Chapter 9. Factoring

Ex. 9.6

Answer 1CU.

A trinomial is a perfect square trinomial of its first term is perfect square

Last term is perfect square

And the second term must be twice the product of the square roots of the first and last terms.

Then it is a perfect square trinomial.

Answer 2CU.

Consider the following statement

$$a^2 - 2ab - b^2 = (a-b)^2, \quad a \neq 0$$

The objective is to check the given statement is true or false.

$a^2 - 2ab - b^2 = (a-b)^2$

$a^2 - 2ab - b^2 = a^2 - 2ab + b^2$

$a^2 - 2ab - b^2 - a^2 = a^2 - 2ab + b^2 - a^2$ (Subtract a^2 on both sides)

$-2ab - b^2 = -2ab + b^2 + 2ab$ (Simplify)

$b - b^2 + 2ab = -2ab + b^2 + 2ab$ (Add $2ab$ on bothsides)

$-b^2 = b^2$ (Simplify)

$-b^2 + b^2 = b^2 + b^2$ (Add b^2 on both sides)

$0 = 2b^2$

$2b^2 = 0$

$\frac{2b^2}{2} = \frac{0}{2}$ (Divide with 2 on both sides)

$b^2 = 0$

$b^2 = 0$ (Only when $b = 0$)

But $b \neq 0$ (Given)

Hence the given statement is true only when

$b = 0$

Therefore,

$\boxed{a^2 - 2ab - b^2 = (a-b)^2, \; b \neq 0}$ is never true.

Answer 3CU.

Consider the polynomial $2x^3+3x^2-32x-48$

Now factor the given polynomial

$$2x^3+3x^2-32x-48=\left(2x^3+3x^2\right)+\left(-32x-48\right) \quad \begin{bmatrix}\text{Graup the terms having}\\ \text{common factors}\end{bmatrix}$$

$$=\left(2x^2\cdot x+3x^2\right)+\left(16\cdot 2x+\left(-16\cdot 3\right)\right)$$

$$=x^2\left(2x+3\right)+\left(-16\right)\left(2x+3\right) \quad [\text{Factor the GCF}]$$

$$=\left(x^2-16\right)\left(2x+3\right) \quad \begin{bmatrix}\text{By distributive}\\ (b+c)a=ba+ca\end{bmatrix}$$

The difference of square property is $a^2-b^2=(a+b)(a-b)$

$$=\left(x^2-4^2\right)\left(2x+3\right) \quad \left[\text{Since; } 16=4^2\right]$$

$$=\left(x+4\right)\left(x-4\right)\left(2x+3\right) \quad [\text{By difference of squares property}]$$

Here, to factor the polynomial, use the grouping property and difference of square property, distributive property.

Answer 4CU.

Consider the trinomial $y^2+8y+16$.

The objective is to factor the given trinomial, if it is a perfect square.

Since a trinomial is perfect then

Its first term is a perfect square.

Last term is a perfect square.

Middle term is equal to twice the product of the square roots of first and last terms.

The first term $y^2=(y)^2$ (Perfect square)

Las term $16=4\cdot 4$

$=(4)^2$ (Perfect square)

Middle term $=8y$

$=2\cdot 4\cdot y$

= Twice the product of square not of first and last terms.

Therefore, $y^2+8y+16$ is a perfect square trinomial.

$$y^2+8y+16=(y)^2+2\cdot y\cdot 4+(4)^2$$

$=(y+4)^2$ [Because $a^2+2ab+b^2=(a+b)^2$]

Therefore the factorization of $\boxed{y^2+8y+16}$ is $\boxed{(y+4)^2}$.

Answer 5CU.

Consider the trinomial $9x^2 - 30x + 10$.

The objective is to factor the given trinomial if it is a perfect square trinomial.

Since a trinomial is said to be perfect square, if its first term is perfect square.

Last term is perfect square and middle term is equal to twice the product of the square roots of first and cast terms.

First term $= 9x^2$

$= 3 \cdot 3 \cdot x \cdot$

$= (3x)^2$ (Perfect square)

Last term $= 10$

$= 2.5$ (No. It is not a perfect square)

Since the last term 10 of $9x^2 - 30x + 10$ is not a perfect square.

<u>The given trinomial is not a perfect square</u>.

Answer 6CU.

Consider the polynomial $2x^2 + 18$.

The objective is to factor the given polynomial

$2x^2 + 18 = 2 \cdot x^2 + 2 \cdot 9$

$= 2(x^2 + 9)$ (Factor $GCF(2x^2, 18) = 2$)

Since the binomial $x^2 + 9$ is not factored further.

Therefore, $2x^2 + 18 = 2(x^2 + 9)$

$2x^2 + 18$ is not factored $\boxed{2x^2 + 18}$ is prime.

Answer 7CU.

Consider the polynomial $c^2 - 5c + 6$.

The objective is to factor $c^2 - 5c + 6$.

Since c^2 is $c^2 - 5c + 6$ is trinomial, in this c^2 is perfect square but the lost term 6 is not perfect square.

So perfect square method is not applicable here.

Compare $c^2 - 5c + 6$ with $x^2 + bx + c$

Where $b=-5$ and

$c=6$

$$c^2-5c+6=(c+m)(c+n)$$
$$=c^2+(m+n)(c+n)$$

That is $m+n=-5$ is negative and

$mn=6$ is positive that mean both m and n are negative.

Now find two negative, whose sum is

$m+n=-5$, product is

$mn=6$.

For this first list all the negative factors of 6.

Factors of 6	Sum of factors
$-1,-6$ $-2,-3$	-7 -5

The connect factors are $-2,-3$.

$$c^2-5c+6=(c+m)(c+n)$$
$$=(c+(-2))(c+(-3)) \quad (m=-2,n=-3)$$
$$=(c-2)(c-3)$$

Check: To check the factors multiply factors using $FOIL$ Method

$$(c-2)(c-3)=\overset{F}{c\cdot c}+\overset{O}{(-3)\cdot c}+\overset{I}{(-2)\cdot c}+\overset{L}{(-2)(-3)}$$

($FOIL$ Method)

$=c^2-3c-2c+6$ (Simplify)

$=c^2-5c+6$ (True)

Therefore, the factorization of $\boxed{c^2-5c+6}$ is $\boxed{(c-2)(c-3)}$.

Answer 8CU.

Consider the polynomial $5a^3-80a$.

The objective is to factor the given polynomial.

$5a^3-80a=5\cdot a\cdot a\cdot a-5\cdot 16\cdot a$ (Because $a^3=a\cdot a\cdot a, 80=5\cdot 60$)

$=5a(a\cdot a-16)$ (Factor $GCF(5a^3,80a)=5a$)

$=5a(a^2-4^2)$ (Because $16=4\cdot 4$)

$=5a(a+4)(a-4)$ [Because $a^2-b^2=(a+b)(a-b)$]

Therefore, the factorization of $\boxed{5a^3-80a}$ is $\boxed{5a(a+4)(a-4)}$.

Answer 9CU.

Consider the polynomial $8x^2-18x-35$

The objective is to factor the given polynomial

Compare $8x^2-18x-35$ with ax^2+bx+c

Here

$$a=8,$$
$$b=-18,$$
$$c=-35$$

We have to find two numbers m and n such that,

$m+n=-18$ and

$$mn=ac$$
$$=8\cdot(-35)$$
$$=-270$$

Since $m+n=-18$ is negative and

$mn=-280$ is negative, either m or n must be negative but not both.

List all the factors of -280 in those choose a pair of factors whose sum is -18.

Factors of -280	Sum
$-1\cdot 280$	279
$1\cdot -280$	-279

[illegible]	[illegible]
–2·140	138
2·–140	–138
–4·70	66
4·–70	–66
–5·56	51
5·–56	–51
–7·40	33
7·–40	–33
–8·35	27
8·–35	–27
–10·28	18
10·–28	–18
–14·20	6
14·–20	–6

The correct factors are $10, -28$.

Therefore,

$$8x^2 - 18x - 35 = 8x^2 + mx + nx - 35$$

$$= 8x^2 + 10x - 28x - 35 \quad (m = 10, n = -28)$$

$$= 2 \cdot 4 \cdot x \cdot x + 2 \cdot 5 \cdot x - 7 \cdot 4 \cdot x - 7 \cdot 5$$

(Simplify)

$$= 2x(4x+5) + (-7)4x + (-7)5$$

(Factor $GCF(8x^2, 10x) = 2x$)

$$= 2x(4x+5) + (-7)(4x+5)$$

(Factor $GCF(-28x, -38) = -7$)

$$= (2x + (-7))(4x+5)$$ (Because $(b+c)a = ba + ca$)

$$= (2x-7)(4x+5)$$

Check: To check the proposed factors use $FOIL$ Method to multiply the factors

$$(2x-7)(4x+5) = \overset{F}{2}x \cdot 4x + \overset{O}{2}x \cdot (5) + (\overset{I}{-}7)4x + (\overset{L}{-}7) \cdot 5$$

(FGL Method)

$$= 8x^2 + 10x - 28x - 35$$ (Simplify)

$$= 8x^2 - 18x - 35$$ (True)

Therefore the factorization of given polynomial is $\boxed{(2x-7)(4x+5)}$.

Answer 10CU.

Consider the polynomial $9g^2 + 12g - 4$

Compare $9g^2 + 12g - 4$ with $ac^2 + bx + c$

$a = 9,$
$b = 12,$
$c = -4$

Find two numbers m, n such that,

$m + n = b$
$= 12$ Positive and

$m \cdot n = ac$

$m \cdot n = ac$

$= 9 \cdot (-4)$ negative

$= -36$

$9g^2 + 12g - 4 = 9g^2 + mg + ng - 4$

Since $m+n$ is positive and mn is negative, then either m or n must be negative but not both

For this list all the factors of -36, in those choose a pair of factors whose sum is 12.

Factors of -36	Sum of factors
-1.36	35
$1.-36$	-35
-2.18	14
$2.-18$	-14
-3.12	9
$3.-12$	-9
-4.9	5
-6.6	0

There exists no pair of factors whose sum is 12.

Therefore $9g^2 + 12g - 4$ is not factored.

Therefore, given polynomial is prime.

Answer 11CU.

Consider the polynomial $3m^3+2m^2n-12m-8n$

The objective is to factor the given polynomial.

$$3m^3+2m^2n-12m-8n=\left(3m^3+2m^2n\right)-12m-8n$$

(Factors the terms)

$$=\left(3\cdot m\cdot m^2+2m^2\cdot n\right)+-4\cdot 3\cdot m+-4\cdot 2\cdot n$$

$$=m^2\left(3m+2n\right)+-43\cdot m+-4\cdot 2\cdot n$$

($GCF\left(3m^2,2m^2n\right)=m^2$)

$$=m^2\left(3m+2n\right)+-4\left(3n+2n\right)$$

(Factor $GCF\left(-12m,-8n\right)=-4$)

$$=\left(m^2-4\right)\left(3m+2n\right)$$

(Because $\left(a+b\right)c=ac+bc$)

$$=\left(m^2-2^2\right)\left(3m+2n\right)$$

(Because $4=2^2$)

$$=\left(m+2\right)\left(m-2\right)\left(3m+2n\right)$$

(Because $a^2-b^2=\left(a+b\right)\left(a-b\right)$)

Therefore, the factorization of given polynomial is $\boxed{\left(m+2\right)\left(m-2\right)\left(3m+2n\right)}$.

Answer 12CU.

Consider the equation

$$4y^2+24y+36=0$$

The objective is to solve the given equation

$$4y^2+24y+36=0$$

To solve the given equation first factor $4y^2+24y+36$.

Since the first term $=4y^2$

$$=2^2\cdot y^2$$

$=\left(2y\right)^2$ (Perfect square of $2y$)

Last term $=36$

$=6^2$ (Perfect square of 6)

Twice the square root of product of first and last term

$=2\cdot 2y\cdot 6$
$=4\cdot y\cdot 6$

$=24y$ (Middle term)

Therefore $4y^2+24y+36$ is perfect square

$4y^2+24y+36=(2y)^2+2\cdot 2y\cdot 6+6^2$

$=(2y+6)^2$ (Because $a^2+2ab+b^2=(a+b)^2$)

$4y^2+24y+36=(2y+6)(2y+6)$

$4y^2+24y+36=0$

$2y+6=0$

Or $2y+6=0$ (By zero product property)

Now solve each equation separately.

$2y+6=0$

$2y+6-6=0-6$ (Subtract 6 on each side)

$2y=-6$

$\frac{2y}{2}=\frac{-6}{2}$ (Divide with 2 on each side)

$y=-3$

$2g+6=0$

$2y+6-6=0-6$ (Subtract 6 on each sides)

$2y=-6$

$\frac{2y}{2} = \frac{-6}{2}$ (Divide with 2 on both sides)

$y = -3$

The solution set is $\{-3, -3\}$.

Check: To check the proposed solutions substitute each solution in the given equation.

$4y^2 + 24y + 36 = 0$

$4(-3)^2 + 24(-3) + 36 = 0$ (Put $y = -3$)

$4 \cdot 9 + (-72) + 36 = 0$ (Simplify)

$36 - 72 + 36 = 0$

$0 = 0$ True

Therefore the solution set of given equation is $\boxed{\{-3, -3\}}$.

Answer 13CU.

Consider the equation

$3n^2 = 48$

The objective is to find the solution set of given equation.

$3n^2 = 48$

$3n^2 - 48 = 48 - 48$ (Subtract 48 on both sides)

$3n^2 - 48 = 0$

$3 \cdot n^2 - 3 \cdot 16 = 0$ $(3 \cdot 16 = 48)$

$3(n^2 - 16) = 0$ [Factor $GCF(3n^2, 48) = 3$]

$3(n^2 - 4^2) = 0$ $(16 = 4^2)$

$3(n+4)(n-4) = 0$ (Because $a^2 - b^2 = (a+b)(a-b)$)

The zero product property is if

$ab = 0$ then

$a = 0$ or

$b = 0$ or both

$3(n+4)(n-4) = 0$

$n + 4 = 0$

Or $n - 4 = 0$

Now solve each equation separately

$n + 4 = 0$

$n + 4 - 4 = 0 - 4$ (Subtract 4 on each side)

$n = -4$

$n - 4 = 0$

$n - 4 + 4 = 0 + 4$ (Add 4 on each side)

$n = 4$

The solution set is $\{-4, 4\}$.

Check: To check the proposed solution set, substitute each solution in the given equation.

For $n=-4$,

$3n^2=48$

$3(-4)^2=48$ (Put $n=-4$)

$3\cdot 16=48$

$48=48$ True

For $n=4$,

$3n^2=48$

$3\cdot 4^2=48$ (Put $n=4$)

$3\cdot 16=48$

$48=48$ True.

The solution set of given equation is $\boxed{\{-4,4\}}$.

Answer 14CU.

Consider the equation

$a^2-6a+9=16$

The objective is to solve the given equation.

$a^2-6a+9=16$

$a^2-6a+3^2=16$ (Because $3^2=9$)

$a^2-2\cdot 3\cdot a+3^2=16$

$(a-3)^2=16$ (Since $a^2-2ab+b^2=(a-b)^2$)

$(a-3)^2=16$

Since for any $n>0$,

$x^2=n$, then

$x=\pm\sqrt{n}$

$(a-3)^2=16$,

$16>0$

$a-3=\pm\sqrt{16}$ (Square root property)

$a-3+3=3\pm\sqrt{4^2}$ (Add 3 on each side)

$a=3\pm 4$

Therefore $a=3+4$

Or, $a=3-4$

$a=7$

Or, $a=-1$

The solution set is $\{-1,7\}$.

Check: To check the proposed solution set, substitute each equation in the given equation.

For $a=-1$,

$a^2-6a+9=16$

$(-1)^2-6(-1)+9=16$ (Put $a=-1$)

$1+6+9=16$

$16=16$ True

For $a=7$,

$a^2-6a+9=16$

$7^2-6(7)+9=16$ (Put $a=7$)

$49-42+9=16$ (Simplify)

$7+9=16$

$16=16$ True

Therefore the solution set of given equation is $\boxed{\{-1,7\}}$.

Answer 15CU.

Consider the equation

$(m-5)^2=13$

The objective is to find the solution set of given equation

Since the square root property is,

For $n>0$,

$x^2=n$ then

$x=\pm\sqrt{n}$

$(m-5)^2=13$

$m-5=\pm\sqrt{13}$ (By square root property)

$m-5+5=5\pm\sqrt{3}$ (Add 5 on each side)

$m=5\pm\sqrt{13}$

$m=5\pm\sqrt{13}$

Or, $m=5-\sqrt{13}$

The solution set is $\{5+\sqrt{13},5-\sqrt{13}\}$.

Check:- To check the proposed solution, substitute each solution in the given equation.

For $m=5+\sqrt{13}$,

$(m-5)^2=13$

$\left(5+\sqrt{13}-5\right)^2=13$ (Put $m=5+\sqrt{13}$)

$\left(\sqrt{13}\right)^2=13$

$13=13$ True

For $m=5-\sqrt{13}$,

$(m-5)^2=13$

$\left(5-\sqrt{13}-5\right)^2=13$ (Put $m=5-\sqrt{13}$)

$\left(-\sqrt{13}\right)^2=13$

$13=13$ True

The solution set of given equation is $\boxed{\left\{5+\sqrt{13},5-\sqrt{13}\right\}}$.

Answer 17PA.

Consider the trinomial $x^2+9x+81$

The objective is to factor given trinomial if it is a perfect square.

Since if a trinomial is perfect then

Its first term is a perfect square.

Last term is a perfect square.

The middle term is equal to twice the product of the square roots of the first and last terms.

Here first term $=x^2$ – perfect square

Last term $=81$

$=9\cdot 9$

$81=9^2$ (Perfect square)

Middle term $=9x$

But $2\times$Product of square roots of first and last terms

$=2\times x\times 9$

$=18x$

The middle term is not equal to twice the product of square roots of first and last terms.

The given trinomial is not a perfect square.

Answer 18PA.

Consider the trinomial $a^2 + 24a + 144$

The objective is to factor given trinomial, if it is a perfect square.

Since a trinomial is perfect then, its

First term is a perfect square

Last term is a perfect square.

Middle term is equal to twice the product of the square roots of the first and last terms.

The first term $= a^2$ (Perfect square of a)

Last term $= 144$

$= 12 \cdot 12$

$= (12)^2$ (Perfect square of 12)

Middle term $= 24a$

$= 2 \cdot 12 \cdot a$

= Twice the product of square roots of first and last terms

Therefore $a^2 - 24a + 144$ is a perfect square

$$a^2 - 24a + 144 = a^2 - 2 \cdot 12 \cdot a + (12)^2$$

$= (a-12)^2$ [Because $a^2 - 2ab + b^2 = (a-b)^2$]

Therefore $\boxed{a^2 - 24a + 144}$ is a perfect square and its factorization is $\boxed{(a+12)^2}$.

Answer 19PA.

Consider the trinomial $4y^2 - 44y + 121$.

The objective is to factor the given trinomial, if it is a perfect square.

Its first term is perfect square

Last term is perfect square

Middle term is equal to twice the product of the square roots of the first and last terms.

First term $= 4y^2$

$= 2 \cdot 2 \cdot y^2$

$= (2y)^2$ (Perfect square of $2y$)

Last term $= 121$

$= 11 \cdot 11$

$= 11^2$ (Perfect square of 11)

Twice the product of square roots of first and last terms is

$2 \cdot 2y \cdot 11 = 44y$

$=$ Middle term

Therefore $4y^2 - 44y + 121$ is perfect square.

$4y^2 - 44y + 121 = (2y)^2 - 2 \cdot 2y \cdot 11 + 121$

$= (2y - 11)^2$ (Since $a^2 - 2ab + b^2 = (a-b)^2$)

Therefore the given trinomial is perfect square, and its factorization is $\boxed{(2y-11)^2}$.

Answer 20PA.

Consider the trinomial $2c^2 + 10c + 25$

The objective is to factor the given trinomial if it is a perfect square

Since if a trinomial is a perfect square, then

First term is a perfect square.

Last term is a perfect square.

Middle term is equal to twice the product of the square roots of the first and last terms.

The first term $= 2c^2$ – It is not a perfect square

Since the first term of $2c^2 + 10c + 25$ is not a perfect square.

Therefore $\boxed{2c^2 + 10c + 25}$ is not a perfect square.

Answer 21PA.

Consider the trinomial $9n^2+49+42n$.

The objective is to factor the given trinomial if it is a perfect square.

$$9n^2+49+42n=9n^2+42n+49$$

Since if a trinomial is a perfect square, then

First term is a perfect square

Last term is a perfect square

Middle term is equal to twice the product of the square roots of the first and last terms.

The first term $=9n^2$

$=3\cdot 3\cdot n\cdot n$

$=(3n)^2$ (Perfect square to $3n$)

Last term $=49$

$=7\cdot 7$

$=7^2$ (Perfect square of 7)

Twice the product of square roots of first and last terms is

$=2\cdot 3n\cdot 7$

$=6\cdot n\cdot 7$

$=42n$

$=$Middle term

Therefore $9n^2+49+42n$ is a perfect square

$$9n^2+42n+49=(3n)^2+2\cdot 3n\cdot 7+7^2$$

$=(3n+7)^2$ (Because $a^2+2ab+b^2=(a+b)^2$)

Therefore $\boxed{9n^2+49+42n}$ is perfect square and its factorization is $\boxed{(3n+7)^2}$.

Answer 22PA.

Consider the trinomial $25a^2 - 120ab + 144b^2$

The objective is to factor the given trinomial if it is a perfect sqare

If a trinomial is perfect then, its

First term is a perfect square.

Last term is a perfect square.

Middle term is equal of twice the product of the square root of the first and last terms.

The first term $= 25a^2$

$= 5 \cdot 5 \cdot a^2$

$= (5a)^2$ (Perfect square of $5a$)

Last term $= 144b^2$

$= 12^2 \cdot b^2$ (Because $144 = 12 \cdot 12$)

$= (12b)^2$ (Perfect square of $12b$)

Twice the product of square root of first and last terms is

$$2 \cdot 5a \cdot 12b = 10a \cdot 12b$$
$$= 120ab$$

$=$ Middle term

Therefore $25a^2 - 120ab + 144b^2$ is a perfect square.

$$25a^2 - 120ab + 144b^2$$
$$= (5a)^2 - 2 \cdot 5a \cdot 12b + (12b)^2$$

$= (5a - 12b)^2$ (Because $a^2 - 2ab + b^2 = (a-b)^2$)

Therefore, $\boxed{25a^2 - 120ab + 144b^2}$ is a perfect square and it factorization is $\boxed{(5a - 12b)^2}$

Answer 23PA.

Consider that, the area of a circle is $\left(16x^2+80x+10\right)\pi$ square inches.

The objective is to find the diameter of circle

Since Area of circle with radius 'r' is

$$A=\pi\left(16x^2+80x+100\right)$$

$$=\pi\left(4^2x^2+80x+100\right)$$ (Because $16=4^2$)

$$=\pi\left((4x)^2+2\cdot 40\cdot x+10^2\right)\ \left(100=10^2\right)$$

$$=\pi\left((4x)^2+2\cdot 4\cdot 10\cdot x+10^2\right)$$

$$=\pi\left((4x)^2+2\cdot 4x\cdot 10+10^2\right)$$

$$=\pi(4x+10)^2$$ (Because $(a+b)^2=a^2+2ab+b^2$)

Therefore Area

$$A=\pi r^2$$

$$=\pi(4x+10)^2$$

$$\Rightarrow \quad r^2=(4x+10)^2$$

Take square root on both sides

$$\sqrt{r^2}=\pm\sqrt{(4x+10)^2}$$

$$r=\pm(4x+10)$$

$r=4x+10$ (Since radius always positive)

Also diameter of a circle

$$=2r$$

$$=2(4x+10)$$ (Put $r=4x+10$)

$$=8x+20$$

Therefore Diameter of circle is $\boxed{8x+20\text{ inches}}$.

Answer 24PA.

Consider that the area of a square $81-90x+25x^2$

Also x is a positive integer.

The objective is to find the least possible perimeter measures for the square.

Given area of square $= 81 - 90x + 25x^2$

Here first term $= 81$

$= 9 \cdot 9$

$= (9)^2$ [Perfect square]

Last term $= 25x^2$

$= 5 \cdot 5x^2$ $[5 \cdot 5 = 25]$

$= (5x)^2$ [Perfect square]

Twice the product of square roots of first and last terms

$= 2 \cdot 9 \cdot 5x$

$= 90x$ [Middle term]

Therefore, $81 - 90x + 25x^2 = (9 - 5x)^2$

Thus, it is in perfect square form $(a-b)^2 = a^2 - 2ab + b^2$

$= (9 - 5x)^2$

Thus, $81 - 90x + 25x^2 = (9 - 5x)^2$

Since area of square $= 9^2$ [Where a is side of square]

$= (9 - 5x)^2$

$a^2 = (9 - 5x)^2$

$a = 9 - 5x$

Since *x* is a least positive integer.

x takes

Then for the least perimeter, $x = 0$

Then side of square $a = 9 - 5x$

$= 9 - 5(0)$ $[x = 0]$

$= 9$

Perimeter $= 4a$

$= 4(9)$

$= 36$

Therefore, least possible perimeter measures for square $= \boxed{36\text{ meters}}$

Answer 25PA.

Consider the polynomial $4k^2 - 100$.

The objective is to factor the given polynomial

$4k^2 - 100 = 2^2 \cdot k^2 - 100$ (Since $4 = 2^2$)

$= (2k)^2 - 100$ $\left(x^m \cdot y^m = (xy)^m\right)$

$= (2k)^2 - 10^2$ (Since $10^2 = 100$)

$= (2k+10)(2k-10)$ (Since $a^2 - b^2 = (a+b)(a-b)$)

$= (2k+2.5)(2k-2.5)$

$= 2(k+5)2(k-5)$ (Factor $GCF(2k,10) = 2$)

$= 4(k+5)(k-5)$

Therefore the factorization of $\boxed{4k^2 - 100}$ is $\boxed{4(k+5)(k-5)}$.

Answer 26PA.

Consider the polynomial $9x^2 - 3x - 20$

The objective is to factor the given polynomial

Compare $9x^2 - 3x - 20$ with $ax^2 + bx + c$

Here $a = 9$,

$b = -3$,

$c = -20$

Now

$9x^2 - 3x - 20 = 9x^2 + mx + nx - 20$

That is we have to find two numbers m, n such that

$m + n = -3$ negative and

$mn = -180$ negative

Since $m+n, mn$ are both negative one of m, n must be negative but not both.

For this list all the factors of

$mn = -180$, in which one factor is negative in those select a pair whose sum is -3.

Factors of -180	Sum of factors
-1.180	179
$1.-180$	-179
-2.90	88
$2.-90$	-88
-3.60	57
$3.-60$	
-4.45	41
$4.-45$	-41

-5.36	29
$5.-36$	-29
-6.30	24
$6.-30$	-24
-9.20	11
$9.-20$	-11
-10.18	8
$10.-18$	-8
-12.15	3
$12.-15$	-3

The correct factors are $12,-15$.

$9x^2-3x-20=9x^2+mx+nx-20$

$=9x^2+12x-15x-20 \quad (m=12, n=-15)$

$=3x\cdot 3x+3x\cdot 4-5\cdot 3x+5\cdot 4$

$=3x(3x+4)-5(3x+4)$ (Factor $GCF(9x^2,12x)=3x,$

$GCF(-15x,-20)=-5$)

$=(3x-5)(3x+4)$ (Because $(b+c)a=ba+ca$)

Check: To check the factorization, multiply the factors by $FOIL$ method.

$$(3x-5)(3x+4)=\overset{F}{3x\cdot 3x}+\overset{O}{4\cdot 3x}+\overset{I}{(-5)\cdot 3x}+\overset{L}{(-5)\cdot 4}$$

($FOIL$ method)

$=9x^2+12x-15x-20$

$=9x^2-3x-20$ True

Therefore, the factorization of $\boxed{9x^2-3x-20}$ is $\boxed{(3x-5)(3x+4)}$.

Answer 27PA.

Consider the polynomial x^2+6x-9.

The objective is to factor the given polynomial.

Compare x^2+6x-9 with x^2+bx+c

Here $b=6,$

$c=-9$.

$$x^2+6x-9=(x+m)(x+n)$$
$$=x^2+(m+n)x+mn$$

Now find two numbers m,n such that

$m+n=-6$ and

$mn=-9$, $m+n$ is positive and mn is negative either m or n must be negative but not both.

List all the pair of factors of

$mn=-9$, in those choose a pair of factors whose sum is 6.

Factors of -9	Sum of factors
$-1,9$	8
$1,-9$	-8
$-3,3$	0

There is no such pair of factors such that

$m+n=6$

Hence x^2+6x-9 is not factored.

Therefore the given polynomial is prime.

Answer 28PA.

Consider the polynomial $50g^2+40g+8$

The objective is to factor the given polynomial.

$$50g^2+40g+8=2\cdot25g^2+2\cdot20g+2\cdot4$$

$$=2\left(25g^2+20g+4\right)$$ (Factor $GCF,4$)

Now compare $25g^2+20g+4$ with ax^2+bx+c

Here $a=25$,

$b=20,$

$c=4$

Now

$$25g^2+20g+4=25g^2+mg+ng+4$$

Where $mn=25\cdot4$
$=100$

$m+n=20$

Since $m+n=20$ positive add

$mn=100$ is positive.

Now list all factors of 100, in those choose a pair whose sum is 20.

Factors of 100	Sum of factors
1,100	101
2,50	52
4,25	29
5,20	25
10,10	20

The correct factors are $10,10$

$$2\left(25g^2+20g+4\right)$$

$$=2\left(25g^2+mg+ng+4\right)$$

$$=2\left(25g^2+10g+10g+4\right)\quad(m=10,n=10)$$

$$=2\left[5g(5g+2)+2(5g+2)\right]$$

$$=2(5g+2)(5g+2)$$

Therefore, the factorization of $\boxed{5g^2+40g+8}$ is $\boxed{2(5g+2)(5g+2)}$.

Answer 29PA.

Consider the polynomial $9t^3+66t^2-48t$.

The objective is to factor the given polynomial

$9t^3+66t^2-48t=3\cdot3\cdot t\cdot t^2+3\cdot22\cdot t\cdot t-3\cdot16\cdot t$

$=3t\left[3t^2+22t-16\right]$ [Factor $GCF\left(9t^3,66t^2,48t\right)=3t$]

Now factor $3t^2+22t-16$.

Compare

$3t^2+22t-16=ax^2+bx+c$

Here $a=3$,

$b=+22,$

$c=-16$

$3t^2+22t-16=3t^2+3t+nt-16$

Now find two numbers m,n such that

$m+n=22$ positive

$mn=3\cdot(-16)$ negative,

$=-48$

Since $m+n=22$ positive and

$mn=-48$ negative

Hence either m or n negative but not both.

List all pairs factor of -48 in those choose a pair whose sum is $+22$.

Factors of -48	Sum of factors
$-1,48$	47
$1,-48$	-47
$-2,24$	22
$2,-24$	-22

$-3,16$	13
$3,-16$	-13
$-4,12$	8
$4,-12$	-8
$-6,8$	2
$6,-8$	-2

The correct factors are $-2,24$.

$3t\left[3t^2+22t-16\right]$

$$=3t\left[3t^2+mt+nt-16\right]$$

$=3t\left[3t^2-2t+24t-16\right]$ $(m=-2, n=24)$

$=3t\left[t(3t-2)+8(3t-2)\right]$

$=3t\left[(3t-2)(t+8)\right]$ (By distributive)

Therefore, the factorization of $\boxed{9t^3+66t^2-48t}$ is $\boxed{3t(3t-2)(t+8)}$.

Answer 30PA.

Consider the polynomial is $4a^2-36b^2$

The objective is to factor the given polynomial

$4a^2-36b^2=2\cdot 2a^2-36b^2$ (Because $4=2\cdot 2$)

$=2^2\cdot a^2-6^2\cdot b^2$ (Because $36=6^2$)

$=(2a)^2-(6b)^2$ (Because $a^m\cdot b^m=(ab)^m$)

The difference of squares property is

$$a^2-b^2=(a+b)(a-b)$$

$4a^2-36b^2=(2a)^2-(6b)^2$

$=(2a+6b)(2a-6b)$

(Because $a^2-b^2=(a+b)(a-b)$)

Therefore the factors of $\boxed{4a^2-36b^2}$ is $\boxed{(2a+6b)(2a-6b)}$.

Answer 31PA.

Consider the polynomial $20n^2+34n+6$.

The objective is to factor the given polynomial

$20n^2+34n+6=2\cdot 10n^2+2\cdot 17n+2\cdot 3$

$=2\left[10n^2+17n+3\right]$ (Factor $GCF\left(20n^2,34n,6\right)=2$)

Now compare $10n^2+17n+3$ with ax^2+bx+c

$a=10$

$b=17,$

$c=3$

Now find two numbers m,n such that

$m+n=b$
$\quad =17$ positive and

$mn=a\cdot c$

$\quad =10\cdot 3$

$\quad =30$

Positive m and n must be positive

List all pair of factors of 30 in those choose a pair of factors of sum 17.

Factors of 30	Sum of factors
1.30	31
2.15	17
3.10	13
5.6	11

The correct factors are $2,15$.

$$2\left[10n^2+17n+3\right]=2\left[10n^2+15n+2n+3\right]$$
$$=2\left[5n(2n+3)+(2n+3)\right]$$
$$=2(5n+1)(2n+3)$$ (By distributive $(a+b)c=ac+bc$)

Therefore the factorization of $\boxed{20n^2+34n+6}$ is $\boxed{2(5n+1)(2n+3)}$.

Answer 32PA.

Consider the polynomial $5y^2-90$

The objective is to factor the given polynomial.

$$5y^2-90=5\cdot y^2-5\cdot 18$$
$$=5\left(y^2-18\right)\quad\left(GCF\left(5y^2,90\right)=5\right)$$
$$=5\left[(y)^2-\left(\sqrt{18}\right)^2\right]$$
$$=5\left(y+\sqrt{18}\right)\left(y-\sqrt{18}\right)$$ [Because $a^2-b^2=(a+b)(a-b)$]

Therefore,

$$\boxed{5y^2-90=5\left(y+\sqrt{18}\right)\left(y-\sqrt{18}\right)}.$$

Answer 33PA.

Consider the polynomial $24x^3 - 78x^2 + 45x$.

The objective is to factor the given polynomial.

$$24x^3 - 78x^2 + 45x = 3 \cdot 8 \cdot x \cdot x^2 - 3 \cdot 26 \cdot x \cdot x + 3 \cdot 15 \cdot x$$

$= 3x\left[8x^2 - 26x + 15\right]$ (Factor the GCF)

Now $8x^2 - 26x + 15 = ax^2 + bx + c$

Here $a = 8,$

$b = -26,$

$c = 15$

$$8x^2 - 26x + 15 = 8x^2 + mx + nx + 15$$

Find two numbers m, n such that

negative

$m \cdot n = a \cdot c$

$= 8 \cdot 15$ positive,

$= 120$

Since $m + n$ negative and mn positive, both m and n are negative.

Now list all the pair of negative factors of 120 in those choose a pair whose sum is -26.

Factors of 120	Sum of factors
$-1.-120$	-121
$-2.-60$	-62
$-3.-40$	-43
$-4.-30$	-34
$-5.-24$	-29
$-6.-20$	-26
$-8.-15$	-23
$-10.-12$	-120

The correct factors are $-6,-20$.

Therefore

$$3x\left[8x^2-26x+15\right]=3x\left[8x^2-6x-20x+15\right]$$

$$=3x\left[2x(4x-3)-5(4x-3)\right]$$

(Factor GCF)

$$=3x\left[(2x-5)(4x-3)\right]$$ (By distributive $(b+c)a=ba+ca$)

Therefore the factorization of $\boxed{24x^3-78x^2+45x}$ is $\boxed{3x(2x-5)(4x-3)}$.

Answer 34PA.

Consider the polynomial $18y^2-48y+32$

The objective is to factor the given polynomial.

$$18y^2-48y+32=2\cdot9y^2-2\cdot24y+2\cdot16$$

$$=2\left[9y^2-24y+16\right]$$

$$=2\left[(3y)^2-2\cdot1\cdot2\cdot y+16\right]\ \left(9y^2=(3y)^2\right)$$

$$=2\left[(3y)^2-2\cdot3\cdot4\cdot y+4\cdot4\right]\ (24=2\cdot3\cdot4)$$

$$=2\left[(3y)^2-2\cdot3y\cdot4+(4)^2\right]\ \left(4^2=4\cdot4\right)$$

$=2(3y-4)^2$ [Since $a^2-2ab+b^2=(a-b)^2$]

Therefore the factorization of $\boxed{18y^2-48y+32}$ is $\boxed{2(3y-4)^2}$.

Answer 35PA.

Consider the polynomial $90g-27g^2-75$

The objective is to factor the given polynomial.

$$90g-27g^2-75=-27g^2+90g-75$$

$=-3\cdot9g^2-3\cdot(-30g)-75$ (Because $90=-3\cdot-30,-27=-3\cdot9$)

$$=-3\cdot9g^2-3\cdot-30g-3\cdot(25)\ (-75=-3\cdot25)$$

$=-3\left[9g^2-30g+25\right]$ (Factor $GCF-3$)

$$=-3\left[(3g)^2-2\cdot15g+25\right]\ \left(9g^2=(3g)^2,30=2\cdot15\right)$$

$$=-3\left[(3g)^2-2\cdot3\cdot5\cdot g+5\cdot5\right]$$

$$=-3\left[(3g)^2-2\cdot3g\cdot5+(5)^2\right]\ \left(5^2=5\cdot5\right)$$

$=-3\left[(3g-5)^2\right]$ [Because $a^2-2ab+b^2=(a-b)^2$]

Therefore the factorization of $\boxed{90g-27g^2-75}$ is $\boxed{-3(3g-5)^2}$.

Answer 36PA.

Consider the polynomial $45c^2 - 32cd$

The objective is to factor the given polynomial

$45c^2 - 32cd = 45 \cdot c \cdot c - 32\, c \cdot d$

$= c(45c - 32d)$ [$GCF(45c^2, 32cd) = c$]

Therefore the factorization of $\boxed{45c^2 - 32cd}$ is $\boxed{c(45c - 32d)}$.

Answer 37PA.

Consider the polynomial $4a^3 + 3a^2b^2 + 8a + 6b^2$

The objective is to factor the given polynomial.

$4a^3 + 3a^2b^2 + 8a + 6b^2 = 4a^3 + 8a + 3a^2b^2 + 6b^2$

$= (4a^3 + 8a) + (3a^2b^2 + 6b^2)$

(Group the terms with common factors)

$= (4 \cdot a \cdot a^2 + 4 \cdot 2 \cdot a) + (3 \cdot a^2 \cdot b^2 + 3 \cdot 2 \cdot b^2)$

$(a^3 = a \cdot a^2, 6 = 3 \cdot 2)$

$= 4a(a^2 + 2) + 3b^2(a^2 + 2)$

(Factor GCF)

$= (4a + 3b^2)(a^2 + 2)$

(By distributive $(b + c)a = ba + ca$)

Therefore the factorization of given polynomial is $\boxed{(4a + 3b^2)(a^2 + 2)}$.

Answer 38PA.

Consider the polynomial $5a^2+7a+6b^2-4b$

The objective is to factor the given polynomial.

$$5a^2+7a+6b^2-4b=\left(5a^2+7a\right)+\left(6b^2-4b\right)$$
$$=(5\cdot a\cdot a+7\cdot a)+(2\cdot 3\cdot b\cdot b-2\cdot 2\cdot b)$$

$=a(5a+7)+2b(3b-2)$ (Factor GCF)

It is not factored further.

Hence the given polynomial is not factored.

Therefore the given polynomial is prime.

Answer 39PA.

Consider the polynomial $x^2y^2-y^2-z^2+x^2z^2$

The objective is to factor the given polynomial

$$x^2y^2-y^2-z^2+x^2z^2=\left(x^2y^2-y^2\right)+\left(x^2-z^2-z^1\right)$$

(Group the terms with common factors)

$$=y^2\left(x^2-1\right)+z^2\left(x^2-1\right)$$

(Factor GCF)

$$=\left(y^2+z^2\right)\left(x^2-1\right)$$

(Since $(b+c)a=ba+ca$)

Therefore, the factorization of $\boxed{x^2y^2-y^2-z^2+x^2+z^2}$ is $\boxed{\left(y^2+z^2\right)\left(x^2-1\right)}$.

Answer 40PA.

Consider the polynomial $4m^4n+6m^3n-16m^2n^2-24mn^2$.

The objective is to factor the given polynomial.

$$4m^4n+6m^3n-16m^2n^2-24mn^2$$
$$=\left(4m^4n-16m^2n^2\right)+\left(6m^3n-24mn^2\right)$$

(Group the terms with common factors)

$$=\left(4\cdot m^2\cdot m^2\cdot n-4\cdot 4\cdot m^2\cdot n\cdot n\right)+\left(6\cdot m\cdot m^2\cdot n-6\cdot 4\cdot m\cdot n\cdot n\right)$$

$$=4m^2\left(m^2-4n\right)+6mn\left(m^2-4n\right)$$

(Factor GCF)

$$=\left(4m^2n+6mn\right)\left(m^2-4n\right)$$

(Since $(b+c)a=ba+ca$)

$$=(2\cdot 2\cdot m\cdot n\cdot n+2\cdot 3mn)\left(m^2-4n\right)$$

$$=2mn(m+3)\left(m^2-4n\right)$$

(Factor $GCF\ 2mn$)

Therefore the factorization of given polynomial is $\boxed{2mn(m+3)\left(m^2-4n\right)}$.

Answer 41PA.

Consider that the volume of a rectangular prism is

$x^3y-63y^2+7x^2-9xy^3$ Cube meters.

The objective is to find the dimensions of the prim.

For this first factor the given polynomial completely.

$$\begin{aligned}7x^2-9xy^3 &= \left(x^3y+7x^2\right)-63y^2-9xy^3\\ &= \left(x^2\cdot xy+7\cdot x^2\right)+\left(-9\cdot 7y^2-9\cdot x\cdot y^2\cdot y\right)\\ &= x^2(xy+7)-9y^2(7+xy) \quad \text{[Factor the GCF]}\\ &= \left(x^2-9y^2\right)(xy+7) \quad \left[\text{By distributive } (b+c)a=ba+ca\right]\end{aligned}$$

$$\begin{aligned}&= \left(x^2-3^2y^2\right)(xy+7) \quad [\text{Since } 9=3\cdot 3]\\ &= \left[x^2-(3y)^2\right](xy+7) \quad \left[a^mb^m=(ab)^m\right]\\ &= (x+3y)(x-3y)(xy+7) \quad \left[\text{Since } a^2-b^2=(a+b)(a-b)\right]\\ s &= (x-3y)(x+3y)(xy+7)\end{aligned}$$

Since the volume of a rectangular prism is $v=lbh$

Where $l=\text{length}, b=\text{bredth}, h=\text{height of prism}$

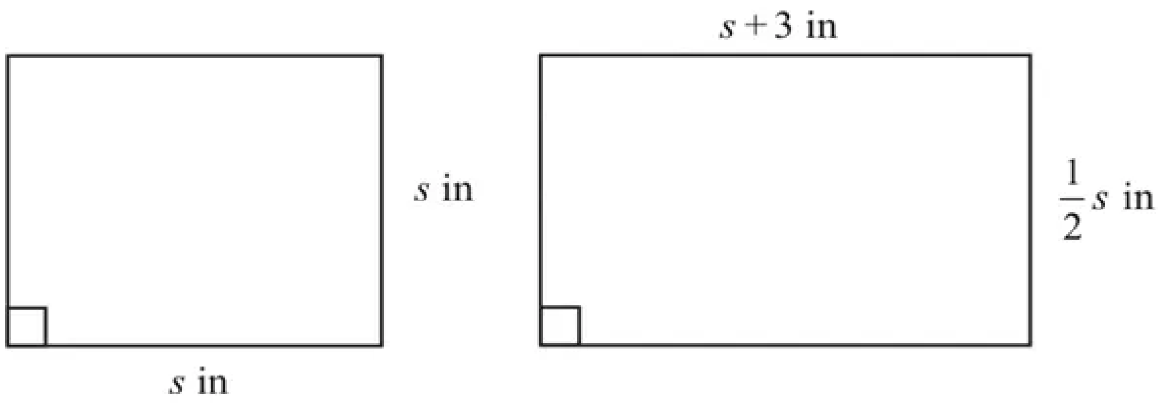
By comparing $v=x^3y-63y^2+7x^2-9xy^3$ with $v=lbh$

$$\begin{aligned}lbh &= x^3y-63y^2+7x^2-9xy^3\\ &= (x-3y)(x+3y)(xy+7)\end{aligned}$$

Therefore, the dimensions of the prism are $\boxed{x-3ym, x+3ym, xy+7m}$

Answer 42PA.

Consider the following figures



Also given the area of square is $16x^2 - 56x + 49$ square inches

The objective is to find the area of rectangle.

Since area of square $= a^2$ $[a \text{ is side of square}]$

$= s^2$

$= 16x^2 - 56x + 49$

i.e. $s^2 = 16x^2 - 56x + 49$

Since the first term of polynomial is $16x^2 = 4^2 \cdot x^2$

$= (4x)^2$ perfect square

Last term= 49

$= 7 \cdot 7$

$= 7^2$ perfect aquare

Twice the product of the square root of first and last terms

$= 2 \cdot 4x \cdot 7$

$= 56x$ middle term

Therefore, $16x^2 - 56x + 49$ is in perfect square trinomial term

Since $(a-b)^2 = a^2 - 2ab + b^2$

Here $a = 4x, b = 7$

$$s^2 = 16x^2 - 56x + 49$$
$$= (4x-7)^2 \quad \left[\text{By } (a-b)^2 = a^2 - 2ab + b^2\right]$$

Also the square root property is if $n > 0, x^2 = n$ then $x = \pm\sqrt{n}$

$$s^2 = (4x-7)^2$$
$$s = \pm\sqrt{(4x-7)^2}$$
$$s = \pm(4x-7)$$

Since lengths always positive

$$s = (4x-7)$$

Area of rectangle = $\text{length} \cdot \text{breadth}$

$= (s+3)\left(\frac{1}{2}s\right)$ [From figure]

$= [(4x-7+3)]\left[\frac{1}{2}(4x-7)\right]$ [put $s = 4x-7$]

$= [(4x-4)]\left[\frac{1}{2}(4x-7)\right]$

$= 4(x-1)\left[\frac{1}{2}(4x-7)\right]$ [Factor 4]

$= 2(x-1)(4x-7)$

$= (2x-2)(4x-7)$

$= 2x \cdot 4x + 2x \cdot (-7) + (-2) \cdot (4x) + (-2)(-7)$ [By FOIL method]

$= 8x^2 - 14x - 8x + 14$ [Simplify]

$= 8x^2 - 22x + 14$

Therefore, the area of rectangle in *x* is $\boxed{(8x^2 - 22x + 14) \text{ square inches}}$

Answer 43PA.

Consider the equation $3x^2 + 24x + 48 = 0$

The objective is to find the solution set of given equation.

For this first factor $3x^2 + 24x + 48$.

$3x^2 + 24x + 48 = 3 \cdot x^2 + 3 \cdot 8 \cdot x + 3 \cdot 16$

$= 3(x^2 + 8x + 16)$ (Factor $GCF = 3$)

$= 3\left[x^2 + 2 \cdot 4 \cdot x + (4)^2\right]$

$= 3(x+4)^2$ (Because $a^2 + 2ab + b^2 = (a+b)^2$)

Therefore

$3x^2 + 24x + 48 = 0$

$\Rightarrow \quad 3(x+4)^2 = 0$

The zero product property is if

$ab = 0$ then

$a=0$ or

$b=0$ or both.

$3(x+4)^2=0$

$\Rightarrow \quad (x+4)^2=0$

$\Rightarrow \quad x+4=0$

Or, $x+4=0$

$x+4=0$

$x+4-4=0-4$ (Subtract 4 on each side)

$x=-4$

Check: To check the proposed solutions substitute the solution in the given equation.

Given equation is,

$3x^2+24x+48=0$

$3(-4)^2+24(-4)+48=0$ (Put $x=-4$)

$3\cdot16-96+48=0$ (Simplify)

$48-96+48=0$

$0=0$ True

The solution set of given equation is $\boxed{\{-4\}}$.

Answer 44PA.

Consider the equation

$7r^2=70r-175$

The objective is to find the solution set of given equation

$7r^2=70r-175$

$7r^2=7\cdot10r-7\cdot25$

$7r^2=7(10r-25)$ (Factor 7)

$\frac{7r^2}{7}=\frac{7(10r-25)}{7}$ (Divide with 7 on both sides)

$r^2=10r-25$

$r^2-10r=10r-25+10r$ (Subtract $10r$ on both sides)

$r^2 - 10r = -25$

$r^2 - 10r + 25 = -25 + 25$ (Add 25 on both sides)

$r^2 - 10r + 25 = 0$

$r^2 - 10r + (5)^2 = 0$ [$25 = 5 \cdot 5 = (5)^2$]

$r^2 - 2 \cdot 5 \cdot r + (5)^2 = 0$

$(r-5)^2 = 0$ [Because $a^2 - 2ab + b^2 = (a-b)^2$]

The zero product property is of

$ab = 0$ then

$a = 0$ or

$b = 0$ or both.

$(r-5)^2 = 0$

$\Rightarrow \quad (r-5) = 0$

Or, $r - 5 = 0$

$r - 5 = 0$

$r - 5 + 5 = 0 + 5$ (Add 5 on both sides)

$r = 5$

Check: To check the proposed solution substitute r within the given equation.

Given equation is

$7r^2 = 70r - 175$

$7(5)^2 = 70(5) - 175$ (Put $r = 5$)

$7 \cdot 25 = 350 - 175$ (Simplify)

$175 = 175$ True

Therefore the solution set of given equation is $\boxed{\{5\}}$.

Answer 45PA.

Consider the equation

$49a^2 + 16 = 56a$

The objective is to solve the given equation.

$49a^2 + 16 = 56a$

$49ah2 + 16 - 56a = 56a - 56a$ (Subtract $56a$ on both sides)

$49a^2 - 56a + 16 = 0$

To solve above first factor left side polynomial of equation.

$49a^2 - 56a + 16 = 0$

$7 \cdot 7a^2 - 56a + 16 = 0$ (Because $7 \cdot 7 = 49$)

$7^2 a^2 - 56a + 4 \cdot 4 = 0$ (Because $4 \cdot 4 = 16$)

$(7a)^2 - 2 \cdot 28 \cdot a + 4^2 = 0$ (Because $56 = 2 \cdot 28$)

$(7a)^2 - 2 \cdot 7 \cdot 4 \cdot a + 4^2 = 0$ (Because $28 = 7 \cdot 4$)

$(7a)^2 - 2 \cdot (7a)4 + 4^2 = 0$

$(7a - 4)^2 = 0$

Since the zero product property is if

$ab = 0$ then

$a = 0$ or

$b = 0$ or both.

$(7a - 4)(7a - 4) = 0$

$7a - 4 = 0$

Or, $7a - 4 = 0$

$7a - 4 = 0$

$7a-4+4=0+4$ (Add 4 on each side)

$7a=4$

$\frac{7a}{7}=\frac{4}{7}$ (Divide with $\frac{4}{7}$ on both sides)

$a=\frac{4}{7}$

Therefore the solution set is $\left\{\frac{4}{7}\right\}$.

Check:- To check the proposed solution set substitute a with $\frac{4}{7}$ in the given equation.

Given equation is

$49a^2+16=56a$

$49\cdot\left(\frac{4}{7}\right)^2+16=56\cdot\frac{4}{7}$ (Put $a=\frac{4}{7}$)

$49\cdot\frac{16}{49}+16=8\cdot 4$ (Simplify)

$32=32$ True

Therefore, the solution set of given equation is $\boxed{\left\{\frac{4}{7}\right\}}$.

Answer 46PA.

Consider the equation

$18y^2+24y+8=0$

The objective is to find the solution set of given equation.

$18y^2+24y+8=0$

$\Rightarrow \quad 2\cdot 9y^2+2\cdot 12\cdot y+2\cdot 4=0$

$\Rightarrow \quad 2\left[9y^2+12y+4\right]=0$ [Because take GCF of 2]

Now factor $9y^2+12y+4$

Since $9y^2+12y+4=3\cdot 3\cdot y^2+2\cdot 3\cdot 2\cdot y+2\cdot 2$

$= 3^2 y^2 + 2 \cdot 3y \cdot 2 + 2^2$

(Simplify)

$= (3y)^2 + 2 \cdot 3y \cdot 2 + 2^2$

$= (3y+2)^2$ [Because $a^2 + 2ab + b^2 = (a+b)^2$]

Therefore,

$18y^2 + 24y + 8 = 0$

$\Rightarrow \quad 2\left[9y^2 + 12y + 4\right] = 0$

$\Rightarrow \quad 2(3y+2)^2 = 0$

The zero product property is if

$ab = 0$ then

$a = 0$ or

$b = 0$ or both

$2(3y+2)^2 = 0$

$\Rightarrow \quad 2(3y+2)(3y+2) = 0$

$3y + 2 = 0$

Or. $3y + 2 = 0$ (Since $2 \neq 0$)

Now solve $3y + 2 = 0$ completely

$3y + 2 = 0$

$3y+2-2=0-2$ (Subtract 2 on both sides)

$3y=-2$

$\frac{3y}{3}=\frac{-2}{3}$ (Divide with 3 on both sides)

$y=\frac{-2}{3}$

The solution set is $\left\{\frac{-2}{3}\right\}$.

Check: To check the proposed solution set substitute the solution in the given equation y by $\frac{-2}{3}$.

Given equation is

$18y^2+24y+8=0$

$18\left(\frac{-2}{3}\right)^2+24\left(\frac{-2}{3}\right)+8=0$ (Put $y=\frac{-2}{3}$)

$18\left(\frac{4}{9}\right)+24\cdot\frac{-2}{3}+8=0$ (Simplify)

$2\cdot9\cdot\frac{4}{9}+\frac{3\cdot8\cdot(-2)}{3}+8=0$ (Simplify)

$8+(-16)+8=0$

$16-16=0$

$0=0$ True

Therefore the solution set of given equation is $\boxed{\left\{\frac{-2}{3}\right\}}$.

Answer 47PA.

Consider the equation

$y^2-\frac{2}{3}y+\frac{1}{9}=0$

The objective is to solve the given equation.

For this write the given equation in perfect square form

$y^2-\frac{2}{3}y+\frac{1}{9}=0$

$\Rightarrow \quad y^2 - 2\cdot\frac{1}{3}\cdot y + \left(\frac{1}{3}\right)^2 = 0$ (Because $\frac{1}{9} = \frac{1}{3^2}$)

$\Rightarrow \quad \left(y - \frac{1}{3}\right)^2 = 0$ (Because $a^2 - 2ab + b^2 = (a-b)^2$)

Also the zero product property,

If $ab = 0$ then

$a = 0$

Or, $b = 0$ or both

$\left(y - \frac{1}{3}\right)^2 = 0$

$\left(y - \frac{1}{3}\right)\left(y - \frac{1}{3}\right) = 0$

$y - \frac{1}{3} = 0$

Or, $y - \frac{1}{3} = 0$ (By zero product property)

$y - \frac{1}{3} = 0$

$y - \frac{1}{3} + \frac{1}{3} = 0 + \frac{1}{3}$ (Add $\frac{1}{3}$ on both sides)

$y = \frac{1}{3}$

The solution set is $\left\{\frac{1}{3}\right\}$.

Check: To check the proposed solution substitute y by $\frac{1}{3}$, in the given equation.

Given equation is

$$y^2-\frac{2}{3}y+\frac{1}{9}=0$$

$$\left(\frac{1}{3}\right)^2-\frac{2}{3}\cdot\frac{1}{3}+\frac{1}{9}=0 \text{ (Put } y=\frac{1}{3}\text{)}$$

$$\frac{1}{9}-\frac{2}{9}+\frac{1}{9}=0 \text{ (Simplify)}$$

$$\frac{2}{9}-\frac{2}{9}=0$$

$0=0$ True

Therefore, the solution set is $\left\{\frac{1}{3}\right\}$.

Answer 48PA.

Consider the equation

$$a^2+\frac{4}{5}a+\frac{4}{25}=0$$

The objective is to find the solution set of given equation.

For this write the given equation in perfect square form.

$$a^2+\frac{4}{5}a+\frac{4}{25}=0$$

$$a^2+2\cdot\frac{2}{3}\cdot a+\left(\frac{2}{5}\right)^2=0 \text{ (Because } 4=2^2, 25=5^2\text{)}$$

$$a^2+2\cdot a\cdot\frac{2}{5}+\left(\frac{2}{5}\right)^2=0$$

$$\left(a+\frac{2}{5}\right)^2=0 \text{ (Because } a^2+2ab+b^2=(a+b)^2\text{)}$$

By zero product property, if

$ab=0$ then

$a=0$ or

$b=0$ or both.

$$\left(a+\frac{2}{5}\right)^2=0$$

$$\left(a+\frac{2}{5}\right)\left(a+\frac{2}{5}\right)=0$$

$$a+\frac{2}{5}=0$$

Or $\left(a+\frac{2}{5}\right)=0$ (By zero product property)

$$a+\frac{2}{5}=0$$

$a+\frac{2}{5}-\frac{2}{5}=0-\frac{2}{5}$ (Subtract $\frac{2}{5}$ on both sides)

$$a=\frac{-2}{5}$$

The solution set is $\left\{\frac{-2}{5}\right\}$.

Check: To check the proposed solution substitute a with $\frac{-2}{5}$ in the given equation.

Given equation is

$$a^2+\frac{4}{5}a+\frac{4}{25}=0$$

$\left(\frac{-2}{5}\right)^2+\frac{4}{5}\cdot\frac{-2}{5}+\frac{4}{25}=0$ (Put $a=\frac{-2}{5}$)

$\frac{4}{25}-\frac{8}{25}+\frac{4}{25}=0$ (Simplify)

$\frac{8}{25}-\frac{8}{25}=0$ (Simplify)

$0=0$ True

Therefore, the solution set is $\boxed{\left\{\frac{-2}{5}\right\}}$.

Answer 49PA.

Consider the equation

$z^2 + 2z + 1 = 16$

The objective is to find the solution set of given equation.

$z^2 + 2z + 1 = 16$

$z^2 + 2 \cdot z \cdot 1 + 1^2 = 16$

$(z+1)^2 = 16$ (Because $a^2 + 2ab + b^2 = (a+b)^2$)

The square root property is of

$n > 0$ and

$x^2 = n$ then

$x = \pm\sqrt{n}$

$(z+1)^2 = 16$

$\Rightarrow \quad z + 1 = \pm\sqrt{16}$ (By square root property)

$\Rightarrow \quad z + 1 = \pm\sqrt{(4^2)}$ (Because $16 = 4^2$)

$\Rightarrow \quad z + 1 = \pm 4$

$z + 1 - 1 = -1 \pm 4$ (Subtract 1 on both sides)

$z = -1 \pm 4$

$z = -1 + 4$

Or $z = -1 - 4$

$z = 3$

Or, $z = -5$

The solution set is $\{-5, 3\}$.

Check: To check the proposed solution set substitute each solution in the given equation.

Given equation is

$z^2 + 2z + 1 = 16$

$(-5)^2 + 2(-5) + 1 = 16$ (Put $z = -5$)

$25 - 10 + 1 = 16$ (Simplify)

$16 = 16$ True

For $z = 3$,

$z^2 + 2z + 1 = 16$

$(3)^2 + 2(3) + 1 = 16$ (Put $z = 3$)

$9 + 6 + 1 = 16$

$16 = 16$ True

Therefore, the solution set of given equation is $\boxed{\{-5, 3\}}$.

Answer 50PA.

Consider the equation

$x^2 + 10x + 25 = 81$

The objective is to find the solution set of given equation.

$x^2 + 10x + 25 = 81$

$x^2 + 2 \cdot 5 \cdot x + 5^2 = 81$ (Since $25 = 5^2, 10 = 2 \cdot 5$)

$(x+5)^2 = 81$ [Because $a^2 + 2ab + b^2 = (a+b)^2$]

The square root property is, of

$n > 0$, and

$x^2 = n$ then

$x = \pm\sqrt{n}$

$(x+5)^2 = 81$

$a + 5 = \pm\sqrt{81}$ (By square root property)

$x + 5 - 5 = -5 \pm \sqrt{81}$ (Subtract 5 on both sides)

$x = -5 \pm \sqrt{(9^2)}$ $(81 = 9^2)$

$x = -5 \pm 9$

$x = -5 + 9$

Or, $x = -5 - 9$

$x = 4$

Or, $x = -14$

The solution set is $\{-14, 4\}$.

Check:- To check the proposed solution set substitute each solution in the given equation.

For $x=-14$,

$x^2+10x+25=81$

$(-14)^2+10(-14)+25 = 81$ (Put $x=-14$)

$196-140+25=81$ (Simplify)

$56+25=81$

$81=81$ True

For $x=4$,

$x^2+10x+25=81$

$4^2+10(4)+25=81$ (Put $x=4$)

$16+40+25=81$

$81=81$ True

The solution set is $\boxed{\{-14,4\}}$.

Answer 51PA.

Consider the equation

$(y-8)^2=7$

The objective is to find the solution set of given equation.

Since the square root property is of

$n>0$, and

$x^2=n$ then

$x=\pm\sqrt{n}$

$(y-8)^2=7$

$y-8=\pm\sqrt{7}$ (By square root property)

$y=8\pm\sqrt{7}$

Therefore $y=8\pm\sqrt{7}$

Or, $y=8-\sqrt{7}$

The solution set is $\{8+\sqrt{7},8-\sqrt{7}\}$.

Check: To check the proposed solution set substitute each solution in the given equation.

Given equation is

$$(y-8)^2=7$$

$$\left(8+\sqrt{7}-8\right)^2=7$$ (Put $y=8+\sqrt{7}$)

$$\left(\sqrt{7}\right)^2=7$$ (Simplify)

$7=7$ True

For $y=8-\sqrt{7}$,

$$(y-8)^2=7$$

$$\left(8-\sqrt{7}-8\right)^2=7$$ (Put $y=8-\sqrt{7}$)

$$\left(-\sqrt{7}\right)^2=7$$ (Simplify)

$7=7$ True

Therefore, the set of given equation is $\boxed{\left\{8+\sqrt{7},8-\sqrt{7}\right\}}$.

Answer 52PA.

Consider the equation

$$(w+3)^2=2$$

The objective is to find the solution set of given equation

$$(w+3)^2=2$$

Since the square root property is

$n>0$, if

$x^2=n$ then

$x=\pm\sqrt{n}$

Here $2>0$,

$(w+3)^2=2$ then

$$w+3=\pm\sqrt{2}$$

$w+3-3=-3\pm\sqrt{2}$ (Subtract 3 on each side)

$w+3-3=-3\pm\sqrt{2}$ (Subtract 3 on each side)

$w=-3\pm\sqrt{2}$

$w=-3+\sqrt{2},$

$w=-3-\sqrt{2}$

The solution set is $\left\{-3+\sqrt{2},-3-\sqrt{2}\right\}$.

Check:- To check the proposed solution set substitute each solution in the given equation.

Given equation is

$(w+3)^2=2$

$\left(-3+\sqrt{2}+3\right)^2=2 \quad \left(w=-3+\sqrt{2}\right)$

$\left(\sqrt{2}\right)^2=2$

$2=2$ True

$(w+3)^2=2$

$\left(-3-\sqrt{2}+3\right)^2=2$ (Put $w=-3-\sqrt{2}$)

$\left(-\sqrt{2}\right)^2=2$ (Simplify)

$2=2$ True

Therefore the solution set is $\boxed{\left\{-3+\sqrt{2},-3-\sqrt{2}\right\}}$.

Answer 53PA.

Consider the equation

$p^2+2p+1=6$

The objective is to find the solution set of given equation.

$p^2+2p+1=6$

$p^2+2\cdot p\cdot 1+1^2=6$

$(p+1)^2=6$ (Since $(a+b)^2=a^2+2ab+b^2$)

The square root property is of

$n>0$, and

$x^2 = n$ then

$x = \pm\sqrt{n}$.

$(p+1)^2 = 6$

$p+1 = \pm\sqrt{6}$ (By square root property)

$p+1-1 = -1\pm\sqrt{6}$ (Subtract 1 on each side)

$p = -1\pm\sqrt{6}$

$p = -1\pm\sqrt{6}$,

Or, $p = -1-\sqrt{6}$

The solution set is $\left\{-1+\sqrt{6}, -1-\sqrt{6}\right\}$.

Check:- To check the proposed solution set, substitute each solution in the given equation.

For $p = -1+\sqrt{6}$,

$p^2 + 2p + 1 = 6$

$(p+1)^2 = 6$

$\left(-1-\sqrt{6}+1\right)^2 = 6$ (Put $p = -1-\sqrt{6}$)

$\left(-\sqrt{6}\right)^2 = 6$

$6 = 6$ True

Therefore, the solution set is $\boxed{\left\{-1+\sqrt{6}, -1-\sqrt{6}\right\}}$.

Answer 54PA.

Consider the equation

$x^2 - 12x + 36 = 11$

The objective is to find the solution set of given equation.

$x^2 - 12x + 36 = 11$

$x^2 - 2 \cdot 6 \cdot x + 6 \cdot 6 = 11$ (Since $6 \cdot 6 = 36, 2 \cdot 6 = 12$)

$x^2 - 2 \cdot 6 \cdot x + 6^2 = 11$

$(x-6)^2 = 11$ [Since $a^2 - 2ab + b^2 = (a-b)^2$]

The square root property is of

$n > 0$,

$x^2 = n$ then

$x = \pm\sqrt{n}$.

$(x-6)^2 = 11$

$x - 6 = \pm\sqrt{11}$ (By square root property)

$x - 6 + 6 = 6 \pm \sqrt{11}$ (Add 6 on both sides)

$x = 6 \pm \sqrt{11}$

$x = 6 \pm \sqrt{11}$

Or, $x = 6 - \sqrt{11}$

The solution set is $\{6 + \sqrt{11}, 6 - \sqrt{11}\}$.

Check: To check the proposed solution set, substitute each solution in the given equation.

Given equation,

$$x^2 - 12x + 36 = 11$$

$$(x-6)^2 = 11$$

$$\left(6+\sqrt{11}-6\right)^2 = 11$$ (Put $x = 6+\sqrt{11}$)

$$\left(\sqrt{11}\right)^2 = 11$$

$11 = 11$ True

For $x = 6-\sqrt{11}$,

$$x^2 - 12x + 36 = 11$$

$$(x-6)^2 = 11$$

$$\left(6-\sqrt{11}-6\right)^2 = 11$$ (Put $x = 6-\sqrt{11}$)

$$\left(-\sqrt{11}\right)^2 = 11$$

$11 = 11$ True

The solution set of given equation is $\boxed{\left\{6+\sqrt{11}, 6-\sqrt{11}\right\}}$.

Answer 55PA.

Consider that "one of the most commonly used formula for estimating Board feet is the Doyle log rule

$$B = \frac{L}{16}\left(D^2 - 80 + 16\right)$$

Where B is the number of Board feet

D is the Diameter in inches.

L is the length of the long in feet.

The objective is the factor the given formula

$$B = \frac{L}{16}\left(D^2 - 8D + 16\right)$$

In $D^2 - 8D + 16,$ first term $= D^2$ perfect square

Last term $= 16$

$= 4^2$ Perfect square

Twice the product of square roots of first and last term is

$$\begin{aligned} &= 2\sqrt{D^2} \cdot \sqrt{4^2} \\ &= 2 \cdot D \cdot 4 \\ &= 8D \quad \text{middle term} \end{aligned}$$

$D^8 - 8D + 16$ is a perfect square trinomial

Also $(a-b)^2 = a^2 - 2ab + b^2$

$$\begin{aligned} D^2 - 8D + 16 &= D^2 - 2 \cdot 4 \cdot D + 4^2 \\ &= (D-4)^2 \\ B &= \frac{L}{16}\left(D^2 - 8D + 16\right) \\ &= \frac{L}{16}(D-4)^2 \end{aligned}$$

Therefore, the factored form of given formula is $\boxed{\frac{L}{16}(D-4)^2}$

Answer 60PA.

Consider the perfect square trinomial $x^2+kx+64$

The objective is to find all value of k so that $x^2+kx+64$ is a perfect square trinomial.

Since a trinomial is perfect square.

If its first term must be perfect square

Last terms must be perfect square

Middle term must be twice the product of the square roots of the first and last terms.

First term $=x^2=(x)^2 \rightarrow \text{perfect square}$

Last term $=64=8\cdot 8=8^2 \rightarrow \text{Perfect aquare}$

Middle term $=kx=$ twice the product of square roots of the first and last terms.

$kx=2\cdot x\cdot(\pm 8)$ $\quad$ $\left[\text{Since } \sqrt{64}=\pm 8\right]$

$kx=\pm 16x$

Thus $k=\pm 16$

Therefore, the possible values of k are $\boxed{\pm 16}$

Answer 61PA.

Consider the perfect square trinomial $4x^2+kx+1$

The objective is to find all value of k so that $4x^2+kx+1$ is a perfect square trinomial.

Since a trinomial is perfect square, of its

First term must be perfect square

Last term must be perfect square

Middle term must be twice the product of the square root of the first and last term.

First term $=4x^2$

$=2\cdot 2x^2$ $\quad$ $[\text{Since } 4=2\cdot 2]$

$=2^2\cdot x^2$ $\quad$ $\left[2\cdot 2=2^2\right]$

$=(2x)^2 \rightarrow \text{Perfect square}$

Last term $=1\cdot 1$

$=1^2$

Middle term = twice the products of square roots of the first and last terms.

$kx=2\cdot 2x\cdot(\pm 1)$ $\quad$ $\left[\sqrt{1^2}=\pm 1\right]$

$kx=\pm 4x$ $\quad$ $[\text{Simplify}]$

By comparing $k=\pm 4$

Therefore, the possible values of k are $\boxed{\pm 4}$

Answer 62PA.

Consider the perfect square trinomial $25x^2 + kx + 49$

The objective is to find all values of *k* so that $25x^2 + kx + 49$ is a perfect square trinomial.

Since a trinomial is perfect square if its

First term must be perfect square.

Last term must be perfect square

Middle term must be twice the product of the square root of the first and last terms.

First term $= 25x^2$

$= 5^2 \cdot x^2 \qquad [25 = 5 \cdot 5 = 5^2]$

$= (5x)^2 \rightarrow \qquad \text{Perfect square}$

Last term $= 49$

$= 7 \cdot 7$

$= 7^2 \qquad \rightarrow \text{perfect square}$

Middle term $= kx =$ twice the product of square roots of the first and last terms.

$kx = 2 \cdot 5x(\pm 7) \qquad \left[\sqrt{7^2} = \pm 7\right]$

$kx = \pm 70x$

By comparing, $k = \pm 70$

Therefore, the possible values of k are $\boxed{\pm 70}$

Answer 63PA.

Consider the perfect square trinomial x^2+8x+k

The objective is to find all values of *k* so that x^2+8x+k is a perfect square trinomial.

Since a trinomial is perfect square if its

First term must be perfect square.

Last term must be perfect square

Middle term must be twice the product of the square root of the first and last terms.

First term $=x^2-\text{Perfect square}$

Last term $=k$

Middle term = twice the product of square roots of the first and last terms.

$$\begin{aligned}
8x &= 2\cdot x\cdot\sqrt{k} \\
2\cdot 4\cdot x &= 2x\cdot\sqrt{k} \\
\frac{2\cdot 4\cdot x}{2x} &= \frac{2x\cdot\sqrt{k}}{2x} && [\text{Divide with } 2x \text{ on both sides}] \\
4 &= \sqrt{k} \\
\sqrt{k} &= 4 \\
\left(\sqrt{k}\right)^2 &= (4)^2 && [\text{Squaring on both sides}] \\
k &= 16
\end{aligned}$$

Therefore, the value of *k* is $\boxed{16}$

Answer 64PA.

Consider the perfect square trinomial $x^2 - 18x + k$

The objective is to find all values of *k* so that $x^2 - 18x + k$ is a perfect square.

Since a trinomial is perfect square if its

First term must be perfect square.

Last term must be perfect square

Middle term must be twice the product of the square root of the first and last terms.

First term $= x^2 - \text{Perfect square}$

Last term $= k$

Middle term = twice the product of square roots of the first and last terms.

$$-18x = 2 \cdot x \cdot \sqrt{k}$$

$$\frac{-18x}{2x} = \frac{2x \cdot \sqrt{k}}{2x} \qquad [\text{Divide with } 2x \text{ on both sides}]$$

$$-9 = \sqrt{k}$$

Squaring on both sides

$$(-9)^2 = \left(\sqrt{k}\right)^2$$

$$81 = k$$

Therefore, the value of *k* is $\boxed{81}$

Answer 65PA.

Consider the perfect square trinomial $x^2 + 20x + k$

The objective is to find all values of *k* so that $x^2 + 20x + k$ is a perfect square trinomial.

Since a trinomial is perfect square if its

First term must be perfect square.

Last term must be perfect square

Middle term must be twice the product of the square root of the first and last terms.

First term $= x^2 - \text{Perfect square}$

Last term $= k - \left(\sqrt{k}\right)^2$

Middle term = twice the product of square roots of the first and last terms.

$$20x = 2 \cdot x \cdot \sqrt{k}$$

$$20x = 2\sqrt{k} \cdot x$$

$$\frac{20x}{2x} = \frac{2\sqrt{k}x}{2x} \qquad [\text{Divide with } 2x \text{ on both sides}]$$

$$10 = \sqrt{k}$$

Squaring on both sides

$$10^2 = \left(\sqrt{k}\right)^2$$

$$100 = k$$

Therefore, the value of *k* is $\boxed{100}$

Answer 67PA.

Consider that, during an experiment, a ball is dropped off a bridge from a height of 205 feet.

The formula $205 = 16t^2$ can be used to approximate the amount of time, in seconds; it takes for the ball to reach the surface of the water of the river below the bridge.

The objective is to find the time it takes the ball to reach the water to the nearest tenth of a second.

$$205 = 16t^2$$
$$16t^2 = 205$$
$$\frac{16t^2}{16} = \frac{205}{16} \quad [\text{divide with 16 on both sides}]$$
$$t^2 = 12.8125$$

The square root property is if $n > 0, x^2 = n$ then $x = \pm\sqrt{n}$

$$t^2 = 12.8125$$
$$t = \pm\sqrt{12.8125} \; [\text{By square root property}]$$
$$= \pm 3.579$$
$$\approx 3.6$$

Since time is always nearness in positive

$$t \approx 3.6 \text{ sec}$$

Therefore, the time taken by the ball to reach the water is $\boxed{3.6 \text{ s}}$

Answer 68PA

Consider that $\sqrt{a^2 - 2ab + b^2} = a - b$

Since $\sqrt{a^2 - 2ab + b^2} = \sqrt{(a-b)^2}$

$$= a - b$$

If $a < b$ then $a - b < 0, a - b$ is a negative number

If $a \le b$ then $a - b \le 0, a - b$ is negative or zero

If $a > b$ then $a - b > 0, a - b$ is a positive number

If $a \ge b$ then $a - b \ge 0, a - b$ is a positive number or zero

Since the square root is not a negative number

Thus the best describes for *a* and *b* is $a \ge b$.

Therefore, the relationship between *a* and *b* is $\boxed{a \ge b}$

Answer 69MYS.

Consider the equation

$s^2 = 25$

The objective is to find the solution set of given equation

$s^2 = 25$

$s^2 - 25 = 25 - 25$ (Subtract 25 on both sides)

$s^2 - 25 = 0$

$s^2 - 5 \cdot 5 = 0$ (Since $5 \cdot 5 = 25$)

$s^2 - 5^2 = 0$

The difference of squares property is

$a^2 - b^2 = (a+b)(a-b)$

$s^2 - 5^2 = 0$

$(s+5)(s-5) = 0$ [Because $a^2 - b^2 = (a+b)(a-b)$]

The zero product property is if

$ab = 0$ then

$a = 0$ or

$b = 0$ or both.

$(s+5)(s-5) = 0$

$s + 5 = 0$

Or, $s - 5 = 0$ (By zero product property)

$s + 5 = 0$

$s + 5 - 5 = 0 - 5$ (Subtract 5 on both sides)

$s = -5$

$s - 5 = 0$

$s - 5 + 5 = 0 + 5$ (Add 5 on both sides)

$s = 5$

The solution set is $\{-5, 5\}$.

Check: To check the proposed solutions, substitute each solution in the given equation.

Given equation is

$s^2 = 25$

$(-5)^2 = 25$ (Put $s = -5$)

$25 = 25$ True

For $s = 5$,

$s^2 = 25$

$\Rightarrow \quad 5^2 = 25$ (Put $s = 5$)

$25 = 25$ True

Therefore, the solution set of given equation is $\boxed{\{-5, 5\}}$.

Answer 70MYS.

Consider the equation

$9x^2 - 16 = 0$

The objective is to solve the given equation.

The difference of square property is

$a^2 - b^2 = (a+b)(a-b)$

$9x^2 - 16 = 0$

$\Rightarrow \quad 3 \cdot 3x^2 - 4 \cdot 4 = 0$ (Because $9 = 3 \cdot 3, 16 = 4 \cdot 4$)

$\Rightarrow \quad 3^2 x^2 - 4^2 = 0$

$\Rightarrow \quad (3x)^2 - 4^2 = 0$ (Because $a^m \cdot b^m = (ab)^m$)

$\Rightarrow \quad (3x+4)(3x-4) = 0$ (By difference of square property)

The zero product property is if

$ab = 0$ then

$a = 0$ or

$b = 0$ or both

$(3x+4)(3x-4) = 0$

$\Rightarrow \quad 3x + 4 = 0$

Or, $3x - 4 = 0$ (Zero product property)

Now solve each equation

$3x + 4 = 0$

$3x + 4 - 4 = 0 - 4$ (Subtract 4 on both sides)

$3x = -4$

$\frac{3x}{3} = \frac{-4}{3}$ (Divide with 3 on both sides)

$x = \frac{-4}{3}$

$3x - 4 = 0$

$3x - 4 + 4 = 0 + 4$ (Add 4 on both sides)

$3x = 4$

$\frac{3x}{3} = \frac{4}{3}$ (Divide with 3 on both sides)

$x = \frac{4}{3}$

The solution set is $\left\{\frac{-4}{3},\frac{4}{3}\right\}$.

Check: To check the proposed solutions substitute each solution in the given equation.

Given equation is

$$9x^2-16=0$$

$$9\left(\frac{-4}{3}\right)^2-16=0 \text{ (Put } x=\frac{-4}{3}\text{)}$$

$$9\frac{16}{9}-16=0$$

$$16-16=0$$

$0=0$ True

For, $x=\frac{4}{3}$,

$$9x^2-16=0$$

$$9\left(\frac{4}{3}\right)^2-16=0 \text{ (Put } x=\frac{4}{3}\text{)}$$

$$9\cdot\frac{16}{9}-16=0$$

$$16-16=0$$

$0=0$ True

The solution set of given equation is $\boxed{\left\{\frac{-4}{3},\frac{4}{3}\right\}}$.

Answer 71MYS.

Consider the equation

$$49m^2=81$$

The objective is to find the solution set of given equation

The difference of squares properly is

$$a^2-b^2=(a+b)(a-b)$$

$$49m^2=81$$

$$7\cdot 7m^2=81$$

$$7^2\cdot m^2=81$$

$(7m)^2 = 81$ (Because $a^m b^m = (ab)^m$)

$(7m)^2 - 81 = 81 - 81$ (Subtract 81 on both sides)

$(7m)^2 - 9 \cdot 9 = 0$

$(7m)^2 - 9^2 = 0$

$(7m+9)(7m-9) = 0$ (By difference of squares method)

The zero product property is of

$ab = 0$ then

$a = 0$ or

$b = 0$ or both.

$7m + 9 = 0$

Or, $7m - 9 = 0$

Now solve each equations separately

$7m + 9 = 0$

$7m + 9 - 9 = 0 - 9$ (Subtract 9 on both sides)

$7m = -9$

$\frac{7m}{7} = \frac{-9}{7}$ (Divide 7 on both sides)

$m = \frac{-9}{7}$

$7m - 9 = 0$

$7m - 9 + 9 = 0 + 9$ (Add 9 on both sides)

$7m = 9$

$\frac{7m}{7} = \frac{9}{7}$ (Divide with 7 on both sides)

$m = \frac{9}{7}$

The solution set is $\left\{\frac{-9}{7}, \frac{9}{7}\right\}$.

Check: To check the proposed solution, substitute each solution in the given equation.

Given equation is

$49m^2 = 81$

$49\left(\frac{-9}{7}\right)^2 = 81$ (Put $m = \frac{-9}{7}$)

$49 \cdot \frac{81}{49} = 81$ (Simplify)

$81 = 81$ True

For $m = \frac{9}{7}$,

$49m^2 = 81$

$49 \cdot \left(\frac{9}{7}\right)^2 = 81$ (Put $m = \frac{9}{7}$)

$49 \cdot \frac{81}{49} = 81$ (Simplify)

$81 = 81$ True

Therefore, the solution set of given equation is $\boxed{\left\{\frac{-9}{7}, \frac{9}{7}\right\}}$.

Answer 72MYS.

Consider the equation

$8k^2 + 22k - 6 = 0$

The objective is to solve the given equation completely.

$8k^2 + 22k - 6 = 0$

$2 \cdot 4k^2 + 2 \cdot 11k - 2 \cdot 3 = 0$

$2\left(4k^2 + 11k - 3\right) = 0$ (Factor LCM the 2)

Now compare

$4k^2 + 11k - 3 = 0$ with

$ax^2 + bx + c = 0$

$a = 4,$

$b = 11,$

$c = -3$

Now factor $4k^2 + 11k - 3$

$4k^2 + 11k - 3 = 4k^2 + mk + nk - 3$

That is we have to find two numbers m, n such that

$m + n = 11$ and

$$\begin{aligned} m \cdot n &= a \cdot c \\ &= 4 \cdot (-3) \\ &= -12 \end{aligned}$$

Since $m + n$ is positive and mn is negative, then either m or n must be negative.

List all the pair of factors of -12 in which one factor is negative ad choose a pair of factors whose sum is 11.

Factors of -12	Sum of factors
$1, -12$	-11
$-1, 12$	11
2.6	4

-2.6	4
$2.-6$	-4
-3.4	1
$3.-4$	-1

The correct factors are $-1,12$.

Therefore,

$2\left(4k^2+11k-3\right)=0$

$2\left(4k^2+mk+nk-3\right)=0$

$2\left(4k^2-k+12k-3\right)=0 \quad (m=-1, n=12)$

$2\left[k(4k-1)+3(4k-1)\right]$
$=0$

$2(k+3)(4k-1)=0$ (Because $(b+c)a=ba+ca$)

The zero product property is if

$ab=0$ then

$a=0$ or

$b=0$ or both

$2(k+3)(4k-1)=0$

$k+3=0$

Or $4k-1=0$ (By zero product property)

Now solve equation separately,

$k+3=0$

$k+3-3=0-3$ (Subtract 3 on both sides)

$k=-3$

$4k-1=0$

$4k-1+1=0+1$ (Add 1 on both sides)

$4k-1+1=0+1$ (Add 1 on both sides)

$4k=1$

$\frac{4k}{4}=\frac{1}{4}$ (Divide with 4 on both sides)

$k=\frac{1}{4}$

The solution set is $\left\{-3,\frac{1}{4}\right\}$.

Check: To check the proposed solution set, substitute each solution in the given equation.

Given equation is

$8k^2+22k-6=0$

$8(-3)^2+22(-3)-6=0$ (Put $k=-3$)

$8\cdot 9-66-6=0$

$72-72=0$

$72=72$ True

For $k=\frac{1}{4}$,

$8k^2+22k-6=0$

$8\left(\frac{1}{4}\right)^2+22\left(\frac{1}{4}\right)-6=0$ (Put $k=\frac{1}{4}$)

$8\cdot\frac{1}{16}+22\cdot\frac{1}{4}-6=0$

$\frac{1}{2}+\frac{11}{2}-6=0$ (Simplify)

$\frac{12}{2}-6=0$

$6-6=0$

$0=0$ True

Therefore the solution set of given equation is $\boxed{\left\{-3,\frac{1}{4}\right\}}$.

Answer 73MYS.

Consider the equation

$$12w^2 + 23w = -5$$

The objective is to factor the given equation.

$$12w^2 + 23w = -5$$

$$12w^2 + 23w + 5 = -5 + 5$$ (Add 5 on both sides)

$$12w^2 + 23w + 5 = 0$$

To solve the above equation, first factor $12w^2 + 23w + 5$.

Compare

$$12w^2 + 23w + 5 = ax^2 + bx + c$$

$$a = 12,$$
$$b = 23,$$
$$c = 5$$

$$12w^2 + 23w + 5 = 12w^2 + mw + nw + 5$$

Now find two numbers m, n such that

$$m + n = b \\ = 23$$ and

$$mn = ac \\ = 12 \cdot 5 \\ = 60$$

Since $m+n, mn$ are positive, m, n must be positive

List all the pair of factors of 60, in those choose a pair whose sum is 23.

Factors of 60	Sum of factors
$1, 60$	61
$2, 60$	32
$3, 20$	23

4,15	19
4,12	17
6,10	60

The correct factors are $3, 20$

$12w^2 + 23w + 5 = 12w^2 + mw + nw + 5$

$= 12w^2 + 3w + 20w + 5$

$(m = 3, n = 20)$

$= (3 \cdot 4 \cdot w \cdot w + 3w) + (5 \cdot 4w + 5)$

$= 3w(4w + 1) + 5(4w + 1)$

$= (3w + 5)(4w + 1)$

(Because $(b + c)a = ba + ca$)

Therefore,

$12w^2 + 23w + 5 = 0$

$\Rightarrow \quad (3w + 5)(4w + 1)$

$= 0$

The zero product property is, if

$ab = 0$ then

$a = 0$ or

$b = 0$ or both.

$3w + 5 = 0$

Or, $4w + 1 = 0$ (By zero product property)

Now solve each equation separately

$3w + 5 = 0$

$3w + 5 - 5 = 0 - 5$ (Subtract 5 on both sides)

$3w = -5$

$3w \quad -5$ (Divide with 3 on both sides)

$\frac{\quad}{3}=\frac{\quad}{3}$ (Divide with 3 on both sides)

$w=\frac{-5}{3}$

$4w+1=0$

$4w+1-1=0-1$ (Subtract 1 on both sides)

$4w=-1$

$\frac{4w}{4}=\frac{-1}{4}$ (Divide with 4 on each side)

$w=\frac{-1}{4}$

The solution set is $\left\{\frac{-5}{3},\frac{-1}{4}\right\}$.

Check: To check the proposed solution set, substitute each solution in the give equation.

Given equation is

$12w^2+23w=-5$

$12\left(\frac{-5}{3}\right)^2+23\left(\frac{-5}{3}\right)=-5$ (Put $w=\frac{-5}{3}$)

$12\frac{25}{9}-\frac{115}{3}=-5$ (Simplify)

$4\cdot\frac{25}{3}-\frac{115}{3}=-5$

$\frac{100-115}{3}=-5$ (Simplify)

$\frac{-15}{3}=-5$

$-5=-5$ True

For $w=\frac{-1}{4}$,

$12w^2+23w=-5$

$$12\left(\frac{-1}{4}\right)^2 + 23\left(\frac{-1}{4}\right) = -5 \text{ (Put } w = \frac{-1}{4}\text{)}$$

$$12 \cdot \frac{1}{16} - \frac{23}{4} = -5 \text{ (Simplify)}$$

$$\frac{3}{4} - \frac{23}{4} = -5 \text{ (Simplify)}$$

$$-\frac{20}{4} = -5$$

$-5 = -5$ True

The solution set is $\boxed{\left\{\frac{-5}{3}, \frac{-1}{4}\right\}}$.

Answer 74MYS.

Consider the equation

$$6z^2 + 7 = 17z$$

The objective is to solve the given equation

$$6z^2 + 7 = 17z$$

$6z^2 + 7 - 17z = 17z - 17z$ (Subtract $17z$ on both sides)

$6z^2 - 17z + 7 = 0$ (Combine like terms)

To solve $6z^2 - 17z + 7 = 0$ first factor $6z^2 - 17z + 7$

Compare $6z^2 - 17z + 7$ with $ax^2 + bx + c$

Here $a = 6$,

$b = -17$,

$c = 7$

$$6z^2 - 17z + 7 = 6z^2 + mz + nz + 7$$

Now find two numbers m, n such that

$m + n = -17$ and

$$\begin{aligned} mn &= 6 \cdot 7 \\ &= 42 \end{aligned}$$

Since $m + n$ is negative and mn is positive, therefore both m and n are negative.

List all the negative factors of 42, in those choose a pair of factors whose sum is -17.

List all the negative factors of 42, in these choose a pair of factors whose sum is -17.

Factors of 42	Sum of factors
$-1,-42$	-43
$-2,-21$	-23
$-3,-14$	-17
$-6,-7$	-13

The correct factors are $-3,-14$.

$6z^2-17z+7=6z^2+mz+nz+7$

$=6z^2-3z+14z+7$

$(m=-3,n=-14)$

$=3z(2z-1)-7(2z-1)$

$=(3z-7)(2z-1)$

(Because $(b+c)a=ba+ca$)

$$6z^2-17z+7=0$$
$$(3z-7)(2z-1)=0$$

$3z-7=0$

Or $2z-1=0$ (By zero product property)

Now solve each equation separately

$3z-7=0$

$3z-7+7=0+7$ (Add 7 on both sides)

$3z=7$

$\frac{3z}{3}=\frac{7}{3}$ (Divide with 3 on both sides)

$z=\frac{7}{3}$

$2z-1=0$

$2z-1+1=0+1$ (Add 1 on both sides)

$2z=1$

$\frac{2z}{2}=\frac{1}{2}$ (Divide with 2 on both sides)

$z=\frac{1}{2}$

The solution set is $\left\{\frac{7}{3},\frac{1}{2}\right\}$.

Check: To check the proposed solution set substitute each solution in the given equation.

Given equation is

$$6z^2+7=17z$$

$$6\left(\frac{7}{3}\right)^2+7=17\left(\frac{7}{3}\right)$$ (Put $z=\frac{7}{3}$)

$$6\cdot\frac{49}{9}+7=\frac{119}{3}$$

$$2\cdot\frac{49}{3}+\frac{7\cdot 3}{3}=\frac{119}{3}$$ (Equate the denominators)

$$\frac{98+21}{3}=\frac{119}{3}$$

$$\frac{119}{3}=\frac{119}{3}$$ True

For $z=\frac{1}{2}$,

$$6z^2+7=17z$$

$$6\cdot\left(\frac{1}{2}\right)^2+7=17\cdot\frac{1}{2}$$ (Put $z=\frac{1}{2}$)

$$6\cdot\frac{1}{4}+7=\frac{17}{2}$$

$$\frac{3}{2}+\frac{7}{2}\cdot 2=\frac{17}{2}$$ (Equate the denominators)

$$\frac{3+14}{2}=\frac{17}{2}$$

$$\frac{17}{2}=\frac{17}{2}$$ True

The solution set of given equation is $\boxed{\left\{\frac{7}{3},\frac{1}{2}\right\}}$.

Answer 75MYS.

Consider the line equation $y=2x-1$

Point is (1.4)

The objective is to find the equation of line which passing through given point $(1,4)$ and perpendicular to the given line

$y=2x-1$ in slope intercept form.

The slope intercept form of a line is

$y = mx + c$

$y = 2x - 1$ is in slope intercept form.

Slope of $y = 2x - 1$ is

$m = 2$

Since two lines are perpendicular then the product of their slopes is -1.

Therefore, let m_1, be slope of required line.

Therefore,

$$m \cdot m_1 = -1$$
$$2 \cdot m_1 = -1$$

$\frac{2m_1}{2} = \frac{-1}{2}$ (Divide with 2 on both sides)

$m_1 = \frac{-1}{2}$

Equation of a line passing through (x_1, y_1) and having slope is m is

$y - y_1 = m(x - x_1)$

Equation of a line passing through $(1,4)$ and slope $\frac{-1}{2}$ is

$y - 4 = -\frac{1}{2}(x - 1) \quad \left(x_1 = 1, y_1 = 4, m = \frac{-1}{2}\right)$

$y - 4 = \frac{-1}{2}x + \frac{1}{2}$

$y - 4 + 4 = \frac{-1}{2}x + \frac{1}{2} + 4$ (Add 4 on each side)

$y = \frac{-1}{2}x + \frac{9}{2}$ (Simplify)

Therefore, equation of the line in slope – intercept form is

$\boxed{y = \frac{-1}{2}x + \frac{9}{2}}$.

Answer 76MYS.

Consider the line equation $y = \frac{-2}{3}x + 7$

The point is $(-4, 7)$

The objective is to find the equation of line which passing through the point $(-4, 7)$ and perpendicular to the line

$$y = \frac{-2}{3}x + 7$$

The slope – intercept form of a line is

$$y = mx + c$$

$y = \frac{-2}{3}x + 7$ is in slope – intercept form.

Slope $m = \frac{-2}{3}$

Since two lines are perpendicular then the product of their slopes is -1.

Let m_1 be slope of required line.

$$m \cdot m_1 = -1$$

$$\frac{-2}{3}m_1 = -1$$

$$m_1 = -1 \cdot \frac{3}{-2}$$

$$m_1 = \frac{3}{2}$$

Therefore equation of a line passing through (x_1, y_1) and having slope m is

$y - y_1 = m(x - x_1)$

Here $(x_1, y_1) = (-4, 7)$,

$m = \frac{3}{2}$

$y - 7 = \frac{3}{2}(x - (-4))$

$y - 7 = \frac{3}{2}(x + 4)$

$y - 7 = \frac{3}{2}x + \frac{3}{2} \cdot 4$ (Because $a(b+c) = ab + ac$)

$y - 7 = \frac{3}{2}x + 3 \cdot 2$

$y - 7 = \frac{3}{2}x + 6$

$y - 7 + 7 = \frac{3}{2}x + 6 + 7$ (Add 7 on both sides)

$y = \frac{3}{2}x + 13$

Equation of the line in slope – Intercept form is

$\boxed{y = \frac{3}{2}x + 13}$.

Answer 78MYS.

Consider the sequence $17, 13, 9, 5, \ldots$

The objective is to find the next three terms

Since the given sequence is arithmetic,

The first term $= a_1$

$= 17$, second term

$a = 13$.

Common difference

$$\begin{aligned} d &= a_2 - a_1 \\ &= 13 - 17 \\ &= -4 \end{aligned}$$

The general term of arithmetic sequence is

$$a_n = a_1 + (n+1)d$$

$$\begin{aligned} a_5 &= a_1 + (5-1)d \\ &= 17 + 4 \cdot (-4) \\ &= 17 - 16 \\ &= 1 \end{aligned}$$

$$\begin{aligned} a_6 &= a_1 + (6-1)d \\ &= 17 + 5(-4) \\ &= 17 - 20 \\ &= -3 \end{aligned}$$

$$\begin{aligned} a_7 &= a_1 + (7-1)d \\ &= 17 + (6)(-4) \\ &= 17 - 24 \\ &= -7 \end{aligned}$$

Therefore, the next three terms of arithmetic sequence are $\boxed{1, -3, -7}$.

Answer 79MYS.

Consider the sequence $-5,-4.5,4,-3.5,...$

The objective is to find the next three terms.

The first term is

$a_1 = -5$, second term

$a_2 = -4.5$

Common difference

$$\begin{aligned} d &= a_2 - a_1 \\ &= -4.5-(25) \\ &= -4.5+5 \\ &= 0.5 \end{aligned}$$

The general term of arithmetic sequence is

$$\begin{aligned} a_n &= a_1 + (n-1)d \\ &= -5+(n-1)0.5 \end{aligned}$$

$n = 5$

$$\begin{aligned} a_5 &= -5+(5-1)0.5 \\ &= -5+4(0.5) \\ &= -5+2 \\ &= -3 \end{aligned}$$

For $n = 6$,

$$\begin{aligned} a_6 &= -5+(6-1)0.5 \\ &= -5+5(0.5) \\ &= -5+2.5 \\ &= -2.5 \end{aligned}$$

For, $n = 7$,

$$\begin{aligned} a_7 &= -5+(7-1)0.5 \\ &= -5+6(0.5) \\ &= -5+3 \\ &= -2 \end{aligned}$$

The next three terms of arithmetic sequence is $\boxed{-3,-2.5,-2}$.

Answer 80MYS.

Consider the arithmetic sequence $45, 54, 63, 72, ...$

The objective is to find the next three terms of the sequence.

The first term is $a_1 = 45$

Second term is $a_2 = 54$

Common difference $d = a_2 - a_1$

$= 54 - 45$
$= 9$

The general term of arithmetic sequence is

$$a_n = a_1 + (n-1)d$$
$$= 45 + (n-1)9$$

The fifth term $a_5 = 45 + (n-1)9$

$= 45 + (5-1)9$
$= 45 + 4.9$
$= 45 + 36$
$= 81$

The sixth term $a_6 = 45 + (n-1)9$

$= 45 + (6-1)9 \qquad (n = 6)$
$= 45 + 5 \cdot 9$
$= 45 + 45$
$= 90$

The seventh term $a_7 = 45 + (n-1)9$

$= 45 + (7-1)9 \qquad (n = 7)$
$= 45 + 6 \cdot 9$
$= 45 + 54$
$= 99$

The next three terms of arithmetic sequence is $\boxed{81, 90, 99}$.