#### GRADE: 10

#### Introduction

Euclid was a mathematician who lived around 300 BC in Greece. He is famous for his monumental work: Elements which consists of 13 books dealing in numbers, elementary algebra and geometry. He organized all the mathematical knowledge that had been compiled since the days of Thales (Greek Mathematician) who lived 300 years before Euclid. Euclid's division algorithm is a technique to find the HCF/GCD/GCF of two numbers. This algorithm is one of the oldest algorithms (step by step procedure to solve a particular problem) in the history of Maths.

What is a dividend? Let us understand it with the help of a simple example.

Can you divide 14 by 6?

 $6)14 \\ -\underline{12} \\ \underline{2}$ 

After division, we get 2 as the quotient and 2 as the remainder.

Thus, we can also write 14 as  $6 \times 2 + 2$ . A dividend can thus be written as:

**Dividend = Divisor × Quotient + Remainder** 

# Can you think of any other number which, when multiplied with 6, gives 14 as the dividend and 2 as the remainder?

Let us try it out with some other sets of dividends and divisors.

- (1) Divide 100 by 20:  $100 = 20 \times 5 + 0$
- (2) Divide 117 by 15:  $117 = 15 \times 7 + 12$
- **(3) Divide 67 by 17:** 67 = 17 × 3 + 16

Thus, if we have a dividend and a divisor, then there will be a unique pair of a quotient and a remainder that will fit into the above equation.

This brings us to **Euclid's division lemma.** 

If *a* and *b* are positive integers, then there exist two unique integers, *q* and *r*,

such that a = bq + r

This lemma is very useful for finding the H.C.F. of large numbers where breaking them into factors is difficult. This method is known as **Euclid's Division Algorithm**.

#### SOLVED EXAMPLES

## Example 1: Find the H.C.F. of 4032 and 262 using Euclid's division algorithm.

#### Solution: Step 1:

First, apply Euclid's division lemma on 4032 and 262.

 $4032 = 262 \times 15 + 102$ 

## Step 2:

As the remainder is non-zero, we apply Euclid's division lemma on 262 and 102.

 $262 = 102 \times 2 + 58$ 

## Step 3:

Apply Euclid's division lemma on 102 and 58.

 $102 = 58 \times 1 + 44$ 

## Step 4:

Apply Euclid's division lemma on 58 and 44.

 $58 = 44 \times 1 + 14$ 

# Step 5:

Apply Euclid's division lemma on 44 and 14.

 $44 = 14 \times 3 + 2$ 

## Step 6:

Apply Euclid's division lemma on 14 and 2.

 $14 = 2 \times 7 + 0$ 

In the problem given above, to obtain 0 as the remainder, the divisor has to be taken as 2.

Hence, 2 is the H.C.F. of 4032 and 262.

## Example 2: Find the H.C.F. of 336 and 90 using Euclid's division algorithm.

## Solution:

As 336 > 90, we apply the division lemma to 336 and 90.

 $336 = 90 \times 3 + 66$ 

Applying Euclid's division lemma to 90 and 66:

 $90 = 66 \times 1 + 24$ 

Applying Euclid's division lemma to 66 and 24:

 $66 = 24 \times 2 + 18$ 

Applying Euclid's division lemma to 24 and 18:

 $24 = 18 \times 1 + 6$ 

Applying Euclid's division lemma to 18 and 6:

 $18 = 6 \times 3 + 0$ 

As the remainder is zero, we need not apply Euclid's division lemma anymore. The divisor (6) is the required H.C.F.

#### ASSIGNMENT

#### Q1. Use Euclid's division Lemma to find the HCF of:

- 1. 135 and 225
- 2. 196 and 38220
- 3. 867 and 255

Q2. Find more information on the contribution and life of Euclid from the internet.