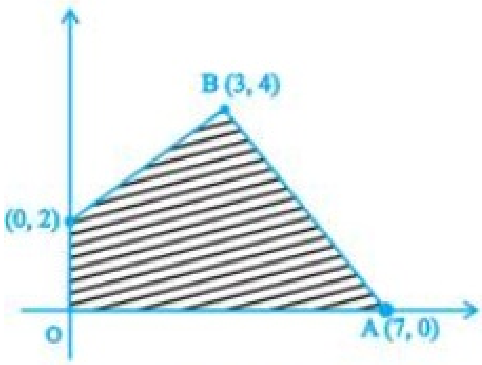
CBSE Test Paper 03
Chapter 12 Linear Programming

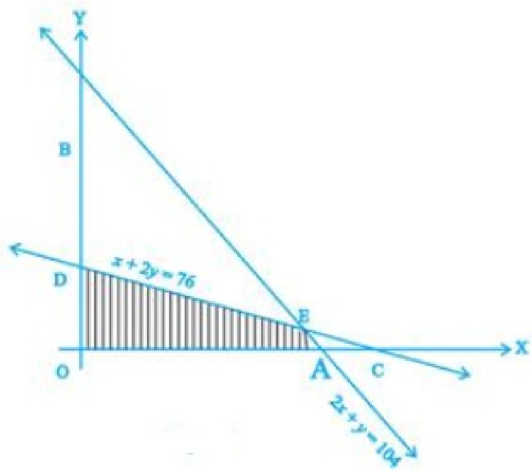
1. If two corner points of the feasible region are both optimal solutions of the same type, i.e., both produce the same maximum or minimum.
   a. then any point on the line segment joining these two points is also an optimal solution of the same type
   b. then no point on the line segment joining these two points is an optimal solution of the same type
   c. then no point on the line segment joining these two points is an optimal solution of the opposite type
   d. then any point on the line segment joining these two points is also an optimal solution of the opposite type
2. A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximize his profit? Determine the maximum profit.
   a. 30 packages of screws A and 22 packages of screws B; Maximum profit = Rs 412
   b. 30 packages of screws A and 20 packages of screws B; Maximum profit = Rs 410
   c. 32 packages of screws A and 22 packages of screws B; Maximum profit = Rs 414
   d. 32 packages of screws A and 20 packages of screws B; Maximum profit = Rs 412
3. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit ?

a. 42 tickets of executive class and 160 tickets of economy class; Maximum profit = Rs 139000
b. 45 tickets of executive class and 160 tickets of economy class; Maximum profit = Rs 156000
c. 44 tickets of executive class and 160 tickets of economy class; Maximum profit = Rs 146000
d. 40 tickets of executive class and 160 tickets of economy class; Maximum profit = Rs 136000

4. In a LPP, the linear inequalities or restrictions on the variables are called
   a. Limits
   b. Inequalities
   c. Linear constraints
   d. Constraints
5. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit?
   a. 820 dolls of type A and 420 dolls of type B; Maximum profit = Rs 16200
   b. 800 dolls of type A and 400 dolls of type B; Maximum profit = Rs 16000
   c. 840 dolls of type A and 404 dolls of type B; Maximum profit = Rs 16500
   d. 830 dolls of type A and 430 dolls of type B; Maximum profit = Rs 16300
6. The maximum or minimum value of an objective function is known as ________.
7. The graph of $x \leq 2$ and $y \geq 2$ will be situated in the first and ________ quadrant.
8. If the feasible region for a LPP is ________, then the optimal value of the objective function Z = ax + by may or may not exist.
9. Feasible region (shaded) for a LPP is shown in Fig. Maximise Z = 5x + 7y.



10. Two tailors A and B, earn Rs 300 and Rs 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.
11. A manufacturing company makes two types of television sets, one is black and white and other is colour. The company has resource to make at most 300 sets a week. It takes Rs.1800 to make a black and white set and Rs.2700 to make a coloured set. The company can spend not more than Rs.648000 a week to make television sets. If it makes a profit of Rs.510 per black and white set and Rs.675 per coloured set, how many sets of each type should be produced so that the company has maximum profit? Formulate this problem as an L.P.P. given that the objective is to maximize the profit.
12. Determine the maximum value of Z = 3x + 4y if the feasible region (shaded) for a LPP is shown in Fig.



13. A man rides his motorcycle at the speed of 50 km/hour. He has to spend Rs 2 per km on petrol. If he rides it at a faster speed of 80 km/hour, the petrol cost increases to Rs 3 per km. He has at most Rs 120 to spend on petrol and one hour time. He wishes to find the maximum distance that he can travel.

    Express this problem as a linear programming problem.

14. D.A manufacture of electronic circuits has a stock of 200 resistors,120 transistors and 150 capacitors and is required to produce two types of circuits A and B .Type A requires 20 resistors,10 transistors and 10 capacitors .Type B requires 10 resistors,20 transistors and 30 capacitors.If the profit on type A circuit is Rs 50 and that an type B circuit is Rs 60,how many of circuits of type A and type B,should be produced by the manufacturer so as to maximize his profit?Determine the maximum profit.
15. Maximize Z = x + 2y subject to the constraints
    $x - y \geq 0, 2y \leq x + 2, x, y \geq 0$
16. A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost Rs 25000 and Rs 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs 70 lakhs and if his profit on the desktop model is Rs 4500 and on portable model is Rs 5000.
17. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hour on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs 5 and that from a shade is Rs 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit?
18. If a youngman rides his motorcycle at 25 km/ hour, he has to spend Rs. 2 per km on petrol. If he rides at a faster speed of 40 km/hour, the petrol cost increases at Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this as an L.P.P. and solve it graphically.
    Which mode of transport you suggest to a young man and why?

Solution

1. a. then any point on the line segment joining these two points is also an optimal solution of the same type
   Explanation: If two corner points of the feasible region are both optimal solutions of the same type, i.e., both produce the same maximum or minimum , then any point on the line segment joining these two points is also an optimal solution of the same type .

2. b. 30 packages of screws A and 20 packages of screws B; Maximum profit = Rs 410
   Explanation: Let number of packages of screws A produced = x
   And number of packages of screws B produced = y
   Therefore , the above L.P.P. is given as :
   Maximise, Z = 7x +10y, subject to the constraints : $4x + 6y \le 240$ and. $6x + 3y \le 240$
   i.e. $2x + 3y \le 120$ and $2x + y \le 80$, $x, y \ge 0$.

Corner points	Z =7 x +10 y
O(0 , 0)	0
D(40,0)	280
A(0,40)	400
B(30,20)	410....................(Max.)

   Here Z = 410 is maximum.
   i.e 30 packages of screws A and 20 packages of screws B; Maximum profit = Rs 410.

3. d. 40 tickets of executive class and 160 tickets of economy class; Maximum profit = Rs 136000
   Explanation: Let number of executive class tickets = x
   And number of economy class tickets = y
   Therefore , the above L.P.P. is given as :
   Minimise , Z = 1000x +600y , subject to the constraints : $x + y \le 200$, $4x - y \le 0$, $x \ge$

20,x,y ≥ 0.,

Corner points	Z =1000x +600 y
C(20 ,80)	68000
B (40,160)	136000
D(20,180)	12800

Here Z = 136000 is maximum.

i.e. 40 tickets of executive class and 160 tickets of economy class; Maximum profit = Rs 136000.

4. c. Linear constraints

Explanation: In a LPP, the linear inequalities or restrictions on the variables are called Linear constraints.

5. b. 800 dolls of type A and 400 dolls of type B; Maximum profit = Rs 16000

Explanation: Here , Maximise Z = 12x + 16y , subject to constraints : : x + y ≤ 1200, x - 2y ≥ 0, x - 3y ≤ 600 , x,y ≥ 0

Corner points	Z = 12x +16 y
C(0, 0)	0
B (600,0)	7200
D(1050,150)	15000
A(800,400)	16000................(Max.)

Here Z = 16000 is maximum.i.e. 800 dolls of type A and 400 dolls of type B; Maximum profit= Rs 16000.

6. optimal value
7. second
8. unbounded
9. The Shaded region is bounded and has coordinate of corner points as (0, 0), (7, 0), (3, 4) and (0, 2). Also, Z = 5x + 7y.

Corner Points	**Corresponding value of Z**
(0, 0)	0
(7, 0)	35

(3, 4)	43 (Maximum)
(0, 2)	14

Hence, the maximum value of Z is 43 at (3, 4).

10. According to the given situation , the given data can be tabularised as following

	Tailor A	**Tailor B**	**Minimum Total No.**
No. of Shirts	6	10	60
No. of Trousers	4	4	32
Wage	Rs 300/day	Rs 400/day	

Let tailor A and tailor B work for x days and y days, respectively.Given that the minimum number of shirts that can be stitched per day is 60. The inequality representing the information is given as

$\therefore$ 6x + 10y $\geq$ 60 $\Rightarrow$(shirt constraint) (dividing by 2 we get)

3x + 5y $\geq$ 30

Given that the minimum number of trousers that can be stitched per day is 32.

$\therefore$ 4x + 4y $\geq$ 32 $\Rightarrow$ (trouser constraint) (dividing throughout by 4 we get

x + y = 8

$\therefore$ x $\geq$ 0, y $\geq$ 0 (non negative constarint which restricts the feasible region in the first quadrant only , since it is real world situation and the variables cannot take negative values.

Let z be the objective function representing the total labour cost. Hence the equation for the cost function is given as z = 300x + 400y

So, the given L P.P. is designed as

z = 300x + 400y

x $\geq$ 0, y $\geq$ 0, 3x + 5y $\geq$ 30 and x + y $\geq$ 8

11. Let x and y denote the number of black and white sets and coloured sets respectively such that $x \geq 0, y \geq 0$

As the company produces at most 300 sets a week

$\Rightarrow \quad x + y \leq 300$

Company cannot spend more than $Rs.648000$,

$1800x + 2700y \leq 648000$ or $2x + 3y \leq 720$

And total profit on x black and white and y coloured sets Rs.$(510x + 675y)$

Let $Z = 510x + 675y$ i.e. objective function.

Maximize $Z = 510x + 675y$

Subject to constraints

$x + y \leq 300$

$2x + 3y \leq 720$

s.t. $x \geq 0, y \geq 0$

For solution of an L.P.P, firstly, we draw the line $x + y = 300$ and $2x + 3y = 720$ and shaded the feasible region w.r.t. the constraint sign. Here, we observed that the feasible region is bounded and corner points are

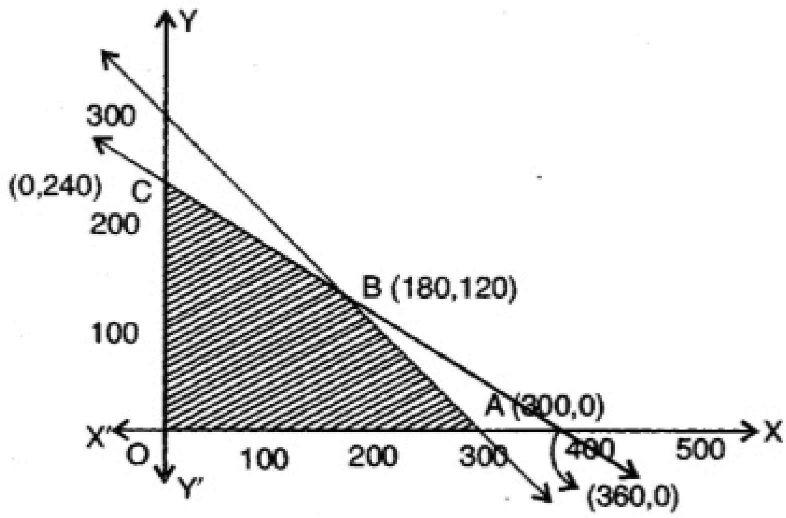
$O(0,0), A(300,0) B(180,120)\ and\ C(0,240).$

Evaluating the 'Z' at each comer point, we have

Corner Points	**Z = 510x + 675y**
$A(300,0)$	$\mathrm{Z} = 510 \times 300 = 153000$
$B(180,120)$	$\mathrm{Z} = 510 \times 180 + 675 \times 120$ $= 91800 + 81000 = 172800$ Maximum
$C(0,240)$	$Z = 0 \times 510 + 240 \times 675 = 16200$

We observed that the maximum profit is $Z = Rs.172800$ at $B(180,120)$.

If the company produces 180 black and white sets and 120 coloured sets then profit of the company will be maximum.



12. As clear from the given graph, the coordinates of the corner points O, A, E and Dare given as (0, 0), (52, 0), (144, 16) and (0, 38), respectively. Also given region is bounded. Given that the objective function, Z = 3x + 4y to be maximised.

∵ Also converting the given inequalities into their equations to find their points of intersection ,

2x + y = 104............(i) and

2x + 4y = 152............(ii)

solving above equations (i) and (ii), we get

⇒ -3y = -48

⇒ y = 16 and x = 44.

Hence the point of intersection of the lines (i) and (ii) is (16,44)

The table below gives the values of the objective function Z, at the corner points of the feasible region.

Corner Points	**Corresponding value of Z**
(0, 0)	Z = 0
(52, 0)	Z = 156
(44, 16)	Z = 196 (maximum)
(0, 38)	Z = 152

Hence, Z attains its maximum at the point (44,16) and its maximum value is 196.

13. Let the man rides to his motorcycle to a distance x km at the speed of 50 km/h and to a distance y km at the speed of 80 km/h.

    Therefore, cost on petrol is 2x + 3y.

    Since, he has to spend Rs 120 at most on petrol.

    $\therefore 2x+3y \leqslant 120$ .... (i)

    Also, he has at most 1 hour time.

    $\therefore \frac{x}{50}+\frac{y}{80} \leqslant 1$

    $\Rightarrow 8x+5y \leqslant 400$ ..... (ii)

    Also, we have $x \geqslant 0, y \geqslant 0$ [non-negative constraints]

    Thus, required LPP to travel maximum distance by him is

    Maximise Z = x + y, subject to $2x+3y \leqslant 120, 8x+5y \leqslant 400, x \geqslant 0, y \geqslant 0$

14. To Maximise the objective function Z = 50x + 60y subject to the constraints

    $2x+y \leqslant 20, x+2y \leqslant 12, x+3y \leqslant 15, x \geqslant 0, y \geqslant 0$

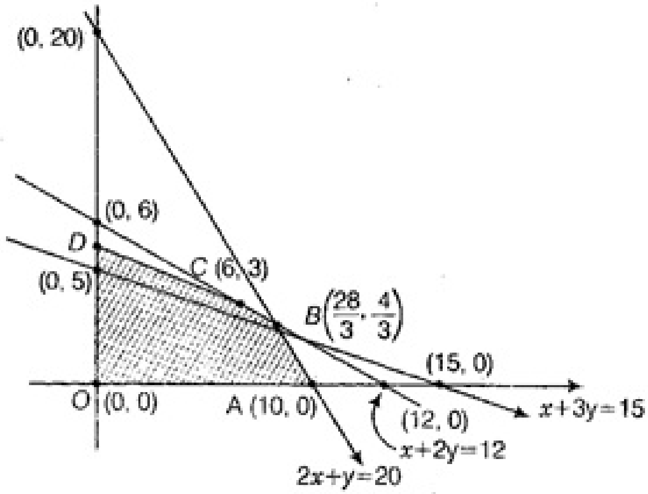
    From the shaded region of the graph, it is clear that the feasible region determined by the system of constraint is OABCDO, and it is bounded and the coordinates of corner points are (0,0) (10,0)

$\left(\frac{28}{3}, \frac{4}{3}\right)$,(6, 3), and (0, 5) respectively.

[Solving, x + 2y = 12 and 2x + y = 20, we get the point of intersection as

$\Rightarrow x=\frac{28}{3}, y=\frac{4}{3}$

and the point of intersection of x + 3y = 15 and x + 2y = 12 is given as y=3 and x=16



Corner Points	Corresponding value of Z = 50x + 60y
(0, 0)	Z = 0
(10, 0)	Z = 500
$\left(\frac{28}{3}, \frac{4}{3}\right)$	Z = $\frac{1400}{3}+\frac{240}{3}=\frac{1640}{3}=546.66(\max imum)$
(6, 3)	Z = 480
(0, 5)	Z = 300

Since, the manufacturer is required to produce two types of circuits A and B and it is clear that parts of resistor, transistor and capacitor cannot be in fraction, So the required maximum profit is Rs. 480 where circuits of type A is 6 and circuits of type B is 3.

15. According to the question,

Maximize $Z=x+2y$

Subject to the constraints

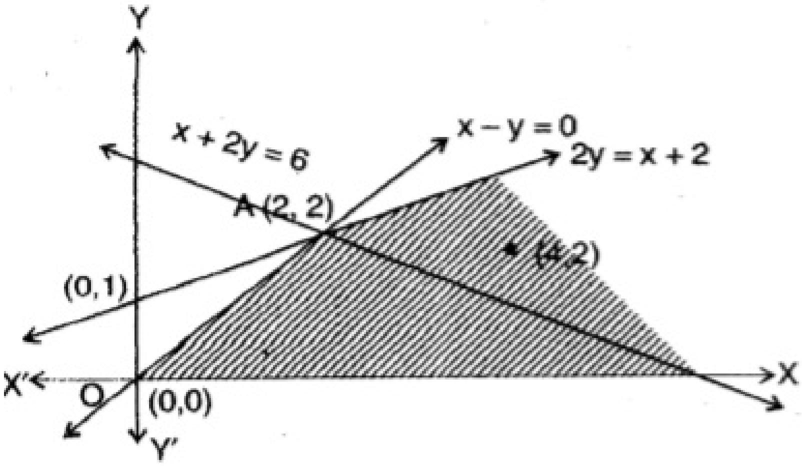
$x-y \geq 0$

$2y \leq x+2$

such that $x \geq 0, y \geq 0$

We draw line $x-y=0$ and $2y=x+2$ and shaded the feasible region with respect to constraints sign.

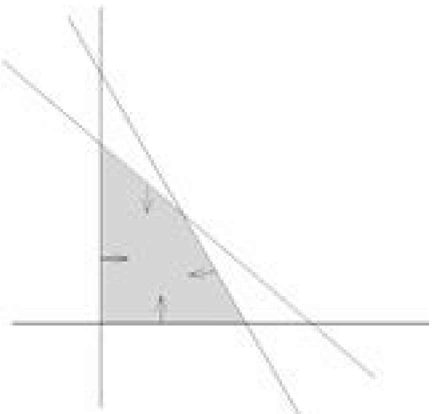
We observe that the feasible region is unbounded and corner points are $O(0,0), A(2,2)$ only. Evaluating Z at all corner points,



Corner Points	Z = x + 2y
$(0,0)$	$Z=0\ (Smallest)$
$(2,2)$	$Z=6\ (largest)$

We are to determine the maximum value. As the feasible region is unbounded we can not say whether the largest value Z = 6 is maximum or not.

We draw the half plane $x+2y>6$ and notice that it has common points with feasible region. There does not exist any maximum value. For testing we let $x=4, y=2$, this point lie in the feasible region and at $(4,2)$ the value of $Z=8$ i.e., greater than '6'.

16. 

Let the merchant stock x desktop computers and y portable computer.

Z = 4500x + 5000y

x + y ≤ 250

25000x + 40000y ≤ 7000000

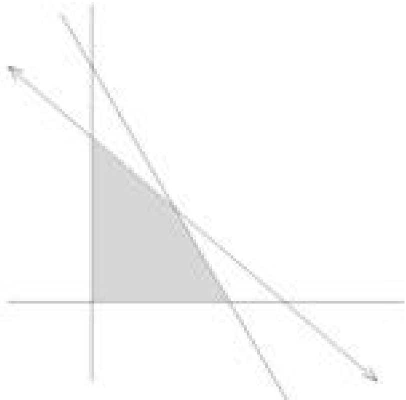
$\Rightarrow$ 5x + 8y ≤ 1400

x ≥ 0, y ≥ 0.

Solving these equations we get x=250 and y=50

Profit is maximum z = 4500 $\times$ 250 + 5000 $\times$ 50 = 1150000

Hence, the merchant should stock 200 units of desktop model and 50 units of a portable model to realise maximum profit.

17. 

Let x be pedestal lamps and y wooden shades

Z = 5x + 3y

2x + y ≤ 12

3x + 2y ≤ 20

x ≥ 0, y ≥ 0

Solving these equations we get x = 4 and y = 4

maximum profit is

$z = 5 \times 4 + 3 \times 4 = 20 + 12 = 32$

Thus, the manufacturer must produce 4 pedestal lamps and 4 wooden shades daily in order to maximize profit.

18. Suppose the youngman travels x km at the speed 25 km/hour and y km at the speed of 40 km/hour.

Speed	Distance (d)	Time	Cost
25 km/hour	x	$\frac{x}{25}$	2x
40 km/hour	y	$\frac{y}{40}$	5y

We have to maximize $d = x + y$ ...(i)

subjects to constraints $x \geq 0, y \geq 0$ ...(ii)

$\frac{x}{25} + \frac{y}{40} \leq 1$ ...(iii)

$2x + 5y \leq 100$ ....(iv)

Consider the line $\frac{x}{25}+\frac{y}{40}=1$

$\Rightarrow 8x+5y=200$ ...(v)

When $x=0, y=40$

and $y=0, x=25$

$A(0,40); B(25,0)$

Consider the line

$2x+5y=100$ ...(vi)

When $x=0, y=20$

and $y=0, x=50$

$C(0,20); D(50,0)$; the lines (v) and (vi) intersect at $\left(\frac{50}{3}, \frac{40}{3}\right)$.

Now, feasible region shaded have corner points

$B(25,0), \mathrm{E}\left(\frac{50}{3}, \frac{40}{3}\right)$ and $C(0,20)$.

Now, corner points of feasible region are examined for the maximum value of d.

At $B(25,0), d=25+0=25$

At $\mathrm{E}\left(\frac{50}{3}, \frac{40}{3}\right), d=\frac{50}{3}+\frac{40}{3}=30$

At $C(0,20), d=0+20=20$

d is maximum, when youngman travel $\frac{50}{3}$ km at a speed of 25 km/hour and $\frac{40}{3}$ km at a speed of 40 km/hour.

Total distance covered = 30 km.

Cycles should be promoted as a mode of transport because it is good for health, saves energy (no petrol) and no pollution.