6. Trigonometric Ratios

Exercise 6.1

1. Question

Find the value of the following:

2 sin 45° . cos 45°

Answer

$$\sin 45° = \frac{1}{\sqrt{2}}$$

$$\cos 45° = \frac{1}{\sqrt{2}}$$

∴ 2 sin 45° .cos 45°

$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$$= 2 \times \frac{1}{2}$$

= 1

2. Question

Find the value of the following:

cos 45° cos 60° – sin 45° sin 60°

Answer

$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$\sin 45° = \frac{1}{\sqrt{2}}$$

$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{1}{2}$$

$\therefore \cos 45° \cos 60° - \sin 45° \sin 60°$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{2} - \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1-\sqrt{3}}{2\sqrt{2}}$$

3. Question

Find the value of the following:

$\sin^2 30° + 2 \cos^2 45° + 3 \tan^2 60°$

Answer

$$\sin 30° = \frac{1}{2}$$

$$\therefore \sin^2 30° = \frac{1}{4}$$

$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$\therefore \cos^2 45° = \frac{1}{2}$$

$\tan 60° = \sqrt{3}$

$$\therefore \tan^2 60° = 3$$

$\therefore \sin^2 30° + 2\cos^2 45° + 3\tan^2 60°$

$$= \frac{1}{4} + 2 \times \frac{1}{2} + 3 \times 3$$

$$= \frac{1}{4} + 1 + 9$$

$$= \frac{41}{4}$$

$$= 10\frac{1}{4}$$

4. Question

Find the value of the following:

$3 \sin 60° - 4 \sin^3 60°$

Answer

Let $\theta = 60°$

then,

$3 \sin 60° - 4 \sin^3 60°$

$= 3\sin\theta - 4\sin^3\theta$

We know that

$\text{Sin}3\theta = 3\sin\theta - 4\sin^3\theta$

$\Rightarrow 3 \sin 60° - 4 \sin^3 60°$

$= \sin(3 \times 60°)$

$= \sin 180°$

$= 0$

5. Question

Find the value of the following:

$$\frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 45°}$$

Answer

$$\cos 60° = \frac{1}{2}$$

$$\therefore \cos^2 60° = \frac{1}{4}$$

$$\sec 30° = \frac{2}{\sqrt{3}}$$

$$\therefore \sec^2 30° = \frac{4}{3}$$

$\tan 45° = 1$

$\therefore \tan^2 45° = 1$

$$\sin 30° = \frac{1}{2}$$

$$\therefore \sin^2 30° = \frac{1}{4}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\therefore \cos^2 45^\circ = \frac{1}{2}$$

$$\therefore \frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 45^\circ}$$

$$= \frac{5 \times \frac{1}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{1}{2}}$$

$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{1}{2}}$$

$$= \frac{\frac{5 \times 3 + 16 \times 4 - 12}{12}}{\frac{1 + 1 \times 2}{4}}$$

$$= \frac{\frac{67}{12}}{\frac{3}{4}}$$

$$= \frac{67}{9}$$

6. Question

Find the value of the following:

$4\cot^2 45^\circ - \sec^2 60^\circ + \sin^2 60^\circ + \cos^2 90^\circ$

Answer

$\cot 45^\circ = 1$

$\therefore \cot^2 45^\circ = 1$

$\sec 60^\circ = 2$

$\therefore \sec^2 60^\circ = 4$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\therefore \sin^2 60^\circ = \frac{3}{4}$$

$\cos 90^\circ = 0$

$\therefore \cos^2 90° = 0$

$\therefore 4\cot^2 45° - \sec^2 60° + \sin^2 60° + \cos^2 90°$

$$= 4 - 4 + \frac{3}{4} + 0$$

$$= \frac{3}{4}$$

7. Question

Find the value of the following:

$$\frac{4}{\cot^2 30°} + \frac{1}{\sin^2 30°} - \cos^2 45°$$

Answer

$\cot 30° = \sqrt{3}$

$\therefore \cot^2 30° = 3$

$$\sin 30° = \frac{1}{2}$$

$$\therefore \sin^2 30° = \frac{1}{4}$$

$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$\therefore \cos^2 45° = \frac{1}{2}$$

$$\therefore \frac{4}{\cot^2 30°} + \frac{1}{\sin^2 30°} - \cos^2 45°$$

$$= \frac{4}{3} + \frac{1}{\frac{1}{4}} - \frac{1}{2}$$

$$= \frac{4}{3} + 4 - \frac{1}{2}$$

$$= \frac{29}{6}$$

8. Question

Find the value of the following:

$$\frac{\tan^2 60° + 4\sin^2 45° + \sin^2 90°}{3\sec^2 30° + \operatorname{cosec}^2 60° - \cot^2 30°}$$

Answer

$\tan 60° = \sqrt{3}$

$\tan^2 60° = 3$

$$\sin 45° = \frac{1}{\sqrt{2}}$$

$$\therefore \sin^2 45° = \frac{1}{2}$$

$\sin 90° = 1$

$\therefore \sin^2 90° = 1$

$$\sec 30° = \frac{2}{\sqrt{3}}$$

$$\therefore \sec^2 30° = \frac{4}{3}$$

$$\operatorname{cosec} 60° = \frac{2}{\sqrt{3}}$$

$$\therefore \operatorname{cosec}^2 60° = \frac{4}{3}$$

$$\frac{\tan^2 60° + 4\sin^2 45° + \sin^2 90°}{3\sec^2 30° + \operatorname{cosec}^2 60° - \cot^2 30°}$$

$$= \frac{3 + 4 \times \frac{1}{2} + 1}{3 \times \frac{4}{3} + \frac{4}{3} - 3}$$

$$= \frac{18}{7}$$

9. Question

Find the value of the following:

$$\frac{\sin 30° - \sin 90° + 2\cos 0°}{\tan 30° \,.\, \tan 60°}$$

Answer

$\sin 90° = 1$

$\cos 0° = 1$

$$\sin 30° = \frac{1}{2}$$

$$\tan 30° = \frac{1}{\sqrt{3}}$$

$\tan 60° = \sqrt{3}$

$$\therefore \frac{\sin 30° - \sin 90° + 2\cos 0°}{\tan 30° . \tan 60°}$$

$$= \frac{\frac{1}{2} - 1 + 2 \times 1}{\sqrt{3} \times \frac{1}{\sqrt{3}}}$$

$$= \frac{3}{2}$$

10. Question

Find the value of the following:

$$\frac{2\tan 30°}{1 - \tan^2 30°}$$

Answer

$$\tan 30° = \frac{1}{\sqrt{3}}$$

$$\tan^2 30° = \frac{1}{3}$$

$$\therefore \frac{2\tan 30°}{1 - \tan^2 30°}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$= \sqrt{3}$

11 A. Question

Find the value of x in the following:

cos x = cos 60° cos 30° + sin 60° sin 30°

Answer

We know that,

Cos(A - B) = cosAcosB + sinAsinB

Let A = 60°, B = 30°

Then,

cos 60° cos 30° + sin 60° sin 30°

= cos(A - B)

= cos(60° - 30°)

= cos30°

∴cos x = cos 60° cos 30° + sin 60° sin 30°

⇒ cos x = cos 30°

⇒ x = 30°

11 B. Question

Find the value of x in the following:

sin 2x = sin 60° cos 30° – cos 60° – sin 30°

Answer

We know that,

sin(A - B) = sinAcosB - cosAsinB

LetA = 60°, B = 30°

Then,

sin 60° cos 30° – cos 60°sin 30°

= sin(60° - 30°)

= sin30°

∴sin 2x = sin 60° cos 30° – cos 60°sin 30°

⇒ sin 2x = sin30°

⇒ 2x = 30°

$$\Rightarrow x = \frac{30°}{2}$$

$\Rightarrow x = 15°$

11 C. Question

Find the value of x in the following:

$\sqrt{3} \tan 2x = \sin 30° + \sin 45° \cos 45° + 2 \sin 90°$

Answer

$$\sin 30° = \frac{1}{2}$$

$$\sin 45° = \frac{1}{\sqrt{2}}$$

$$\cos 45° = \frac{1}{\sqrt{2}}$$

$\sin 90° = 1$

$\therefore \sin 30° + \sin 45° \cos 45° + 2 \sin 90°$

$$= \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + 2 \times 1$$

$= 3$

$\therefore \sqrt{3} \tan 2x = \sin 30° + \sin 45° \cos 45° + 2 \sin 90°$

$\Rightarrow \sqrt{3} \tan 2x = 3$

$$\Rightarrow \tan 2x = \frac{3}{\sqrt{3}}$$

$\Rightarrow \tan 2x = \sqrt{3}$

But, $\tan 60° = \sqrt{3}$

$\Rightarrow \tan 2x = \tan 60°$

$\Rightarrow 2x = 60°$

$$\Rightarrow x = \frac{60}{2}$$

$\Rightarrow x = 30°$

12. Question

Prove that-

$$\frac{\cos 30° + \sin 60°}{1 + \cos 60° + \sin 30°} = \frac{\sqrt{3}}{2}$$

Answer

$$\cos 30° = \frac{\sqrt{3}}{2}$$

$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{1}{2}$$

$$\sin 30° = \frac{1}{2}$$

∴LHS =

$$\frac{\cos 30° + \sin 60°}{1 + \cos 60° + \sin 30°}$$

$$= \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{1 + \frac{1}{2} + \frac{1}{2}}$$

$$= \frac{\sqrt{3}}{2}$$

RHS = $\frac{\sqrt{3}}{2}$

$$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

∴ LHS = RHS

$$\therefore \frac{\cos 30° + \sin 60°}{1 + \cos 60° + \sin 30°} = \frac{\sqrt{3}}{2}$$

Hence Proved

13. Question

Prove that-

$$4\cot^2 45° - \sec^2 60° - \sin^2 30° = -\frac{1}{4}$$

Answer

Cot45° = 1

∴ $\cot^2 45° = 1$

Sec60° = 2

∴ $\sec^2 60° = 4$

$$\sin 30° = \frac{1}{2}$$

$$\therefore \sin^2 30° = \frac{1}{4}$$

∴ LHS = $4\cot^2 45° - \sec^2 60° - \sin^2 30°$

$$= 4 \times 1 - 4 - \frac{1}{4}$$

$$= 0 - \frac{1}{4}$$

$$= -\frac{1}{4}$$

RHS = $-\frac{1}{4}$

$$-\frac{1}{4} = -\frac{1}{4}$$

∴ LHS = RHS

$$\therefore 4\cot^2 45° - \sec^2 60° - \sin^2 30° = -\frac{1}{4}$$

Hence Proved

14. Question

Prove that-

$\sin 30° \tan^2 60° + 3\cos 60° \tan 45° = 2\sec^2 45° - \text{cosec}^2 90°$

Answer

$$\sin 30° = \frac{1}{2}$$

$$\tan 60° = \sqrt{3}$$

$\therefore \tan^2 60 = 3$

$\cos 60° = \frac{1}{2}$

$\tan 45° = 1$

$\sec 45° = \sqrt{2}$

$\therefore \sec^2 45° = 2$

$\text{cosec} 90° = 1$

$\therefore \text{cosec}^2 90° = 1$

$\therefore$ LHS = $\sin 30° \tan^2 60° + 3\cos 60° \tan 45°$

$= \frac{1}{2} \times 3 + 3 \times \frac{1}{2} \times 1$

$= \frac{3}{2} + \frac{3}{2} = 3$

RHS

$2\sec^2 45° - \text{cosec}^2 90°$

$= 2(2) - 1$

$= 3$

RHS = LHS

Henc, Proved!

15. Question

Prove that-

$$\text{cosec}^2 45° . \sec^2 30° \sin^3 90° \cos 60° = \frac{4}{3}$$

Answer

$\text{cosec} 45° = \sqrt{2}$

$\therefore \text{cosec}^2 45° = 2$

$\sec 30° = \frac{2}{\sqrt{3}}$

$\therefore \sec^2 30° = \frac{4}{3}$

$\sin 90° = 1$

$\therefore \sin^3 90° = 1$

$$\cos 60° = \frac{1}{2}$$

$\therefore$ LHS = $\text{cosec}^2 45° . \sec^2 30° \sin^3 90° \cos 60°$

$$= 2 \times \frac{4}{3} \times 1 \times \frac{1}{2}$$

$$= \frac{4}{3}$$

= RHS

$\therefore$ LHS = RHS

$$\therefore \text{cosec}^2 45° . \sec^2 30° \sin^3 90° \cos 60° = \frac{4}{3}$$

Hence Proved

16. Question

Prove that-

$$\frac{\sin 60° + \sin 30°}{\sin 60° - \sin 30°} = \frac{\tan 60° + \tan 45°}{\tan 60° - \tan 45°} =$$

Answer

$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\sin 30° = \frac{1}{2}$$

$\tan 60° = \sqrt{3}$

$\tan 45° = 1$

LHS =

$(\sin 60° + \sin 30°)/(\sin 60° - \sin 30°)$

$$= \frac{\frac{\sqrt{3}}{2} + \frac{1}{2}}{\frac{\sqrt{3}}{2} - \frac{1}{2}}$$

$$= \frac{\frac{\sqrt{3}+1}{2}}{\frac{\sqrt{3}-1}{2}}$$

$$= \frac{(\sqrt{3}\ +\ 1)}{(\sqrt{3}-1)}$$

RHS = $\frac{\tan 60° + \tan 45°}{\tan 60° - \tan 45°}$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$$\frac{\sqrt{3}\ +\ 1}{\sqrt{3}-1} = \frac{\sqrt{3}\ +\ 1}{\sqrt{3}-1}$$

∴ LHS = RHS

$$\therefore \frac{\sin 60° + \sin 30°}{\sin 60° - \sin 30°} = \frac{\tan 60° + \tan 45°}{\tan 60° - \tan 45°} =$$

Hence Proved

17. Question

Prove that-

$2(\cos^2 45° + \tan^2 60°) - 6(\sin^2 45° - \tan^2 30°) = 6$

Answer

$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$\therefore \cos^2 45° = \frac{1}{2}$$

$\tan 60° = \sqrt{3}$

$\therefore \tan^2 60° = 3$

$$\sin 45° = \frac{1}{\sqrt{2}}$$

$$\therefore \sin^2 45° = \frac{1}{2}$$

$$\tan 30° = \frac{1}{\sqrt{3}}$$

$\tan^2 30° = \frac{1}{3}$

∴ LHS =

$2 \times \left(\frac{1}{2} + 3\right) - 6 \times \left(\frac{1}{2} - \frac{1}{3}\right)$

$= 2 \times \frac{7}{2} - 6 \times \frac{1}{6}$

= 7 - 1

= 6

= RHS

∴ LHS = RHS

∴$(\cos^2 45° + \tan^2 60°) - 6(\sin^2 45° - \tan^2 30°) = 6$

Hence Proved

18. Question

Prove that-

$(\sec^2 30° + \text{cosec}^2 45°)$ (2cos 60° + sin 90° + tan 45°) = 10

Answer

$\sec 30° = \frac{2}{\sqrt{3}}$

$\therefore \sec^2 30° = \frac{4}{\sqrt{3}}$

$\text{cosec} 45° = \sqrt{2}$

∴ $\text{cosec}^2 45° = 2$

$\cos 60° = \frac{1}{2}$

sin90° = 1

tan45° = 1

∴ LHS = $(\sec^2 30° + \text{cosec}^2 45°)$ (2cos 60° + sin 90° + tan 45°)

$\left(\frac{4}{3} + 2\right)\left(2 \times \frac{1}{2} + 1 + 1\right)$

$$\left(\frac{10}{3}\right) \times (1 + 1 + 1)$$

$$= \frac{10}{3} \times 3$$

= 10 = RHS

∴ LHS = RHS

∴$(\sec^2 30° + \text{cosec}^2 45°)\ (2\cos 60° + \sin 90° + \tan 45°) = 10$

Hence Proved

19. Question

Prove that-

$$(1 - \sin 45° + \sin 30°)(1 + \cos 45° + \cos 60°) = \frac{7}{4}$$

Answer

$$\sin 45° = \frac{1}{\sqrt{2}}$$

$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$\sin 30° = \frac{1}{2}$$

$$\cos 60° = \frac{1}{2}$$

∴LHS = $(1 - \sin 45° + \sin 30°)(1 + \cos 45° + \cos 60°)$

$$= \left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right)\left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right)$$

$$= \left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right)\left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right)$$

$$= \frac{(3 - \sqrt{2})(3 + \sqrt{2})}{4}$$

$$= \frac{3^2 - (\sqrt{2})^2}{4}$$

$= \frac{9-2}{4}$

$= \frac{7}{4}$ = RHS

∴ LHS = RHS

$\therefore (1-\sin 45° + \sin 30°)(1+\cos 45° + \cos 60°) = \frac{7}{4}$

Hence, Proved

20. Question

Prove that-

$\cos^2 0° - 2\cot^2 30° + 3\operatorname{cosec}^2 90° = 2(\sec^2 45° - \tan^2 60°)$

Answer

$\cos 0° = 1$

$\therefore \cos^2 0° = 1$

$\cot 30° = \sqrt{3}$

$\therefore \cot^2 30° = 3$

$\operatorname{cosec} 90° = 1$

$\therefore \operatorname{cosec}^2 90° = 1$

$\sec 45° = \sqrt{2}$

$\therefore \sec^2 45° = 2$

$\tan 60° = \sqrt{3}$

$\tan^2 60° = 3$

$\therefore$ LHS = $\cos^2 0° - 2\cot^2 30° + 3\operatorname{cosec}^2 90°$

= 1 - 2 × 3 + 3 × 1

= 1 - 6 + 3

= 4 - 6

= - 2

RHS = $2(\sec^2 45° - \tan^2 60°)$

= 2 × (2 - 3)

= 2 × (-1)

= - 2

- 2 = - 2

∴ LHS = RHS

$\therefore \cos^2 0° - 2\cot^2 30° + 3\operatorname{cosec}^2 90° = 2(\sec^2 45° - \tan^2 60°)$

Hence Proved

21 A. Question

If x = 30°, then prove that

$\sin 3x = 3\sin x - 4\sin^3 x$

Answer

For x= 30°,

LHS = sin (3×30°)

= sin (90°)

=1

RHS = 3sin30° - 4× $(\sin 30°)^3$

$= 3 \times \frac{1}{2} - 4 \times \left(\frac{1}{2}\right)^3$

$= \frac{3}{2} - \frac{4}{8}$

$= \frac{3}{2} - \frac{1}{2}$

$= \frac{2}{2} = 1$

Since LHS = RHS

$\sin 3x = 3\sin x - 4\sin^3 x$

Hence proved.

21 B. Question

If x = 30°, then prove that

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Answer

For, x = 30°,

LHS = tan2x = tan(2 × 30°) = tan60° = √3

RHS =

$$\frac{2\tan x}{1-\tan^2 x}$$

$$= \frac{2\tan 30°}{1-\tan^2 30°}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \sqrt{3}$$

√3 = √3

∴ LHS = RHS

Hence Proved

21 C. Question

If x = 30°, then prove that

$$\sin x = \sqrt{\frac{1-\cos 2x}{2}}$$

Answer

For x = 30°

LHS =

$$\sin x = \sin 30° = \frac{1}{2}$$

RHS =

$$\sqrt{\frac{1-\cos 2x}{2}}$$

$$= \sqrt{\frac{1-\cos(2\times 30°)}{2}}$$

$$= \sqrt{\frac{1-\cos 60°}{2}}$$

$$= \sqrt{\frac{1-\frac{1}{2}}{2}}$$

$$= \sqrt{\frac{\frac{1}{2}}{2}}$$

$$= \sqrt{\frac{1}{4}}$$

$$= \frac{1}{2}$$

∴ LHS = RHS

$$\therefore \sin x = \sqrt{\frac{1-\cos 2x}{2}}$$

Hence, Proved

21 D. Question

If x = 30°, then prove that

$\cos 3x = 4\cos^3 x - 3\cos x$

Answer

For x = 30°

LHS = cos(3 × 30°) = cos90° = 0

RHS = $4\cos^3 x - 3\cos x$

= $4\cos^3 30° - 3\cos 30°$

$$\left[\cos 30° = \frac{\sqrt{3}}{2}\right]$$

$\therefore \cos^3 30° = \frac{3\sqrt{3}}{8}$

$\therefore 4\cos^3 30° - 3\cos 30°$

$= 4 \times \frac{3\sqrt{3}}{8} - 3 \times \frac{\sqrt{3}}{2}$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$

= 0

Since, 0 = 0

∴ LHS = RHS

$\therefore \cos 3x = 4\cos^3 x - 3\cos x$

Hence Proved

22. Question

If A = 60° and B = 30° then prove that $\cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$

Answer

For A = 60° and B = 30°

LHS = cot(A - B) = cot(60° - 30°) = cot30° = √3

RHS = $\frac{\cot A \cot B + 1}{\cot B - \cot A}$

$= \frac{\cot 60° \cot 30° + 1}{\cot 30° - \cot 60°}$

$= \frac{\frac{1}{\sqrt{3}} \times \sqrt{3} + 1}{\sqrt{3} - \frac{1}{\sqrt{3}}}$

$= \frac{1 + 1}{\frac{3-1}{\sqrt{3}}}$

$= \frac{2}{\frac{2}{\sqrt{3}}}$

= √3

√3 = √3

∴ LHS = RHS

$$\therefore \cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

Hence Proved

Miscellaneous Exercise 6

1. Question

The value of $\tan^2 60°$ is:

A. 3

B. $\frac{1}{3}$

C. 1

D. ∞

Answer

$\tan 60° = \sqrt{3}$

$\therefore \tan^2 60° = (\tan 60°)^2 = (\sqrt{3})^2 = 3$

2. Question

The value of $2 \sin^2 60° \cos 60°$ will be:

A. $\frac{4}{3}$

B. $\frac{5}{2}$

C. $\frac{3}{4}$

D. $\frac{1}{3}$

Answer

$$\sin 60° = \frac{\sqrt{3}}{2}$$

$\therefore \sin^2 60° = \frac{3}{4}$

$\cos 60° = \frac{1}{2}$

$\therefore 2 \sin^2 60° \cos 60°$

$= 2 \times \frac{3}{4} \times \frac{1}{2}$

$= \frac{3}{4}$

3. Question

If $\text{cosec}\ \theta = \frac{2}{\sqrt{3}}$ then the value of θ is:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{6}$

Answer

$\sin 60° = \frac{\sqrt{3}}{2}$

$\text{cosec}\theta = \frac{1}{\sin\theta}$

$\Rightarrow \text{cosec} 60° = \frac{2}{\sqrt{3}}$

4. Question

The value of $\cos^2 45°$ will be:

A. $\frac{1}{\sqrt{2}}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{1}{2}$

D. $\frac{1}{\sqrt{3}}$

Answer

$\cos 45° = \frac{1}{\sqrt{2}}$

$\cos^2 45° = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

5. Question

If θ = 45° then the value of $\frac{1-\cos 2\theta}{\sin 2\theta}$ is:

A. 0

B. 1

C. 2

D. ∞

Answer

For θ = 45°

cos2θ = cos(2 × 45°) = cos90° = 0

sin2θ = sin(2 × 45°) = sin90° = 1

$\frac{1-\cos 2\theta}{\sin 2\theta}$

$= \frac{1-0}{1}$

= 1

6. Question

Prove that:

$\cos 60° = 2\cos^2 30° - 1$

Answer

$$\cos 60° = \frac{1}{2}$$

$$\cos 30° = \frac{\sqrt{3}}{2}$$

$$\therefore \cos^2 30° = \frac{3}{4}$$

LHS =

$$\cos 60° = \frac{1}{2}$$

RHS =

$2\cos^2 30° - 1$

$$= 2 \times \frac{3}{4} - 1$$

$$= \frac{3}{2} - 1$$

$$= \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2}$$

∴ LHS = RHS

$\therefore \cos 60° = 2\cos^2 30° - 1$

Hence Proved

7. Question

Prove that:

$$\sin 60° = \frac{2\tan 30°}{1 + \tan^2 30°}$$

Answer

$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\therefore \tan^2 30^\circ = \frac{1}{3}$$

∴ LHS

= sin60°

$$= \frac{\sqrt{3}}{2}$$

RHS =

$$\frac{2\tan 30^\circ}{1 + \tan^2 30^\circ}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

∴ LHS = RHS

$$\therefore \sin 60^\circ = \frac{2\tan 30^\circ}{1 + \tan^2 30^\circ}$$

∴ Hence Proved

8. Question

Prove that:

$$\cos 60^\circ = \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ}$$

Answer

$$\cos 60^\circ = \frac{1}{2}$$

$\tan 30° = \frac{1}{\sqrt{3}}$

$\therefore \tan^2 30° = \frac{1}{3}$

∴ LHS =

cos60°

$= \frac{1}{2}$

RHS =

$= \frac{1 - \tan^2 30°}{1 + \tan^2 30°}$

$= \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}}$

$= \frac{\frac{2}{3}}{\frac{4}{3}}$

$= \frac{2}{4}$

$= \frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$

∴ LHS = RHS

Hence Proved

9. Question

Prove that:

$(\sin 45° + \cos 45°)^2 = 2$

Answer

LHS:

$(\sin 45° + \cos 45°)^2$

$= \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)^2$

$= \left(\frac{2}{\sqrt{2}}\right)^2$

$= (\sqrt{2})^2$

= 2

= RHS

Hence Proved!

10. Question

Prove that:

4 tan30° sin45° sin60° sin90° = √2

Answer

$\tan 30° = \frac{1}{\sqrt{3}}$

$\sin 45° = \frac{1}{\sqrt{2}}$

$\sin 60° = \frac{\sqrt{3}}{2}$

sin90° = 1

LHS =

4 tan30° sin45° sin60° sin90°

$= 4 \times \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} \times 1 = \frac{2}{\sqrt{2}}$

= √2

= RHS

Hence Proved!

11. Question

Find the value of $\sin^2 60° \cot^2 60°$.

Answer

$\sin 60° = \frac{\sqrt{3}}{2}$

$\therefore \sin^2 60° = \frac{3}{4}$

$\cot 60° = \frac{1}{\sqrt{3}}$

$\therefore \cot^2 60° = \frac{1}{3}$

$\therefore \sin^2 60° \cot^2 60°$

$= \frac{3}{4} \times \frac{1}{3}$

$= \frac{1}{4}$

12. Question

Find the value of $4\cos^3 30° - 3\cos 30°$.

Answer

For $\theta = 30°$,

$4\cos^3 30° - 3\cos 30° = 4\cos^3\theta - 3\cos\theta$

We know that,

$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\therefore 4\cos^3 30° - 3\cos 30°$

$= \cos(3 \times 30°)$

$= \cos 90°$

$= 0$

13. Question

If $\cot\theta = \frac{1}{\sqrt{3}}$, then prove that $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$

Answer

$\cot\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \cot\theta = \cot 60°$

$\Rightarrow \theta = 60°$

∴ LHS =

$$\frac{1-\cos^2\theta}{2-\sin^2\theta}$$

$$= \frac{1-\cos^2 60°}{2-\sin^2 60°}$$

$$\cos 60° = \frac{1}{2}$$

$$\therefore \cos^2 60° = \frac{1}{4}$$

$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\therefore \sin^2 60° = \frac{3}{4}$$

∴

$$\frac{1-\cos^2 60°}{2-\sin^2 60°}$$

$$= \frac{1-\frac{1}{4}}{2-\frac{3}{4}}$$

$$= \frac{\frac{3}{4}}{\frac{5}{4}}$$

$$= \frac{3}{5}$$

= RHS

∴ LHS = RHS

Hence Proved

14. Question

Prove that $3(\tan^2 30° + \cot^2 30°) - 8(\sin^2 45° + \cos^2 30°) = 0$

Answer

$\tan 30° = \frac{1}{\sqrt{3}}$

$\therefore \tan^2 30° = \frac{1}{3}$

$\cot 30° = \sqrt{3}$

$\therefore \cot^2 30° = 3$

$\sin 45° = \frac{1}{\sqrt{2}}$

$\sin^2 45° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\therefore \cos^2 30° = \frac{3}{4}$

$\therefore$ LHS =

$3(\tan^2 30° + \cot^2 30°) - 8(\sin^2 45° + \cos^2 30°)$

$= 3\left(\frac{1}{3} + 3\right) - 8\left(\frac{1}{2} + \frac{3}{4}\right)$

$= 3 \times \frac{10}{3} - 8 \times \frac{5}{4}$

= 10 - 10

= 0

= RHS

$\therefore$ LHS = RHS

Hence Proved

15. Question

Prove that $4(\sin^4 30° + \cos 60°) - 3(\cos^2 45° - \sin^2 90°) = \frac{15}{4}$

Answer

$\sin 30° = \frac{1}{2}$

$\therefore \sin^4 30° = \frac{1}{16}$

$\cos 60° = \frac{1}{2}$

$\cos 45° = \frac{1}{\sqrt{2}}$

$\therefore \cos^2 45° = \frac{1}{2}$

$\sin 90° = 1$

$\sin^2 90° = 1$

$\therefore$ LHS = $4(\sin^4 30° + \cos 60°) - 3(\cos^2 45° - \sin^2 90°)$

$= 4\left(\frac{1}{16} + \frac{1}{2}\right) - 3\left(\frac{1}{2} - 1\right)$

$= 4 \times \frac{9}{16} - 3 \times \left(-\frac{1}{2}\right)$

$= \frac{9}{4} + \frac{3}{2} = \frac{15}{4}$

RHS = LHS

Hence Proved

16. Question

Prove that $\frac{\cos 30° + \sin 60°}{1 + \cos 60° + \sin 30°} = \frac{\sqrt{3}}{2}$

Answer

$\cos 30° = \frac{\sqrt{3}}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$

$\therefore$ LHS =

$$\frac{\cos 30° + \sin 60°}{1 + \cos 60° + \sin 30°}$$

$$= \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{1 + \frac{1}{2} + \frac{1}{2}}$$

$$= \frac{\sqrt{3}}{2}$$

17. Question

Prove that $2(\cos^2 45° + \tan^2 60°) - 6(\sin^2 45° - \tan^2 30°) = 6$

Answer

$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$\therefore \cos^2 45° = \frac{1}{2}$$

$$\tan 60° = \sqrt{3}$$

$$\tan^2 60° = 3$$

$$\sin 45° = \frac{1}{\sqrt{2}}$$

$$\therefore \sin^2 45° = \frac{1}{2}$$

$$\tan 30° = \frac{1}{\sqrt{3}}$$

$$\therefore \tan^2 30° = \frac{1}{3}$$

$\therefore$ LHS $= 2(\cos^2 45° + \tan^2 60°) - 6(\sin^2 45° - \tan^2 30°)$

$$= 2\left(\frac{1}{2} + 3\right) - 6\left(\frac{1}{2} - \frac{1}{3}\right)$$

$$= 2 \times \frac{7}{2} - 6 \times \frac{1}{6}$$

$= 7 - 1$

$= 6$

$=$ RHS

= Hence Proved.