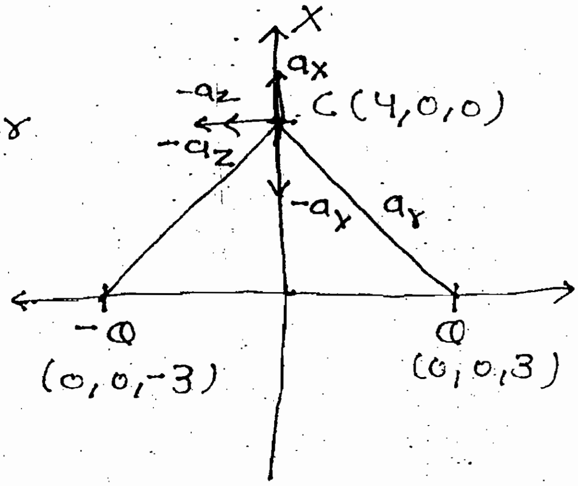
Workbook

$Q \rightarrow (0,0,3)$

$-Q \rightarrow (0,0,-3)$

E at $C\ (4,0,0)$



$$E = \frac{Q}{4\pi\epsilon r^2} a_r$$

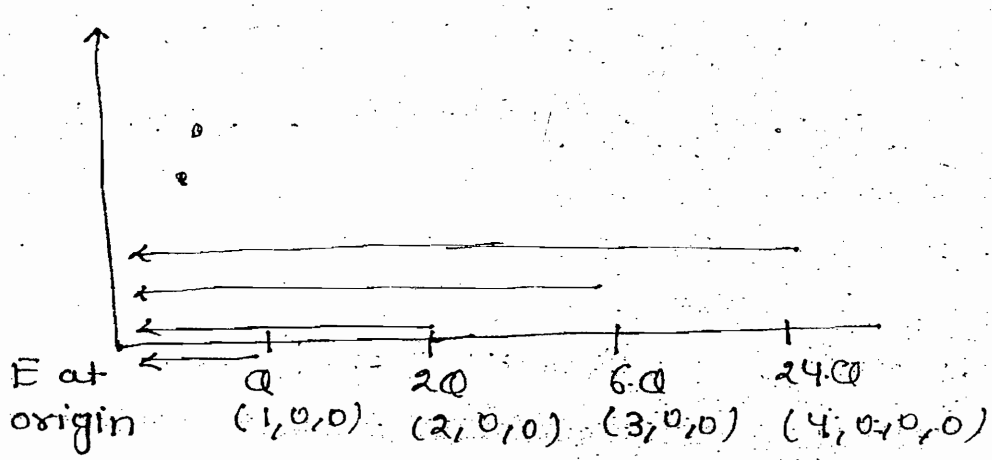
$E_1 = (a_x, -a_z)$
(Q)

$$= \frac{4a_x - 3a_z}{5}$$

$E_2 = (-a_x, -a_z)$
$(-Q)$

$$= \left(\frac{-4a_x - 3a_z}{5}\right)$$

$E_{total} = -a_z$
(direction)



$$E_T = E_1 + E_2 + E_3 + E_4$$

$$E = \frac{Q}{4\pi\epsilon r^2} a_r$$

$$a_r = -a_x$$

$$E_T = \left(\frac{Q}{4\pi\epsilon(1)^2} + \frac{2Q}{4\pi\epsilon(2)^2} + \frac{6Q}{4\pi\epsilon(3)^2} + \frac{24Q}{4\pi\epsilon(4)^2} \right) a_x$$

$$= \frac{Q}{4\pi\epsilon} \left(1 + \frac{1}{2} + \frac{2}{3} + \frac{3}{2} \right) (-a_x)$$

:- Net flux leaving $= \oint D \cdot ds = Q$ = charge enclosed

Sphere $\longrightarrow$ centre = origin
radius = 6m

$2C \longrightarrow (4,8,3) \longrightarrow d_1 = \sqrt{4^2+8^2+3^2} > 6$

$8C \longrightarrow (2,-1,-3) \longrightarrow d_2 = \sqrt{2^2+1^2+3^2} < 6$

$-12C \longrightarrow (-4,0,1) \longrightarrow d_3 = \sqrt{4^2+1^2} < 6$

Net flux leaving $= -4C$

:- Centre $- (2,-3,2)$

$d_1 = \sqrt{2^2+11^2+1^2} > 6 \longrightarrow$ out

$d_2 = \sqrt{0^2+2^2+5^2} < 6 \longrightarrow$ in

$d_3 = \sqrt{6^2+3^2+1^2} > 6 \longrightarrow$ out

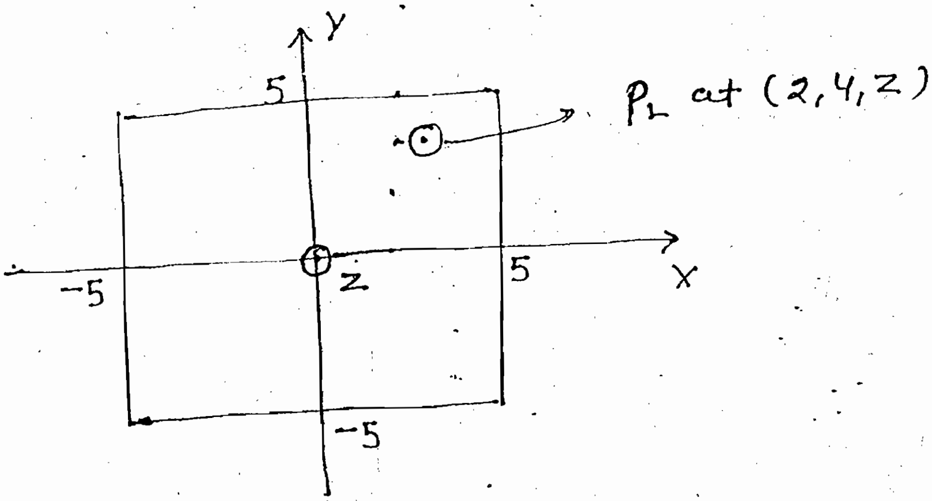
Net flux leaving $= 8C$

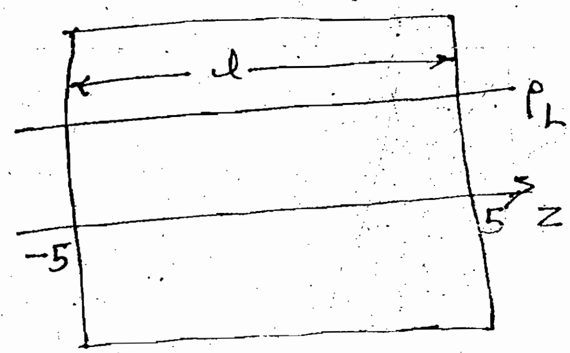
:- Net flux ~~leaving~~ $= Q = \rho_L l$

$\rho_L = 15\,nC/m$

at $x=2, y=4$ for all z

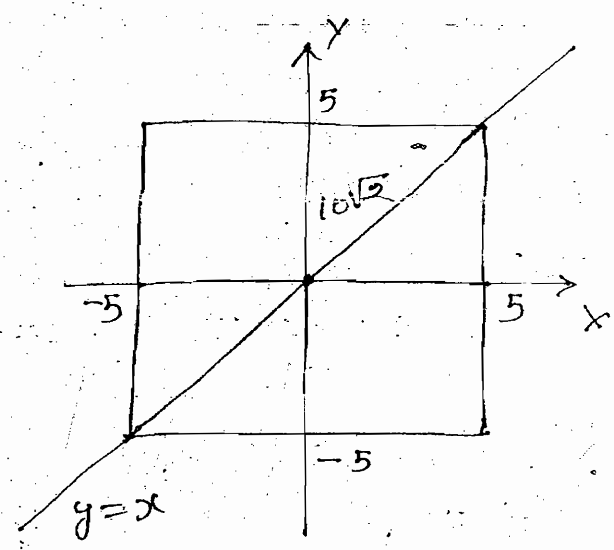
closed surface $= -5 < x < 5$
$-5 < y < 5$
$-5 < z < 5$





$$Q = 15\,\frac{nC}{m} \times 10$$
$$= 150\ nC \quad \text{, Ans}$$

13:-

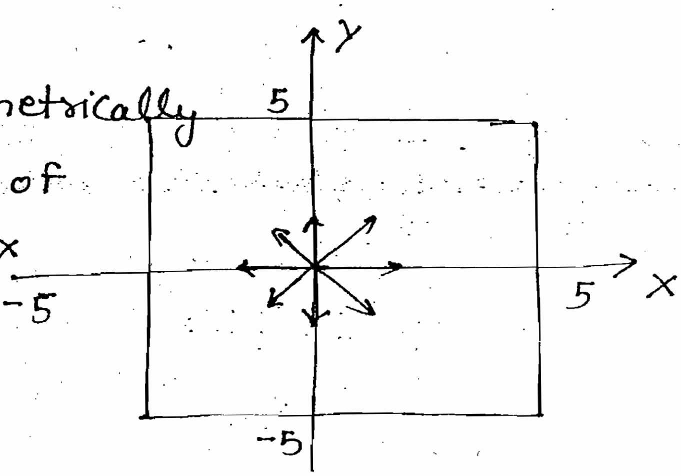


$$Q = 15 \times 10\sqrt{2}$$
$$= 150\sqrt{2}$$

14.:-

$Q = 6\ nC$

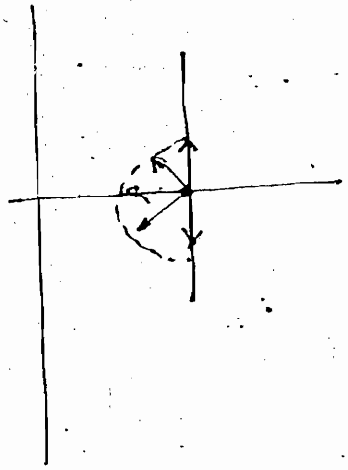
6nC → flux symmetrically crosses 6 sides of cube Inc-flux cross per 1 side



5.

$z = 5m$-plane

$3nC, 14\pi r$



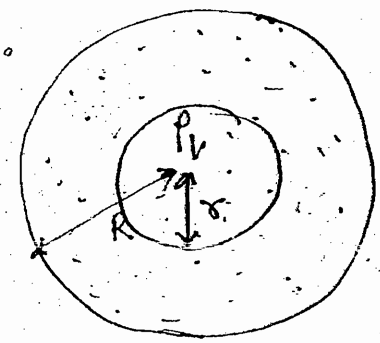
Note:- A point charge and a infinite sheet near by has half the flux crossing through

$$\oint_s D \cdot ds = Q$$

↓ constant

$$D \cdot S = Q$$

$$\Rightarrow D = \frac{Q}{S}$$



$$= \frac{\rho_v \frac{4}{3}\pi r^3}{4\pi r^2} = \frac{\rho_v r}{3}$$

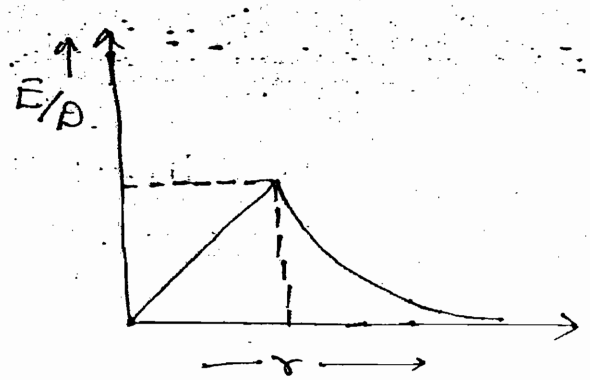
$$E = \frac{\rho_v r}{3\epsilon}$$

$$\boxed{E\left(\frac{R}{2}\right) = \frac{\rho_v R}{6\epsilon}}$$

$$D \cdot S = Q$$

$$\Rightarrow D = \frac{Q}{S}$$

$$= \frac{\rho_v \frac{4}{3}\pi R^3}{4\pi r^2}$$



$$D = \frac{\rho_V R^3}{3r^2}$$

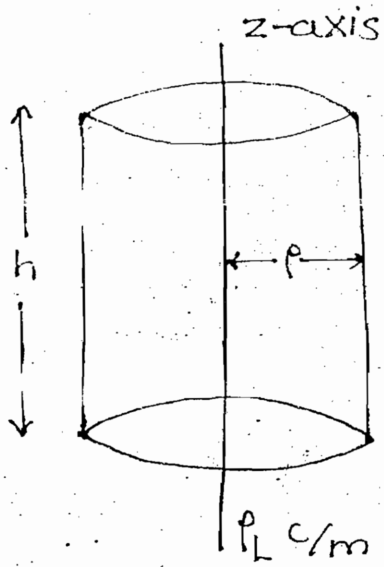
$$E = \frac{\rho_V R^3}{3\epsilon r^2}$$

$$\boxed{E(2R) = \frac{\rho_V R}{12\epsilon}}$$

Gauss Law Application-2 :-

Electric field strength of an infinite length ρ_L c/m line charge :-

Consider a cocentric symmetric Gaussian surface which is a cylinder around the charge



ρ = Constant

$$\oint D \cdot dS = Q$$

a_ρ directed

Constant

$$D \cdot S = Q$$

$$\Rightarrow D = \frac{Q}{S}$$

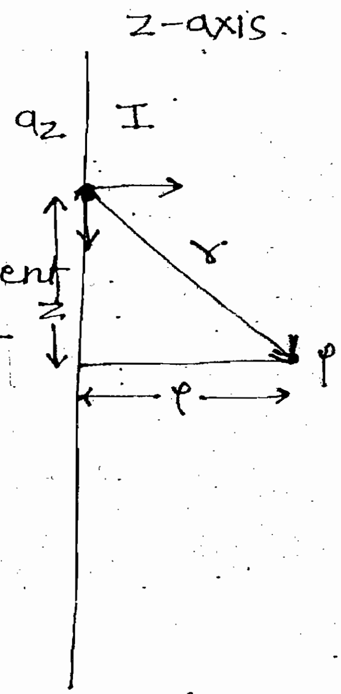
$$\Rightarrow D = \frac{\rho_L \cdot \cancel{h}}{2\pi\rho \cancel{h}}$$

$$\Rightarrow D = \frac{\rho_L}{2\pi\rho} a_\rho$$

$$\Rightarrow \boxed{E = \frac{\rho_L}{2\pi\epsilon\rho} a_\rho}$$

Magnetic field strength of an infinite length I carrying wire :-

$$H = \frac{I\,dl}{4\pi r^2} \times a_r$$



Consider a small dl length I current element at the z point height above the point

$$dl = dz\,a_z$$

$$r = \sqrt{\rho^2+z^2}$$

$$a_r = \frac{\vec{r}}{|r|}$$

$$= \frac{\rho a_\rho - z a_z}{\sqrt{\rho^2+z^2}}$$

$$dH = \frac{I\,dz\,a_z}{4\pi(\rho^2+z^2)} \times \left(\frac{\rho a_\rho - z a_z}{\sqrt{\rho^2+z^2}}\right)$$

$$H = \int_{z=-\infty}^{\infty} dH$$

$$\Rightarrow H = \frac{I\rho}{4\pi}\int_{z=-\infty}^{\infty} \frac{dz\,(a_z \times a_\rho)}{(\rho^2+z^2)^{3/2}}$$

Put $z = \rho\tan\theta$

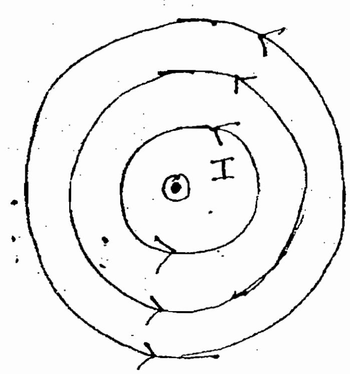
$$dz = \rho\sec^2\theta\,d\theta$$

$$(\rho^2+z^2)^{3/2} = \rho^3\sec^3\theta$$

$$H = \frac{I\rho}{4\pi} \int_{\theta=-\pi/2}^{\pi/2} \frac{\rho \sec^2\theta \, d\theta}{\rho^3 \sec^3\theta} = \frac{I}{4\pi\rho} \int_{-\pi/2}^{\pi/2} \cos\theta \cdot d\theta$$

$$\Rightarrow H = \frac{I}{2\pi\rho}(a_z \times a_\rho) = \frac{I}{2\pi\rho} a_\phi$$

Note :-



$$\oint H \cdot dl = I$$

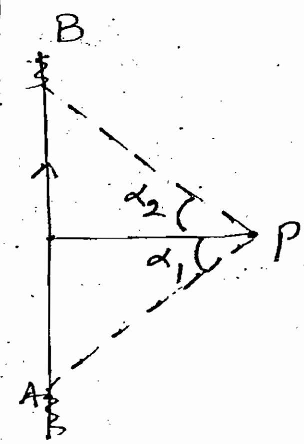
H · Length = current

$$H = \frac{I}{2\pi\rho} a_\phi$$

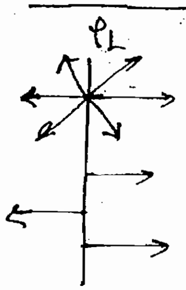
Extension :-

H-field of finite length I wire :-

$$\boxed{H = \frac{I}{4\pi\rho}(\sin\alpha_1 + \sin\alpha_2) a_\phi}$$



Summary :-



$\rho_L \longrightarrow$ Line charge

$E \longrightarrow$ Field

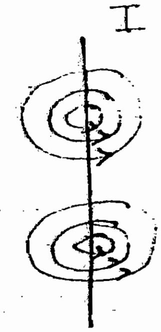
$$D = \frac{\rho_L}{2\pi\rho} a_\rho \qquad E = \frac{\rho_L}{2\pi\epsilon\rho} a_\rho$$

μ

I carrying H-field

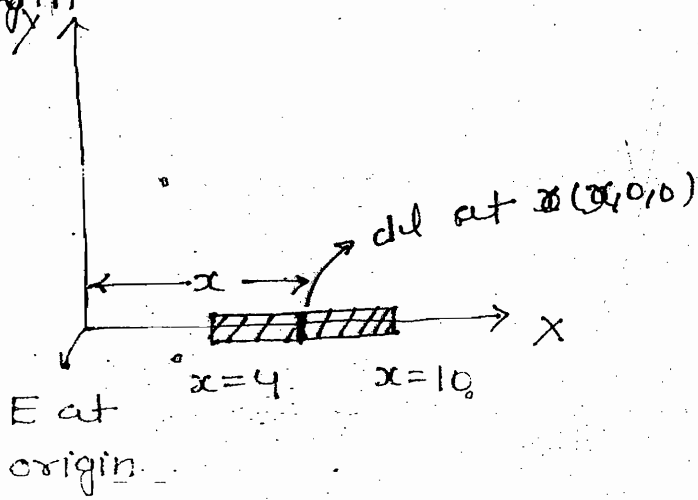
$$H = \frac{I}{2\pi\rho} a_\phi$$

$$B = \frac{\mu I}{2\pi\rho} a_\phi$$



7 :-

consider a dl length on the charge and apply the $\frac{Q}{4\pi\epsilon r^2} a_r$



Field calculation

$Q = \rho_L dx$

$r = x$

$a_r = -a_x$

$$dE = \frac{\rho_L dx}{4\pi\epsilon x^2} (-a_x)$$

$$E = \frac{\rho_L}{4\pi\epsilon} \int_{x=4}^{10} \frac{dx}{x^2} (-a_x)$$

$$= \frac{\rho_L}{4\pi\epsilon} \left(-\frac{1}{x}\right)_4^{10} (-a_x)$$

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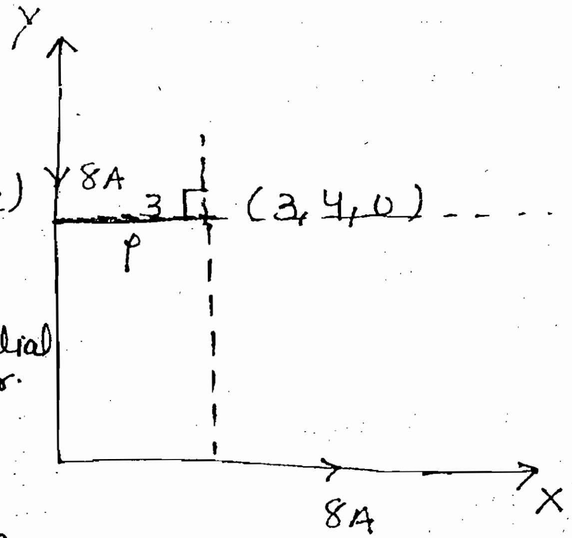
$$\underset{(Y\text{-axis})}{H} = \frac{8}{4\pi 3}\left(1+\frac{4}{5}\right)$$

$(-a_y \times a_x)$



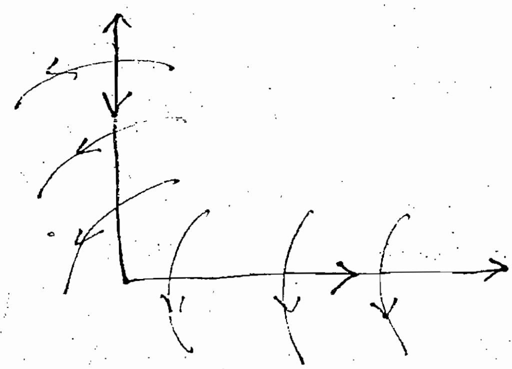
$$a_\phi = a_z \times a_\rho$$

$$H\text{-direction} = I \text{ direction} \times \text{Radial dir.}$$

$$= \frac{8}{4\pi 3} \times \frac{9}{5}\,(a_z)$$

$$\underset{(x\text{-axis})}{H} = \frac{8}{4\pi\cdot 4}\left(\frac{3}{5}+1\right)(a_x \times a_y)$$

$$= \frac{8}{16\pi} \times \frac{8}{5}\,(a_z)$$

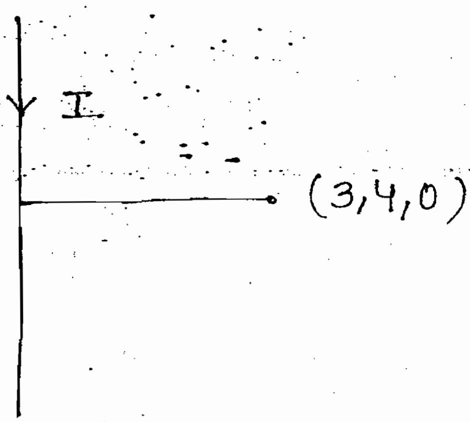


$$\underset{(\text{total})}{H} = \frac{2}{\pi}\,a_z$$

Ques:- Repeat the above problem if current is only entire y-axis

Soln:- $H = \dfrac{8}{2\pi\cdot 3}\,a_z$

$$= \frac{1.33}{\pi}\,a_z$$



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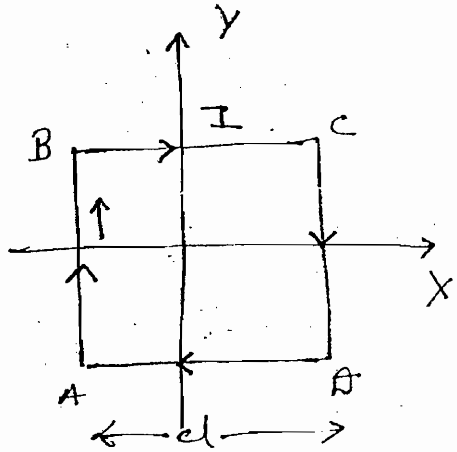
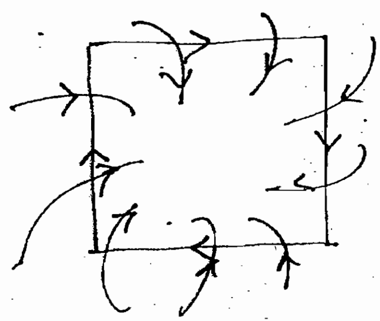
$$H_{AB} = a_y \times a_x = -a_z$$

$$H_{BC} = a_x \times -a_y = -a_z$$



Note:-

→ For any symmetric current distribution H field at geometric centre is always max



→ For any symmetric charge distribution E field at the geometric centre is always zero

$$H = \frac{I}{4\pi \cdot \frac{d}{2}} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) \times 4 \, (-a_z)$$

$$= \frac{2\sqrt{2}\, I}{\pi d} (-a_z) = 0.9 \frac{I}{d} (-a_z)$$

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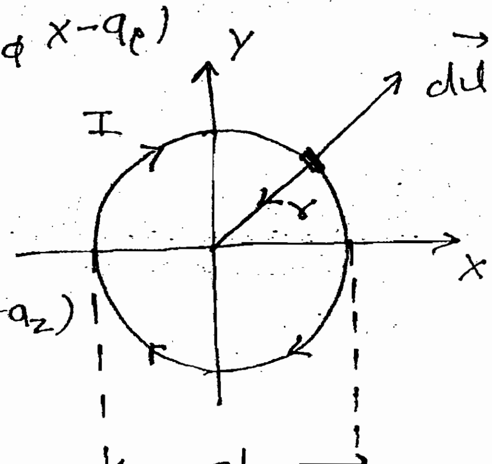
$$dH = \frac{I \, \vec{dl}}{4\pi r^2} (-a_\phi \times -a_\rho)$$

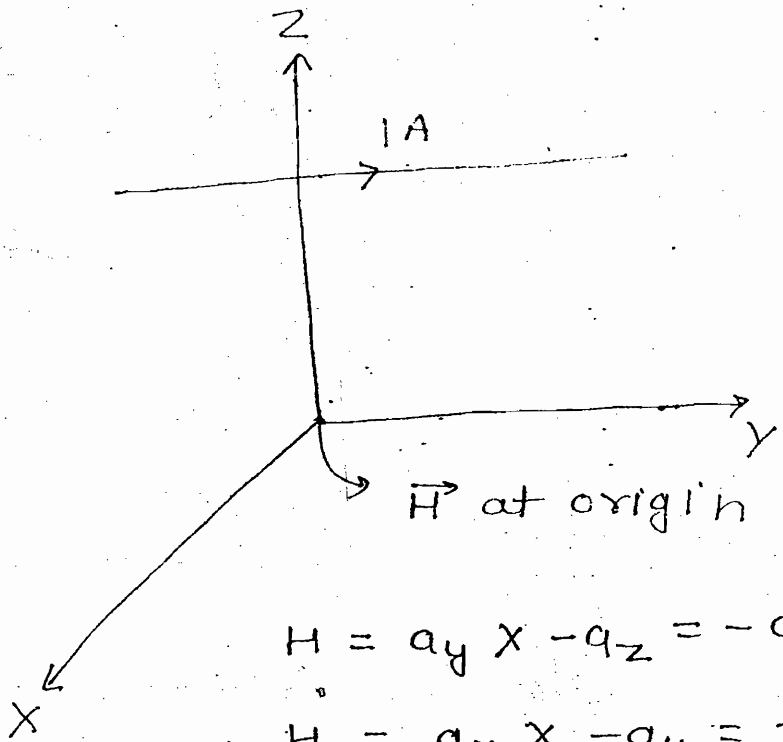


$$H = \int dH$$

$$= \frac{I \, 2\pi r}{4\pi r^2} (-a_z)$$

$$= \frac{I}{2r} = \frac{I}{d} (-a_z)$$

22 :-



$H = a_y \times -a_z = -a_x$

$H = a_x \times -a_y = -a_z$

Ans - d

23.

$H_\phi = a_z \times a_\rho$

Ans - (c)