SAMPLE QUESTION PAPER

BLUE PRINT

Time Allowed : 3 hours **Maximum Marks :** 80

S. No.	Chapter	VSA/Case based (1 mark)	SA-I (2 marks)	SA-II (3 marks)	LA (5 marks)	Total
1.	Relations and Functions	3(3)#	–	1(3)*	–	**4(6)**
2.	Inverse Trigonometric Functions	–	1(2)	–	–	**1(2)**
3.	Matrices	2(2)#	–	–	–	**2(2)**
4.	Determinants	1(1)	1(2)	–	1(5)*	**3(8)**
5.	Continuity and Differentiability	–	2(4)	2(6)	–	**4(10)**
6.	Application of Derivatives	1(4)	1(2)	1(3)	–	**3(9)**
7.	Integrals	2(2)#	1(2)*	1(3)	–	**4(7)**
8.	Application of Integrals	–	1(2)	1(3)	–	**2(5)**
9.	Differential Equations	1(1)	–	1(3)*	–	**2(4)**
10.	Vector Algebra	2(2)	1(2)*	–	–	**3(4)**
11.	Three Dimensional Geometry	3(3)#	1(2)*	–	1(5)*	**5(10)**
12.	Linear Programming	–	–	–	1(5)*	**1(5)**
13.	Probability	2(2)# + 1(4)	1(2)	–	–	**4(8)**
	Total	**18(24)**	**10(20)**	**7(21)**	**3(15)**	**38(80)**

*It is a choice based question.

#Out of the two or more questions, one/two question(s) is/are choice based.

Subject Code : **041**

MATHEMATICS

Time allowed : 3 hours ***Maximum marks : 80***

General Instructions :

1. *This question paper contains two parts A and B. Each part is compulsory. Part-A carries 24 marks and Part-B carries 56 marks.*
2. *Part-A has Objective Type Questions and Part-B has Descriptive Type Questions.*
3. *Both Part-A and Part-B have internal choices.*

Part - A :

1. *It consists of two Sections-I and II.*
2. *Section-I comprises of 16 very short answer type questions.*
3. *Section-II contains 2 case study-based questions.*

Part - B :

1. *It consists of three Sections-III, IV and V.*
2. *Section-III comprises of 10 questions of 2 marks each.*
3. *Section-IV comprises of 7 questions of 3 marks each.*
4. *Section-V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.*

PART - A

Section - I

1. Find x and y such that $\begin{bmatrix} x-y & 3 \\ 2x-y & 2x+1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 12 & 15 \end{bmatrix}$.

OR

For what value of x, the matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ is a skew-symmetric matrix?

2. If $\overrightarrow{PO} + \overrightarrow{OQ} = \overrightarrow{QO} + \overrightarrow{OR}$, show that the points P, Q, R are collinear.

3. Evaluate : $\int \sin^3 x \cos^3 x \, dx$

OR

Evaluate : $\int_0^2 (3x^2 + 2x - 1)\, dx$

4. If $\left(\frac{1}{2}, \frac{1}{3}, n\right)$ are the direction cosines of a line, then find the value of n.

5. Show that the function $f: N \to N$ given by $f(x) = 4x$, is one-one but not onto.

OR

Let $f: R \to R$ be a function defined by $f(x) = x^3 + 4$, then check whether f is a bijection or not.

6. If $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{bmatrix}$ is a singular matrix, then find the value of x.

7. Find the distance of the plane $3x - 6y + 2z + 11 = 0$ from the origin.

OR

Find the value of λ such that the lines $\frac{x}{1} = \frac{y}{3} = \frac{z}{2\lambda}$ and $\frac{x}{-3} = \frac{y}{5} = \frac{z}{2}$ are perpendicular to each other.

8. If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then find $A^T - B^T$.

9. Let A and B be independent events with $P(A) = 1/4$ and $P(A \cup B) = 2P(B) - P(A)$. Find $P(B)$.

OR

A and B are events such that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.5$. Then find $P(B' \cap A)$.

10. Let $R = \{(3, 1), (1, 3), (3, 3)\}$ be a relation defined on the set $A = \{1, 2, 3\}$. check whether R is symmetric, transitive or reflexive.

11. The position vectors of points A, B, C, D are $\vec{a}$, $\vec{b}$, $2\vec{a}+3\vec{b}$ and $\vec{a}-2\vec{b}$ respectively. Find $\overrightarrow{DB}$ and $\overrightarrow{AC}$.

12. Evaluate : $\int_0^{\pi/4} \tan^3 x \, dx$

13. If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, then find the value of $P(A \mid B)$.

14. Find the order and degree for the differential equation $x\frac{dy}{dx} + 2y = xy\frac{dy}{dx}$.

15. Find the equation of a line passing through a point $(2, -1, 3)$ and parallel to the line $\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} + \hat{j} - 2\hat{k})$.

16. Find the number of equivalence relations on the set $\{1, 2, 3\}$ containing $(1, 3)$ and $(3, 1)$.

Section - II

Case study-based questions are compulsory. Attempt any 4 sub parts from each question. Each sub-part carries 1 mark.

17. An owner of a car rental company have determined that if they charge customers ₹ x per day to rent a car, where $50 \le x \le 200$, then number of cars (n), they rent per day can be shown by linear function $n(x) = 1000 - 5x$. If they charge ₹ 50 per day or less they will rent all their cars. If they charge ₹ 200 or more per day they will not rent any car.

Based on the above information, answer the following question.

(i) Total revenue R as a function of x can be represented as

(a) $1000x - 5x^2$ (b) $1000x + 5x^2$ (c) $1000 - 5x$ (d) $1000 - 5x^2$

(ii) If $R(x)$ denote the revenue, then maximum value of $R(x)$ occur when x equals

(a) 10 (b) 100 (c) 1000 (d) 50

(iii) At $x = 220$, the revenue collected by the company is

(a) ₹ 10 (b) ₹ 500 (c) ₹ 0 (d) ₹ 1000

(iv) The number of cars rented per day, if $x = 75$ is

(a) 675 (b) 700 (c) 625 (d) 600

(v) Maximum revenue collected by company is

(a) ₹ 40000 (b) ₹ 45000 (c) ₹ 55000 (d) ₹ 50000

18. In a family, on the occasion of Diwali celebration father, mother, daughter and son line up at random for a family photograph.

(i) Find the probability that son is at one end, given that father and mother are in the middle.

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(ii) Find the probability that mother is at left end, given that son and daughter are together.

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 0

(iii) Find the probability that father and mother are in the middle, given that daughter is at right end.

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

(iv) Find the probability that mother and son are standing together, given that father and daughter are standing together.

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

(v) Find the probability that father and mother are on either of the ends, given that daughter is at second position from right end.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{2}{5}$

PART - B

Section - III

19. Find the value of the constant k so that the function f, defined below, is continuous at $x = 0$.

$$f(x)=\begin{cases}\left(\dfrac{1-\cos 4x}{8x^2}\right), & \text{if } x\neq 0\\ k, & \text{if } x=0\end{cases}$$

20. Evaluate : $\displaystyle\int_0^{2\pi}\frac{dx}{e^{\sin x}+1}$

OR

Evaluate : $\int |x|\,dx$

21. If $0 < x < 1$, then $\sqrt{1+x^2}\left[(x\cos[\cot^{-1}x]+\sin[\cot^{-1}x])^2-1\right]^{1/2}$.

22. Show that the function f given by $f(x) = x^3 - 3x^2 + 4x$, $x \in R$ is increasing on R.

23. If $\vec{a} = \hat{i}+\hat{j}+2\hat{k}$ and $\vec{b} = 3\hat{i}+2\hat{j}-\hat{k}$, then find $(\vec{a}+3\vec{b})\cdot(2\vec{a}-\vec{b})$.

OR

If $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\hat{i}+\hat{j}, \vec{c}=\hat{i}$ and $(\vec{a}\times\vec{b})\times\vec{c}=\lambda\vec{a}+\mu\vec{b}$, then find the value of $\lambda + \mu$.

24. Find the area bounded by the curve $x = 3y^2 - 9$ and the line $x = 0$, $y = 0$ and $y = 1$.

25. Two cards are drawn successively, without replacement, from a well-shuffled pack of 52 cards. Find the probability distribution of number of spades.

26. Differentiate $(\tan^{-1}x^{1/3}+\tan^{-1}a^{1/3})$ w.r.t. x.

27. Given $A=\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1}, if exists.

28. If l_1, m_1, n_1 and l_2, m_2, n_2 be the direction cosines of two lines, show that the direction cosines of the line perpendicular to both of them are proportional to $(m_1n_2 - m_2n_1), (n_1l_2 - n_2l_1), (l_1m_2 - l_2m_1)$.

OR

Find the equation of a line passing through the point $(-3, 2, -4)$ and equally inclined to the axes.

Section - IV

29. Determine the value of the constant k so that the function $f(x)=\begin{cases} \dfrac{\sqrt{1+kx}-\sqrt{1-kx}}{x}, & \text{if } -1\le x<0 \\ \dfrac{2x+1}{x-1}, & \text{if } 0\le x\le 1 \end{cases}$

is continuous at $x = 0$.

30. Find the area of the region bounded by the lines $y = |x - 3|$ and the lines $x = 2$, $x = 4$ and x-axis.

31. Let $A = R - \{3\}$, $B = R - \{1\}$ and $f: A \to B$ be defined by $f(x)=\dfrac{x-2}{x-3}$. Then, prove that f is bijective.

OR

Let $A = \{x : -1 \le x \le 1\}$ and $f: A \to A$ is a function defined by $f(x) = x\,|x|$, then check whether f is a bijection or not.

32. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

33. If $\int \dfrac{\tan x}{1+\tan x+\tan^2 x}\,dx = x-\dfrac{K}{\sqrt{A}}\tan^{-1}\left(\dfrac{K\tan x+1}{\sqrt{A}}\right)+C,$

(where C is a constant of integration), then find the value of ordered pair (K, A).

34. Find the solution of the differential equation $y^2\,dx + (xy + x^2)\,dy = 0$.

OR

Find the particular solution of $\ln\left(\dfrac{dy}{dx}\right)=3x+4y,\ y(0)=0$.

35. Show that $f(x) = |x - 3|$, $\forall\, x \in R$ is continuous but not differentiable at $x = 3$.

Section - V

36. Minimize $z = x + 2y$, subject to $x + 2y \geq 50$, $2x - y \leq 0$, $2x + y \leq 100$, $x, y \geq 0$.

OR

Find the maximum value of $z = 11x + 8y$ subject to $x \leq 4$, $y \leq 6$, $x + y \leq 6$, $x \geq 0$, $y \geq 0$.

37. If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

OR

Find the inverse of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ by using adjoint method, if it exists. Also, find $(\text{adj } A)^2$.

38. Find the equation of the plane passing through the point $2\hat{i} - \hat{k}$ and parallel to the lines $\frac{x}{-3} = \frac{y-2}{4} = z+1$ and $x - 4 = \frac{1-y}{2} = 2z$.

OR

A perpendicular is drawn from the point $P(2, 4, -1)$ to the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$. Find the equation of the perpendicular from P to the given line.

SOLUTIONS

1. We have, $\begin{bmatrix} x-y & 3 \\ 2x-y & 2x+1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 12 & 15 \end{bmatrix}$

$\Rightarrow \ x - y = 5$ ...(i)

and $2x - y = 12$ ...(ii)

Subtracting (i) from (ii), we get $x = 7$

From (i), $y = x - 5 = 7 - 5 = 2$

OR

The matrix $A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ x & -3 & 0 \end{bmatrix}$ is skew-symmetric.

$\therefore \ A' = -A \Rightarrow \begin{bmatrix} 0 & -1 & x \\ 1 & 0 & -3 \\ -2 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & -3 \\ -x & 3 & 0 \end{bmatrix}$

$\Rightarrow \ x = 2$

2. We have, $\overrightarrow{PO} + \overrightarrow{OQ} = \overrightarrow{QO} + \overrightarrow{OR}$

$\Rightarrow \ \overrightarrow{PQ} = \overrightarrow{QR}$ [By triangle law]

Thus, $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ are either parallel or collinear. But, Q is a point common to them.

So, $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ are collinear.

Hence, points P, Q, R are collinear.

3. Let $I = \int \sin^3 x \cos^3 x \, dx$

$\Rightarrow \ I = \frac{1}{8}\int (2\sin x \cos x)^3 dx$

$\Rightarrow \ I = \frac{1}{8}\int \sin^3 2x \, dx \Rightarrow \ I = \frac{1}{8}\int \frac{3\sin 2x - \sin 6x}{4} dx$

$\Rightarrow \ I = \frac{1}{32}\left\{-\frac{3}{2}\cos 2x + \frac{1}{6}\cos 6x\right\} + C$

OR

We have, $I = \int_0^2 (3x^2 + 2x - 1)dx = \left[3\left(\frac{x^3}{3}\right) + 2\left(\frac{x^2}{2}\right) - x\right]_0^2$

$= [x^3 + x^2 - x]_0^2 = (2_3 + 2_2 - 2) - (0_3 + 0_2 - 0) = 10$

4. Since, $\left(\frac{1}{2}, \frac{1}{3}, n\right)$ are the direction cosines of a line

$\therefore \ \left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2 + n^2 = 1 \Rightarrow n^2 = \frac{23}{36} \Rightarrow n = \frac{\pm\sqrt{23}}{6}$

5. For one-one : Consider, $f(x_1) = f(x_2)$

$\Rightarrow \ 4x_1 = 4x_2 \Rightarrow x_1 = x_2$

Hence, f is one-one.

For onto : Let y be any element in N(co-domain), then $f(x) = y \Rightarrow \ 4x = y$

$\Rightarrow \ x = \frac{y}{4}$. But $\forall y \in N, \ \frac{y}{4} \notin N$

Thus, $f(x)$ is not onto.

OR

Given $f(x) = x^3 + 4$. Let $x_1, x_2 \in R$

Now, $f(x_1) = f(x_2) \Rightarrow x_1^3 + 4 = x_2^3 + 4$

$\Rightarrow \ x_1^3 = x_2^3 \Rightarrow x_1 = x_2$

$\therefore \ f(x)$ is one-one. Also it is onto.

Hence it is a bijection.

6. Since the given matrix is singular.

$\therefore \ \begin{vmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{vmatrix} = 0$

$\Rightarrow \ (2 + x)(5 - 2) - 3(-5 - 2x) + 4(1 + x) = 0$

$\Rightarrow \ 13x = -25 \Rightarrow x = -\frac{25}{13}$

7. We have, equation of plane is $3x - 6y + 2z + 11 = 0$. Its distance from origin (0, 0, 0) is

$\left|\frac{3 \times 0 - 6 \times 0 + 2 \times 0 + 11}{\sqrt{3^2 + (-6)^2 + (2)^2}}\right| = \frac{11}{\sqrt{9 + 36 + 4}} = \frac{11}{7}$ units.

OR

Direction ratios of the given lines are $(1, 3, 2\lambda)$ and $(-3, 5, 2)$ respectively. Since, the lines are at right angles, so

$(1) \times (-3) + (3) \times (5) + 2(2\lambda) = 0$

$\Rightarrow \ -3 + 15 + 4\lambda = 0 \Rightarrow \lambda = -3$

8. Given, $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$\Rightarrow \ B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$\therefore \ A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

9. We have, $P(A) = 1/4$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)\,P(B)$ ($\because$ A, B are independent)

$\Rightarrow \ 1/4 + P(B) - (1/4)\,P(B) = 2P(B) - 1/4$ (Given)

$\Rightarrow \ P(B) = 2/5$

OR

Given, $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cup B) = 0.5$.

Clearly, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.4 + 0.3 - 0.5 = 0.2$

Now, $P(B' \cap A) = P(A) - P(A \cap B) = 0.4 - 0.2 = 0.2 = \frac{1}{5}$

10. Given that, $R = \{(3, 1), (1, 3), (3, 3)\}$ be defined on the set $A = \{1, 2, 3\}$.

$\because$ $(2, 2) \notin R$. Therefore, R is not reflexive.

$\because$ $(3, 1) \in R, (1, 3) \in R$. Hence, R is symmetric.

Since, $(1, 3) \in R, (3, 1) \in R$ but $(1, 1) \notin R$

So, R is not transitive.

11. We have, $\overrightarrow{DB}$ = Position vector of B – Position vector of D

$\Rightarrow \overrightarrow{DB} = \vec{b} - (\vec{a} - 2\vec{b}) = 3\vec{b} - \vec{a}$

Similarly, $\overrightarrow{AC} = (2\vec{a} + 3\vec{b}) - \vec{a} = \vec{a} + 3\vec{b}$

12. Let $I = \int_0^{\pi/4} \tan^3 x\,dx = \int_0^{\pi/4} \sec^2 x \tan x\,dx - \int_0^{\pi/4} \tan x\,dx$

Put $\tan x = t$ in first integral $\Rightarrow \sec^2 x\,dx = dt$

$\therefore \quad I = \int_0^1 t\,dt - \int_0^{\pi/4} \tan x\,dx = \left[\frac{t^2}{2}\right]_0^1 - \left[\log|\sec x|\right]_0^{\pi/4}$

$= \left(\frac{1}{2} - 0\right) - \log\left|\sec\frac{\pi}{4}\right| + \log|\sec 0| = \frac{1}{2}(1 - \log 2)$

13. We have, $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{13}}{\frac{9}{13}} = \frac{4}{9}$

14. Given, $(1 - y)x\frac{dy}{dx} + 2y = 0$

Order and degree for the above equation are 1 and 1 respectively.

15. The given line is parallel to the vector $2\hat{i} + \hat{j} - 2\hat{k}$ and the required line is parallel to the given line. So, required line is parallel to the vector $2\hat{i} + \hat{j} - 2\hat{k}$. Thus, the equation of the required line passing through $(2, -1, 3)$ is

$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \mu(2\hat{i} + \hat{j} - 2\hat{k})$

16. Equivalence relations on the set $\{1, 2, 3\}$ containing $(1, 3)$ and $(3, 1)$ are

$A_1 = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$

$A_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 1), (3, 2), (1, 3)\}$

So, only two equivalence relations exist.

17. (i) (a) : Let x be the price charge per car per day and n be the number of cars rented per day.

$R(x) = n \times x = (1000 - 5x)\,x = -5x^2 + 1000x$

(ii) (b) : We have, $R(x) = 1000x - 5x^2$

$\Rightarrow R'(x) = 1000 - 10x$

For $R(x)$ to be maximum or minimum, $R'(x) = 0$

$\Rightarrow -10x + 1000 = 0 \Rightarrow x = 100$

Also, $R''(x) = -10 < 0$

Thus, $R(x)$ is maximum at $x = 100$

(iii) (c) : If company charge ₹ 200 or more, they will not rent any car. Then revenue collected by him will be zero.

(iv) (c) : If $x = 75$, number of cars rented per day is given by

$n = 1000 - 5 \times 75 = 625$

(v) (d) : At $x = 100$, $R(x)$ is maximum.

Maximum revenue $= R(100) = -5(100)^2 + 1000(100)$

$=$ ₹ 50,000

18. Sample space is given by

{$MFSD$, $MFDS$, $MSFD$, $MSDF$, $MDFS$, $MDSF$, $FMSD$, $FMDS$, $FSMD$, $FSDM$, $FDMS$, $FDSM$, $SFMD$, $SFDM$, $SMFD$, $SMDF$, $SDMF$, $SDFM$ $DFMS$, $DFSM$, $DMSF$, $DMFS$, $DSMF$, $DSFM$}

$\therefore \quad n(s) = 24$

(i) (a) : Let A denotes the event that son is at one end.

$\therefore \quad n(A) = 12$

And B denotes the event that father and mother are in middle.

$\therefore \quad n(B) = 4$

Also, $n(A \cap B) = 4$

$\therefore \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{4/24}{4/24} = 1$

(ii) (b) : Let A denotes the event that mother is at left end.

$\therefore \quad n(A) = 6$

And B denotes the event that son and daughter are together.

$\therefore \quad n(B) = 12$

Also, $n(A \cap B) = 4$

$\therefore \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{4/24}{12/24} = \frac{1}{3}$

(iii) (c) : Let A denotes the event that father and mother are in middle.

$\therefore \quad n(A) = 4$

And B denotes the event that daughter is at right end.

$\therefore \quad n(B) = 6$

Also, $n(A \cap B) = 2$

$\therefore \quad P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2/24}{6/24} = \frac{1}{3}$

(iv) (d) : Let A denotes the event that mother and son are standing together.

$\therefore\ n(A) = 12$

And B denotes the event that father and daughter are standing together.

$\therefore\ n(B) = 12$

Also, $n(A \cap B) = 8$

$$\therefore\quad P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{8/24}{12/24} = \frac{2}{3}$$

(v) (a) : Let A denotes the event that father and mother are on other end.

$\therefore\ n(A) = 4$

And B denotes the event that daughter is at second position from right end.

$\therefore\ n(B) = 6$

Also, $n(A \cap B) = 2$

$$\therefore\quad P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{2/24}{6/24} = \frac{1}{3}$$

19. We have, $f(x)$ is continuous at $x = 0$.

Now, $f(0) = k$

and $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{1-\cos 4x}{8x^2}$

$$= \lim_{x\to 0} \frac{2\cdot\sin^2 2x}{8x^2} = \lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)^2 = 1$$

$\because\ f$ is continuous at $x = 0$.

$\therefore\ f(0) = \lim\limits_{x\to 0} f(x) \Rightarrow k = 1$

20. Let $I = \displaystyle\int_0^{2\pi} \frac{dx}{e^{\sin x}+1}$ ...(i)

$$\Rightarrow\ I = \int_0^{2\pi} \frac{dx}{e^{\sin(2\pi - x)}+1}$$

$$\Rightarrow\ I = \int_0^{2\pi} \frac{dx}{e^{-\sin x}+1} \Rightarrow I = \int_0^{2\pi} \frac{e^{\sin x}}{e^{\sin x}+1}\,dx \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{2\pi} 1\cdot dx = 2\pi \Rightarrow I = \pi$$

OR

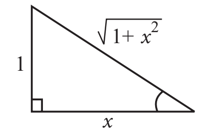
Let $I = \int |x|\cdot 1\,dx$

$$= |x|x - \int \frac{|x|}{x} x\,dx + K = x|x| - \int |x|\,dx + K$$

$\Rightarrow\ I = x|x| - I + K \Rightarrow 2I = x|x| + K$

$$\Rightarrow\ I = \frac{x|x|}{2} + C \quad \left[\text{where } \frac{K}{2} = C\right]$$

21. $\cos(\cot^{-1} x) = \dfrac{x}{\sqrt{1+x^2}}$



$\sin(\cot^{-1} x) = \dfrac{1}{\sqrt{1+x^2}}$

The given expression becomes

$$\sqrt{1+x^2}\left[\left(\frac{x^2+1}{\sqrt{1+x^2}}\right)^2 - 1\right]^{1/2} = x\sqrt{1+x^2}.$$

22. Here $f(x) = x^3 - 3x^2 + 4x$

$\Rightarrow\ f'(x) = 3x^2 - 6x + 4 = 3(x^2 - 2x) + 4$

$= 3(x^2 - 2x + 1) - 3 + 4$

$= 3(x-1)^2 + 1 > 0\ \forall x \in R$

$\Rightarrow\ f$ is increasing on R.

23. Here, $\vec{a} + 3\vec{b} = \hat{i} + \hat{j} + 2\hat{k} + 3(3\hat{i} + 2\hat{j} - \hat{k})$

$= 10\hat{i} + 7\hat{j} - \hat{k}$

and $2\vec{a} - \vec{b} = 2(\hat{i} + \hat{j} + 2\hat{k}) - (3\hat{i} + 2\hat{j} - \hat{k}) = -\hat{i} + 5\hat{k}$

$\therefore\ (\vec{a} + 3\vec{b})\cdot(2\vec{a} - \vec{b}) = (10\hat{i} + 7\hat{j} - \hat{k})\cdot(-\hat{i} + 5\hat{k})$

$= 10 \times (-1) + 7 \times 0 + (-1) \times 5 = -15$

OR

We have, $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -\hat{i} + \hat{j}$

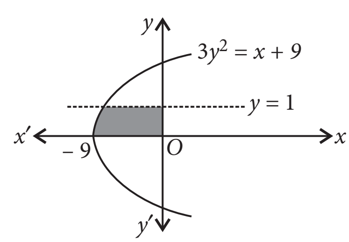
Now, $(\vec{a}\times\vec{b})\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$

$\Rightarrow\ \lambda\vec{a} + \mu\vec{b} = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(0-1)$

$\Rightarrow\ (\lambda+\mu)\hat{i} + (\lambda+\mu)\hat{j} + (\lambda)\hat{k} = -\hat{k}$

On comparing, we get $\lambda = -1$ and $\lambda + \mu = 0$

24. We have, $x = 3y^2 - 9 \Rightarrow 3y^2 = x + 9$



Required area = area of shaded region

$$= \left|\int_0^1 (3y^2 - 9)\,dy\right| = \left|y^3 - 9y\right|_0^1$$

$= |1 - 9| = 8$ sq. units

25. Let the random variable X be defined as the number of spades in a draw of 2 cards successively without replacement, then X can take values 0, 1, 2.

$P(X = 0) = P$(drawing no spade cards)

$$= \frac{^{39}C_2}{^{52}C_2} = \frac{19}{34}$$

$P(X = 1) = P$(drawing one spade and one non-spade card)

$$= \frac{^{13}C_1 \times {^{39}C_1}}{^{52}C_2} = \frac{13}{34}$$

$P(X = 2) = P$(drawing both spade cards)

$$= \frac{^{13}C_2}{^{52}C_2} = \frac{1}{17}$$

$\therefore$ The probability distribution of number of spades is

X	0	1	2
$P(X)$	$\frac{19}{34}$	$\frac{13}{34}$	$\frac{1}{17}$

26. Let $y = \tan^{-1}x^{1/3} + \tan^{-1}a^{1/3}$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{1+(x^{1/3})^2}\left(\frac{1}{3}x^{\frac{1}{3}-1}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1+x^{2/3}}\left(\frac{1}{3x^{2/3}}\right) = \frac{1}{3x^{2/3}\,(1+x^{2/3})}$$

27. We have, $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$$\therefore \; |A| = \begin{vmatrix} 2 & -3 \\ -4 & 7 \end{vmatrix} = 14 - 12 = 2 \neq 0$$

So, A is a non-singular matrix and therefore it is invertible.

$$\therefore \; \text{adj } A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

Hence, $A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

28. Let l, m, n be the direction cosines of the line perpendicular to each of the given lines. Then,

$ll_1 + mm_1 + nn_1 = 0$ ...(i)

and $ll_2 + mm_2 + nn_2 = 0$ ...(ii)

On solving (i) and (ii), we get

$$\frac{l}{m_1n_2 - m_2n_1} = \frac{m}{n_1l_2 - n_2l_1} = \frac{n}{l_1m_2 - l_2m_1}$$

Hence, the direction cosines of the line perpendicular to the given lines are proportional to $(m_1n_2 - m_2n_1)$, $(n_1l_2 - n_2l_1)$, $(l_1m_2 - l_2m_1)$.

OR

Since, the line is equally inclined to the axes.

$\therefore \; l = m = n$ ...(i)

The required equation of line is

$$\frac{x+3}{l} = \frac{y-2}{l} = \frac{z+4}{l} \qquad \text{[using (i)]}$$

$$\Rightarrow \frac{x+3}{1} = \frac{y-2}{1} = \frac{z+4}{1}$$

$\Rightarrow x + 3 = y - 2 = z + 4$

29. We have, L.H.L. (at $x = 0$)

$$= \lim_{x\to 0^-} f(x) = \lim_{x\to 0}\frac{\sqrt{1+kx}-\sqrt{1-kx}}{x}$$

$$= \lim_{x\to 0}\frac{\left(\sqrt{1+kx}-\sqrt{1-kx}\right)}{x}\frac{\left(\sqrt{1+kx}+\sqrt{1-kx}\right)}{\left(\sqrt{1+kx}+\sqrt{1-kx}\right)}$$

$$= \lim_{x\to 0}\frac{1+kx-1+kx}{x\left(\sqrt{1+kx}+\sqrt{1-kx}\right)}$$

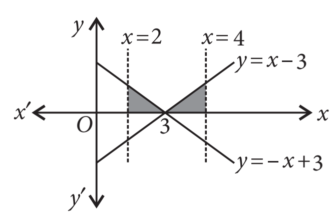
$$= \lim_{x\to 0}\frac{2k}{\sqrt{1+kx}+\sqrt{1-kx}} = \frac{2k}{\sqrt{1}+\sqrt{1}} = \frac{2k}{2} = k$$

R.H.L. (at $x = 0$)

$$= \lim_{x\to 0^+} f(x) = \lim_{x\to 0}\frac{2x+1}{x-1} = -1 \text{ and } f(0) = -1$$

Since $f(x)$ is continuous at $x = 0$. $\therefore \; k = -1$.

30. We have, $y = \begin{cases} x-3, & \forall\, x \geq 3 \\ -x+3, & \forall\, x < 3 \end{cases}$



$$\therefore \; \text{Required area} = \int_2^3 -(x-3)\,dx + \int_3^4 (x-3)\,dx$$

$$= \left[3x - \frac{x^2}{2}\right]_2^3 + \left[\frac{x^2}{2} - 3x\right]_3^4 = \frac{1}{2} + \frac{1}{2} = 1 \text{ sq. unit}$$

31. Let x and y be two arbitrary elements in A.

Then, $f(x) = f(y) \Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$

$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$

$\Rightarrow x = y, \forall\, x, y \in A$

So, f is an injective mapping.

Again, let y be an arbitrary element in B, then $f(x) = y$

$$\Rightarrow \frac{x-2}{x-3} = y \Rightarrow x = \frac{3y-2}{y-1}$$

Clearly, $\forall\ y \in B,\ x = \dfrac{3y-2}{y-1} \in A$, thus for all $y \in B$, there exists $x \in A$ such that

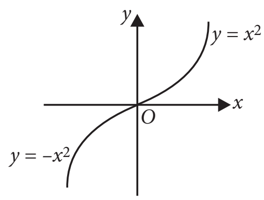
$$f(x) = f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3} = y$$

Thus, every element in the co-domain B has its pre-image in A, so f is a surjective. Hence, $f: A \to B$ is bijective.

OR

$$f(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$$

The graph shows $f(x)$ is one-one, as any straight line parallel to x-axis cuts only at one point.



Here, range of $f(x) \in [-1, 1]$.
Thus range = co-domain.
Hence, $f(x)$ is onto.
Therefore $f(x)$ is one-one and onto, *i.e*, bijective.

32. Let dimensions of the rectangle be x and y (as shown).

$\therefore$ Perimeter of window,

$P = 2y + x + \pi x/2 = 10 \Rightarrow \quad y = 5 - \dfrac{x}{2} - \dfrac{\pi x}{4}$ ...(i)

Area of window, $A = xy + \dfrac{1}{2}\pi\dfrac{x^2}{4}$

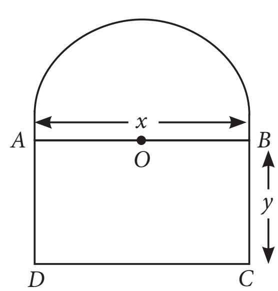
$$\Rightarrow A = x\left[5 - \frac{x}{2} - \frac{\pi x}{4}\right] + \frac{1}{2}\pi\frac{x^2}{4}$$

$$= 5x - \frac{x^2}{2} - \frac{\pi x^2}{8}$$

$$\therefore \frac{dA}{dx} = 5 - x - \frac{\pi x}{4}$$

Now, $\dfrac{dA}{dx} = 0 \Rightarrow x = \dfrac{20}{4+\pi}$

$$\frac{d^2A}{dx^2} = -\left(1 + \frac{\pi}{4}\right) < 0$$



Thus, A is maximum for $x = \dfrac{20}{4+\pi}$

From (i), $y = \dfrac{10}{4+\pi}$ m

So, $x = \dfrac{20}{4+\pi}$ m, $y = \dfrac{10}{4+\pi}$ m will give maximum light.

33. We have, $\displaystyle\int \frac{\tan x}{1+\tan x+\tan^2 x}dx$

$$= \int \frac{1+\tan x - 1}{1+\tan x+\tan^2 x}dx = \int \frac{1+\tan x+\tan^2 x - \sec^2 x}{1+\tan x+\tan^2 x}dx$$

$$= \int\left(1 - \frac{\sec^2 x}{1+\tan x+\tan^2 x}\right)dx = x - \int \frac{\sec^2 x}{1+\tan x+\tan^2 x}dx$$

$$= x - \int \frac{1}{1+t+t^2}dt \text{ (Putting } \tan x = t \Rightarrow \sec^2 x dx = dt)$$

$$= x - \int \frac{dt}{\left(t+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = x - \frac{1}{\sqrt{3}/2}\tan^{-1}\left(\frac{t+\frac{1}{2}}{\sqrt{3}/2}\right) + C$$

$$= x - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x + 1}{\sqrt{3}}\right) + C$$

$\therefore$ $K = 2$, $A = 3$

34. We have, $y^2 dx + (xy + x^2)dy = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{-y^2}{xy+x^2}$

Put $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$$\therefore\ v + x\frac{dv}{dx} = \frac{-v^2x^2}{vx^2 + x^2} \Rightarrow x\frac{dv}{dx} = \frac{-v^2}{v+1} - v$$

$$\Rightarrow\ x\frac{dv}{dx} = -\left(\frac{2v^2+v}{v+1}\right) \Rightarrow \int\left(\frac{v+1}{v(2v+1)}\right)dv = -\int\frac{dx}{x}$$

$$\Rightarrow\ \int\left(\frac{1}{v} - \frac{1}{2v+1}\right)dv = -\log x + \log c$$

$$\Rightarrow\ \log v - \frac{1}{2}\log|2v+1| + \log x = \log c$$

$$\Rightarrow\ \log\left|\frac{v^2x^2}{2v+1}\right| = \log c^2 \Rightarrow \frac{v^2x^2}{2v+1} = c^2$$

$$\Rightarrow\ y^2 = c^2\left(\frac{2y}{x}+1\right) \Rightarrow xy^2 = c^2(x+2y)$$

OR

We have, $\ln\left(\dfrac{dy}{dx}\right) = 3x + 4y$

$$\Rightarrow\ \frac{dy}{dx} = e^{3x}\cdot e^{4y} \Rightarrow e^{-4y}dy = e^{3x}dx$$

On integration, we get

$$-\frac{1}{4}e^{-4y} = \frac{e^{3x}}{3} + C$$

At $x = 0$, $y = 0$; we have

$$-\frac{1}{4} = \frac{1}{3} + C \Rightarrow C = -\frac{7}{12}$$

$\therefore$ Solution is $\dfrac{e^{-4y}}{4}+\dfrac{e^{3x}}{3}=\dfrac{7}{12} \Rightarrow 3e^{-4y}+4e^{3x}=7$

35. We have, $f(x)=\begin{cases}-(x-3), & \text{if } x<3\\ x-3, & \text{if } x\ge 3\end{cases}$

Test for continuity :

$$\text{L.H.L.} = \lim_{x\to 3^-} f(x) = \lim_{x\to 3} -(x-3) = -(3-3)=0$$

$$\text{R.H.L.} = \lim_{x\to 3^+} f(x) = \lim_{x\to 3} (x-3) = 3-3=0$$

Also, $f(3) = 3 - 3 = 0$

$\therefore$ L.H.L. = R.H.L. = $f(3)$

Hence, $f(x)$ is continuous at $x = 3$.

Test for differentiability :

$$Lf'(3)=\lim_{h\to 0}\frac{f(3-h)-f(3)}{-h}$$

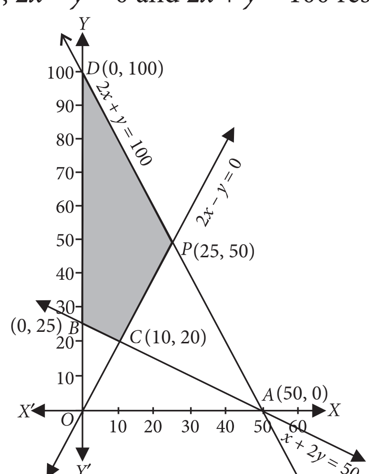
$$=\lim_{h\to 0}\frac{-(3-h-3)-0}{-h}=-1$$

$$Rf'(3)=\lim_{h\to 0}\frac{f(3+h)-f(3)}{h}=\lim_{h\to 0}\frac{(3+h-3)-0}{h}=1$$

Thus, $Lf'(3) \ne Rf'(3)$

Hence, $f(x)$ is not differentiable at $x = 3$.

36. First we draw the lines whose equations are $x + 2y = 50$, $2x - y = 0$ and $2x + y = 100$ respectively.



The feasible region is $BCPDB$ which is shaded in the figure.

The vertices of the feasible region are $B(0, 25)$, $C(10, 20)$, $P(25, 50)$ and $D(0, 100)$.

The values of the objective function $z = x + 2y$ at these vertices are given below.

Corner points	Value of $z = x + 2y$
$B(0, 25)$	50 (minimum)
$C(10, 20)$	50 (minimum)
$P(25, 50)$	125
$D(0, 100)$	200

$\therefore$ z has minimum value 50 at two consecutive vertices B and C.

$\therefore$ z has minimum value 50 at every point of segment joining the points $B(0, 25)$ and $C(10, 20)$.

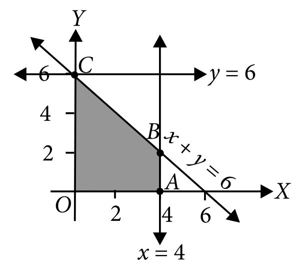
Hence, there are infinite number of optimal solutions.

OR

Convert the inequations into equations and draw the corresponding lines.

$x + y = 6$, $x = 4$, $y = 6$

As $x, y \ge 0$, the solution lies in the first quadrant.



We have seen that O, A, B, C are the corner points. Hence maximum value of the objective function z will occur at one of the corner points.

B is the point of intersection of the lines $x + y = 6$ and $x = 4$ *i.e.*, $B(4, 2)$

We have points $A(4, 0)$, $B(4, 2)$ and $C(0, 6)$

Now, $z = 11x + 8y$

$\therefore$ $z(A) = 11(4) + 8(0) = 44$

$z(B) = 11(4) + 8(2) = 60$

$z(C) = 11(0) + 8(6) = 48$

$z(O) = 11(0) + 8(0) = 0$

$\therefore$ z has maximum value 60 at $B(4, 2)$.

37. We have, $|A|=\begin{vmatrix}1 & \tan x\\ -\tan x & 1\end{vmatrix}=1+\tan^2 x\ne 0$

So, A is invertible.

$$\therefore \text{adj}A=\begin{bmatrix}1 & \tan x\\ -\tan x & 1\end{bmatrix}^T=\begin{bmatrix}1 & -\tan x\\ \tan x & 1\end{bmatrix}$$

Now, $A^{-1}=\dfrac{1}{|A|}\text{adj}A$

$$\Rightarrow A^{-1}=\begin{bmatrix}\dfrac{1}{1+\tan^2 x} & \dfrac{-\tan x}{1+\tan^2 x}\\ \dfrac{\tan x}{1+\tan^2 x} & \dfrac{1}{1+\tan^2 x}\end{bmatrix}$$

$$\therefore A^T A^{-1}=\begin{bmatrix}1 & -\tan x\\ \tan x & 1\end{bmatrix}\begin{bmatrix}\dfrac{1}{1+\tan^2 x} & \dfrac{-\tan x}{1+\tan^2 x}\\ \dfrac{\tan x}{1+\tan^2 x} & \dfrac{1}{1+\tan^2 x}\end{bmatrix}$$

$$\Rightarrow \quad A^T A^{-1} = \begin{bmatrix} \frac{1-\tan^2 x}{1+\tan^2 x} & \frac{-2\tan x}{1+\tan^2 x} \\ \frac{2\tan x}{1+\tan^2 x} & \frac{1-\tan^2 x}{1+\tan^2 x} \end{bmatrix}$$

$$\Rightarrow \quad A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}.$$

OR

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$|A| = 2(-1-0) - 0(0-2) + 1(0+1) = -2 + 1 = -1 \neq 0$

So, A^{-1} exists.

$$\therefore \quad \text{adj } A = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -4 \\ 1 & 0 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = -1\begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -4 \\ 1 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -2 & -1 & 4 \\ -1 & 0 & 2 \end{bmatrix}$$

$$\text{Now, } (\text{adj } A)^2 = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -4 \\ 1 & 0 & -2 \end{bmatrix}\begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -4 \\ 1 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & -3 \\ -4 & 1 & 6 \\ -3 & 0 & 5 \end{bmatrix}$$

38. The required plane passes through the point with position vector $\vec{a} = 2\hat{i} - \hat{k}$ *i.e.*, the point (2, 0, –1) and is parallel to the lines $\frac{x}{-3} = \frac{y-2}{4} = \frac{z+1}{1}$ and $\frac{x-4}{1} = \frac{y-1}{-2} = \frac{z}{1/2}$

i.e. parallel to the lines whose direction ratios are

–3, 4, 1 and 1, –2, $\frac{1}{2}$ *i.e.*, –3, 4, 1 and 2, –4, 1

$\therefore$ The equation of the required plane is

$(\vec{r} - \vec{a}) \cdot (\vec{b} \times \vec{c}) = 0$.

$$\Rightarrow \begin{vmatrix} x-2 & y-0 & z-(-1) \\ -3 & 4 & 1 \\ 2 & -4 & 1 \end{vmatrix} = 0$$

$\Rightarrow (x-2)(4+4) - y(-3-2) + (z+1)(12-8) = 0$

$\Rightarrow 8(x-2) + 5y + 4(z+1) = 0$

$\Rightarrow 8x + 5y + 4z - 12 = 0.$

Its vector equation is $\vec{r} \cdot (8\hat{i} + 5\hat{j} + 4\hat{k}) - 12 = 0$

OR

Let M be the foot of the perpendicular drawn from the point $P(2, 4, -1)$ to the given line.

The coordinates of any point on the line

$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ are $M(\lambda - 5, 4\lambda - 3, -9\lambda + 6)$

Direction ratios of PM are

$\lambda - 7, 4\lambda - 7, -9\lambda + 7$

The direction ratios of the given line are 1, 4, –9

Since PM is perpendicular to the given line.

$\therefore \quad 1(\lambda - 7) + 4(4\lambda - 7) - 9(-9\lambda + 7) = 0$

$\Rightarrow 98\lambda - 98 = 0 \Rightarrow \lambda = 1$

Putting $\lambda = 1$, we have

$M \equiv (-4, 1, -3)$

Now, equation of PM = equation of the perpendicular from P to the given line

i.e., $\frac{x-2}{-4-2} = \frac{y-4}{1-4} = \frac{z+1}{-3+1}$

i.e., $\frac{x-2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$

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