be one-third of the first, then the second number is :

(A) 48 (B) 54

(C) 72 (D) 84

Q.30 The sum of two numbers is 22. Five times one number is equal to 6 times the other. The bigger of the two numbers is :

(A) 10 (B) 12

(C) 15 (D) 16

Q.31 One-fifth of a number is equal to $\frac{5}{8}$ of another number. If 35 is added to the first number, it becomes four times of the second number. The second number is:

(A) 25 (B) 40

(C) 70 (D) 125

Q.32 The sum of two numbers is 25 and their difference is 13, Find their product :
(A) 104 (B) 114
(C) 315 (D) 325

Q.33 If the sum of two numbers is 33 and their difference is 15, the smaller number is :
(A) 9 (B) 12
(C) 15 (D) 18

Q.34 The sum of two numbers is 40 and their difference is 4. The ratio of the numbers is :
(A) 11 : 9 (B) 11 : 18
(C) 21 : 19 (D) 22 : 9

Q.35 The product of two number is 192 and the sum of these two numbers is 28. What is the smallerof these two numbers ?
(A) 12 (B) 14
(C) 16 (D) 18

Q.36 Sachin is younger than Rahul by 4 years. If their ages are in the respective ratio of 7 : 9, how old is Sachin ?
(A) 16 years
(B) 18 years
(C) 28 years
(D) None of these

Q.37 The ratio between the present ages of P and Q is 6 : 7. If Q is 4 years old than P, what will be the ratio of the ages of P and Q after 4 years?
(A) 3 : 4 (B) 3 : 5
(C) 4 : 3 (D) None of these

Q.38 The ratio between the present ages of P and Q is 5 : 7 respectively. If the difference between Q's present age and P's age after 6 years is 2, what is the total of P's and Q's present ages ?
(A) 48 years (B) 52 years
(C) 56 years (D) None of these

Q.39 At present, the ratio between the ages of Arun and Deepak is 4 : 3. After 6 years, Arun's age will be 26 years. What is the age of Deepak at present?
(A) 12 years (B) 15 years
(C) $19\frac{1}{2}$ years (D) 21 years

Q.40 Present ages of X and Y are in the ratio 5 : 6 respectively. Seven years hence this ratio will be become 6 : 7 respectively. What is X's present age in years ?
(A) 35 (B) 42
(C) 49 (D) Cannot be determind

Q.41 Present ages of Sameer and Anand are in the ratio of 5 : 4 respectively. Three years hence, the ratio of their ages will become 11 : 9 resepectively. What is Anand's present age in years?
(A) 24 (B) 27
(C) 40 (D) Cannot be determind

Q.42 Six years ago, the ratio of the ages of Kunal and Sagar was 6 : 5. Four years hence, the ratio of their ages will be 11 : 10. What is Sagar's age at present ?
(A) 16 years (B) 18 years
(C) 20 years (D) Cannot be determind

Q.43 The total of the ages of Jayant, Prem and Saransh is 93 years. Ten years ago, the ratio of their ages was 2 : 3 : 4. What is the present age of Saransh?
(A) 24 years (B) 32 years
(C) 34 years (D) 38 years

Q.44 The ratio of the present ages of two brothers is 1 : 2 and 5 years back, the ratio was 1 : 3. What will be the ratio of their ages after 5 years ?
(A) 1 : 4 (B) 2 : 3
(C) 3 : 5 (D) 5 : 6

Q.45 Hitesh is 40 years old and Ronnie is 60 years old. How many years ago was the ratio of their ages 3 : 5 ?
(A) 5 years (B) 10 years
(C) 20 years (D) 37 years

Q.46 The ratio of the father's age to his son's age is 7 : 3. The product of their ages is 756. The ratio of their ages after 6 years will be :
(A) 5 : 2 (B) 2 : 1
(C) 11 : 7 (D) 13 : 9

Q.47 The present ages of three persons are in proportions 4 : 7 : 9. Eight years ago, the sum of their ages was 56. Find their present ages (in yeras).
(A) 8, 20, 28 (B) 16, 28, 36
(C) 20, 35, 45 (D) None of these

Q.48 The ratio of the ages of a man and his wife is 4 : 3. After 4 years, this ratio will be 9 : 7. If at the time of marriage, the ratio was 5 : 3, then how many years ago were they married ?
(A) 8 years (B) 10 years
(C) 12 years (D) 15 years

Q.49 The ratio between the school ages of Neelam and Shaan is 5 : 6 respectively. If the ratio between the one-third age of Neelam and half of Shaan's age is 5 : 9, then what is the school age of Shaan?
(A) 25 years (B) 30 years
(C) 36 years (D) Cannot be determined

Q.50 The ratio between the present ages of A and B is 5 : 3 respectively. The ratio between A's age 4 years ago and B's age 4 years hence 1 : 1. What is the ratio between A's age 4 years hence and B's age 4 years ago?
(A) 1 : 3 (B) 2 : 1
(C) 3 : 1 (D) 4 : 1

Q.51 Ten years ago, A was half of B in age. If the ratio of their present ages is 3 : 4, what will be the total of their present ages ?
(A) 20 years (B) 30 years
(C) 45 years (D) None of these

Q.52 A is two years older than B who is twice as old as C. If the total of the ages of A, B and C be 27, then how old is B ?
(A) 7 (B) 8
(C) 9 (D) 10

Q.53 A man is 24 years older than his son. In two years, his age will be twice the age of his son. The present age of the son is:
(A) 14 (B) 18 years
(C) 20 years (D) 22 years

Q.54 Eighteen years ago, a father was three times as old as his son. Then the sum of the present ages of the son and the father is:
(A) 54 (B) 72
(C) 105 (D) 108

Q.55 A person's present age is two-fifth of the age of his mother. After 8 years, he will be one-half of the age of his mother. How old is the mother at present?
(A) 32 years (B) 36 years
(C) 40 years (D) 48 years

Q.56 Tanya's grandfather was 8 times older to her 16 years ago. He would be 3 times of her age 8 years from now. Eight years ago, what was the ratio of Tanya's age to that of her grandfather?
(A) 1 : 2
(B) 1 : 5
(C) 3 : 8
(D) None of these

Q.57 The age of father 10 years ago was thrice the age of his son. Ten years hence, father's age will be twice that of his son. The ratio of their present ages is :
(A) 5: 2 (B) 7 : 3
(C) 9 : 2 (D) 13 : 4

Q.58 Four years ago, the father's age was three times the age of his son. The total of the ages of the father and the son after four years, will be 64 years. What is the father's age at present?
(A) 32 years (B) 36 years
(C) 44 years (D) None of these

Q.59 One year ago, Promila was four times as old as her daughter Sakshi. Six years hence, Promila age will exceed her daughter age by 9 year the ratio of the present ages of Promila and her daugther is:
(A) 9 : 2 (B) 11 : 3
(C) 12 : 5 (D) 13 : 4

Q.60 The sum of the present ages of a father and his son is 60 years. Six years ago, father's age was five times the age of the son. After 6 years, son's age will be:
(A) 12 years (B) 14 years
(C) 18 years (D) 20 years

ANSWER KEY

Q.No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	C	B	D	C	C	C	A	C	D	A	D	C	C	B	D	B	B	A	A	A
Q.No	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	C	A	C	B	A	C	A	C	C	B	B	B	A	A	A	D	D	A	B	A
Q.No	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	A	A	D	C	B	B	B	C	D	C	D	D	D	D	C	D	B	D	D	D

HINTS & SOLUTION - 1

Sol.1 Let the number be x. Then,
$x - 36 = 86 - x \Leftrightarrow 2x = 86 + 36 = 122 \Leftrightarrow x = 61$
Hence, the required number is 61.

Sol.2 Let the number be x. Then,
$7x - 15 = 2x + 10 \Leftrightarrow 5x = 25 \Leftrightarrow x = 5$
Hence, the required number is 5.

Sol.3 Let the number be x.

Then, $x + \frac{1}{x} = \frac{13}{6} \Leftrightarrow \frac{x^2+1}{x} = \frac{13}{6}$

$\Leftrightarrow 6x^2 - 13x + 6 = 0$

$\Leftrightarrow 6x^2 - 9x - 4x + 6 = 0$

$\Leftrightarrow (3x - 2)(2x - 3) = 0$

$\Leftrightarrow x = \frac{2}{3}$ or $x = \frac{3}{2}$.

Hence, the required number is $\frac{2}{3}$ or $\frac{3}{2}$.

Sol.4 Let the numbes be x and (184 – x). Then,

$\frac{x}{3} - \frac{(184-x)}{7} = 8 \Leftrightarrow 7x - 3(184 - x) = 168$

$\Leftrightarrow 10x = 720 \Leftrightarrow x = 72$

So, the number are 72 and 112.
Hence, smaller number = 72.

Sol.5 Let the numbers be x and y. Then,

$x - y = 11$ ... (i)

and $\frac{1}{5}(x + y) = 9 \Rightarrow x + y = 45$... (ii)

Adding (i) and (ii), we get : $2x = 56$ or $x = 28$.
Putting $x = 28$ in (i), we get : $y = 17$.
Hence, the numbers are 28 and 17.

Sol.6 Let the numbers be x and y. Then, $x + y = 42$ and $xy = 437$.

$x - y = \sqrt{(x+y)^2 - 4xy} = \sqrt{(42)^2 - 4 \times 437}$

$= \sqrt{1764 - 1748} = \sqrt{16} = 4.$

$\therefore$ Required difference = 4.

Sol.7 Let the numbers be x and (15 – x).
Then, $x^2 + (15 - x)^2 = 113$

$\Leftrightarrow x^2 + 225 + x^2 - 30x = 113$

$\Leftrightarrow 2x^2 - 30x + 112 = 0$

$\Leftrightarrow x^2 - 15x + 56 = 0$

$\Leftrightarrow (x - 7)(x - 8) = 0$

$\Leftrightarrow x = 7$ or $x = 8$

So, the numbers are 7 and 8.

Sol.8 Let the four consecutive even numbers be
$x, x + 2, x + 4$ and $x + 6$
So $x + (x + 2) + (x + 4) + (x + 6) = 108$
or $4x = 96$
or $x = 24$.
$\therefore$ Largest number $= (x + 6) = 30$.

Sol.9 Let the numbers be x, $x + 2$ and $x + 4$.
Then, $x^2 + (x + 2)^2 + (x + 4)^2 = 2531$
$\Leftrightarrow 3x^2 + 12x - 2511 = 0$
$\Leftrightarrow x^2 + 4x - 837 = 0 \Leftrightarrow (x - 27)(x + 31) = 0$
$\Leftrightarrow x = 27$
Hence, the required numbers are 27, 29 and 31.

Sol.10 Let the numbers be x and y, such that $x > y$.
Then, $3x - 4y = 5$ ....(i) and $(x + y) - 6(x - y) = 6 \Rightarrow -5x + 7y = 6$ .....(2)
Solving (1) and (2) we get $x = 59$ and $y = 43$
Hence, the required numbers are 59 and 43.

Sol.11 Let the ten's digit be x. Then, unit's digit $= (x + 3)$.
Sum of the digits $= x + (x + 3) = 2x + 3$.
Number $= 10x + (x + 3) = 11x + 3$.

$\therefore \frac{11x+3}{2x+3} = \frac{4}{1} \Leftrightarrow 11x + 3 = 4(2x + 3)$
$\Leftrightarrow 3x = 9 \Leftrightarrow x = 3$.
Hence, required number $= 11x + 3 = 36$.

Sol.12 Let the ten's digit be x. Then, unit's digit $= (9 - x)$.
Number $= 10x + (9 - x) = 9x + 9$
Number obtained by reversing the digits
$= 10(9 - x) + x = 90 - 9x$
$\therefore (9x + 9) - 63 = 90 - 9x \Leftrightarrow 18x = 144 \Leftrightarrow x = 8$
So, ten's digit = 8 and unit's digit = 1
Hence, the required number is 81.

Sol.13 Let the required fraction be $\frac{x}{y}$. Then,

$\frac{x+1}{y+1} = \frac{2}{3} \Rightarrow 3x - 2y = -1$ .... (i)

and $\frac{x-1}{y-1} = \frac{1}{2} \Rightarrow 2x - y = 1$ ....(ii)

Solving (i) and (ii), we get : $x = 3$, $y = 5$.

$\therefore$ Required fraction $= \frac{3}{5}$

Sol.14 Let the two parts be x and $(50 - x)$.

Then, $\frac{1}{x} + \frac{1}{50-x} = \frac{1}{12} \Leftrightarrow \frac{50-x+x}{x(50-x)} = \frac{1}{12}$
$\Rightarrow x^2 - 50x + 600 = 0$
$\Rightarrow (x - 30)(x - 20) = 0 \Rightarrow x = 30$ or $x = 20$.
So, the parts are 30 and 20.

Sol.15 Let the numbers be x, y and z. Then,

$x + y = 10$ ... (i)

$y + z = 19$ ... (ii)

$x + z = 21$ ... (iii)

Adding (i), (ii) and (iii),

we get : $2(x + y + z) = 50$ or $(x + y + z) = 25$.

Thus, $x = (25 - 19) = 6$; $y = (25 - 21) = 4$;

$z = (25 - 10) = 15$

Hence, the required numbers are 6, 4, and 15.

Sol.16 Let Rajeev's present age be x years. Then,

Rajeev's afe after 15 years = $(x + 15)$ years.

Rajeev's age 5 years back = $(x - 5)$ years.

$\therefore x + 15 = 5(x - 5) \Leftrightarrow x + 15 = 5x - 25$

$\Leftrightarrow 4x = 10 \Leftrightarrow x = 10.$

Hence, Rajeev's present age = 10 years.

Sol.17 Let the age of the younger person be x years.

Then, age of the elder person = $(x + 16)$ years.

$\therefore 3(x - 6) = (x + 16 - 6) \Leftrightarrow 3x - 18 = x + 10$

$\Leftrightarrow 2x = 28 \Leftrightarrow x = 14.$

Hence, their present ages are 14 years and 30 years.

Sol.18 Let Ankit's age be x years. Then, Nikita's age

$= \frac{240}{x}$ years.

$\therefore 2 \times \frac{240}{x} - x = 4 \Leftrightarrow 480 - x^2 = 4x$

$\Leftrightarrow x^2 + 4x - 480 = 0$

$\Leftrightarrow (x + 24)(x - 20) = 0 \Leftrightarrow x = 20.$

Hence, Nikita's age = $\left(\frac{240}{20}\right)$ years = 12 years.

Sol.19 Let the son's present age be x years. The, father's present age = $(3x + 3)$ years.

$\therefore (3x + 3 + 3) = 2(x + 3) + 10$

$\Leftrightarrow 3x + 6 = 2x + 16 \Leftrightarrow x = 10.$

Hence, father's present age = $(3x + 3)$

$= (3 \times 10 + 3)$years = 33 years.

Sol.20 Let son's age 8 years ago be x years. Then, Rohit's age 8 years ago = 4x years.

Son's age after 8 years = (x + 8) + 8

= (x + 16)years.

Rohit's age after 8 years = (4x + 8) + 8

= (4x + 16) years.

$\therefore$ 2 (x + 16) = 4x + 16 $\Leftrightarrow$ 2x = 16 $\Leftrightarrow$ x = 8.

Hence, son's present age = (x + 8) = 16 years.

Rohit's present age = (4x + 8) = 40 years.

Sol.21 Let Gaurav's and Sachin's ages one years ago be 6x and 7x years respectively. Then, Gaurav's age 4 years hence = (6x + 1) + 4 = (6x + 5) years.

Sachin's age 4 years hence = (7x + 1) + 4 = (7x + 5) years.

$\therefore \frac{6x+5}{7x+5} = \frac{7}{8} \Leftrightarrow 8(6x + 5) = 7(7x + 5)$

$\Leftrightarrow$ 48x + 40 = 49x+ 35 $\Leftrightarrow$ x = 5.

Hence, Sachin's present age = (7x + 1) = 36 years.

Sol.22 Let the ages of Abhay and his father 10 years ago be x and 5x years respectively. Then, Abahy's age after 6 years = (x + 10) + 6

= (x + 16) years.

Father's age after 6 years

= (5x + 10) + 6 = (5x + 16) years.

$\therefore$ (x + 16) = $\frac{3}{7}$ (5x + 16) $\Leftrightarrow$ 7(x + 16)

= 3 (5x + 16) $\Leftrightarrow$ 7x + 112 = 15x + 48

$\Leftrightarrow$ 8x = 64 $\Leftrightarrow$ x = 8.

Hence, Abhay's father's present age = (5x + 10)

= 50 years.

HINTS & SOLUTION -2

Sol.1 Let the number be x. Then, $x - \frac{3}{5}x = 50$

$$\Leftrightarrow x = \left(\frac{50\times 5}{2}\right) = 125.$$

Sol.2 Let the number x. Then, $\frac{x-4}{6} = 8 \Leftrightarrow x - 4 = 48$

$$\Leftrightarrow x - 4 = 48 \Leftrightarrow x = 52.$$

$$\therefore \frac{x-2}{5} = \frac{52-2}{5} = \frac{50}{5} = 10.$$

Sol.3 Let the number be x. Then, $\frac{1}{3}$ of $\frac{1}{4}$ of $x = 15$

$$\Leftrightarrow x = 15 \times 12 = 180.$$

So,required number $= \left(\frac{3}{10}\times 180\right) = 54.$

Sol.4 Let the number be x. Then, $3(2x + 9) = 75$

$$\Leftrightarrow 2x + 9 = 25 \Leftrightarrow 2x = 16 \Leftrightarrow x = 8.$$

Sol.5 Let the number be x. Then , $\frac{3}{4}x - \frac{1}{3}x = 60$

$$\Leftrightarrow \frac{5x}{12} = 60 \Leftrightarrow x = \left(\frac{60\times 12}{5}\right) = 144.$$

Sol.6 Let the number be x. Then,

$$x - 24 = \frac{4}{7}x \Leftrightarrow x - \frac{4}{7}x = 24 \Leftrightarrow \frac{3}{7}x = 24$$

$$\Leftrightarrow x = \left(\frac{24\times 7}{3}\right) = 56.$$

$\therefore$ Sum of the digits $= (5 + 6) = 11.$

Sol.7 Let the number be x. Then, $15x - x = 196$

$$\Leftrightarrow 14x = 196 \Leftrightarrow x = 14.$$

Sol.8 Let the number be x. Then, $\frac{x}{4} = x - 21$

$$\Leftrightarrow x = 4x - 84 \Leftrightarrow 3x = 84 \Leftrightarrow x = 28.$$

Sol.9 Let the numer be x. Then, $\left(\frac{1}{5}x+4\right) = \left(\frac{1}{4}x-10\right) \Leftrightarrow \frac{x}{20} = 14 \Leftrightarrow x = 14 \times 20 = 280.$

Sol.10 Let the larger number be x.

Then, $x - 12 = 20\%$ of $x \Leftrightarrow x - \frac{x}{5} = 12$

$\Leftrightarrow \frac{4x}{5} = 12 \Leftrightarrow x = \left(\frac{12\times5}{4}\right) = 15.$

Sol.11 Let the number be x. Then, $\frac{1}{7}x - \frac{1}{11}x = 100 \Leftrightarrow \frac{4x}{77} = 100 \Leftrightarrow x = \frac{7700}{4} = 1925.$

Sol.12 Let the number be x.

Then, $\left(\frac{1}{2}x+\frac{1}{5}x\right) - \frac{1}{3}x = \frac{22}{3} \Leftrightarrow \frac{11x}{30} = \frac{22}{3}$

$\Leftrightarrow x = \left(\frac{22\times30}{3\times11}\right) = 20.$

Sol.13 Let the number be x. Then, $2x + 20 = 8x - 4$

$\Leftrightarrow 6x = 24 \Leftrightarrow x = 4.$

Sol.14 Let the number be x.

Then, $\frac{2}{3}x - 50 = \frac{1}{4}x + 40 \Leftrightarrow \frac{2}{3}x - \frac{1}{4}x = 90$

$\Leftrightarrow \frac{5x}{12} = 90 \Leftrightarrow x = \left(\frac{90\times12}{5}\right) = 216.$

Sol.15 Let the number be x.

Then, $x + x^2 = 182$

$\Leftrightarrow x^2 + x - 182 = 0$

$\Leftrightarrow (x + 14)(x - 13) = 0 \Leftrightarrow x = 13$.

Sol.16 Let the integer be x.

Then, $x^2 - 20x = 96$

$\Leftrightarrow x^2 - 20x - 96 = 0$

$\Leftrightarrow (x + 4)(x - 24) = 0$

$\Leftrightarrow x = 24.$

Sol.17 Let the number be x.

Then, $3x^2 - 4x = x + 50$

$\Leftrightarrow 3x^2 - 5x - 50 = 0$

$\Leftrightarrow (3x + 10)(x - 5) = 0$

$\Leftrightarrow x = 5.$

Sol.18 Let the number be x. Then, $x + \frac{1}{x} = \frac{34}{8}$

$\Leftrightarrow \frac{x^2+1}{x} = \frac{34}{8} \Leftrightarrow 8x^2 - 34x + 8 = 0$

$\Leftrightarrow 4x^2 - 17x + 4 = 0 \Leftrightarrow (4x - 1)(x - 4) = 0$

$\Leftrightarrow x = 4.$

$\left[\text{neglecting } x = \frac{1}{4}, \text{as x is a natural no.}\right]$

$\therefore$ Required number $= 4 \times \sqrt{4} = 4 \times 2 = 8.$

Sol.19 Let the number be x.

Then, $\frac{2}{3}x = \frac{25}{216} \times \frac{1}{x} \Leftrightarrow x^2 = \frac{25}{216} \times \frac{3}{2} = \frac{25}{144} \Leftrightarrow x = \sqrt{\frac{25}{144}} = \frac{5}{12}.$

Sol.20 Let the number be x.

Then, $x + 17 = \frac{60}{x} \Leftrightarrow x^2 + 17x - 60 = 0$

$\Leftrightarrow (x + 20)(x - 3) = 0 \Leftrightarrow x = 3.$

Sol.21 Let the number be x.

Then, $x - 4 = \frac{21}{x} \Leftrightarrow x^2 - 4x - 21 = 0$

$\Leftrightarrow (x - 7)(x + 3) = 0 \Leftrightarrow x = 7.$

Sol.22 Let the number be x. Then, $x + \frac{1}{x} = 3\left(x - \frac{1}{x}\right)$

$\Leftrightarrow \frac{x^2+1}{x} = 3\left(\frac{x^2-1}{x}\right)$

$\Leftrightarrow x^2 + 1 = 3x^2 - 3 \Leftrightarrow 2x^2 = 4 \Leftrightarrow x^2 = 2$

$\Leftrightarrow x = \sqrt{2}.$

Sol.23 Let the number be a and b. Then, ab = 17

$\Leftrightarrow$ a = 1 and b = 17.

So, $\frac{1}{a^2} + \frac{1}{b^2} = \frac{a^2+b^2}{a^2b^2} = \frac{1^2+(17)^2}{(1\times17)^2} = \frac{290}{289}.$

Sol.24 Let the number be x. Then,

$$\frac{4\frac{1}{2}\left(x + 2\frac{1}{2}\right) + 3}{1\frac{1}{5}} = 25 \Leftrightarrow \frac{\frac{9}{2}\left(x + \frac{5}{2}\right) + 3}{\frac{6}{5}} = 25$$

$\Leftrightarrow \frac{9x}{2} + \frac{45}{4} + 3 = 25 \times \frac{6}{5} = 30$

$\Leftrightarrow \frac{9x}{2} = 30 - \frac{57}{4} \Leftrightarrow \frac{9x}{2} = \frac{63}{4}$

$\Leftrightarrow x = \left(\frac{63}{4} \times \frac{2}{9}\right) = \frac{7}{2} = 3\frac{1}{2}.$

Sol.25 Let the numbers be 4x, 5x and 6x. Then, $\frac{4x+5x+6x}{3} = 25 \Leftrightarrow 5x = 25 \Leftrightarrow x = 5.$

$\therefore$ Largest number = 6x = 30.

Sol.26 Let the numbers be 3x, 4x and 6x.

Then, $3x \times 4x \times 6x = 1944 \Leftrightarrow 72x^3 = 1944$

$\Leftrightarrow x^3 = 27 \Leftrightarrow x = 3.$

$\therefore$ Largest number 6x = 18.

Sol.27 Let the numbers be 3x and 4x. Then,

$(4x)^2 = 8 \times (3x^2) - 224 \Leftrightarrow 16x^2 \times 72x^2 - 224$

$\Leftrightarrow 56x^2 = 224 \Leftrightarrow x^2 = 4 \Leftrightarrow x = 2.$

So, the numbers are 6 and 8

Sol.28 Let the number be 4x and 7x.

Then, $\frac{4x+4}{7x+4} = \frac{3}{5}$

$\Leftrightarrow 5(4x + 4) = 3\ (7x + 4) \Leftrightarrow x = 8.$

$\therefore$ Larger number = 7x = 56.

Sol.29 Let the second number be x. Then, first number = 2x and third number = $\frac{2x}{3}$

$\therefore 2x + x + \frac{2x}{3} = 264 \Leftrightarrow \frac{11x}{3} = 264$

$\Leftrightarrow x = \left(\frac{264\times 3}{11}\right) = 72.$

Sol.30 Let the numbers be x and (22 – x).

Then, $5x = 6\ (22 - x) \Leftrightarrow 11x = 132 \Leftrightarrow x = 12.$

So, the numbers are 12 and 10.

Sol.31 Let the numbers be x and y. Then, $\frac{1}{5}x = \frac{5}{8}y$

$\Leftrightarrow y = \frac{8}{25}x.$

Now, $x + 35 = 4y \Leftrightarrow x + 35 = \frac{32}{25}x \Leftrightarrow \frac{7}{25}x = 35$

$\Leftrightarrow x = \left(\frac{35\times 25}{7}\right) = 125.$

$\therefore$ Second number = $y = \frac{8}{25}x = \left(\frac{8}{25}\times 125\right) = 40.$

Sol.32 Let the numbers be x and y. Then, $x + y = 25$ and $x - y = 13.$

$4xy = (x + y)^2 - (x - y)^2 = (25)^2 - (13)^2$

$= 625 - 169 = 456 \Rightarrow xy = 114.$

Sol.33 Let the numbers be x and y.
Then, $x + y = 33$ ....(i) and $x - y = 15$....(ii)
Solving (i) and (ii), we get : $x = 24$, $y = 9$.
$\therefore$ Smaller number = 9.

Sol.34 Let the numbers be x and y.
Then,

$$\frac{x+y}{x-y} = \frac{40}{4} = 10 \Leftrightarrow (x + y) = 10\,(x - y)$$

$$\Leftrightarrow 9x = 11y \Leftrightarrow \frac{x}{y} = \frac{11}{9}.$$

Sol.35 Let the numbers be x and $(28 - x)$. Then,
$x(28 - x) = 192 \Leftrightarrow x^2 - 28x + 192 = 0$
$\Leftrightarrow (x - 16)\,(x - 12) = 0.$
$\Leftrightarrow x = 16$ or $x = 12$.
So, the numbers are 16 and 12

Sol.36 Let Rahul's age be x years. Then, Sachin's age = $(x - 7)$ years.

$$\therefore \frac{x-4}{x} = \frac{7}{9} \Leftrightarrow 9x - 36 = 7x$$

$\Leftrightarrow 2x = 36 \Rightarrow x = 18$
Hence, Sachin's age $(x - 4) = 14$ years.

Sol.37 Let P's age and Q's age be 6x years and 7x respectively.
Then, $7x - 6x = 4 \Leftrightarrow x = 4$.
$\therefore$ Required ratio = $(6x + 4) : (7x + 4) = 28 : 32$
$= 7 : 8.$

Sol.38 Let the present ages of P and Q be 5x years and 7x years respectively.
Then, $7x - (5x + 6) = 2 \Leftrightarrow 2x = 8 \Leftrightarrow x = 4$.
$\therefore$ Required sum = $5x + 7x = 12x = 48$ years.

Sol.39 Let the present ages of Arun and Deepak be 4x years and 3x years respectively.
Then, $4x + 6 = 26 \Leftrightarrow 4x = 20 \Leftrightarrow x = 5$.
$\therefore$ Deepak's age = $3x = 15$ years.

Sol.40 Let the present ages of X and Y be 5x years and 6x years respectively.

$$\text{Then, } \frac{5x+7}{6x+7} = \frac{6}{7} \Leftrightarrow 7(5x + 7) = 6\,(6x + 7)$$

$$\Leftrightarrow x = 7.$$

$\therefore$ X's present age = $5x = 35$ years.

Sol.41 Let the present ages of Sameer and Anand be 5x years and 4x years respectively.

$$\text{Then, } \frac{5x+3}{4x+3} = \frac{11}{9} \Leftrightarrow 9(5x + 3) = 11(4x + 3)$$

$$\Leftrightarrow x = 6.$$

$\therefore$ Anand's present age = $4x = 24$ years.

Sol.42 Let the ages of Kunal and Sagar 6 years ago be 6x and 5x years respectively.

Then, $\frac{(6x+6)+4}{(5x+6)+4} = \frac{11}{10} \Leftrightarrow 10(6x + 10)$

$= 11(5x + 10) \Leftrightarrow 5x = 10 \Leftrightarrow x = 2.$

$\therefore$ Sagar's present age = (5x + 6) = 16 years.

Sol.43 Let the ages of Jayant, Prem and Saransh 10 years ago be 2x, 3x and 4x years respectively.

Then, $(2x + 10) + (3x + 10) + (4x + 10) = 93$

$\Leftrightarrow 9x = 63 \Leftrightarrow x = 7.$

$\therefore$ Saransh's present age = (4x + 10) = 38 years.

Sol.44 Let the present ages of the two brothers be x years and 2x years respectively.

Then, $\frac{x-5}{2x-5} = \frac{1}{3} \Leftrightarrow 3(x - 5) = (2x -)5 \Leftrightarrow x = 10.$

$\therefore$ Required ratio = (x + 5) : (2x + 5) = 15 : 25

= 3 : 5

Sol.45 Suppose, the ratio was 3 : 5, x years ago.

Then, $\frac{40-x}{60-x} = \frac{1}{3} \Leftrightarrow 5(40 - x) = 3\ (60 - x)$

$\Leftrightarrow 2x = 20 \Leftrightarrow x = 10.$

Sol.46 Let the present ages of the father and son be 7x and 3x years respectively.

Then, $7x \times 3x = 756 \Leftrightarrow 21x^2 = 756 \Leftrightarrow x^2 = 36$

$\Leftrightarrow x = 6.$

$\therefore$ Required ratio = (7x + 6) : (3x + 6) = 48 : 24

= 2 : 1

Sol.47 Let their present ages be 4x, 7x and 9x years respectively.

Then, $(4x - 8) + (7x - 8) + (9x - 8) = 56$

$\Leftrightarrow 20x = 80 \Leftrightarrow x = 4$

$\therefore$ Their present ages are 16 years, 28 years and 36 years respectively.

Sol.48 Let the present ages of the man and his wife be 4x and 3x years respectively.

Then, $\frac{4x+4}{3x+4} = \frac{9}{7} \Leftrightarrow 7(4x + 4) = 9(3x + 4)$

$\Leftrightarrow x = 8.$

So, their present ages are 32 years and 24 years respectively.

Suppose they were married z years ago.

Then, $\frac{32-z}{24-z} = \frac{5}{3} = 3(32 - z) = 5(24 - z)$

$\Leftrightarrow 2z = 24 \Leftrightarrow z = 12.$

Sol.49 Let the school ages of Neelam and Shaan be 5x and 6x years respectively. Then, $\frac{\frac{1}{3}\times 5x}{\frac{1}{2}\times 6x} = \frac{5}{9}$

$\Leftrightarrow \left(\frac{1}{3}\times 9\times 5x\right) = \left(\frac{5}{2}\times 6x\right) \Leftrightarrow 15 = 15.$

Thus, Shaan's are cannot be determind.

Sol.50 Let the present ages of A and B be 5x and 3x years respectively.

Then, $\frac{5x-4}{3x+4} = \frac{1}{1} \Leftrightarrow 5x - 4 = 3x + 4$

$\Leftrightarrow 2x = 8 \Leftrightarrow x = 4.$

$\therefore$ Required ratio = $(5x + 4): (3x - 4)$

$= 24 : 8 = 3 : 1.$

Sol.51 Let the ages of A and B 10 years ago be x and 2x years respectively.

Then, $\frac{x+10}{2x+10} = \frac{3}{4} \Leftrightarrow 4(x + 10) = 3(2x + 10)$

$\Leftrightarrow 2x = 10 \Leftrightarrow x = 5.$

$\therefore$ Sum of their present ages

$= (x + 10) + (2x + 10) = (3x + 20) = 35$ years.

Sol.52 Let C's age be x years . Then, B's age = 2x years, A's age = $(2x + 2)$ years.

$\therefore (2x + 2) + 2x + x = 27 \Leftrightarrow 5x = 25 \Leftrightarrow x = 5.$

Hence, B's age = $2x = 10$ years.

Sol.53 Let the son's present age be x years. Then, man's present age = $(x + 24)$ years.

$\therefore \quad (x + 24) + 2 = 2(x + 2)$

$\Leftrightarrow \quad x + 26 = 2x + 4 \Leftrightarrow x = 22.$

Sol.54 Let the present ages of the father and son be 2x and x years respectively.

Then, $(2x - 18) = 3(x - 18) \Leftrightarrow x = 36.$

$\therefore$ Required sum = $(2x + x) = 3x = 108$ years.

Sol.55 Let the mother's present age be x years.Then, the person's present age = $\left(\frac{2}{5}x\right)$ years

$\therefore \left(\frac{2}{5}x+8\right) = \frac{1}{2}(x + 8) \Leftrightarrow 2(2x + 40) = 5(x + 8) \Leftrightarrow x = 40.$

Sol.56 16 years ago, let T = x years and G = 8x years.

After 8 years from now, T = $(x + 16 + 8)$ years and G = $(8x + 16 + 8)$ years.

$\therefore 8x + 24 = 3 (x + 24) \Leftrightarrow 5x = 48.$

8 years ago, $\frac{T}{G} = \frac{x+8}{8x+8} = \frac{\frac{48}{5}+8}{8\times\frac{48}{5}+8} = \frac{88}{424} = \frac{11}{53}.$

Sol.57 Let the ages of father and son 10 years ago be 3x and x years respectively.

Then, $(3x + 10) + 10 = 2\ [(x + 10) + 10] \Leftrightarrow 3x + 20 = 2x + 40 \Leftrightarrow x = 20.$

$\therefore$ Required rtaio = $(3x + 10) : (x + 10)$

$\Leftrightarrow 70 : 30 = 7 : 3.$

Sol.58 Let the ages of father and son 4 years ago be 3x and x years respectively.

Then, $[(3x + 4) + 4] + [(x + 4) + 4] = 64$

$\Leftrightarrow 4x = 48 \Leftrightarrow x = 12.$

$\therefore$ Father's present age = $3x = 36$ years.

Sol.59 Let ages of Promila and Sakshi 1 year ago be 4x and x years respectively.

Then, $[(4x + 1) + 6] - [(x + 1) + 6] = 9$

$\Leftrightarrow 3x = 9 \Leftrightarrow x = 3.$

$\therefore$ Required ratio $= (4x + 1) : (x + 1) = 13 : 4.$

Sol.60 Let the present ages of son and father be x and $(60 - x)$ years respectively.

Then, $(60 - x) - 6 = 5(x - 6) \Leftrightarrow 54 - x$

$= 5x - 30 \Leftrightarrow 6x = 84 \Leftrightarrow x = 14.$

$\therefore$ Son's age after 6 year $= (x + 6) = 20$ years.