CHAPTER 07

Seepage Through Soils

7.1 Introduction

Seepage is the flow of water under gravitational forces in a permeable medium. Flow of water takes place from a point of high head to a point of low head. In one dimensional flow, where Darcy's law is valid, the velocity of flow is taken constant at every point over a cross section normal to the direction of flow. However many practical situations (like flow through earth dams, sheet piles) the flow of water is not unidirectional and velocity of water does not remain constant. In such cases, the quantity of seepage and other parameters such as hydraulic gradient and pore water pressure are estimated by the help of flow nets. The concept and construction of flow nets are based on Laplace's equation of continuity.

7.2 Type of Head

There are three types of head available in fluid flow:

(i) Velocity head (ii) Pressure head (iii) Datum or elevation head

(i) Velocity head

- It is equal to $\frac{V^2}{2g}$.
- Since laminar flow occurs during the seepage and velocity is very small during laminar flow. Hence in seepage analysis, velocity head may be neglected.

(ii) Pressure head

- It is equal to $\frac{P}{\gamma_w}$.
- If a piezometer or an open stand pipe is inserted at a point of flow, water would stand at a particular height inside the piezometer.
- The actual height of rise of water column in the piezometer, represents the pressure head.

(iii) Datum or elevation head

- It is represented by 'Z'.
- The elevation or datum head at a point is the vertical distance of that point measured from an assumed datum. Generally, datum is assumed at tail water level.

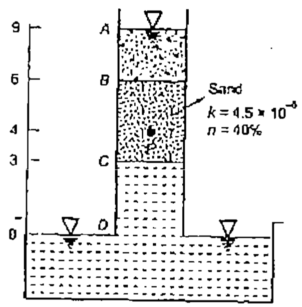
7.3 Total Head

- Total head = velocity head + pressure head + elevation head
- Here, Total head = $0 + \frac{P}{\gamma_w} + z$
- If we insert a stand pipe at a point of flow, the elevation of water level in stand pipe with reference to the datum is equal to total head.
- In seepage analysis, velocity head is neglected. Hence total head and piezometric head both are equal.

7.3.1 Head Loss

- The difference in total head between two points in a soil through which flow is occurring is represented by the head loss during the flow between these points.
- If flow is occurring through the soil sample, the total head is assumed to reduce during flow.

Example 7.1 Determine the pressure head, elevation head and total head at the entering end, exit end and point *P* of the sample. What are the discharge (superficial) and seepage velocity of flow?



Solution:

Assume datum is taken at *EL* 0(*m*).

Elevation of point (*m*)	Elevation Head (*m*)	Pressure Head (*m*)	Total Head *E.H + P.H* (*m*)	Head Loss (*m*)
A	9	0	9	0
B	6	3	9	0
C	3	−3	0	9
D	0	0	0	9
P	4	−1	3	6

It is convenient to first determine the total head loss due to sand specimen which is equal to the difference of initial and final water level. Now determine the elevation and total heads, and then calculate the pressure head by substracting elevation head from the total head. This procedure can be adopted for any point within the soil.

Head loss due to sand specimen, $\Delta H = 9 - 0 = 9\text{m}$

For point *P*, the head loss upto *P* is calculated proportionately as $\frac{9}{3} \times 2 = 6$ m. Thus the total head at *P* is equal to total head at *B* (9 m) minus the head loss upto *P* (6 m).

i.e. $9 - 6 = 3$ m

The pressure head is computed last; it is the total head − elevation head

i.e. $3 - 4 = -1$ m

We know, discharge (superficial) velocity is given by

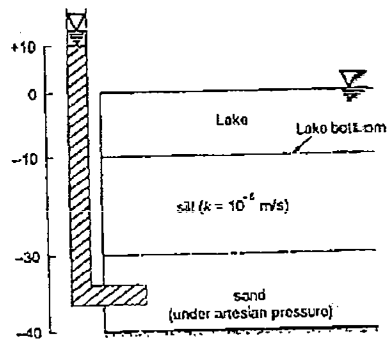
$$v = k.i = k.\frac{\Delta H}{3} = 4.5 \times 10^{-5} \times \frac{9}{3}$$

$$= 1.35 \times 10^{-4} \text{ m/s or } 0.135 \text{ mm/sec}$$

The seepage velocity, v_s is given by the equation,

$$v_s = \frac{v}{n} = \frac{0.135}{0.4} = 0.3375 \text{ mm/sec}$$

Example 7.2 The soil profile below a lake with water level at elevation = 0 m and lake bottom at elevation = 10 m is shown in the figure. A piezometer (stand pipe) installed in the same layer shows a reading of +10 elevation. Assume that the piezometric head is uniform in the sand layer. The quantity of water in cum per day, flowing into the lake from the sand layer through the silt layer per unit area of the lake bed would be?



Solution:

Given,
$k = 10^{-6}$ m/s
$= 10^{-6} \times 60 \times 60 \times 24$ m/day
$= 0.0864$ m/day
$L = 20$ m,
$A = 1\ \text{m}^2$

Total head available at the bottom of silt stratum
= Elevation head + Pressure head
= 10 + 40 = 50 m

Total head available at the top of silt stratum
= Elevation head + Pressure head
= 30 + 10 = 40 m

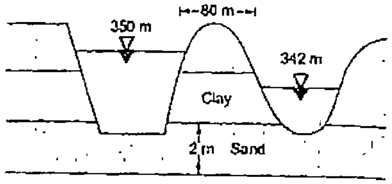
∴ Head loss due to silt stratum,
$\Delta H = 50 - 40 = 10$ m

Hydraulic gradient, $i = \frac{\Delta H}{L} = \frac{10}{20} = 0.5$

By Darcy's law $Q = k.i.A$
$= 0.0864 \times 0.5 \times 1$ cum/day $= 4.32 \times 10^{-2}$ cum/day

Example 7.3 A canal and a river run parallel with an average distance of 80 m apart. The elevation of the water surface in the canal is at 350 m and in the river at 342 m. A stratum of sand intersect both the river and the canal below their water levels. The sand is 2 m thick and is sandwiched between strata of impervious clay. Compute the seepage loss from the canal in m³/day-km, if the permeability of sand is 4.5×10^{-5} m/s.

Solution:



Head loss, $H_L = 350 - 342 = 8\ \text{m}$

$\therefore$ Hydraulic gradient, $i = \frac{H_L}{L} = \frac{8}{80} = \frac{1}{10}$

Using Darcy's equation, $Q = k.i.A$

$$= 4.5\times10^{-5}\left(\frac{86,400\,s}{day}\right)\times\frac{1}{10}\times(2\ \text{m}\times1000\ \text{m})$$

$$= 777.6\ \text{m}^3/\text{day-km}$$

7.4 Seepage Pressure and its Effect on Effective Stress

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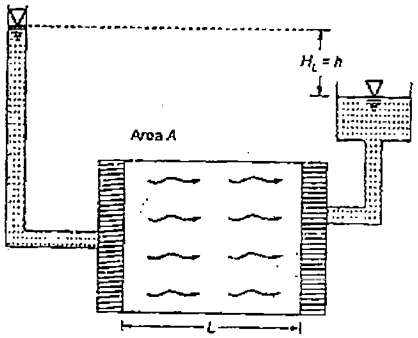


Fig. 7.1 Seepage pressure in Soil

- **Seepage Pressure:** When water flows through the saturated soil mass, it exerts the pressure over the solids by the virtue of viscous friction and is referred as seepage pressure.

  Hence, seepage pressure is the pressure exerted by the water over the soil solids in the mass through which it percolates.

- If 'h' is the hydraulic head (i.e Head loss) under which flow is taking place, then seepage is given by

$$p_s = h\gamma_w$$

Also, $$p_s = \frac{h}{z}z\gamma_w$$

or $$p_s = iz\gamma_w$$

If A be the area cross-section, then seepage force is given by

$$P_s = \text{seepage pressure } (p_s) \times A$$

$$= i\,z\,\gamma_w \times A$$

$$= i.(z \times A).\gamma_w$$

$$= i.V.\gamma_w$$

Seepage per unit volume is known as "specific seepage force" which is given by

$$P_{ss} = \frac{P_s}{V} = \frac{i.V.\gamma_w}{V} = i\gamma_w$$

NOTE

Seepage pressure always acts in the direction of flow. Hence vertical pressure (effective stress) at any given section in flow condition, may either increase or decrease.

$\therefore$ $$\bar{\sigma}' = \bar{\sigma} \pm P_s$$

where $\bar{\sigma}$ = effective stress under no flow condition

P_s = seepage pressure under flow condition

7.4.1 No Flow Condition

- The figure shows a tank filled with submerged soil. Since the valve at the base is closed, no seepage will occur.

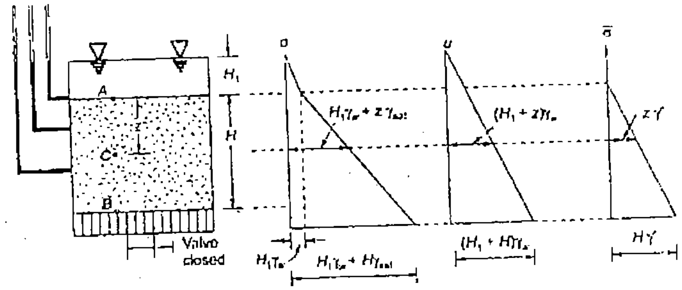


Fig. 7.2

- Pressure head, datum head and total head are tabulated below.

Point	A	B	C
Pressure head	H_1	$H_1 + H$	$H_1 + Z$
Datum head	H	0	$H - Z$
Total head	$H_1 + H$	$H_1 + H$	$H_1 + H$

- Total stress, pore water pressure and effective stress and tabulated below:

Point	A	B	C
Total stress (σ)	$\gamma_w H_1$	$\gamma_{sat} H + \gamma_w H_1$	$\gamma_{sat} H + \gamma_w H_1$
Pore water pressure (u)	$\gamma_w H_1$	$\gamma_w(H_1 + H)$	$\gamma_w(H_1 + Z)$
Effective stress ($\bar{\sigma} = \sigma - u$)	0	$\gamma_{sat} H - \gamma_w H = \gamma' H$	$\gamma_{sat} Z - \gamma_w Z = \gamma' Z$

The variations of σ, u and $\bar{\sigma}$, are shown in figure above.

7.4.2 Downward Flow

- Figure shows a tank filled with submerged soil, since the valve at the base is open and downward seepage is allowed. The downward seepage increases in effective stress.

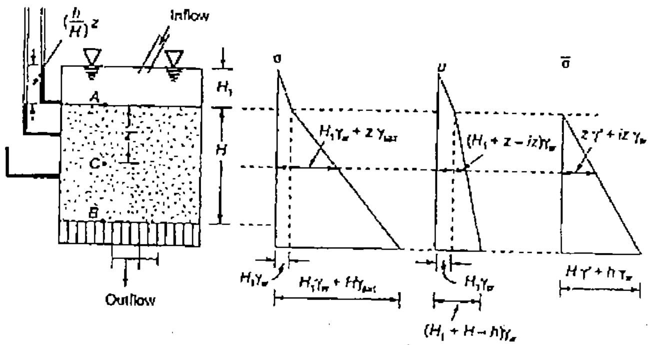


Fig.7.3 Variation of σ, u and $\bar{\sigma}$ with depth downward flow

- Pressure head, datum head and total head are tabulated below:

Point	A	B	C
Pressure head	H_1	$H_1 + H - h$	$H_1 + Z - iZ$
Datum head	H	0	$H - Z$
Total head	$H_1 + H$	$H_1 + H - h$	$H_1 + H - iZ$

where, h = hydraulic head (head loss) under which flow takes place from A to B

Hydraulic gradient, $i = \dfrac{h}{H}$

- The total vertical stress at any point in soil mass is due submerged weight of soil mass and seepage pressure depending upon the direction of flow.
- Total stress, pore water pressure and effective stress are tabulated below:

Point	A	B	C
Total stress (σ)	$\gamma_w H_1$	$\gamma_{sat} H + \gamma_w H_1$	$\gamma_{sat} Z + \gamma_w H_1$
Pore water pressure (u)	$\gamma_w H_1$	$\gamma_w(H_1 + H - h)$	$\gamma_w(H_1 + Z - iZ)$
Effective stress ($\bar{\sigma} = \sigma - u$)	0	$= (\gamma_{sat} - \gamma_w) H + \gamma_w h$ $= \gamma' H + \gamma_w h$	$= (\gamma_{sat} - \gamma_w) Z + \gamma_w iZ$ $= \gamma' Z + \gamma_w iZ$

The variations of σ, u and $\bar{\sigma}$ are shown in figure above

7.4.3 Upward Flow

- Figure shows, valve at the bottom of tank is open and upward seepage is allowed.

Point	A	B	C
Pressure head	H_1	$H_1 + H + h$	$H_1 + Z + iZ$
Datum head	H	0	$H - Z$
Total head	$H_1 + H$	$H_1 + H + h$	$H_1 + H + iZ$

Where, h = hydraulic head (head loss) under which flow takes place from A to B

Hydraulic gradient, $i = \dfrac{h}{H}$

Total stress, pore water pressure and effective stresses are tabulated below:

Point	A	B	C
Total stress (σ)	$\gamma_w H_1$	$\gamma_{sat} H + \gamma_w H_1$	$\gamma_{sat} Z + \gamma_w H_1$
Pore water pressure (u)	$\gamma_w H_1$	$\gamma_w(H_1 + H + h)$	$\gamma_w(H_1 + Z - iZ)$
Effective stress ($\bar{\sigma} = \sigma - u$)	0	$= (\gamma_{sat} - \gamma_w) H - \gamma_w h$ $= \gamma' H - \gamma_w h$ $= \gamma' H - iZ\gamma_w$	$= (\gamma_{sat} - \gamma_w) H - \gamma_w h$ $= \gamma' H - \gamma_w h$ $= \gamma' H - iZ\gamma_w$

The variations of σ, u and $\bar{\sigma}$ are shown in figure.

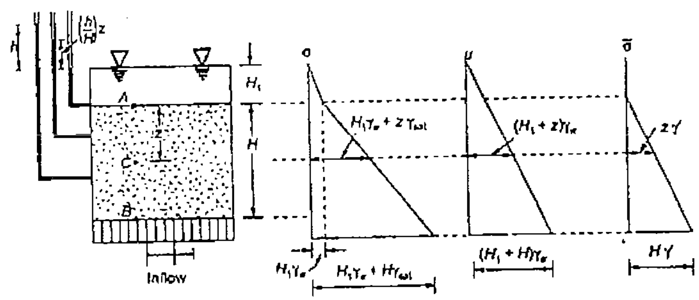


Fig. 7.4 Variation of σ, u and $\bar{\sigma}$ with depth for upward flow

7.5 Quick Sand Condition

For upward flow condition, effective stress at any point within soil mass is given by

$$\bar{\sigma} = \bar{\sigma} - p_s$$
$$= \gamma' z - p_s$$

It is clear from above equation, upward seepage pressure decreases effective stresses in the soil mass.

- If the seepage pressure is such that it equals the submerged weight of the soil mass, then effective stresses at that location reduces to zero. Under such condition, cohesionless soil mass loses all shear strength. Now soil mass has a tendency to move also with the flowing water in the upward direction.

- This process in which soil particles are lifted over the soil mass is called quick sand condition. It is also known as 'boiling of sand' as the surface of sand looks it is boiling.
- At quick sand condition, net effective stress is reduced to zero. i.e.

$$\bar{\sigma} = 0$$

$$\therefore \quad \gamma' z = p_s = 0 \qquad \text{[where } p_s = \text{seepage pressure]}$$

$$\text{or} \quad \gamma' z - iz\gamma_w = 0$$

$$i = \frac{\gamma'}{\gamma_w} = i_{cr}$$

- The hydraulic gradient under which quick sand condition occurs is termed as critical hydraulic gradient. If void ratio and specific gravity of soil is known, then i_{cr} may be given as

$$i_{cr} = \frac{\gamma'}{\gamma_w} = \frac{\frac{(G-1)\gamma_w}{1+e}}{\gamma_w}$$

$$\therefore \quad i_{cr} = \frac{G-1}{1+e}$$

- For fine sand and silts for which specific gravity $\approx$ 2.65 and void radio $e \simeq 0.65$,

$$i_{cr} = \frac{2.65-1}{1+0.65} \simeq 1$$

- At quick sand condition, cohesionless particles of fine sand may start flowing with the water which may result in piping failure below the hydraulic structure.
- In order to prevent quick sand or piping failure, the hydraulic gradient should be less then critical hydraulic gradient. Hence factor of safety against quick sand failure or piping failure is

$$\text{F.O.S.} = \frac{i_c}{i} \qquad \text{where,} \quad i = \frac{h_L}{L}$$

NOTE

- Quick sand is not a type of sand. It is a flow condition which exists in cohesionless soil mass where effective stresses are reduced to zero in upward flow conditions.
- Quick sand conditions is found only in fine sand and coarse silts and it is not being observed in the case of gravels, coarse sands and clays.
- In cohesive soils which posses inherent cohesion, shear strength is not reduced to zero even when effective stresses are reduced to zero.
  For cohesive soils,

$$\tau_f = c' + \bar{\sigma}\tan\phi'$$

$$\text{If} \quad p_s = \gamma' z$$

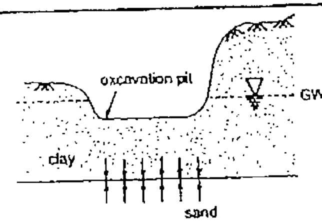
$$\text{then,} \quad \bar{\sigma} = \gamma' z - p_s = 0$$

$$\text{or} \quad \tau_f = c' + 0$$

- Quick sand condition is not observed in gravels and coarse sands which are highly permeable soils. As per Darcy's law a large discharge is required to generate critical hydraulic gradient.

$$Q = k.i.A; \qquad i = \frac{Q}{k.A}$$

Remember Quick sand condition is generally observed when excavation is done below the GWT and water is pumped out to keep the excavated area free from it or it is observed when sand is under artesian pressure and overlain by a clay layer.



Example 7.4 A soil sample (sand) obtained from trench has water content of 38%. The specific gravity of solid is 2.65. Determine the critical hydraulic gradient for the soil. Also determine the maximum permissible upward gradient, if a factor of safety of 3 is considered.

Solution:

Given, $w = 38\%$, $G = 2.65$

For saturated soil,

$$1 \times e = wG$$

$$\Rightarrow \quad e = 0.38 \times 2.65 = 1.007$$

Now, critical hydraulic gradient, $i_{cr} = \dfrac{G-1}{1+e}$

$$\therefore \quad i_{cr} = \frac{2.65-1}{1+1.007} = 0.822$$

If the factor of safety is 3, then

Maximum permissible upward gradient, $i = \dfrac{i_{cr}}{F.O.S} = \dfrac{0.822}{3} = 0.274$

Example 7.5 A 3 m thick soil stratum has coefficient of permeability 3×10^{-7} m/sec. A separate test have porosity 40% and bulk unit weight 21 kN/m^3 at a moisture content of 31%. Determine the head at which upward seepage will cause quick sand condition. What is the flow required to maintain critical conditions?

Solution:

Given $k = 3 \times 10^{-7}$ m/s, $n = 0.40$

$\gamma = 21$ kN/m^3, $w = 0.31$

Using, $n = \dfrac{e}{1+e}$

$$0.40 = \frac{e}{1+e}$$

$$\therefore \quad e = 0.667$$

Using, $\gamma_d = \dfrac{\gamma}{1+w}$

We get $\gamma_d = \dfrac{21}{1+0.31} = 16.03$ kN/m^3

Also, $\gamma_d = \dfrac{G\gamma_w}{1+e}$

$\therefore$ $16.03 = \dfrac{G\times 9.81}{1+0.667}$

or $G = \dfrac{16.03\times 1.667}{9.81} = 2.724$

Now, we can use, $i_{cr} = \dfrac{G-1}{1+e} = \dfrac{2.724-1}{1+0.667} = 1.034$

Also, $i_{cr} = \dfrac{H_L}{L}$

$\therefore$ $\dfrac{H_L}{3} = 1.034$

or $H_L = 1.034 \times 3 = 3.1$ m

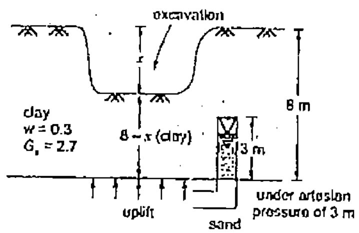
Hence, 3.1 m of head is sufficient to cause quick sand condition.

The flow required to cause quick sand condition can be obtain by Darcy's equation.

$$Q = k.i.A$$
$$= 3\times 10^{-7}\times 1.034\times 1 \quad \text{(Assume unit area)}$$
$$= 3.1\times 10^{-7}\ \text{m}^3/\text{sec/m}^2$$

Example 7.6 At a given location, 8 m thick saturated clay; natural water content = 30%, G_S = 2.7 is under-lain by sand. The sand layer is under artesian pressure equivalent to 3 m of water head. It is proposed to make an open excavation in the clay. How deep can this excavation be made before the bottom heaves?

Solution:



Let the safe depth of excavation = x m

$\therefore$ Thickness of clay overlaying sand at excavation site $= (8-x)$ m

The bottom of excavation will remain stable as long as the downward stresses due to clay stratum are more than upward pressure due to artesian head in sand.

For maximum depth of excavation, the downward and upward pressure should be just equal

$\therefore$ $(8-x)\gamma_{sat,clay} = 3\times\gamma_w$ ...(i)

where $\gamma_{sat\,clay} = \dfrac{G+e}{1+e}\gamma_w$

where $e = wG = 0.3\times 2.7 = 0.81$

$\therefore$ $\gamma_{sat,clay} = \dfrac{(2.7+0.81)}{1+0.81}\times 9.81 = 19.02\ \text{kN/m}^3$

On substituting values of $\gamma_{sat\,clay}$ and γ_w, we get

$$(8-x)\times 19.02 = 3\times 9.81$$
$$8-x = 1.55$$
$$x = 8-1.55 = 6.45\ \text{m}$$

$\therefore$ Maximum depth of cut = 6.45 m

Alternative approach:

Head loss during flow, $H_L = 3-(8-x)$

$\therefore$ Hydraulic gradient, $i = \dfrac{3-(8-x)}{(8-x)}$

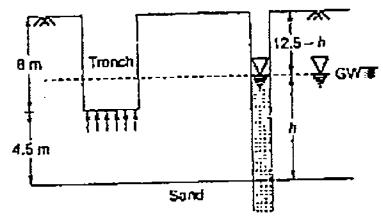
For maximum depth of cut, $i = i_{cr}$

$\therefore$ $\dfrac{3-(8-x)}{(8-x)} = \dfrac{G-1}{1+e}$

$$\frac{3-(8-x)}{(8-x)} = \frac{2.7-7}{1+0.81} = 0.939$$
$$x-5 = 7.514-0.939x$$
$$1.939x = 12.514$$
$$x = 6.45$$

Example 7.7 A layer of clay of thickness 12.5 m is underlain by sand. The saturated unit weight of the clay is 18.5 kN/m³. When the depth of an open trench excavated in the clay reached a depth of 8 m, the bottom cracked and the water started entering the trench from below. Determine the height to which water would have risen from the top of sand in a bore hole it were drilled into sand prior to the excavation. Take γ_w = 10 kN/m³

Solution:



Let h be the height of the water table above the top of the sand as shown in figure.

We know quick sand condition will start in sand when net effective stress become zero at top of sand.

Total stress on top of sand below trench,

$$\sigma = 4.5\,\gamma_{sat}$$

Let h_w be the artesian pressure head

$\therefore$ Pore pressure due to artesian head.

$$u = h_w\gamma_w = h_w\times 10$$

$\therefore$ Effective stress, $\bar{\sigma} = \sigma - u$
$$= 4.5\,\gamma_{sat} - 10\,h_w$$

For quick sand condition, $\bar{\sigma} = 4.5\,\gamma_{sat} - 10\,h_w = 0$

$$h_w = \frac{4.5\gamma_{sat}}{10} = \frac{4.5\times 18.5}{10} = 8.325\ \text{m}$$

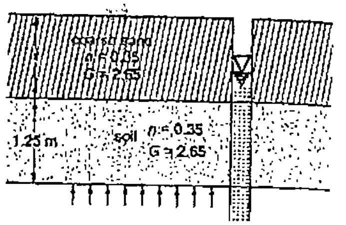
Example 7.8 A 1.25 m layer of soil having the porosity 0.35 and specific gravity 2.65 is subjected to an upward seepage head of 1.85 m. What depth of coarse sand would be required above the existing soil to provide a factor of safety of 2 against piping? Assume that coarse sand has the same porosity and specific gravity as the soil and there is negligible head loss in sand.

Solution:

Let x be the depth of coarse sand required above existing soil.

Using, $n = \dfrac{e}{1+e} = 0.35$

$$0.35 + 0.35e = e$$
$$e = 0.5385$$



Now,
$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w$$
$$= \left(\frac{2.65+0.5385}{1+0.5385}\right)\times 9.81$$
$$= 20.33 \text{ kN/m}^3$$

$\therefore$
$$\gamma_{sub} = \gamma_{sat} - \gamma_w$$
$$= 20.33 - 9.81$$
$$= 10.52 \text{ kN/m}^3$$

$$\text{F.O.S} = \frac{\text{Downward force}}{\text{Upward force}}$$

or
$$\frac{\text{Downward force due to wt of soil}}{\text{Upward force due to 1.85 m seepage head}} = 2$$

$\therefore$
$$\frac{(1.25+x)\times\gamma_{sub}}{1.85\times\gamma_w} = 2$$
$$\frac{(1.25+x)\times 10.52}{1.85\times 9.81} = 2$$
$$(1.25+x) = 3.45$$
$$x = 2.20 \text{ m}$$

7.6 Laplace Equations

- When flow takes place in 2-D then Darcy's equation cannot be used. In such case of seepage, Laplace equations are used which represents the loss of energy head in any resistive medium like soil.
- For isotropic medium

$$\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2} = 0$$

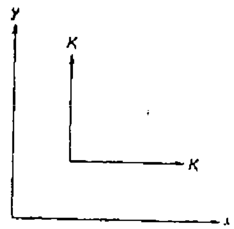


Fig. 7.5 Isotropic medium

- A soil is said to be non isotropic when it has different permeability in two-mutually perpendicular direction. For non-isotropic medium, Laplace equation becomes

$$k_x\frac{\partial^2 H}{\partial x^2}+k_y\frac{\partial^2 H}{\partial y^2} = 0$$

where
$\frac{\partial H}{\partial x}$ = loss of energy head in x-direction

$\frac{\partial H}{\partial y}$ = loss of energy head in y-direction

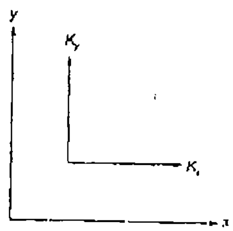


Fig. 7.6 Non-isotropic medium

- The solution of two Laplace equations for the potential and flow functions takes the form of two families of orthogonal curves.
- One set of curve (ψ-lines) represent the trajectories of seepage and are termed as flow lines. The space between two adjacent flow lines are known as flow channel.
- The another set of curve (ϕ-lines) represents lines of equal head and are termed equipotential lines. The head loss caused by water crossing two adjacent equipotential lines is known as potential drop.

Assumptions made in Laplace Equations:

- Soil is homogeneous and fully saturated.
- Flow is Laminar and Darcy's Law is valid
- Pore water and soil solids are assumed to be incompressible.

7.7 Flow Nets

The entire pattern of flow lines and equipotential lines is referred as a flow net. It is the solution of Laplace's equation for relevant boundary conditions. Thus, a flow net is a graphical representation of the head and direction of seepage at every point.

7.7.1 Properties of Flow Nets

- Flow lines (ψ-lines) and equipotential (ϕ-lines) meet each other othogonally.
- Flow lines and equipotential lines are smooth continuous curves, being either elliptical or parabolic in shape.
- There can be no flow across the flow line and velocity of flow is always perpendicular to equipotential lines.
- Area bounded between two adjacent flow lines is called flow channel or flow path and quantity of water flowing through each channel is same.
- Area bounded between two adjacent equipotential lines and adjacent flow lines is referred to a flow field.
- For isotropic medium, flow field is approximately square which may either be linear or curvilinear.
- For non-isotropic medium, flow field is rectangular which may either be linear or curvilinear.

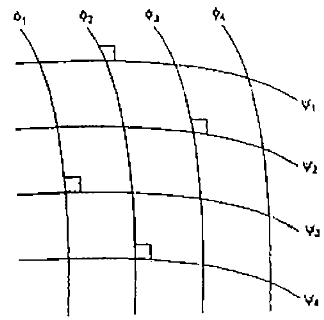


Fig. 7.7

- Loss of head between two equipotential lines i.e equipotential drop remains constant for all the equipotential lines.
- Flow net remain unchanged if boundary conditions are not altered i.e flow net remain same for the same set of boundary conditions.

7.7.2 Application of Flow Nets

Flow net can be used for the determination of seepage, seepage pressure, hydrostatic pressure and exit gradient.

1. **Determination of Seepage**

Let Δq be the discharge through each flow channel under the total head of H.

(a) For Isotropic Medium:

- For isotropic medium, flow fields are square in shape. Let '$b \times b$' be the size of flow field

$\therefore \quad \Delta q = k.i.A = k.\frac{\Delta h}{l}(b \times 1)$

But for isotropic medium, $l = b$

$\therefore \quad \Delta q = k.\Delta h$ ...(i)

If 'H' be the total head loss and N_d be the number of potential drops.

then, $\quad \Delta h = \frac{H}{N_d}$

Therefore, equation (i) becomes

$$\Delta q = k.\frac{H}{N_d}$$

Let N_f be the total number of flow channels.

The total discharge through the complete flow net per unit length is given as

$$q = \Delta q \times N_f = k.\frac{H}{N_d}.N_f$$

$$= kH.\frac{N_f}{N_d}$$

- The ratio $\frac{N_f}{N_d}$ is independent of k and H and is characteristic of the flow net. This is called shape factor of the flow net.

(b) For Non-Isotropic Medium:

- In this case the equation of continuity assumes the form

$$k_x\frac{\partial^2 H}{\partial x^2} + k_y\frac{\partial^2 H}{\partial y^2} = 0 \quad \text{...(i)}$$

- Solution of above equation cannot be given as it is not in Laplacian form, so we have to transform it by substituting a transformed coordinate x_T for x such that

$$x_T = x\sqrt{\frac{k_y}{k_x}}$$

or

$$x = x_T\sqrt{\frac{k_x}{k_y}}$$

$$\therefore \quad k_x\frac{\partial^2 H}{\partial^2_{x_T}\left(\sqrt{\frac{k_x}{k_y}}\right)} + k_y\frac{\partial^2 H}{\partial y^2} = 0$$

$$k_x.\frac{\partial^2 H}{\partial x_T^2}.\frac{k_y}{k_x} + k_y\frac{\partial^2 H}{\partial y^2} = 0$$

$$\frac{\partial^2 H}{\partial x_T^2} + \frac{\partial^2 H}{\partial y^2} = 0 \quad \text{...(ii)}$$

Equation (ii) is in standard laplacian form, flow net (flow field) for which can be drawn for transformed section. On this transformed scale, flow field will be a square

Let, equivalent permeability on transformed scale be k_e. Also let Δ_{q_T} and Δ_{q_A} be the quantity of flow through transformed and actual section respectively.

For transformed section, $\quad \Delta_{q_T} = k_e\frac{\Delta h}{l}(l \times 1)$

For actual section, $\quad \Delta_{q_A} = k_x.\frac{\Delta h}{l\sqrt{\frac{k_x}{k_y}}}(l \times 1)$

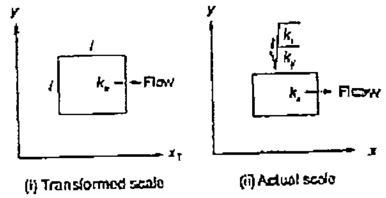


Fig. 7.8

But $\quad \Delta_{q_T} = \Delta_{q_A}$

Therefore, $\quad k_e.\frac{\Delta h}{l}\times l = k_x.\frac{\Delta h}{l\sqrt{\frac{k_x}{k_y}}}.l$

$$\therefore \quad k_e = \sqrt{k_x k_y}$$

Thus seepage quantity, $\quad \Delta q = k_e H\frac{N_f}{N_d}$

Note: If flow is in 3-D, $\quad k_e = \sqrt[3]{k_x k_y k_z}$

Example 7.9 A concrete gravity dam, 120 m long and 75 m wide, lies on a permeable soil with a coefficient of permeability of 40×10^{-3} mm/s. The head of water is maintained at 32 m upstream and zero at the tail-end. The soil is under lain by an impervious stratum. The depth from the base of the dam to impervious stratum is 38 m. A flow net constructed for this condition yielded 8 flow channel and 18 equipotential drops. What is the seepage loss per day under the dam, considering a two-dimensional flow.

Solution:

Given,

Coefficient of permeability,	$k = 40 \times 10^{-3}$ mm/s
	$= \frac{40 \times 10^{-3}}{1000}$ m/s
Head loss,	$H = 32$ m
No. of flow channels,	$N_f = 8$
No. of flow channel,	$N_d = 18$

For homogenous isotropic soil, $\quad q = kH.\frac{N_f}{N_d} = \frac{40 \times 10^{-3}}{1000} \times 32 \times \frac{8}{18}$ m³/s/m

$$= \frac{40 \times 10^{-3}}{1000} \times 32 \times \frac{8}{18} \times (60 \times 60 \times 24) \text{ m}^3\text{/d/m}$$

$= 49.152$ m³/d/m

Example 7.10 An earth dam is built on an impervious foundation with a horizontal filter at the base near the toe. The permeability of the soil in the horizontal and vertical directions are 3.2×10^{-2} mm/sec and 1.5×10^{-2} mm/sec respectively. The full reservoir level is 30 m above the filter. A flow net constructed for the transformed section of the dam, consist of 5 flow channels and 13 potential drops. Estimate the seepage loss per m length level of the dam.

Solution:

Given:

Horizontal permeability, $k_H = 3.2 \times 10^{-2}$ mm/s

Vertical permeability, $k_V = 1.5 \times 10^{-2}$ mm/s

$\therefore$ Equivalent permeability, $k_e = \sqrt{k_H . k_V} = \sqrt{3.2\times10^{-2}\times1.5\times10^{-2}}$

$= 2.19 \times 10^{-2}$ mm/s

Available Head above filter, $H = 30$ m

No. of flow channels, $N_f = 5$

No. of potential drop, $N_d = 13$

$\therefore$ Seepage per unit length of dam, $q = k_e H . \dfrac{N_f}{N_d} = \dfrac{2.19\times10^{-2}}{1000}\times30\times\dfrac{5}{13}$

$= 2.527 \times 10^{-4}$ m³/sec/m or 0.2527 l/sec

Example 7.11 A sand deposit 12 m thick overlie on impervious soil. A vertical sheet pile penetrate 5 m into the sand deposit. The water level on U/S and D/S is 3.0 m and 0.75 m above the ground surface respectively. The horizontal and vertical permeability for sand are 4 m/day and 1.25 m/day respectively. A flow net construction reveals that there are 12 flow channels and 26 potential drops. Determine the seepage flow per day.

Solution:

Equivalent permeability, $k_e = \sqrt{k_H k_V}$

$= \sqrt{4\times1.25} = 2.236$ m/day

Head loss, $H = 3.0 - 0.75 = 2.25$ m

No. of flow channels, $N_f = 12$

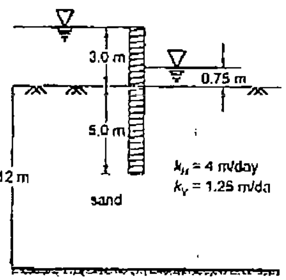
No. of potential drops, $N_d = 26$

$\therefore$ Seepage per day per unit length of sheet pile,

$$q = k_e H . \frac{N_f}{N_d}$$

$$= 2.236\times2.25\times\frac{12}{26}$$

$$= 3.77 \text{ m}^3\text{/day}$$



2. Determination of Seepage Pressure

- Let n_d be the number of potential drop (each of value Δh) by a water particle before reaching a point where seepage pressure is required. Let H be total head governing flow, hence net available head at point under consideration will be,

$$h_1 = H - n_d \Delta h$$

Hence seepage pressure,

$$p_s = h_1 \gamma_w$$

or

$$p_s = (H - n_d \Delta H)\gamma_w$$

- This pressure acts in the direction of flow.

3. Determination of Uplift Pressure

- Hydrostatic pressure at any point after 'n_d' equipotential drop is given by

$$u = h_w \gamma_w$$

where h_w is piezometric head.

$$h_w = h_1 \pm z \text{ or } (H - n_d \Delta H) \pm z$$

where z is elevation head

- Generally the downstream water level is usually considered as the datum and all points above the datum are considered as positive.

4. Determination of Exit gradient

Exit gradient is the hydraulic gradient at the D/S end of the flow line where the percolating water leaves the soil mass and emerges out as free water.

$$i_e = \frac{\Delta h}{l}$$

where l is length of the smallest square in the last flow field and Δh is potential drop.

7.8 Flow Through Non-Homogeneous Section

- If there is a change in soil conditions, the flow lines are deflected at the interface of the soil with varying permeability, k_1 and k_2.
- Let the potential drop from P to Q and from R to S be ΔH, then

$$\Delta q = k_1\left(\frac{\Delta H}{PR}\right)PQ$$

$$= k_2\left(\frac{\Delta H}{QS}\right)RS$$

But $\tan\alpha_1 = \dfrac{PR}{PQ}$

and $\alpha_2 = \dfrac{QS}{RS}$

$\therefore$ $\dfrac{k_1}{\tan\alpha_1} = \dfrac{k_2}{\tan\alpha_2}$

or $\dfrac{k_1}{k_2} = \dfrac{\tan\alpha_1}{\tan\alpha_2}$

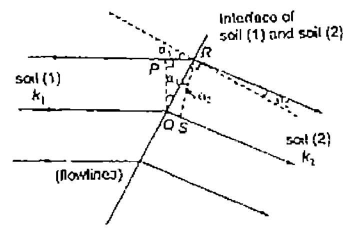


Fig. 7.9 Flow through non-homogenous section

- If $k_1 > k_2$, than $\alpha_1 > \alpha_2$, flow get deflected towards normal.
- If $k_1 < k_2$, than $\alpha_1 < \alpha_2$, flow get deflected away from normal.