Work, Energy and Power

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. In a giant vacuum chamber heavy and light objects are dropped at the same time to see which will hit the ground faster. Though some objects, like feathers, seem to fall slower because of air resistance. The bowling ball and feathers are released, falling gracefully to the ground with neither accelerating farther than the other. The objects both stay in unison as they descend at the exact same time. Original experiments made by Galileo and Sir Isaac Newton lead to the theory of gravitation, though it also became the foundation of Albert Einstein's theory of relativity.

(A) A light and a heavy body have equal momenta. Which one has greater kinetic energy?

(B) What is the final velocity of the ball if it is dropped from a certain height and takes 10 s to reach the ground. The air resistance is not taken into account? [take $g = 10\ m/s^2$]

(C) The potential energy of a freely falling object decreases progressively. Does this violate the law of conservation? Why?

Ans. (A) Since, momenta of heavy body and light body are same, thus velocity of light body is greater than that of heavy body. Also, kinetic energy is proportional to the square of velocity. Hence, lighter body has greater kinetic energy.

(B) Applying first equation of motion under free fall,

$u = 0$ m/s

$a = g = 10$ m/s^2

$t = 10$ s

As we know that:

$v = u + at$

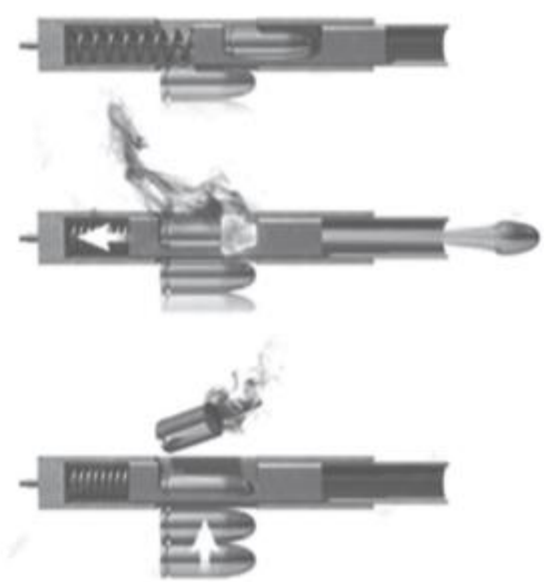
$v = gt$

$v = 10 \times 10$

$v = 100$ m/s

(C) No, the process does not violate the law of conservation of energy because when the body falls from a height, its potential energy changes into kinetic energy progressively. A decrease in the potential energy is equal to an increase in the kinetic energy of the body. During the process, the total mechanical energy of the body remains conserved.

2. Kinetic energy depends upon the frame of reference and is always positive. A bullet fired from a gun has very high kinetic energy and so, it can easily penetrate any object. This is because of the large amount of velocity possessed by the bullet. Although the mass of the bullet is less, its high speed renders double the amount of kinetic energy.



(A) A bullet of mass 50 g is moving with a velocity of 500 ms-^{1}. It penetrates 10 cm into a still target and comes to rest. Kinetic energy possessed by the bullet is:

(a) 6250 joule

(b) 7000 joule

(c) 6500 joule

(d) 7250 joule

(B) Using the values in the above question, the average retarding force offered by the target will be:

(a) 60250 N

(b) 62500 N

(c) 60000 N

(d) 70000 N

(C) A bullet of mass 10 g moving with a velocity of 100 m/s hits a wooden log and penetrates it up to a thickness of 5 cm. The resistance force of log is:

(a) 200 N

(b) 500 N

(c) 1000 N

(d) 600 N

(D) A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest, assuming that it faces constant resistance to motion?

(a) 0.5 cm

(b) 10 cm

(c) 5 cm

(d) 1 cm

(E) A bullet of mass 10 g is fired horizontally with a velocity 1000 ms-1 from a rifle situated at a height 50 m above the ground. If the bullet reaches the ground with a velocity 500 m/s, the work done against air resistance (magnitude) in the trajectory of the bullet in Joule ($g = 10\ m/s^2$) is:

(a) 5005 J

(b) 3755 J

(c) 3750 J

(d) 17.5 J

Ans. (A) (a) 6250 joule

Explanation: $K.E. = \frac{1}{2} \times \frac{50}{1000} \times (500)^2$

$= \frac{25000}{4}$ Joule

$= 6250$ Joul

(B) (b) 62500 N

Explanation: $v^2 = u^2 + 2as$

$$0 = (500)^2 + 2a\ \frac{10}{100}$$

$$a = -12{,}50000\ m/s^2$$

Average retarding force

$$F_{av} = ma$$

$$= \frac{50}{1000} \times 12{,}50000$$

$$F_{av} = 62500\ N$$

(C) (c) 1000 N

Explanation: Since finally, the bullet stops. Therefore, loss in kinetic energy Initial kinetic energy Final kinetic energy Let F be the average resisting force,

m = 10 g or 0.01 kg

s = 5 cm = 0.05 m

(D) (d) 1 cm

Explanation: Given that: s = 3 cm

$$2as = \left[\frac{u}{2}\right]^2 - u^2$$

$$2.a.3 = \left[\frac{u}{2}\right]^2 - u^2$$

$$\Rightarrow \quad \frac{-3u^2}{4} = 2.a.3$$

$$a = \frac{-u^2}{8}$$

$$0 - \left[\frac{u}{2}\right]^2 = 2.a.x$$

$$\frac{-u^2}{4} = 2\left[\frac{-u^2}{8}\right] \times x$$

$$x = 1\ cm$$

(E) (b) 3755 J

Explanation: Work done against air resis- tance is the energy tends to, Given that:

m = 10 g

u = 1000 m/s

v = 500 m/s

$g = 10\ m/s^2$

which is $W = (mgh + \frac{1}{2}\ mu^2) - \frac{1}{2}\ mv^2$

$$= \frac{1}{2}\ m\ (u^2 - v^2) + mgh$$

$$= 10^{-2}\ (\frac{1}{2}\ (1000^2 - 500^2) + 10 \times 50)$$

$$= 3755\ J$$

3. In billiards if the cue ball strikes a stationary billiard ball straight on, then the cue ball will stop moving after the collision. It will have transferred all of its kinetic energy to the other ball, which will move forward with the same velocity that the cue ball had before the collision. Collisions can only be elastic if the masses are equal. The masses of billiard balls are the same, which can make some collisions close to elastic.

(A) Two spheres A and B of masses m_1 and m_2 respectively collide. A is at rest initially and B is moving with velocity v along

x-axis. After collision B has a velocity $\frac{v}{2}$

in a direction perpendicular to the original direction. The mass A moves after collision in the direction:

(a) $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ to the x-axis

(b) $\theta = \tan^{-1}\left(-\frac{1}{2}\right)$ to the y-axis

(c) Same as that of B

(d) Opposite to that of B

(B) Two bodies with kinetic energies in the ratio of 4 : 1 are moving with equal linear momentum. The ratio of their masses is:

(a) 4:1

(b) 1:1

(c) 1:2

(d) 1:4

(C) Two balls of masses m each are moving at right angle to each other with velocities 6 m/s and 8 m/s respectively. If collision between them is perfectly inelastic, the velocity of combined mass is:

(a) 15 m/s

(b) 10 m/s

(c) 5 m/s

(d) 2.5 m/s

(D) Body A of mass 4 m moving with speed u collides with another body B of mass 2 m at rest. The collision is head on and elastic in nature. After the collision the fraction of energy lost by the colliding body A is:

(a) $\frac{1}{9}$ (b) $\frac{8}{9}$

(c) $\frac{4}{9}$ (d) $\frac{5}{9}$

(E) A moving block having mass m, collides with another stationary block having mass 4 m. The lighter block comes to rest after collision. When the initial velocity of the lighter block is v, then the value of coefficient of restitution (e) will be:

(a) 0.5

(b) 0.25

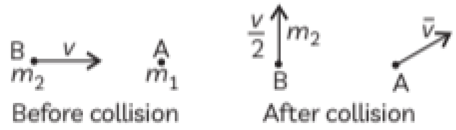
(c) 0.4

(d) 0.8

Ans. (A)

(a) $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ *to the x-axis*

Explanation:



$$m_2 v\hat{i} + 0 = -m_2 \frac{v}{2}\hat{j} + m_1 \bar{v}$$

Using momentum conservation,

$$m_1 \bar{v} = m_2 v\hat{i} + m_2 \frac{v}{2}\hat{j}$$

$$\theta = \tan^{-1}\left(\frac{v}{2v}\right) = \tan^{-1}\left(\frac{1}{2}\right)$$

Angle is from x-axis.

(B) (d) 1:4

Explanation:

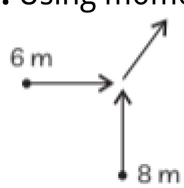
$$\frac{K_1}{K_2} = \frac{P_1^2 2m_2}{2m_1 P_2^2} \qquad (P_1 = P_2 \text{ given})$$

$$\Rightarrow \quad \frac{K_1}{K_2} = \frac{m_2}{m_1} = \frac{4}{1}$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{1}{4}$$

(C) (c) 5 m/s

Explanation: Using momentum conser- vation,



$$m\sqrt{6^2 + 8^2} = 2mv'$$

$$v' = 5 \text{ m/s}$$

(D)

(b) $\frac{8}{9}$

Explanation: Fractional loss of KE of colliding body.

$$\frac{\Delta KE}{KE} = \frac{4(m_1 m_2)}{(m_1+m_2)^2}$$

$$= \frac{4(4m)2m}{(4m+2m)^2}$$

$$= \frac{32m^2}{36m^2} = \frac{8}{9}$$

(E) (b) 0.25

Explanation: According to law of conservation of linear momentum,

Mv+ 4m x 0 = 4m' +0

$$v' = \frac{v}{4}$$

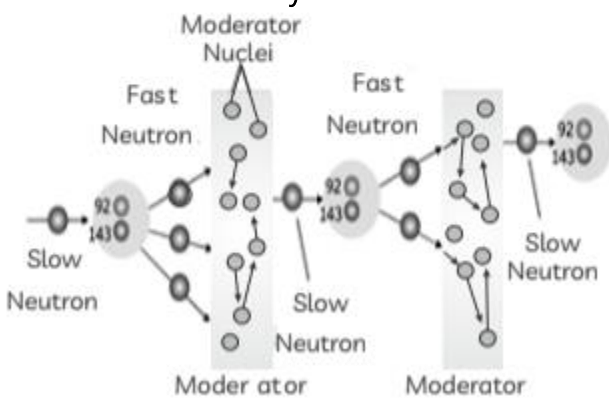
$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of appoach}}$$

$$= \frac{\frac{v}{4}}{v}$$

$$e = \frac{1}{4}$$

$$= 0.25$$

4. Hydrogenic material is used as a moderator in nuclear reactors to slow down the neutrons. As we know that when two bodies of same mass undergo elastic collision, their velocities are interchanged after collision. Water and heavy water are hydrogenic materials containing protons having approximately the same mass as that of a neutron. When fast moving neutrons collide with protons, the neutrons come to rest and protons move with the velocity of neutrons.



(A) A molecule in a gas container hits a horizontal wall with speed and angle 30° with the

normal, and rebounds with the same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic?
(B) If two bodies stick together after collision will the collision be elastic or inelastic?
(C) If two objects collide and one is initially at rest (i) is it possible for both to be at rest after collision? (ii) is it possible for any one to be at rest after collision?

Ans. (A) Yes; Collision is elastic. The momentum of the gas molecule remains conserved whether the collision is elastic or inelastic. The gas molecule moves with a velocity of 200 m/s and strikes the stationary wall of the container, rebounding with the same speed. It shows that the rebound velocity of the wall remains zero. Hence, the total kinetic energy of the molecule remains conserved during the collision. The given collision is an example of an elastic collision.
(B) Inelastic collision
(C) (i) No, because momentum will not be conserved in that case
(ii) Yes, when masses of two objects are equal and collision is perfectly elastic.