Short Answer Questions – II (PYQ)

Q. 1. (a) Using Bohr's second postulate of quantization of orbital angular momentum show that the circumference of the electron in the n^th orbital state in hydrogen atom is n times the de Broglie wavelength associated with it.
[CBSE (F) 2017]

(b) The electron in hydrogen atom is initially in the third excited state. What is the maximum number of spectral lines which can be emitted when it finally moves to the ground state?

OR

(a) State Bohr's quantization condition for defining stationary orbits. How does de Broglie hypothesis explain the stationary orbits?

(b) Find the relation between the three wavelengths λ_1, λ_2 and λ_3 from the energy level diagram shown below. [CBSE Delhi 2016]

Ans. (a) Only those orbits are stable for which the angular momentum of revolving electron is an integral multiple of $\left(\frac{h}{2\pi}\right)$ where h is the planck's constant.

According to Bohr's second postulate

$$\mathrm{mvr}_n = n\frac{h}{2\pi} \qquad \Rightarrow \qquad 2\pi r_n = \frac{\mathrm{nh}}{\mathrm{mv}}$$

$$\text{But } \frac{h}{\mathrm{mv}} = \frac{h}{p} = \lambda \quad \text{(By de Broglie hypothesis)}$$

$$\therefore \qquad 2\pi r_n = n\lambda$$

(b) For third excited state, $n = 4$

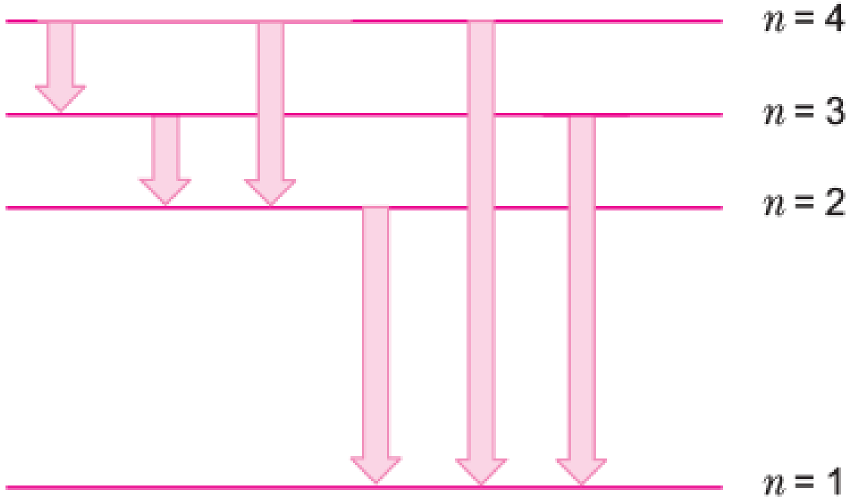
For ground state, $n = 1$

Hence possible transitions are

$n_i = 4$ to $n_f = 3, 2, 1$

$n_i = 3$ to $n_f = 2, 1$

$n_i = 2$ to $n_f = 1$



Total number of transitions = 6

$$E_C - E_B = \frac{hc}{\lambda_1} \qquad ...(1)$$

$$E_B - E_A = \frac{hc}{\lambda_2} \qquad ...(2)$$

$$E_C - E_A = \frac{hc}{\lambda_3} \qquad ...(3)$$

Adding (1) and (2), we have

$$E_C - E_A = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} \qquad ...(4)$$

From (3) and (4), we have

$$\frac{hc}{\lambda_3} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2} \quad \Rightarrow \quad \frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

$$\lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

Q. 2. Answer the following questions.

(i) State Bohr postulate of hydrogen atom that gives the relationship for the frequency of emitted photon in a transition.

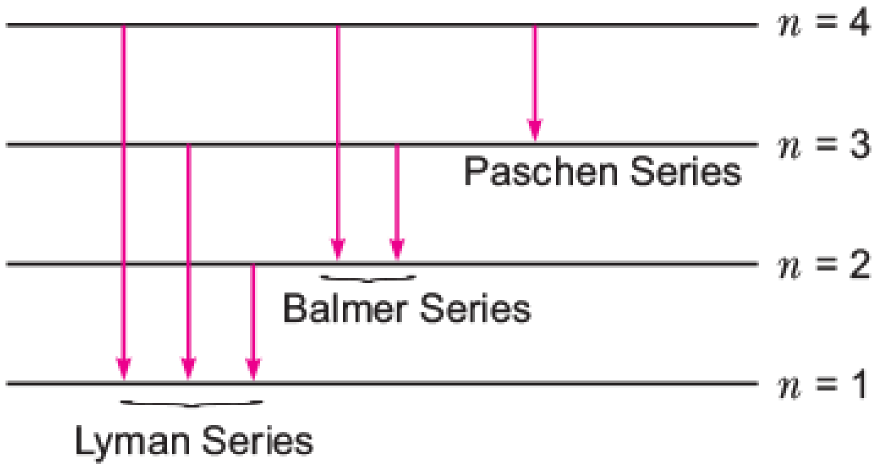
(ii) An electron jumps from fourth to first orbit in an atom. How many maximum number of spectral lines can be emitted by the atom? To which series these lines correspond? [CBSE (F) 2016]

Ans. (i) Bohr's third postulate: It states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states. The frequency of the emitted photon is given by

$h\nu = E_i - E_f$

Where E_i and E_f are the energies of the initial and final states and $E_i > E_f$.

(ii) Electron jumps from fourth to first orbit in an atom



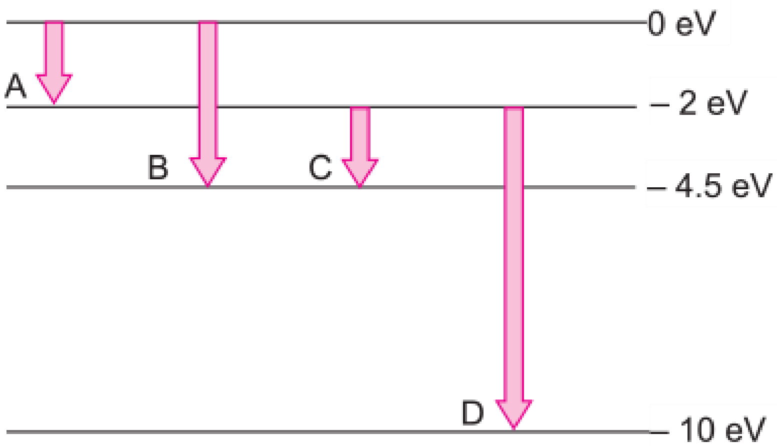
∴ Maximum number of spectral lines can be $^4C_2 = \frac{4!}{2!2!} = \frac{4\times3}{2} = 6$

In diagram, possible way in which electron can jump (above).

The line responds to Lyman series (e^- jumps to 1st orbit), Balmer series (e^- jumps to 2ndorbit), Paschen series (e^- jumps to 3rd orbit).

Q. 3. The energy levels of a hypothetical atom are shown below. Which of the shown transitions will result in the emission of a photon of wavelength 275 nm?

Which of these transitions correspond to emission of radiation of (i) maximum and (ii) minimum wavelength? [CBSE Delhi 2011]



Ans. Energy of photon wavelength 275 nm

$$E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{275 \times 10^{-9} \times 1.6 \times 10^{-19}} \text{ eV} = 4.5 \text{ eV}.$$

This corresponds to transition '*B*'.

i. $\Delta E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\Delta E}$ For maximum wavelength ΔE should be minimum. This corresponds to transition *A*.

ii. For minimum wavelength ΔE should be maximum. This corresponds to transition *D*.

Q. 4. The energy level diagram of an element is given below. Identify, by doing necessary calculations, which transition corresponds to the emission of a spectral line of wavelength 102.7 nm. [CBSE Delhi 2008]

Ans.

$$\Delta E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{102.7\times10^{-9}} J$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{102.7\times10^{-9}\times1.6\times10^{-19}} \ eV$$

$$= \frac{66\times3000}{1027\times16} = 12.04 \ eV$$

$$\text{Now, } \Delta E = |-13.6 - (-1.50)|$$

$$= 12.1 \ eV$$

Hence, transition shown by arrow D corresponds to emission of λ = 102.7 nm.

Q. 5. Using de Broglie's hypothesis, explain with the help of a suitable diagram, Bohr's second postulate of quantisation of energy levels in a hydrogen atom. [CBSE (AI) 2011, Patna 2015]

OR

Show mathematically how Bohr's postulate of quantisation of orbital angular momentum in hydrogen atom is explained by de-Broglie's hypothesis. [CBSE East 2016]

Ans. According to de Broglie's hypothesis,

$$\lambda = \frac{h}{mv} \qquad ...(i)$$

According to de Broglie's condition of stationary orbits, the stationary orbits are those which contain complete de Broglie wavelength.

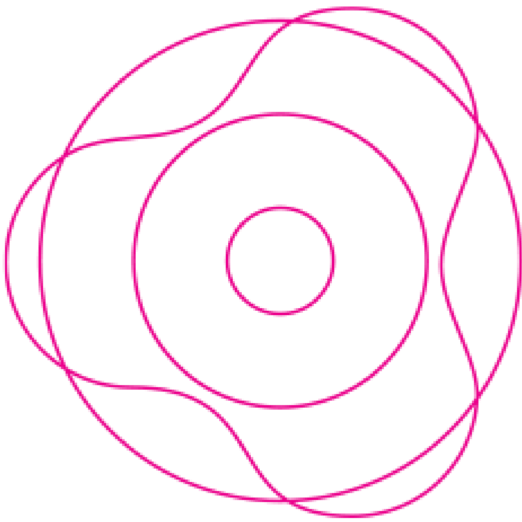
$$2\pi r = n\lambda \qquad ...(ii)$$

Substituting value of λ from (ii) in (i), we get

$$2\pi r = n\frac{h}{\mathrm{mv}}$$

$$\Rightarrow \qquad \mathrm{mvr} = n\frac{h}{2\pi} \qquad \ldots(iii)$$

This is Bohr's postulate of quantisation of energy levels.



Q. 6. Determine the distance of closest approach when an alpha particle of kinetic energy 4.5 MeV strikes a nucleus of Z = 80, stops and reverses its direction. [CBSE Ajmer 2015]

Ans. Let r be the centre to centre distance between the alpha particle and the nucleus (Z = 80). When the alpha particle is at the stopping point, then

$$K = \frac{1}{4\pi\varepsilon_0}\frac{(\mathrm{Ze})(2e)}{r}$$

or $$r = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\,\mathrm{Ze}^2}{K}$$

$$= \frac{9\times10^9\times2\times80\ e^2}{4.5\ \mathrm{MeV}} = \frac{9\times10^9\times2\times80\times(1.6\times10^{-19})^2}{4.5\times10^6\ \times\ 1.6\times10^{-19}\ J}$$

$$= \frac{9\times160\times1.6}{4.5}\times10^{-16} = 512\times10^{-16}m$$

$$= 5.12\times10^{-14}\ \mathrm{m}$$

Q. 7. A 12.3 eV electron beam is used to bombard gaseous hydrogen at room temperature. Upto which energy level the hydrogen atoms would be excited?

Calculate the wavelengths of the second member of Lyman series and second member of Balmer series. [CBSE Delhi 2014]

Ans. The energy of electron in the nth orbit of hydrogen atom is

$$E_n = -\frac{13.6}{n^2}\ \mathrm{eV}$$

When the incident beam of energy 12.3 eV is absorbed by hydrogen atom. Let the electron jump from n = 1 to n = n level.

$E = E_n - E_1$

$$12.3 = -\frac{13.6}{n^2} - \left(-\frac{13.6}{1^2}\right)$$

$$\Rightarrow \quad 12.3 = 13.6\left[1 - \frac{1}{n^2}\right] \quad \Rightarrow \quad \frac{12.3}{13.6} = 1 - \frac{1}{n^2}$$

$$\Rightarrow \quad 0.9 = 1 - \frac{1}{n^2} \quad \Rightarrow \quad n^2 = 10 \quad \Rightarrow \quad n = 3$$

That is the hydrogen atom would be excited upto second excited state.

For Lyman Series

$$\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$\Rightarrow \quad \frac{1}{\lambda} = 1.097 \times 10^7 \left[\frac{1}{1} - \frac{1}{9}\right] \quad \Rightarrow \quad \frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{8}{9}$$

$$\Rightarrow \quad \lambda = \frac{9}{8 \times 1.097 \times 10^7} = 1.025 \times 10^{-7} = 102.5\,\text{mm}$$

For Balmer Series

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left[\frac{1}{4} - \frac{1}{16}\right] \quad \Rightarrow \quad \frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{3}{16}$$

$$\Rightarrow \lambda = 4.86 \times 10^{-7}\,\text{m} \quad \Rightarrow \quad \lambda = 486\text{ nm}$$

Q. 8. The ground state energy of hydrogen atom is – 13.6 eV. If an electron makes a transition from an energy level – 1.51 eV to – 3.4 eV, calculate the wavelength of the spectral line emitted and name the series of hydrogen spectrum to which it belongs.
[CBSE (AI) 2017]

Ans. Energy difference = Energy of emitted photon

= $E_2 - E_1$

= – 1.51 – (–3.4) = 1.89 eV = 1.89 × 1.6 × 10^{-19} J

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left[\frac{1}{4} - \frac{1}{16}\right] \quad \Rightarrow \quad \frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{3}{16}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.89 \times 1.6 \times 10^{-19}} = \frac{19.8}{3.024} \times 10^{-7}$$

$$= 6.548 \times 10^{-7}\text{ m} = 6548\text{ Å}$$

This wavelength belongs to Balmer series of hydrogen spectrum.

Q. 9. A hydrogen atom initially in its ground state absorbs a photon and is in the excited state with energy 12.5 eV. Calculate the longest wavelength of the radiation emitted and identify the series to which it belongs.

[Take Rydberg constant R = 1.1 × 10^7 m^{-1}]   [CBSE East 2016]

Ans. Let n_i and n_f are the quantum numbers of initial and final states, then we have

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

The energy of the incident photon = 12.5 eV.

Energy of ground state = –13.6 eV

∴ Energy after absorption of photon can be –1.1 eV.

This means that electron can go to the excited state n_i =3. It emits photon of maximum wavelength on going to n_f =2, therefore,

$$\frac{1}{\lambda_{max}} = \left\{\frac{1}{2^2} - \frac{1}{3^2}\right\}R$$

$$\lambda_{max} = \frac{36}{5R} = \frac{36}{5\times1.1\times10^7} = 6.555 \times 10^{-7}\ \text{m} = 6555\ \text{Å}$$

It belongs to Balmer Series.

Q. 10. The short wavelength limit for the Lyman series of the hydrogen spectrum is 913.4 Å. Calculate the short wavelength limit for Balmer series of the hydrogen spectrum.   [CBSE (AI) 2017]

Ans.

$$\lambda_{max} = \frac{36}{5R} = \frac{36}{5 \times 1.1 \times 10^7}$$

$$\lambda_L = \frac{1}{R} = 913.4 \text{ Å}$$

For short wavelength of Lyman series, $n_1 = 1$, $n_2 = \infty$

$$\therefore \quad \frac{1}{\lambda_L} = R\left(\frac{1}{1^2} - \frac{1}{\infty}\right) = R$$

For short wavelength of Balmer series, $n_1 = 2$, $n_2 = \infty$

$$\lambda_L = \frac{1}{R} = 913.4 \text{ Å}$$

$$\therefore \quad \lambda_B = \frac{4}{R} = 4 \times 913.4 \text{ Å} = 3653.6 \text{ Å}$$

Q. 11. A 12.5 eV electron beam is used to excite a gaseous hydrogen atom at room temperature. Determine the wave lengths and the corresponding series of the lines emitted. [CBSE (AI) 2017]

Ans. It is given that the energy of the electron beam used to bombard gaseous hydrogen at room temperature is 12.5 eV.

Also, the energy of the gaseous hydrogen in its ground state at room temperature is –13.6 eV.

When gaseous hydrogen is bombarded with an electron beam, the energy of the gaseous hydrogen becomes –13.6 + 12.5 eV = –1.1 eV.

Orbital energy related to orbit level (n) is

$$E = \frac{-13.6}{(n)^2} \text{eV}$$

For $n = 3$,

$$E = \frac{-13.6}{(3)^2} \text{ eV} = \frac{-13.6}{9} \text{ eV} = -1.5 \text{ eV}$$

This energy is approximately equal to the energy of gaseous hydrogen.

This implies that the electron has jumped from n = 1 to n = 3 level.

During its de-excitation, electrons can jump from n = 3 to n = 1 directly, which forms a line of the Lyman series of the hydrogen spectrum.

Relation for wave number for the Lyman series is

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$$

For first member $n = 3$

$$\therefore \quad \frac{1}{\lambda_1} = R\left[\frac{1}{1^2} - \frac{1}{(3)^2}\right] = R\left[\frac{1}{1} - \frac{1}{9}\right]$$

$$\therefore \quad \frac{1}{\lambda_1} = 1.097 \times 10^7 \left[\frac{9-1}{9}\right] \quad \text{(where Rydberg constant } R = 1.097 \times 10^7 \text{ m}^{-1}\text{)}$$

$$\therefore \quad \frac{1}{\lambda_1} = 1.097 \times 10^7 \times \frac{8}{9} \quad \Rightarrow \quad \lambda_1 = 1.025 \times 10^{-7} \text{ m}$$

For $n = 3$,

$$\therefore \quad \frac{1}{\lambda_2} = R\left[\frac{1}{1^2} - \frac{1}{(2)^2}\right] = R\left[\frac{1}{1} - \frac{1}{4}\right]$$

$$\therefore \quad \frac{1}{\lambda_2} = 1.097 \times 10^7 \left[\frac{4-1}{4}\right] \quad \text{(where Rydberg constant } R = 1.097 \times 10^7 \text{ m}^{-1}\text{)}$$

$$\therefore \quad \frac{1}{\lambda_2} = 1.097 \times 10^7 \times \frac{3}{4} \quad \Rightarrow \quad \lambda_2 = 1.215 \times 10^7 \text{ m}$$

Relation for wave number for the Balmer series is

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$$

For first member, $n = 3$

$$\therefore \quad \frac{1}{\lambda_3} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = 1.097 \times 10^7 \times \left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\Rightarrow \quad \lambda_3 = 6.56 \times 10^{-7} \text{ m}$$

Short Answer Questions – II (OIQ)

Q. 1. Calculate the ratio of energies of photons produced due to transition of electron of hydrogen atom from its

(i) Second permitted energy level to the first level, and
(ii) Highest permitted energy level to the second permitted level.

Ans. (i)

Energy of electron in permitted level $E_n = \frac{\text{Rhc}}{n^2}$

When an electron jumps from the second to the first permitted energy level,

Energy of photon = $E_{2-1} = \text{Rhc}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}\text{Rhc}$

(ii) When an electron jumps from the highest permitted level ($n = \infty$) to the second permitted level (n=2)

$$E_{\infty-2} = \text{Rhc}\left(\frac{1}{2^2} - \frac{1}{\infty}\right) = \frac{\text{Rhc}}{4}$$

$$\therefore \quad \text{Ratio } \frac{E_{2-1}}{E_{\infty-2}} = \frac{3\,\text{Rhc}/4}{\text{Rhc}/4} = \frac{3}{1};\ \text{Ratio} = 3:1$$

Q. 2. The spectrum of a star in the visible and the ultraviolet region was observed and the wavelength of some of the lines that could be identified were found to be:

824 Å, 970 Å, 1120 Å, 2504 Å, 5173 Å, 6100 Å

Which of these lines cannot belong to hydrogen atom spectrum? (Given Rydberg constant R = 1.03 × 10^7 m^{-1} and $\frac{1}{R}$ = 970 Å. Support your answer with suitable calculations.

Ans. For hydrogen atom, the wave number (i.e., reciprocal of wavelength) of the emitted radiation is given by

$$\bar{\nu} = \frac{1}{\lambda} = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

$$\therefore \qquad \lambda = \frac{\frac{1}{R}}{\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)} = \frac{970Å}{\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)}$$

For Lyman series of hydrogen spectrum, we take n_2=1. Hence the permitted values of λ can be given as:

$$\lambda = \frac{970Å}{3/4}, \frac{970Å}{8/9}, \frac{970Å}{15/16} \cdots\cdots\cdots \frac{970Å}{1} \qquad (\text{taking } n_1 = 2, 3, 4, \ldots\ldots\ \infty)$$

= 1293.3 Å, 1091 Å, 1034.6 Å, ... 970 Å

For Balmer series of hydrogen spectrum, we take n_2 = 2. Hence the possible values of λ can be given as:

$$\lambda = \frac{970Å}{5/36}, \frac{970Å}{3/16}, \frac{970Å}{21/100} \cdots\cdots\cdots \frac{970Å}{1/4} \qquad (\text{taking } n_1 = 3, 4, 5, \ldots\ldots\ \infty)$$

=698 Å, 5173.3 Å, 4619 Å, ... 3880 Å

Hence λ = 824 Å, 1120 Å, 2504 Å, 6100 Å, of the given lines, cannot belong to the hydrogen atom spectrum.