7. Alternating Current

7.1. Using a Rogowski loop (see Problem 5.30), one can measure the effective value I_{eff} of an alternating current flowing in a conductor. The loop has a rectangular cross section with N turns. The dimensions and the position of the loop are shown in the figure. Determine the effective emf generated in the loop by the alternating current.

7.2. The figure shows the vector diagram of reactances and resistances in an AC circuit. Construct a similar dia-

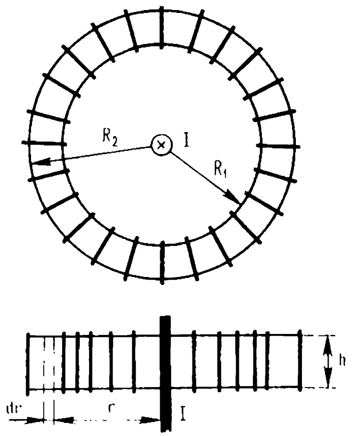

Fig. 7.1

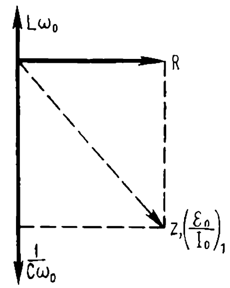

Fig. 7.2

gram for a circuit in which the current frequency is doubled and the emf amplitude is the same, and determine how the current will change as a result of this.

7.3. What is the frequency dependence of the current, of the phase shift between voltage and current, and of the consumed power for a circuit consisting of a resistance and an inductance connected in series provided that the emf amplitude remains constant?

7.4. What is the frequency dependence of the current, of the phase shift between current and voltage, and of the consumed power for a circuit consisting of a resistance and a capacitance connected in series provided that the emf amplitude remains constant?

7.5. A circuit (Figure (a)) contains an alternating emf, a resistance, and a reactive element (only a capacitance or only an inductance). What is this element if the time dependences of the current in the circuit and the emf of the source are those as shown in Figure (b)?

7.6. Are the readings of the ammeter *A3* equal to the sum of the readings of the ammeters *A1* and *A2* for the cases depicted in Figures (a) and (b)?

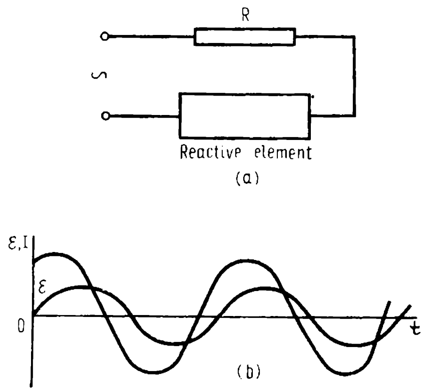

Fig. 7.5

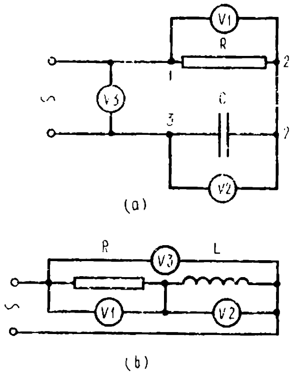

Fig. 7.7

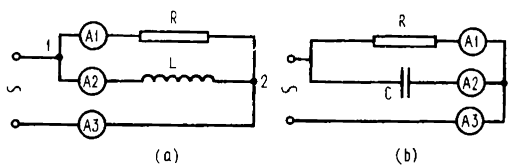

Fig. 7.6

7.7. Are the readings of the voltmeter *V3* equal to the sum of the readings of the voltmeters *V1* and *V2* for the cases depicted in Figures (a) and (b)?

7.8. The current flowing through the resistance in an AC circuit shown in Figure (a), where a resistance R, a capacitance C, and an inductance L are connected in series, is $I = \mathscr{E}/R$. What will be the current in the AC circuit when the inductance and the capacitance connected in parallel are connected in series with the resistance (Figure (b))?

7.9. The power in an AC circuit varies with time according to the curve in the figure. How, knowing the maximal and minimal values of the power, to determine the numerical value of the phase shift between voltage and

current? What is the period of variation of the power?
7.10. To demagnetize watches that have been accidentally magnetized, they are placed inside a solenoid connected to an AC source. The watches are then slowly removed from the solenoid. Explain why the watches become demagnetized as a result of such manipulations.

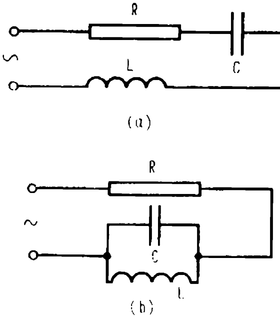

Fig. 7.8

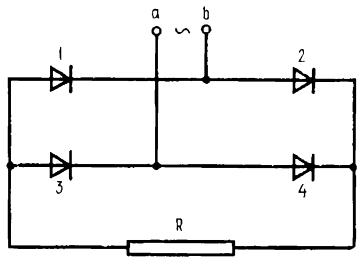

Fig. 7.11

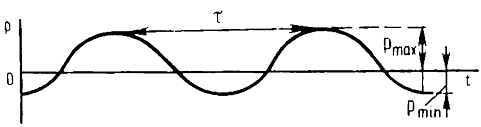

Fig. 7.9

7.11. A full-wave rectifier (the circuit is shown in the figure) rectifies the current that flows continuously in one direction. Sketch the time dependence of the current, ignoring all losses, and, assuming that the load of the rectifier constitutes a resistance, calculate the average value of the current. If the rectifier is loaded to a primary winding of a transformer, is a constant emf generated in the secondary winding?
7.12. In the circuit shown in the Figure, a capacitor of capacitance C is connected in parallel with a resistor R. How will this influence the time dependence of the current?
7.13. Two semiconductor diodes in opposition to each other in series are connected to the primary winding of a transformer. Draw the oscillograms of the current in the primary winding and of the emf generated in the secondary winding.

7.14. Two vacuum diodes in opposition to each other in parallel are connected to the primary winding of a transformer. The amplitude of the emf applied to the primary

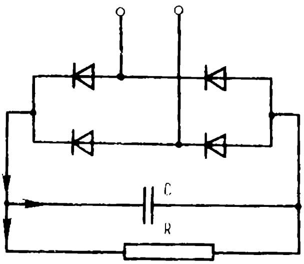

Fig. 7.12

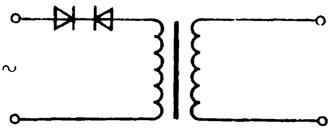

Fig. 7.13

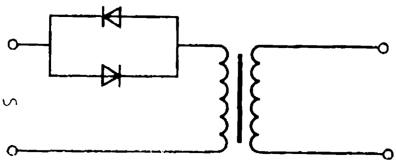

Fig. 7.14

winding exceeds considerably the voltage at which the diodes go into the saturation mode. Draw the oscillograms of the current in the primary winding and of the emf generated in the secondary winding.

7. Alternating Current

7.1. The segment of the cross section of the loop of width dr and height h is penetrated by a magnetic flux whose instantaneous value is

$$d\Phi = Bh\,dr,$$

where $B = \mu_0 I/2\pi r$. Whence

$$\Phi = \frac{\mu_0 h I}{2\pi} \int_{R_1}^{R_2} \frac{dr}{r} = \frac{\mu_0 h I}{2\pi} \ln \frac{R_2}{R_1}.$$

The flux coupled with the loop is

$$\Psi = \frac{\mu_0 h I N}{2\pi} \ln \frac{R_2}{R_1}.$$

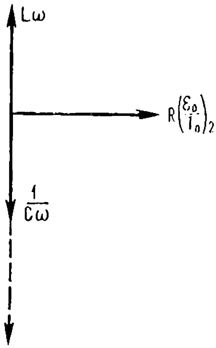

Fig. 7.2

The current in the conductor is $I = I_0 \cos \omega t$. The emf induced in the loop is

$$\mathscr{E}_i = -\frac{d\Psi}{dt} = \frac{\mu_0 h I_0 N \omega}{2\pi} \ln \frac{R_2}{R_1} \sin \omega t.$$

Finally, the effective value of this emf is

$$\mathscr{E}_{i\ \text{eff}} = \frac{\mu_0 h N \omega I_{\text{eff}}}{2\pi} \ln \frac{R_2}{R_1}.$$

7.2. The figure accompanying the problem shows that the capacitive reactance is four times the inductive reactance. If the frequency is doubled, the first quantity will decrease by a half and the second will double, which means they will become equal. As shown by the figure accompanying the answer, the ratio $\mathscr{E}_0/I_0$ will decrease, and since $\mathscr{E}_0$ must remain unchanged, the current grows. The same result can be obtained analytically. The amplitude of the current in the circuit is

$$I_0 = \frac{\mathscr{E}_0}{\sqrt{R^2 + \left(\frac{1}{C\omega} - L\omega\right)^2}}.$$

Prior to the change in frequency, $1/C\omega > L\omega$, and hence

$$\left(\frac{1}{C\omega} - L\omega\right)^2 > 0.$$

After the frequency is doubled, $1/C\omega = L\omega$. Here $I_0 = \mathscr{E}_0/R$.

7.3. The current in the circuit containing a resistance and an inductance connected in series is

$$I = I_0 \sin(\omega t + \varphi),$$

where the amplitude value of the current is

$$I_0 = \frac{\mathscr{E}_0}{R\cos\varphi - L\omega\sin\varphi}, \quad \text{or} \quad I_0 = \frac{\mathscr{E}_0}{\sqrt{R^2 + L^2\omega^2}},$$

and the tangent of the phase of the current with respect to the voltage is

$$\tan\varphi = -L\omega/R.$$

From these expressions it follows that as the frequency grows the lag of the current phase in relation to the voltage phase increases, which results in a decrease in the current. The average power in the circuit is defined thus:

$$P = \frac{1}{2}\mathscr{E}_0 I_0 \cos\varphi.$$

As the frequency grows, the amplitude of the current decreases and so does the power factor, which is the cosine of the phase shift between voltage and current. The power will also decrease as a result.

7.4. The current in the circuit containing a resistance and a capacitance connected in series is

$$I = I_0 \sin(\omega t + \varphi),$$

where the amplitude value of the current is

$$I_0 = \frac{\mathscr{E}_0}{R\cos\varphi + (1/C\omega)\sin\varphi}, \quad \text{or} \quad I_0 = \frac{\mathscr{E}_0}{\sqrt{R^2 + 1/C^2\omega^2}},$$

and the tangent of the phase shift of the current with respect to the voltage is

$$\tan \varphi = 1/RC\omega.$$

From these expressions it follows that as the frequency grows the phase shift by which the current leads the voltage decreases and tends to zero, while the current grows. The average power in the circuit, defined as

$$P = \frac{1}{2} \mathscr{E}_0 I_0 \cos \varphi,$$

increases with frequency, since $\cos \varphi$ tends to unity, and so does the amplitude value of the current.

7.5. The figure accompanying the problem shows that the current leads the voltage in the phase by $0 < \varphi < \pi/2$. This happens if a capacitance is connected in series with the resistance.

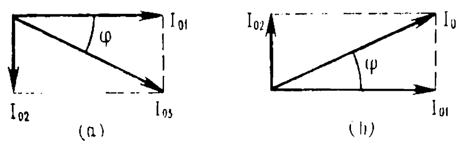

Fig. 7.6

7.6. For the case shown in Figure (a) accompanying the problem, we can write (if we ignore the resistances of the ammeters)

$$I_1 = \frac{U_0}{R} \sin \omega t = I_{01} \sin \omega t,$$

$$I_2 = \frac{U_0}{L\omega} \sin\left(\omega t - \frac{\pi}{2}\right) = -I_{02} \cos \omega t,$$

where U_0 is the amplitude value of the voltage between points *1* and *2*. The current I_3 flowing through ammeter *A3* is the sum of currents I_1 and I_2:

$$I_3 = I_{03} \sin(\omega t + \varphi)$$

(the vector diagram of currents is depicted in Figure (a) accompanying the answer). The amplitude value of current I_3 is

$$I_{03} = \sqrt{I_{01}^2 + I_{02}^2},$$

and the phase shift of the current in relation to the voltage is

$$\tan \varphi = -I_{02}/I_{01}.$$

Since the ammeters measure the effective value of the current, $I_{\text{eff}} = I_0/\sqrt{2}$, we have

$$I_{3\text{eff}} = \sqrt{I_{1\text{eff}}^2 + I_{2\text{eff}}^2}.$$

In the case shown in Figure (b) accompanying the problem, just like in the previous one, the currents that flow through the resistance and the capacitance differ in phase by $\pi/2$, the only difference being that here the current flowing through the capacitance leads the applied voltage, while the current flowing through the inductance lags behind the voltage. The corresponding vector diagram is depicted in Figure (b) accompanying the answer. The currents measured by ammeters *A1* and *A2* are

$$I_1 = (U_0/R) \sin \omega t = I_{01} \sin \omega t,$$

$$I_2 = U_0 C\omega \sin (\omega t + \pi/2) = I_{02} \cos \omega t.$$

The amplitude of the current measured by ammeter *A3* is

$$I_{03} = \sqrt{I_{01}^2 + I_{02}^2},$$

while the tangent of the phase shift is

$$\tan \varphi = I_{02}/I_{01}.$$

The current measured by ammeter *A3* is

$$I_{3\text{eff}} = \sqrt{I_{1\text{eff}}^2 + I_{2\text{eff}}^2} < I_{1\text{eff}} + I_{2\text{eff}}.$$

7.7. For the case depicted in Figure (a) accompanying the problem, the voltage between points *1* and *2* is

$$U_1 = I_0 R \sin \omega t = U_{01} \sin \omega t,$$

while that between points *2* and *3* is

$$U_2 = \frac{I_0}{C\omega} \sin \left(\omega t - \frac{\pi}{2}\right) = -\frac{I_0}{C\omega} \cos \omega t = -U_{02} \cos \omega t$$

(see the vector diagram in Figure (a) accompanying the answer). The voltage between points *1* and *3* is the sum of U_1 and U_2:

$$U_3 = U_1 + U_2 = U_{03} \sin (\omega t + \varphi).$$

Its amplitude value is

$$U_{03} = \sqrt{U_{01}^2 + U_{02}^2},$$

while the phase shift with respect to the applied voltage is given by the following formula:

$$\tan \varphi = -U_{02}/U_{01}.$$

Since the voltmeters measure the effective value $U_{eff} = U_0/\sqrt{2}$, we have

$$U_{3eff} = \sqrt{U_{1eff}^2 + U_{2eff}^2} < U_{1eff} + U_{2eff}.$$

For the circuit depicted in Figure (b) accompanying the problem, just like in the previous case, the voltages

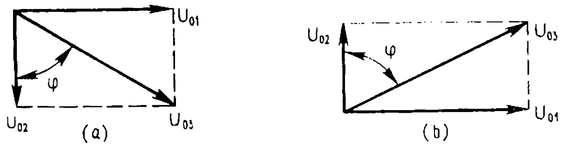

Fig. 7.7

across the resistance and the inductance differ in phase by $\pi/2$, the only difference being that here the current flowing through the inductance lags behind the voltage, while in the previous case the current flowing through the capacitance leads the voltage (and, hence, the phase shift between the voltages across the resistance and across the capacitance is $-\pi/2$). The respective voltages are

$$U_1 = I_0 R \sin \omega t = U_{01} \sin \omega t,$$

$$U_2 = I_0 L\omega \sin(\omega t + \pi/2) = U_{02} \cos \omega t$$

and

$$U_3 = U_3 \sin(\omega t + \varphi)$$

(see the vector diagram depicted in Figure (b) accompanying the answer). The amplitude value of the voltage is

$$U_{03} = \sqrt{U_{01}^2 + U_{02}^2}.$$

The effective voltages measured by the voltmeters are related thus:

$$U_{3eff} = \sqrt{U_{1eff}^2 + U_{2eff}^2} < U_{1eff} + U_{2eff}.$$

The tangent of the phase of the voltages is

$$\tan \varphi = U_{02}/U_{01}.$$

7.8. In the first case we have resonance, at which the voltages across the capacitor and the inductance,

$$U_C = \frac{I_0}{C\omega} \sin(\omega t - \pi/2) \text{ and}$$

$$U_L = I_0 L\omega \sin(\omega t + \pi/2), \qquad (7.8.1)$$

are equal in magnitude and opposite in phase. From Eq. (7.8.1) and the fact that a capacitor and an inductance

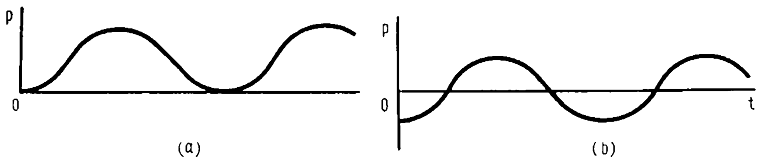

Fig. 7.9

connected in series do not change the current it follows that

$$1/C\omega = L\omega.$$

For the case where a capacitance and an inductance are connected in parallel, in each of these elements there flows a current

$$I_C = U_0 C\omega \sin(\omega t - \pi/2) \text{ and } I_L = \frac{U_0}{\omega L} \sin(\omega t + \pi/2).$$

The total current is

$$I = U_0 (C\omega - 1/L\omega) \cos \omega t,$$

and, since $C\omega = 1/L\omega$, we have

$$I = I_C + I_L = 0.$$

7.9. If the voltage varies according to the law

$$U = U_0 \sin \omega t$$

and there is a definite phase shift between voltage and current, so that

$$I = I_0 \sin(\omega t + \varphi)$$

(where the phase difference φ may be either positive or negative), then the instantaneous value of the power is

$$P = U_0 I_0 \sin(\omega t + \varphi) \sin \omega t.$$

If we write

$$\sin(\omega t + \varphi) \sin \omega t = \sin^2 \omega t \times \cos \varphi + \sin \omega t \times \cos \omega t \times \cos \varphi$$

$$= \frac{1}{2} [(1 - \cos 2\omega t) \cos \varphi + \sin 2\omega t \times \sin \varphi],$$

we get

$$P = \frac{1}{2} [\cos \varphi - \cos(2\omega t + \varphi)] U_0 I_0. \qquad (7.9.1)$$

The maximal value of the power is

$$P_{max} = \frac{1}{2} U_0 I_0 (\cos \varphi + 1),$$

while the minimal value is

$$P_{min} = \frac{1}{2} U_0 I_0 (\cos \varphi - 1).$$

Whence, the power factor is

$$\cos \varphi = \frac{P_{max} + P_{min}}{P_{max} - P_{min}}$$

(bear in mind that P_{min} is negative).

Formula (7.9.1) shows that the frequency of power variation is twice the frequency of the applied voltage. During one period of voltage variation the power passes twice through the maximum and the minimum.

Here are some particular cases.

(1) $\varphi = 0$. The load is a purely active resistance. In this case (Figure (a) accompanying the answer) $P_{min} = 0$ and $P_{max} = U_0 I_0$.

(2) $\varphi = \pm\pi/2$. The circuit contains only a reactive element, that is, a capacitance or an inductance. Since in this case $\cos \varphi = 0$, we have (see Figure (b) accompanying the answer)

$$P_{max} = -P_{min}.$$

The work performed by the AC source over one period of variation of the power is zero. This means that during one half of the period the energy flows from the AC source to the reactive element in the form of the electrostatic energy of the capacitance or the magnetic energy of the inductance, while during the other half the energy is returned to the AC source.

7.10. When a watch is inside the solenoid, the magnetic field generated by the solenoid forces the steel parts of the watch to change periodically their magnetization, following the hysteresis loop. When the watch is slowly removed from the solenoid, the magnetic field acting on the watch gradually decreases, and as the periods pass, the hysteresis loop shrinks. Each second 50 hysteresis loops are traversed, each being smaller than the previous one (the number "50" appears because the frequency of the AC source is usually 50 Hz). This process is roughly sketched in the figure. When the watch is completely removed from the magnetic field, it proves to be completely demagnetized.

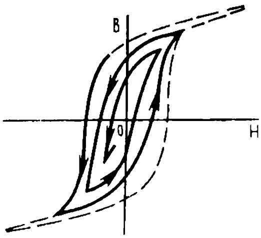

Fig. 7.10

7.11. At the moment when the "plus" of the voltage is at terminal a (see the figure accompanying the problem),

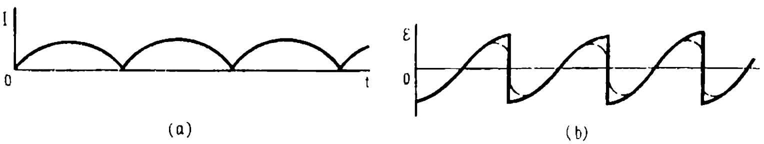

Fig. 7.11

the current passes through diode *2*, resistor R, diode *3*, and returns to the AC source through terminal b, which has the "minus" of the voltage at that moment. After the applied voltage changes sign, the current from terminal b passes through diode *4*, resistor R, diode *1*, and returns to the AC source via the negative terminal a. Thus, the current that passes through R consists of a series of alternating halves of sinusoids (Figure (a)). The average value of the current over one or any integral number of half-periods is

$$I_{av} = \frac{2I_0}{T\omega}\int_0^{T/2} \sin\omega t\, dt = \frac{4I_0}{T\omega} = \frac{2I_0}{\pi} \approx 0.637 I_0.$$

In carrying out this calculation in accordance with the conditions of the problem, it was assumed that the voltage drop across the diodes is negligible and that the rectification process does not alter the sinusoidal nature of the emf. As for the emf that is generated in the secondary winding of a transformer whose primary winding is the load resistance R, it must have two opposite symmetric sections, since each half-period of the pulsating current has an ascending section and a descending section. An idealized curve of the voltage in the secondary winding of a transformer is shown in Figure (b). Actually, the curve is much smoother because of the inductance of the transformer, which plays the role of a choke coil, the interturn capacitance, and other factors. The approximate shape of the voltage curve on the transient sections is depicted by a dashed curve.

7.12. After the rectifier the current branches out (see the arrows in the figure accompanying the problem). A

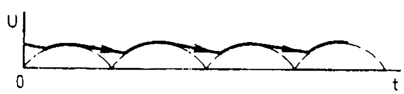

Fig. 7.12

fraction of the current flows through resistor R and a fraction is used to charge the capacitor. If the internal resistance of the source (together with the diodes) is low, then the voltage across the capacitor is equal to the voltage at the "out" terminals. This occurs as long as the voltage is lower than the maximum of the pulsating voltage. After the voltage passes the maximum, it falls off and becomes lower than the voltage across the capacitor. Because of this the capacitor will begin to discharge through the resistor, with the voltage across the capacitor decaying according to the law

$$U = U_0 \exp(-t/RC)$$

(the discharge current is designated by arrows in the figure accompanying the answer). The greater the capacitance, the slower the decay, which continues until the voltage across the capacitor becomes equal to the growing voltage in the following half-wave. Then the capacitor is charged to the maximum of the voltage anew. The process continues in this manner. Thus, a capacitor in the circuit makes the "out" voltage smoother, and the higher the capacitance the stronger the effect. The curve representing the time variation of the current flowing through the resistor follows the voltage curve in parallel.

7.13. For both directions of the emf applied to the transformer, the current is limited by the diode introduced into the circuit in the blocking direction. This current is

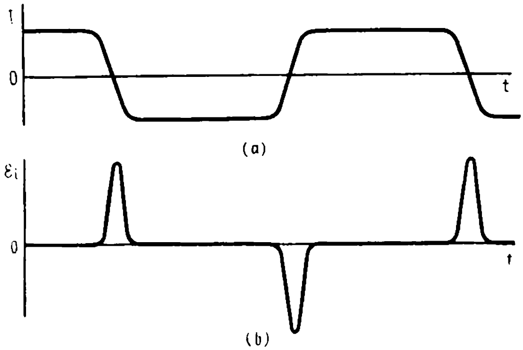

Fig. 7.13

caused by the motion of the minority (intrinsic) charge carriers and reaches a plateau very rapidly as the voltage is increased. The diode introduced in the conducting direction does not limit the current. For this reason, the oscillogram of the current in the primary circuit has the form shown in Figure (a). Accordingly, the greater fraction of time in each half-period (in each direction) the emf induced in the secondary winding is zero. Only over small time intervals when the current passes through zero does an emf emerge, first in one direction and then in the other (Figure (b)). The oscillograms here are, of course, only rough sketches, since they do not take into account the inductances in the transformer circuits. Note that in modern semiconductor diodes the reverse current is negligible, with the result that the problem is of purely academic interest.

7.14. In some respects this problem resembles the previous one. Here, too, the current in the primary circuit is limited to the saturation current in one of the diodes, introduced into the circuit in the conducting direction rather than in the blocking. In contrast to Problem 7.13, the present one possesses a special feature that manifests itself in the initial section near the zero of the current in the circuit. While in a semiconductor diode the current increases with voltage almost linearly in the initial section, in a vacuum diode the voltage dependence of the

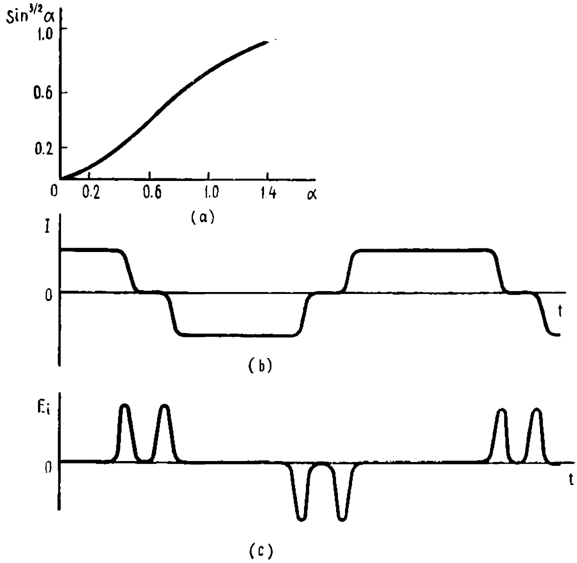

Fig. 7.14

current is described with sufficient accuracy by the three-halves power law $I = KU_0^{3/2}$.
The constant K incorporates universal constants and the distance between the electrodes in the diode. Since the voltage varies with time according to the sinusoidal law, the current flowing through the diode on the initial section of the voltage increase must be written in the form

$$I = KU_0^{3/2} \sin^{3/2} \omega t$$

(the function $f(\alpha) = \sin^{3/2} \alpha$ is depicted in Figure (a)). Allowing for this dependence, we obtain the oscillograms of current in the primary circuit (Figure (b)) and of the emf in the secondary circuit (Figure (c)). Just as in the previous problem, we have not allowed for the effects associated with the presence of inductances in the transformer circuits.