CBSE Test Paper 03

Chapter 8 Introduction to Trigonometry

1. The value of $2\ tan^2 45^\circ +\ cos^2 30^\circ\ -\ sin^2 60^\circ$ is **(1)**
   a. 0
   b. 1
   c. 2
   d. -2
2. Choose the correct option and justify your choice: $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$ **(1)**
   a. tan 90°
   b. 1
   c. sin 45°
   d. 0
3. The value of $cosec^4 A\ -\ 2\ cosec^2 A\ +\ 1$ is **(1)**
   a. $tan^4 A$
   b. $sec^4 A$
   c. $coseec^4 A$
   d. $cot^4 A$
4. Choose the correct option. Justify your choice.(sec A + tan A) (1 – sin A) **(1)**
   a. cos A
   b. sec A
   c. sin A
   d. cosec A
5. $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta) =$ **(1)**
   a. $\tan^2\theta + \cos^2\theta$
   b. $\tan^2\theta - \cos^2\theta$
   c. $\tan^2\theta + \sin^2\theta$

d. $\tan^2\theta - \sin^2\theta$

6. If sin (20° + θ) = cos 30°, then find the value of θ. **(1)**

7. Write the value of 4tan$^2\theta$ - $\frac{4}{\cos^2\theta}$. **(1)**

8. Find the value of x, if $2\sin 3x = \sqrt{3}$. **(1)**

9. Prove that: $\frac{1}{(cosec\theta - \cot\theta)}$ = (cosecθ + cotθ) **(1)**

10. If cos θ = $\frac{2}{3}$, then find the value of (4 + 4 tan$^2\theta$). **(1)**

11. Find the value of θ if $\sqrt{3}$ tan 2θ - 3 = 0. **(2)**

12. Prove that tan 1° tan 11° tan 21° tan 69° tan 79° tan 89° = 1 **(2)**

13. Prove that: $\sec 50^\circ \sin 40^\circ + \cos 40^\circ \cos ec\ 50^\circ = 2$ **(2)**

14. If $\tan\theta + \frac{1}{\tan\theta} = 2$, find the value of $\tan^2\theta + \frac{1}{\tan^2\theta}$ **(3)**

15. Prove the trigonometric identity:
$\left(\tan\theta + \frac{1}{\cos\theta}\right)^2 + \left(\tan\theta - \frac{1}{\cos\theta}\right)^2 = 2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$ **(3)**

16. In $\triangle$ABC right angles at B, if $\tan A = \frac{1}{\sqrt{3}}$ find value of: sinA cosC + cosA sinC **(3)**

17. If $\sin\theta = \frac{3}{5}$, evaluate $\frac{\cos\theta - \frac{1}{\tan\theta}}{2\cot\theta}$. **(3)**

18. Prove that: (sin θ + cosec θ)^2 + (cos θ + sec θ)^2 = 7 + tan^2 θ + cot^2 θ. **(4)**

19. Prove that : $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}}$ = 2 cosecθ. **(4)**

20. Prove that: $\frac{1+\cos^2 A}{\sin^2 A} = 2\cos ec^2 A - 1$ **(4)**

CBSE Test Paper 03
Chapter 8 Introduction to Trigonometry

Solution

1. c. 2

   Explanation: Given: $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

   = $2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$ = 2

2. d. 0

   Explanation: $= \frac{1 - tan^2 45^0}{1 + tan^2 45^0} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

   Hence the correct answer is 0.

3. d. $cot^4 A$

   Explanation: Given: $\cos ec^4 \mathrm{A} - 2\cos ec^2 \mathrm{A} + 1$

   = $\left(\cos ec^2 \mathrm{A} - 1\right)^2$

   = $\left(\cot^2 \mathrm{A}\right)^2$

   = $\cot^4 \mathrm{A}$

4. a. cos A

   Explanation: $(\sec A + \tan A)(1 - \sin A)$

   $= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$

   $= [\frac{1+SinA}{CosA}] \times (1 - SinA) = \frac{1 - Sin^2 A}{CosA}$

   $= \frac{\cos^2 A}{\cos A} = \cos A$

   Hence, the correct choice is cos A

5. c. $\tan^2\theta + \sin^2\theta$

   Explanation: Given: $(\sec\theta + \cos\theta)(\sec\theta - \cos\theta)$

   = $\left(\sec^2\theta - \cos^2\theta\right)$

   = $\left(1 + \tan^2\theta - 1 + \sin^2\theta\right)$

   = $\left(\tan^2\theta + \sin^2\theta\right)$

6. sin (20° + θ) = cos 30°

   cos (90° - (20° + θ) = cos 30° [∵cos (90° - θ) = sin θ]

   cos (90° - 20° - θ) = cos 30°

cos (70° - θ) = cos 30°

70° - θ = 30°

70° - 30° = θ

40 ° = θ

Hence, the value of θ is 40°.

7. 4tan$^2\theta$ - $\frac{4}{\cos^2\theta}$
   $= 4\frac{\sin^2\theta}{\cos^2\theta} - \frac{4}{\cos^2\theta}$
   $= \frac{4\sin^2\theta - 4}{\cos^2\theta}$
   $= \frac{4(\sin^2\theta - 1)}{\cos^2\theta} = \frac{-4(1-\sin^2\theta)}{\cos^2\theta}$
   $= \frac{-4\times(\cos^2\theta)}{\cos^2\theta}$
   = -4

8. Given,
   $2\sin 3x = \sqrt{3}$
   $\Rightarrow \sin 3x = \frac{\sqrt{3}}{2}$
   $\Rightarrow \sin 3x = \sin 60^\circ \left[\because \sin 60^\circ = \frac{\sqrt{3}}{2}\right]$
   $\Rightarrow 3x = 60^\circ$
   $\Rightarrow x = \frac{60^\circ}{3} = 20^\circ$

9. We have,
   L.H.S. = $\frac{1}{(cosec\theta - \cot\theta)} = \frac{1}{(cosec\theta - \cot\theta)} \times \frac{(cosec\theta + \cot\theta)}{(cosec\theta + \cot\theta)}$
   $= \frac{(cosec\theta + \cot\theta)}{(cosec^2\theta - \cot^2\theta)} = cosec\theta + \cot\theta$ = R.H.S. [$\therefore$ cosec$^2\theta$ - cot$^2\theta$ = 1]
   $\therefore$ L.H.S. = R.H.S.

10. cos θ = $\frac{2}{3}$ $\therefore$ cos$^2\theta$ = $\frac{4}{9}$.
    Now, 4+4tan$^2\theta$ = 4(1+tan$^2\theta$) = 4sec$^2\theta$ = 4(1/cos$^2\theta$)
    =4 $\times$ $\frac{1}{4/9}$
    =9

11. $\sqrt{3}$ tan 2θ - 3 = 0
    $\Rightarrow$ $\sqrt{3}$ tan 2θ = 3

$\Rightarrow$ tan $2\theta = \frac{3}{\sqrt{3}}$

$\Rightarrow$ tan $2\theta = \sqrt{3}$ ...(i)

Also, tan 60° $= \sqrt{3}$ ...(ii)

$\therefore$ on equating (i) and (ii), we get

2θ = 60°

$\Rightarrow \theta$ = 30°

12. LHS = tan 1° tan 11° tan 21° tan 69° tan 79° tan 89°

= tan(90° - 89°) tan(90° - 79°) tan(90° - 69°) tan 69° tan 79° tan 89°

= cot 89° cot 79° cot 69°tan 69° tan 79° tan 89°

= (cot 89° tan 89°) (cot 79° tan 79°) (cot 69° tan 69°)

= $1 \times 1 \times 1$

= 1 =RHS.

13. We have,

$\sec 50^\circ \sin 40^\circ + \cos 40^\circ \cos ec 50^\circ$

$= \sec(90^\circ - 40^\circ) \sin 40^\circ + \cos 40^\circ \cos ec\,(90^\circ - 40^\circ)$

$= \cos ec 40^\circ \sin 40^\circ + \cos 40^\circ \sec 40^\circ \left[\because \begin{array}{c} \sec(90^\circ - \theta) = \cos ec\theta \\ \cos ec\,(90^\circ - \theta) = \sec\theta \end{array}\right]$

$= \frac{\sin 40^\circ}{\sin 40^\circ} + \frac{\cos 40^\circ}{\cos 40^\circ}$ = 1 + 1 = 2.

14. We have,

$\tan\theta + \frac{1}{\tan\theta} = 2$

Squaring both sides, we get

$\Rightarrow \left(\tan\theta + \frac{1}{\tan\theta}\right)^2 = 2^2$

$\Rightarrow \quad \tan^2\theta + \frac{1}{\tan^2\theta} + 2 \times \tan\theta \times \frac{1}{\tan\theta} = 4$

$\Rightarrow \quad \tan^2\theta + \frac{1}{\tan^2\theta} + 2 = 4$

$\Rightarrow \quad \tan^2\theta + \frac{1}{\tan^2\theta} = 2$

Alternate method, We have

$\tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow \quad \tan^2\theta + 1 = 2\tan\theta$

$\Rightarrow \quad \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow \quad (\tan\theta - 1)^2 = 0$

$\Rightarrow \quad \tan\theta = 1$

$\therefore \quad \tan^2\theta + \frac{1}{\tan^2\theta} = 1 + 1 = 2$

15. LHS $= \left(\tan\theta + \frac{1}{\cos\theta}\right)^2 + \left(\tan\theta - \frac{1}{\cos\theta}\right)^2$

$= \left(\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)^2 + \left(\frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta}\right)^2 \left(\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

$= \frac{(\sin\theta+1)^2}{\cos^2\theta} + \frac{(\sin\theta-1)^2}{\cos^2\theta}$

$= \frac{(\sin\theta+1)^2+(\sin\theta-1)^2}{\cos^2\theta}$

$= \frac{\sin^2\theta+1+2\sin\theta+\sin^2\theta+1-2\sin\theta}{\cos^2\theta}$

$= \frac{2\sin^2\theta+2}{\cos^2\theta}$

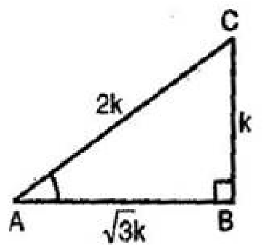
$= \frac{2(\sin^2\theta+1)}{1-\sin^2\theta} \left[\because \cos^2\theta = 1 - \sin^2\theta\right]$

$= 2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$

= RHS.

Hence proved

16. Consider a triangle ABC in which $\angle$ B $=90^\circ$



Let BC = k and AB = $\sqrt{3}k$

Then, using Pythagoras theorem,

AC = $\sqrt{(BC^2) + (AB)^2} = \sqrt{(k)^2 + (\sqrt{3}k)^2}$

$= \sqrt{k^2 + 3k^2} = \sqrt{4k^2} = 2k$

$\therefore$ sin A$= \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

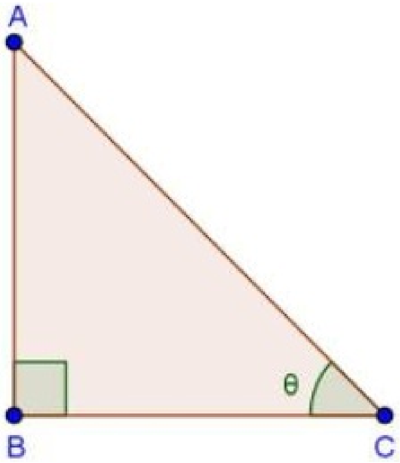
For $\angle$ C, Base = BC, Perpendicular = AB and Hypotenuse = AC

$\therefore \sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$cosC = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

sinA cosC + cosA sinC $= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4}$ = 1

17.



Given $\sin\theta = \frac{3}{5} = \frac{AB}{AC}$

Let $AB = 3K$

and, $AC = 5K$

In ΔABC, by Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$(3K)^2 + BC^2 = 25K^2$

$BC^2 = 25K^2 - 9K^2$

$BC^2 = 16K^2$

$BC = \sqrt{16K^2} = 4K$

$\therefore \cos\theta = \frac{BC}{AC} = \frac{4K}{5K} = \frac{4}{5}$

$\tan\theta = \frac{AB}{BC} = \frac{3K}{4K} = \frac{3}{4}$

$\cot\theta = \frac{BC}{AB} = \frac{4K}{3K} = \frac{4}{3}$

Now $\frac{\cos\theta - \frac{1}{\tan\theta}}{2\cot\theta}$

$= \frac{\frac{4}{5} - \frac{4}{3}}{2 \times \frac{4}{3}}$

$= \frac{\frac{12-20}{15}}{\frac{8}{3}}$

$= \frac{\frac{-8}{15}}{\frac{8}{3}}$

$= \frac{-8}{15} \times \frac{3}{8}$

$= \frac{-1}{5}$

18. = $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

= $\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\operatorname{cosec}\theta + \cos^2\theta + \sec^2\theta + 2\cos\theta\sec\theta$

= $sin^2\theta + cos^2\theta + cosec^2\theta + 2sin\theta \times \frac{1}{\sin\theta} + sec^2\theta + 2\cos\theta \times \frac{1}{\cos\theta}$

= 1 + 1 + $\cot^2\theta$ + 2 + 1+ $\tan^2\theta$ + 2

$= 1 + 1 + 2 + 1 + 2 + tan^2\theta + cot^2\theta$

= $7 + tan^2\theta + cot^2\theta$

L.H.S = R.H.S

19. LHS = $\sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} + \sqrt{\frac{\sec\theta + 1}{\sec\theta - 1}}$

Rationalise the denominator and we get,

$= \sqrt{\frac{(sec\theta - 1)^2}{sec^2\theta - 1}} + \sqrt{\frac{(sec\theta + 1)^2}{sec^2\theta - 1}}$

= $\frac{(\sec\theta - 1) + (\sec\theta + 1)}{\sqrt{(\sec\theta + 1)(\sec\theta - 1)}}$

= $\frac{2\sec\theta}{\sqrt{\sec^2\theta - 1}} = \frac{2\sec\theta}{\sqrt{\tan^2\theta}} = \frac{2\sec\theta}{\tan\theta}$

= $2 \times \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}$

= $2 \times \frac{1}{\sin\theta}$

= 2 cosecθ = RHS (Hence Proved)

20. $\frac{1 + \cos^2 A}{\sin^2 A} = 2\ cosec^2 A - 1$

L.H.S.

$= \frac{1 + \cos^2 A}{\sin^2 A} = \frac{1 + (1 - \sin^2 A)}{\sin^2 A} \left[\because 1 - \sin^2 A = \cos^2 A\right]$

$= \frac{2 - \sin^2 A}{\sin^2 A} = \frac{2}{\sin^2 A} - 1$

$= 2\ cosec^2 A - 1 \left[\because \frac{1}{\sin^2 A} = cosec^2 A\right]$

= R.H.S. (Hence Proved)