Q. 1. In an experimental arrangement of two coils C_1 and C_2 placed coaxially parallel to each other, find out the expression for the emf induced in the coil C_1 (of N1 turns) corresponding to the change of current I_2 in the coil C_2 (of N₂ turns). [CBSE Chennai 2015]

Ans.

Let φ_1 be the flux through coil C_1 (of N_1 turns) when current in coil C_2 is I_2 . Then, we have

$$N_1 \varphi_1 = \mathrm{MI}_2 \qquad ...(i)$$

For current varying with time,

$$\frac{d(N_1 \varphi_1)}{\mathrm{dt}} = \frac{d(\mathrm{MI}_2)}{\mathrm{dt}} \qquad \dots (ii)$$

Since induced *emf* in coil C_1 is given by

$$\varepsilon_{1} = -\frac{d(N_{1} \varphi_{1})}{dt}$$
From (ii), $-\varepsilon_{1} = M\left(\frac{dI_{2}}{dt}\right)$
 $\varepsilon_{1} = -M\frac{dI_{2}}{dt}$ [from (i)]

It shows that varying current in a coil induces emf in the neighbouring coil.

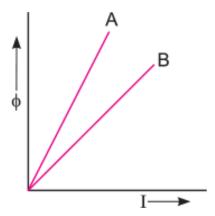
Q. 2. Answer the following questions

(1) How does the mutual inductance of a pair of coils change when

(i) Distance between the coils is increased and

(ii) Number of turns in the coils is increased? [CBSE (AI) 2013]

(2) A plot of magnetic flux (ϕ) versus current (I), is shown in the figure for two inductors A and B. Which of the two has large value of self-inductance? [CBSE Delhi 2010]



(3) How is the mutual inductance of a pair of coils affected when

(i) Separation between the coils is increased?

(ii) The number of turns in each coil is increased?

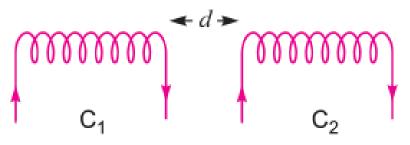
(iii) A thin iron sheet is placed between the two coils, other factors remaining the same?

Justify your answer in each case. [CBSE (AI) 2013]

Ans. (1) (i) Mutual inductance decreases.

(ii) Mutual inductance increases.

Concept: (i) If distance between two coils is increased as shown in figure.



It causes decrease in magnetic flux linked with the coil C_2 . Hence induced emf in coil C_2 decreases by relation Hence mutual inductance decreases.

(ii) From relation $M_{21} = \mu_0 n_1 n_2 AI$, if number of turns in one of the coils or both increases, means mutual inductance will increase.

Q. 3. Define self-inductance of a coil. Show that magnetic energy required to build up the current I in a coil of self-inductance L is given by $\frac{1}{2}$ LI². [CBSE Delhi 2012]

OR

Define the term self-inductance of a solenoid. Obtain the expression for the magnetic energy stored in an inductor of self-inductance L to build up a current I

through it. [CBSE (AI) 2014]

Ans. **Self-inductance** – Using formula φ = LI, if I = 1 **Ampere** then L = φ

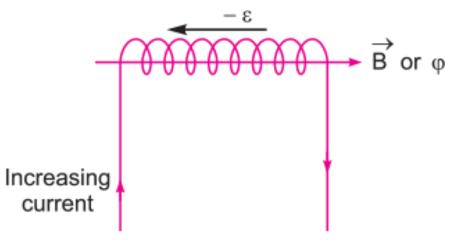
Self-inductance of the coil is equal to the magnitude of the magnetic flux linked with the coil, when a unit current flows through it.

Alternatively

Using formula $|-\varepsilon| = L \frac{\mathrm{dI}}{\mathrm{dt}}$

If
$$\frac{\mathrm{dI}}{\mathrm{dt}} = 1 \; A/s$$
 then $L = \; |-\varepsilon|$

Self-inductance of the coil is equal to the magnitude of induced emf produced in the coil itself, when the current varies at rate 1 A/s.



Expression for magnetic energy

When a time varying current flows through the coil, back emf ($-\varepsilon$) produces, which opposes the growth of the current flow. It means some work needs to be done against induced emf in establishing a current I. This work done will be stored as magnetic potential energy.

For the current I at any instant, the rate of work done is

$$\frac{\mathrm{dW}}{\mathrm{dt}} = (-\varepsilon)I$$

Only for inductive effect of the coil $|-\varepsilon| = L \frac{\mathrm{dI}}{\mathrm{dt}}$

$$\therefore \quad \frac{\mathrm{dW}}{\mathrm{dt}} = L\left(\frac{\mathrm{dI}}{\mathrm{dt}}\right)I \quad \Rightarrow \quad \mathrm{dW} = \mathrm{LI} \ \mathrm{dI}$$

From work-energy theorem

DU = LI dI

$$\therefore \qquad U = \int_{0}^{I} \text{LIdI} = \frac{1}{2} \text{LI}^2$$

Q. 4. Two identical loops, one of copper and the other of aluminium, are rotated with the same angular speed in the same magnetic field. Compare [CBSE (AI) 2010]

(i) The induced emf and

(ii) The current produced in the two coils. Justify your answer.

Ans. (i)

Induced emf,
$$\varepsilon = -\frac{d\varphi}{dt} = -\frac{d}{dt} (BA \cos \omega t)$$

 $= BA \omega \sin \omega t$

As B, A, ω are same for both loops, so induced emf is same in both loops.

(ii)

Current induced,
$$I = \frac{\varepsilon}{R} = \frac{\varepsilon}{\rho l/A} = \frac{\varepsilon A}{\rho l}$$

As area A, length I and emf ϵ are same for both loops but resistivity ρ is less for copper, therefore current I induced is larger in copper loop.

Q. 5. A wheel with 8 metallic spokes each 50 cm long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of the Earth's magnetic field. The Earth's magnetic field at the plane is 0.4 G and the angle of dip is 60°. Calculate the emf induced between the axle and the rim of the wheel.

How will the value of emf be affected if the number of spokes were increased? [CBSE Delhi 2013]

Ans. If a rod of length 'l' rotates with angular speed ω in uniform magnetic field 'B'

$$arepsilon = rac{1}{2} \mathrm{Bl}^2 \, \omega$$

In case of earth's magnetic field $B_{H}=|B_e|\cos \delta$

and $B_V = |B_e| \sin \delta$

$$\therefore \qquad \varepsilon = \frac{1}{2} |B_e| \cos \delta l^2 \omega$$

$$= \frac{1}{2} \times 0.4 \times 10^{-4} \cos 60^o \times (0.5)^2 \times 2\pi \nu$$

$$= \frac{1}{2} \times 0.4 \times 10^{-4} \times \frac{1}{2} \times (0.5)^2 \times 2\pi \times (\frac{120 \text{ rev}}{60s})$$

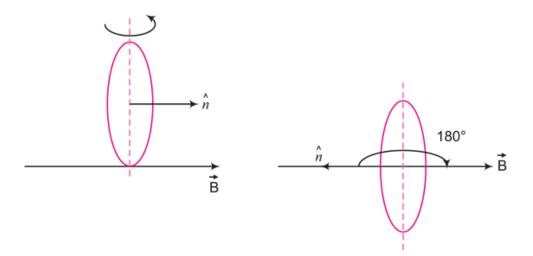
$$= 10^{-5} \times 0.25 \times 2 \times 3.14 \times 2$$

$$= 3.14 \times 10^{-5} \text{ volt}$$

Induced emf is independent of the number of spokes i.e., it remain same.

Q. 6. A circular coil of radius 10 cm, 500 turns and resistance 200Ω is placed with its plane perpendicular to the horizontal component of the Earth's magnetic field. It is rotated about its vertical diameter through 180° in 0.25 s. Estimate the magnitudes of the emf and current induced in the coil. (Horizontal component of the Earth's magnetic field at the place is 3.0×10^{-5} T) [CBSE (F) 2015]

Ans.



Initial magnetic flux, $\phi_i = NBA$

$$= 500 \times 3 \times 10^{-5} \times \pi (0.1)^2$$

$$= 15 \, \pi \times 10^{-5} \, \text{Wb}$$

On turning by 180°.

Final flux, $\varphi_f = -NBA = -15 \ \pi \times 10^{-5} \ Wb$

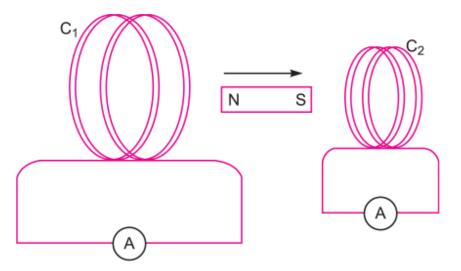
Magnitude of induced emf, $\varepsilon = -\frac{d\varphi}{dt}$

$$=rac{2 imes 15\pi imes 10^{-5}}{0.25}=120\pi imes 10^{-5} ext{ volt}$$

$$=376.8 \times 10^{-5} = 0.038$$
 volt

Induced current,
$$I=rac{arepsilon}{R}=rac{0.038}{200}=19 imes10^{-5}~A$$

Q. 7. A magnet is quickly moved in the direction indicated by an arrow between two coils C_1 and C_2 as shown in the figure. What will be the direction of induced current in each coil as seen from the magnet? Justify your answer. [CBSE Delhi 2011]



Ans. According to Lenz's law, the direction of induced current is such that it opposes the relative motion between coil and magnet.

The near face of coil C_1 will become S-pole, so the direction of current in coil C_1 will be clockwise.

The near face of coil C_2 will also become S-pole to oppose the approach of magnet, so the current in coil C_2 will also be clockwise.

Q. 8. The currents flowing in the two coils of self-inductance $L_1=16$ mH and $L_2=12$ mH are increasing at the same rate. If the power supplied to the two coils are equal, find the ratio of [CBSE (F) 2014]

- (i) Induced voltages
- (ii) The currents and

(iii) The energies stored in the two coils at a given instant.

 $\frac{\varepsilon_1}{\varepsilon_2} = \frac{-L_1 \frac{\mathrm{dI}}{\mathrm{dt}}}{-L_2 \frac{\mathrm{dI}}{\mathrm{dt}}} = \frac{L_1}{L_2} = \frac{16 \,\mathrm{mH}}{12 \,\mathrm{mH}} = \frac{4}{3}$

Ans. (i) Induced voltage (emf) in the coil,

$$arepsilon = -L rac{\mathrm{dI}}{\mathrm{dt}}$$

...

(ii) Power supplied, P= εΙ

Since power is same for both the coils

$$\therefore \ \varepsilon_1 I_1 = \varepsilon_2 I_2 = \frac{I_1}{I_2} = \frac{\varepsilon_2}{\varepsilon_1} = \frac{3}{4}$$

(iii) Energy stored in the coil is given by

$$U = \frac{1}{2} \text{LI}^2$$

$$\therefore \quad \frac{U_1}{U_2} = \frac{\frac{1}{2} L_1 I_1^2}{\frac{1}{2} L_2 I_2^2} \quad = \quad \frac{L_1}{L_2} \times \left(\frac{I_1}{I_2}\right)^2 = \frac{4}{3} \times \left(\frac{3}{4}\right)^2 = \frac{3}{4}$$

Q. 9. Figure shows a rectangular loop conducting PQRS in which the arm PQ is free to move. A uniform magnetic field acts in the direction perpendicular to the plane of the loop. Arm PQ is moved with a velocity v towards the arm RS. Assuming that the arms QR, RS and SP have negligible resistances and the moving arm PQ has the resistance r, obtain the expression for

(i) The current in the loop

(ii) The force and

(iii) The power required to move the arm PQ.

Ans. (i)

Current in the loop PQRS,

$$I = \frac{\varepsilon}{r}$$

Since
$$\varepsilon = \frac{d\varphi}{dt}$$
 Blv So, $I = \frac{\text{Blv}}{r}$

(ii) The force required to keep the arm PQ in

Constant motion

$$F = \mathrm{BI} \; l = B\left(rac{\mathrm{Blv}}{r}
ight) l = rac{B^2 l^2 v}{r}$$

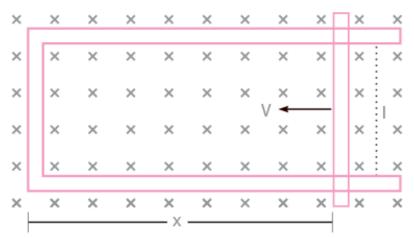
(iii) Power required to move the arm PQ

$$P = F|v| = \left(rac{B^2 l^2 v}{r}
ight)|v| = \left(rac{B^2 l^2 v^2}{r}
ight)$$

Q. 10. Answer the following questions

A rod of length I is moved horizontally with a uniform velocity 'v' in a direction perpendicular to its length through a region in which a uniform magnetic field is

acting vertically downward. Derive the expression for the emf induced across the ends of the rod.



(ii) How does one understand this motional emf by invoking the Lorentz force acting on the free charge carriers of the conductor? Explain. [CBSE (AI) 2014]

Ans. (i) Suppose a rod of length 'l' moves with velocity v inward in the region having uniform magnetic field B.

Initial magnetic flux enclosed in the rectangular space is $\varphi = |B||x|$

As the rod moves with velocity $-v = \frac{dx}{dt}$

Using Lenz's law

$$\varepsilon = -\frac{d\varphi}{dt} = -\frac{d}{dt} \left(\operatorname{Blx} \right) = \operatorname{Bl} \left(-\frac{dx}{dt} \right)$$

 \therefore $\epsilon = Blv$

(ii) Suppose any arbitrary charge 'q' in the conductor of length 'l' moving inward in the field as shown in figure, the charge q also moves with velocity V in the magnetic field B

The Lorentz force on the charge 'q' is F = qvB and its direction is downwards.

So, work done in moving the charge 'q' along the conductor of length I

W = F.I

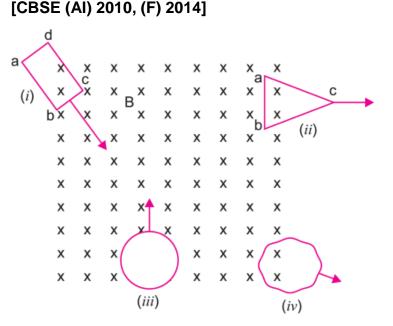
W = qvBI

Since emf is the work done per unit charge

$$\therefore$$
 $\varepsilon = \frac{W}{q} = \mathrm{Blv}$

This equation gives emf induced across the rod.

Q. 11. Figure shows planar loops of different shapes moving out of or into a region of magnetic field which is directed normal to the plane of loops downwards. Determine the direction of induced current in each loop using Lenz's law.



Ans. (a) In Fig. (i) the rectangular loop abcd and in Fig. (iii) circular loop are entering the magnetic field, so the flux linked with them increases; The direction of induced currents in these coils, will be such as to oppose the increase of magnetic flux; hence the magnetic field due to current induced will be upward, i.e., currents induced will flow anticlockwise.

(b) In Fig. (ii), the triangular loop abc and infig. (iv) The zig-zag shaped loop are emerging from the magnetic field, therefore magnetic flux linked with these loops decreases. The currents induced in them will tend to increase the magnetic field in downward direction, so the currents will flow clockwise.

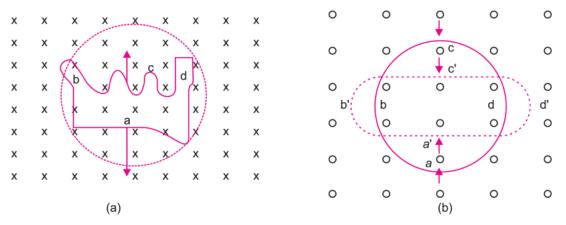
Thus in fig. (i) Current flows anticlockwise,

In fig. (ii) Current flows clockwise,

In fig. (iii) Current flows anticlockwise,

In fig. (iv) Current flows clockwise.

Q. 12. Use Lenz's law to determine the direction of induced current in the situation described by following figs. [CBSE (F) 2014]



(i) A wire of irregular shape turning into a circular shape.

(ii) A circular loop being deformed into a narrow straight wire.

Ans. (i) For the given periphery the area of a circle is maximum. When a coil takes a circular shape, the magnetic flux linked with coil increases, so current induced in the coil will tend to decrease the flux and so will produce a magnetic field upward. As a result the current induced in the coil will flow anticlockwise i.e., along adcb.

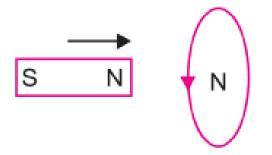
(ii) For given periphery the area of circle is maximum. When circular coil takes the shape of narrow straight wire, the magnetic flux linked with the coil decreases, so current induced in the coil will tend to oppose the decrease in magnetic flux; hence it will produce upward magnetic field, so current induced in the coil will flow anticlockwise i.e., along a' b' c' b'.

Q. 13. Show that Lenz's law is in accordance with the law of conservation of energy.

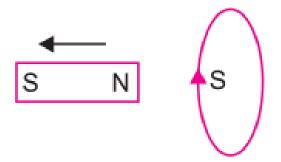
[CBSE (F) 2017]

Ans. Lenz's law: According to this law "the direction of induced current in a closed circuit is always such as to oppose the cause that produces it."

Example: When the north pole of a coil is brought near a closed coil, the direction of current induced in the coil is such as to oppose the approach of North Pole. For this the nearer face of coil behaves as North Pole. This necessitates an anticlockwise current in the coil, when seen from the magnet side [fig. (a)]



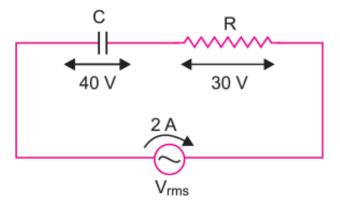
Similarly when North Pole of the magnet is moved away from the coil, the direction of current in the coil will be such as to attract the magnet. For this the nearer face of coil behaves as South Pole. This necessitates a clockwise current in the coil, when seen from the magnet side [fig. (b)].



Conservation of Energy in Lenz's Law: Thus, in each case whenever there is a relative motion between a coil and the magnet, a force begins to act which opposes the relative motion. Therefore to maintain the relative motion, a mechanical work must be done. This work appears in the form of electric energy of coil. Thus Lenz's law is based on principle of conservation of energy.

Short Answer Questions –II (OIQ)

Q. 1. A bar magnet M is dropped so that it falls vertically through the coil C. The graph obtained for voltage produced across the coil versus time is showing in figure (b).



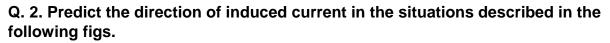
(i) Explain the shape of the graph.

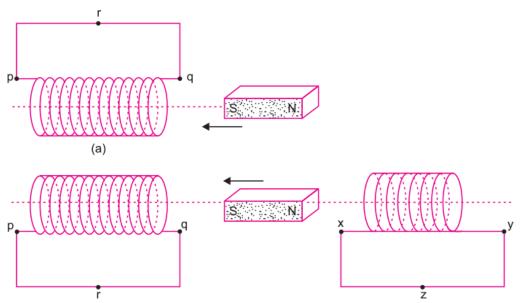
(ii) Why is the negative peak longer than the positive peak?

Ans. (i) When the bar magnet falls through the coil, the magnetic flux linked with the coil changes, so an emf (or pd) is developed across the coil.

Initially, the rate of increase of flux increases, becomes maximum and then it decreases, becomes zero. Now, magnetic flux begins to decrease, the rate of decrease increases becomes maximum and then it decreases and when the magnet is sufficiently far on the other side, the flux becomes zero and so pd induced becomes zero.

(ii) Negative peak is longer than Positive peak because magnet moves out of coil faster than it moves into the coil, so the rate of decrease of magnetic flux is faster than the rate of increase of flux.

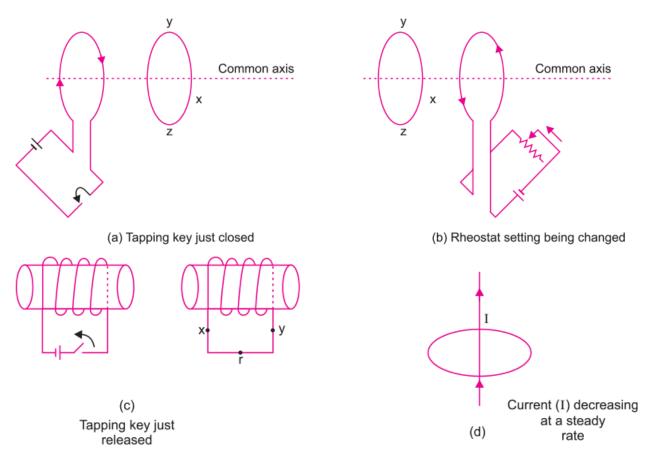




Ans. (a) The direction of current is along qrpq because the current induced in solenoid will oppose the approach of magnet, so from looking on magnet side, the current at nearer face should flow clockwise.

(b) In this case the current induced in coil pq will oppose the approach of magnet while coil xy will oppose the recession of magnet; so nearer faces of coils will act as S-poles. Accordingly the direction of current in coil pq will be along qrp and in coil xy it will be along yzx.

Q. 3. Predict the direction of induced current in the situations described in the following figs.



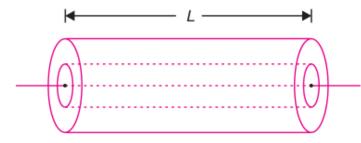
Ans. (a) When the tapping key is just closed, the current produced in the left loop flows clockwise, so magnetic field induced will flow along negative axis; the current induced in right coil will oppose the magnetic field produced, so current in right coil will flow anticlockwise, i.e., direction of current will be along yzx.

(b) The current in coil is anticlockwise. When rheostat setting is being changed, the resistance of the right circuit is decreasing, so current is increasing, the current induced in left loop will oppose the increase of current, so current induced in left coil will flow clockwise i.e., along zyx.

(c) Induced current in the right coil is along xry.

(d) No induced current because magnetic field lines lie in the plane of loop.

Q. 4. Figure shows two long coaxial solenoids, each of length 'L'. The outer solenoid has an area of cross-section A1 and number of turns/length n1 The corresponding values for the inner solenoid are A2 and n2 Write the expression for self-inductance L1, L2 of the two coils and their mutual inductance M. Hence show that $M < \sqrt{L_2} L_2$.



Ans. Self-inductance of a solenoid of length L = μ 0n²AL, where n is number of turns per metre.

$$L_1=\mu_0n_1^2A_1L$$

and
$$L_2 = \mu_0 n_2^2 A_2 L$$

If I_1 is the current in outer solenoid, then magnetic field at axis, $B_1=\mu_0 n_1I_1$ Magnetic flux linked with the secondary coil

$$arphi_2 = (n_2 L) B_1 A_2 = (n_2 L) ((\mu_0 n_1 I_1) A_2 = \mu_0 n_1 n_2 \operatorname{LA}_2 I_1$$
 $M = rac{arphi_2}{I_1} = \mu_0 n_1 n_2 \operatorname{LA}_2 \qquad ... (i)$

Now, $L_1L_2 = (\mu_0 n_1^2 A_1 L), (\mu_0 n_2^2 A_2 L)$

$$\sqrt{L_1 L_2} = \mu_0 n_1 n_2 L \sqrt{A_1 A_2}$$
 ... (*ii*)

$$\frac{M}{\sqrt{L_1 L_2}} = \sqrt{\frac{A_2}{A_1}}$$

As
$$A_2 < A_{1,A_2} < \sqrt{A_1 A_2}$$

$$\therefore M < \sqrt{L_1 L_2}$$