Integers

- **Addition and subtraction of integers**

  Integers are closed under addition and subtraction.
  For two integers, a and b, $a + b$ and $a - b$ are integers.
  For example, $(-14) + 3 = -11$
  $(-7) - (-2) = -7 + 2 = -5$

- Addition is commutative for integers. For integers, a and b, $a + b = b + a$

  For example, $(-7) + 5 = 5 + (-7) = -2$

- Subtraction is not commutative for integers.

  For example, $(-7) - (4) = -11$
  $4 - (-7) = 11$
  $(-7) - (4) \neq (4) - (-7)$

- Addition is associative for integers. For integers, a, b, and c,

  $a + (b + c) = (a + b) + c$
  For example, $(-7) + (4 + (-3)) = ((-7) + 4) + (-3) = -6$

- Subtraction is not associative for integers.
- When 0 is added to any integer, say a, the same integer is obtained. Therefore, 0 is the additive identity of integers.

  $a + 0 = a = 0 + a$

- When $-a$ is added to any integer a, 0 is obtained. Therefore, $-a$ is the additive inverse of the integer a.

  $a + (-a) = 0 = (-a) + a$

- **Multiplication of integers**

  Rules for the product of integers:
  (i)The product of two positive integers is always positive.

(ii) The product of one positive integer and one negative integer is always negative.
For example, $5 \times (-9) = -(5 \times 9) = -45$
(iii) The product of two negative integers is always positive.
(iv) If the number of negative integers in a product is even, then the product is a positive integer. If the number of negative integers in a product is odd, then the product is a negative integer.

For example, $(-1) \times (-2) \times (-3) = -6$, $(-7) \times (-2) = 14$ etc.

- Integers are closed under multiplication. For integers, a and b, $a \times b$ is an integer.

  For example, $(-7) \times (4) = -28$, which is an integer

- Integers are commutative under multiplication.

  For example, $(-2) \times (5) = 5 \times (-2) = -10$

- The product of an integer and zero is zero.

  $(-2) \times 0 = 0$
  $7 \times 0 = 0$

- When an integer, say a, is multiplied by 1, it gives the same integer.

  $1 \times a = a \times 1 = a$
  Therefore, 1 is the multiplicative identity for integers.

- Integers are associative under multiplication. For integers a, b and c,

  $a \times (b \times c) = (a \times b) \times c$
  For instance, $(-25) \times [4 \times 39] = [(-25) \times 4] \times 39 = (-100) \times 39 = -3900$

- Multiplication is distributive over addition and subtraction for integers.

  For integers, a, b, and c,
  $a \times (b + c) = a \times b + a \times c$
  $a \times (b - c) = a \times b - a \times c$

- Commutative, distributive and associative properties can be used to simplify calculations.

For example,
$17 \times 49 = 17 \times (50 - 1)$
$= 17 \times 50 - 17 \times 1$ [Using distributive property]
$= 850 - 17$
$= 833$

- **Division of integers**
  - To divide a positive integer by a negative integer or a negative integer by a positive integer, the division is carried out as in whole numbers and then a negative sign (–) is put before the quotient.

For example, (12) ÷ (–4) = (–12) ÷ 4 = –3

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  - When a negative integer is divided by another negative integer or a positive integer is divided by another positive integer, a positive quotient is obtained.

For example, (–6) ÷ (–3) = 6 ÷ 3 = 2

Properties of Division

Property 1: If *a* and *b* are two integers, then *a* ÷ *b* might not be an integer.

Property 2: If *a* is an integer and *a* ≠ 0, then *a* ÷ *a* = 1.77=1

Property 3: If *a* is an integer and *a* ≠ 0, then *a* ÷ 1 = a–31=−3

Property 4: If *a* is an integer and *a* ≠ 0, then 0 ÷ *a* = 00–8=0

Property 5: If *a* is a non-zero integer, then *a* ÷ 0 is not defined.

Property 6: If *a, b* and *c* are non-zero integers, then
(*a* ÷ *b*) ÷ *c* ≠ *a* ÷ (*b* ÷ *c*) except when *c* = 1
Note: when *c* = 1, (*a* ÷ *b*) ÷ *c* = *a* ÷ (*b* ÷ *c*)

Property 7: If *a, b* and *c* are integers, such that
(i) *a* > *b* and *c* is positive, then (*a* ÷ *c*) > (*b* ÷ *c*)

- Integers are not commutative under division.

For example, (−24) ÷ (−3) = 8, which is not equal to (−3) ÷ (−24).

- For any integer *a*, *a* ÷ 0 is not defined.

- If a is any integer then a ÷ 1 = a always.

For example, 4 5 ÷ 1 = 45

- If a is any integer then $a \div (-1) = (-a)$ always.

  For example, $59 \div (-1) = (-59)$