Surface Areas and Volumes

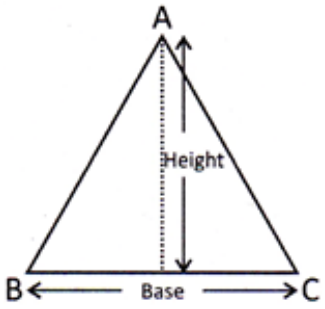
Olympiad Comprehensive Book

NOTES

In this chapter, we will learn about some important formulas related to 2-D and 3-D geometrical shapes.

Area of a Triangle

- Area of a triangle $= \frac{1}{2} \times (\text{Perpendicular}) \times \text{Base}$
- Area of a triangle having lengths of the sides a, b and c is $= \sqrt{s(s-a)(s-b)(s-c)}$ sq. units, where $s = \frac{1}{2}(a+b+c)$



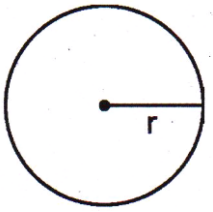
- Area of an equilateral triangle $= \frac{\sqrt{3}}{4}a^2$, where a is the side of the equilateral triangle.

Circle

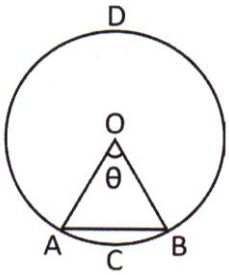
- Circumference of the circle $= 2\pi r$
- Area of the circle $= \pi r^2$
- Area of the semicircle $= \frac{1}{2}\pi r^2$
- Perimeter of the semicircle $= \pi r + 2r$

Length of Arc and Area of a Sector

Let an arc AB an angle 0 < 180° at the center (O) of a circle a^4 radius; Then we have:



- Length of the arc $AB = \frac{2\pi\theta}{360°}$
- Area of the sector $OACB = \frac{\pi r^2 \theta}{360°}$

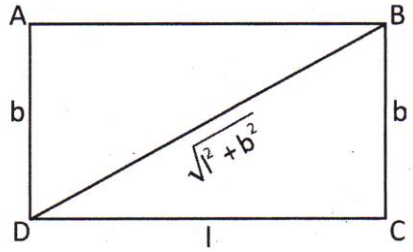


- Area of the minor segments ACBA = area of sector OACB − of the corresponding triangle AOB
- Area of the major segment ADBA = area of the circle-area of the minor segment

Perimeter and Area of a Rectangle

Let ABCD be a rectangle in which length $AB = 1$ units, breadth $BC = b$ units then we have:

- $Area = (l \times b)$ *square units*
- $Length\ (l) = \dfrac{area\ (A)}{breadth\ (B)} units$



- $breadth\ (b) = \dfrac{area\ (A)}{length\ (l)} units$
- Diagonal (d) $= \sqrt{l^2 + b^2}$ *units*
- Perimeter $(p) = 2(l + b)$ units

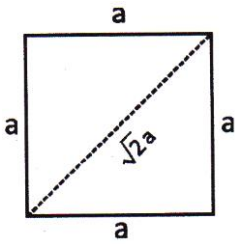
Area of Four Walls of a Room

Let I, b and h are respectively the length/ breadth and height of a room, then area of four walls of the room = $\{2(l+b)\times h\}$ *sq units.*

Perimeter and Area of Square

Let ABCD be a square with each side equal to 'a' units, then

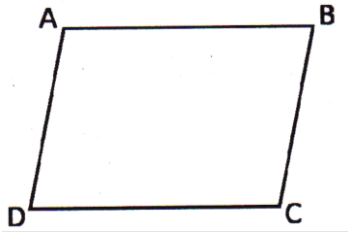
- Area = a^2 sq. units
- $Area = \left(\dfrac{1}{2} \times (diagonal)^2\right)$ *sq. units*



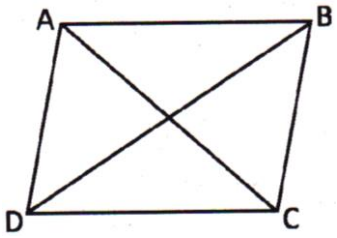
- $Diagonal = a\sqrt{2}$ units
- Perimeter = 4a units

Area of Some Special Types of Quadrilateral

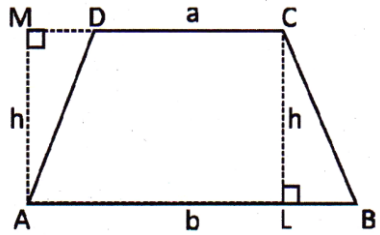
- Area of a parallelogram = (base × height)



- Area of a rhombus $= \frac{1}{2} \times (\text{product of diagonal})$



- Area of a Trapezium $= \frac{1}{2}$(Sum of length of parallel sides) × (distance between then) $= \frac{1}{2}(a+b) \times h$



Solids

The objects having definite shape and size are called solids. A solid occupies a definite space.

Cuboid

For a cuboid of length = l, breadth = b and height = h, we have:

- Volume = $(l \times b \times h)$ cubic units

- Total surface area = $2(lb+bh+lh)$ sq. units
- Lateral surface area $=\left[2(l\ +\ b)\ x\ h\right]$ *sq. units*
- Diagonal of a cuboid = $\sqrt{l^2+b^2+h^2}$

Cube

For a cube having each edge = a units, we have:

- Volume = a^3 cubic units
- Total surface area = $6a^2$ sq. units
- Lateral surface area = $4a^2$ sq. units
- Diagonal of a cube = $a\sqrt{3}$

Cylinder

Solids like jar, circular pencils, circular pipes, road rollers, gas cylinders are of cylindrical shape. For a cylinder of base radius = r units and height = h units, we have:

- Volume = $\pi r^2 h$ cubic units
- Curved surface area = $2\pi rh$ square units
- Total surface area = $(2\pi rh+2\pi r^2)=\ 2\pi r(h+r)$ *sq. units*

Cone

Consider a cone in which base radius = r, height = h and slant height $(l)=\sqrt{h^2+r^2}$, then we have:

- Volume of the cone $\frac{1}{3}\pi r^2 h$
- Curved surface area of the cone = πrl
- Total surface area of the cone $=(curved\ surface\ area)+(area\ of\ the\ base)=\pi rl+\pi r^2=\pi r(l+r)$

Sphere

Objects like a football, a cricket ball, etc. are of spherical shapes. For a sphere of radius r, we have:

- Volume of the sphere $=\frac{4}{3}\pi r^3$
- Surface area of the sphere = $4\pi r^3$

Hemisphere

A plane through the centre of a sphere cuts it into two equal parts, each part is called a hemisphere. For a hemisphere of radius r, we have:

- Volume of the hemisphere $= \frac{2}{3}\pi r^3$
- Curved surface area of the hemisphere $= 2\pi r^2$
- Total surface area of the hemisphere $= 3\pi r^2$

- **Example:**

Find the area of the triangle whose base is 25 cm and height is 10.8 cm.

(a) 125 cm^2 (b) 135 cm^2

(c) 124 cm^2 (d) 199 cm^2

(e) None of these

Ans. (b)

Explanation: Area of the given triangle $= \left(\frac{1}{2} \times 25 \times 10.8\right) cm^2 = 135\ cm^2$

- **Example:**

A chord of a circle of radius 14 cm makes a right angle at the centre. Find the area of the major sector of the circle.

(a) 590 cm^2 (b) 462 cm^2

(c) 595 cm^2 (d) 995 cm^2

(e) None of these

Ans. (b)

Explanation: Area of the major sector $\frac{270}{360^\circ} \times \pi r^2$

$= \frac{270^\circ}{360^\circ} \times \frac{22}{7} \times 14 \times 14 = 462\ \text{cm}^2$

- **Example:**

The length of a rectangular plot of land is twice its breadth. If the perimeter of the plot is 210 m, then find its area.

(a) 2450 m^2 (b) 2251 m^2

(c) 5560 m^2 (d) 9060 m^2

(e) None of these

Ans. (a)

Explanation: Let x metre be the breadth of the triangle, then its length will be 2x metre.

Now, $2(x + 2x) = 210 \Rightarrow x = 35$

$Area = 35 \times 70 = 2450\ m^2$