

CHAPTER-5

Complex numbers and Quadratic Equations

1 Mark Questions

1. Define a complex number. (K)
2. Define Equality of two complex numbers. (K)
3. Define purely real and purely imaginary numbers. (K)
4. Define addition of two complex numbers. (K)
5. Define difference of two complex numbers. (K)
6. Define multiplication of two complex numbers. (K)
7. Define division of two complex numbers. (K)
8. Define modulus of a complex number. (K)
9. Define conjugate of a complex number. (K)
10. What is Polar form of a complex numbers? (K)

Q. Express the following in the form of $a + ib$

- | | |
|---|---|
| 1) $(-5i)\left(\frac{1}{8}i\right)$ (U) | 2) $(-i)(2i)\left(\frac{-1}{8}i\right)^3$ (U) |
| 3) $(5i)\left(\frac{-3}{5}i\right)$ (U) | 4) $i^9 + i^{19}$ (U) |
| 5) i^{-39} (U) | 6) $3(7 + i7) + i(7 + i7)$ (U) |
| 7) $(1 - i) - (-1 + i6)$ (K) | 8) i^{-35} (K) |

2 Marks Questions

Q. Express the following in the form of $a + ib$

- | | | |
|---|---|--|
| 1) $\frac{5 + \sqrt{2}i}{1 - \sqrt{2}i}$ (U) | 2) $(5 - 3i)^3$ (U) | 3) $(1 - i)^4$ (U) |
| 4) $\left(\frac{1}{3} + 3i\right)^3$ (U) | 5) $\left(-2 - \frac{1}{3}i\right)^3$ (U) | 6) $\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3$ (A) |
| 7) $\frac{(3 + i\sqrt{5})(3 - i\sqrt{5})}{(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})}$ (A) | 8) $(-\sqrt{3} + \sqrt{-2})(2\sqrt{3} - i)$ (U) | |

Q. Find the multiplicative inverse of each of the following complex numbers

- 1). $2 - 3i$ (U)
- 2). $4 - 3i$ (U)
- 3). $\sqrt{5} + 3i$ (U)
- 4). $-i$ (U)
- 5). $\frac{1+i}{1-i}$ (A)

$$6). \frac{2+3i}{3+4i} \quad (A)$$

3 Marks Questions

Q. Find the modulus and argument (amplitude) of each of the following complex numbers and express in polar form

- | | |
|---------------------------------|--|
| 1). $1+i\sqrt{3}$ (U) | 2). $-1-i\sqrt{3}$ (U) |
| 3). $-\sqrt{3}+i$ (U) | 4). $\sqrt{3}+i$ (U) |
| 5). $1-i$ (U) | 6). $-1+i$ (U) |
| 7). $-1-i$ (U) | 8). -3 (U) |
| 9). i (U) | 10). $\frac{1+i}{1-i}$ (U) |
| 11). $\frac{1}{1+i}$ (U) | 12). $\frac{i-1}{\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}}$ (U) |
| 13). $\frac{1+7i}{(2-i)^2}$ (U) | 14). $\frac{1+3i}{1-2i}$ (U) |
| 15). $\frac{1+2i}{1-3i}$ (U) | 16). $\frac{-16}{1+i\sqrt{3}}$ (U) |

3 Marks Questions

- If $4x + i(3x - y) = 3 + i(-6)$, where x and y are real numbers, then find the values of x and y (U)
- Find the real numbers x and y if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$. (U)
- If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least positive integral value of m (S)
- If $x - iy = \sqrt{\frac{a-ib}{c-id}}$, prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$ (U)
- If $x + iy = \frac{a+ib}{a-ib}$, prove that $x^2 + y^2 = 1$ (U)
- If $a + ib = \frac{(x+i)^2}{2x^2 + 1}$, prove that $a^2 + b^2 = \frac{(x^2 + 1)^2}{(2x^2 + 1)^2}$ (S)
- If $(x + iy)^3 = u + iv$ then show that $u/x + v/y = 4(x^2 - y^2)$ (S)
- Find real θ such that $\frac{3 + 2i \sin \theta}{1 - 2i \sin \theta}$, is purely real (S)
- Find the conjugate of $\frac{(3 - 2i)(2 + 3i)}{(1 + 2i)(2 - i)}$, (S)
- If $Z_1 = 2 - i$, $Z_2 = 1 + i$ then find $\left| \frac{Z_1 + Z_2 + 1}{Z_1 - Z_2 + 1} \right|$ (S)

11) $Z_1 = 2 - i$, $Z_2 = -2 + i$ find i) $\operatorname{Re}\left(\frac{Z_1 Z_2}{Z_1}\right)$ ii) $\operatorname{Im}\left(\frac{1}{Z_1 Z_1}\right)$ (S)

12) if $(a + i b) (c + i d) (e + i f) (g + i h) = A + i B$ then

Show that $(a^2 + b^2) (c^2 + d^2) (e^2 + f^2) (g^2 + h^2) = A^2 + B^2$ (U)

Quadratic Equations

2 Marks Questions

Q. Solve the following Equations

1). $x^2 + 2 = 0$ (U) 2). $x^2 + x + 1 = 0$ (U)

3). $\sqrt{5}x^2 + x + \sqrt{5} = 0$ (U) 4). $x^2 + 3 = 0$ (U)

5). $2x^2 + x + 1 = 0$ (U) 6). $x^2 + 3x + 9 = 0$ (U)

7). $-x^2 + x - 2 = 0$ (U) 8). $x^2 + 3x + 5 = 0$ (U)

9). $x^2 - x + 2 = 0$ (U) 10). $\sqrt{2}x^2 + x + \sqrt{2} = 0$ (U)

11). $\sqrt{3}x^2 - \sqrt{2}x + 3\sqrt{3} = 0$ (U) 12). $x^2 + x + \frac{1}{\sqrt{2}} = 0$ (U)

13). $x^2 + \frac{x}{\sqrt{2}} + 1 = 0$ (U) 14). $3x^2 - 4x + \frac{20}{3} = 0$ (U)

15). $x^2 - 2x + \frac{3}{2} = 0$ (U) 16). $27x^2 - 10x + 1 = 0$ (U)

17). $21x^2 - 28x + 10 = 0$ (U)
