Exercise 12.5

Answer 1E.

(a)

The given statement is **true**.

Because, the directional vectors of two lines are parallel to the directional vector of the third line, so these two directional vectors are each scalar multiple of the third vector, therefore, these two directional vectors are also scalar multiple of each other. So these two lines are also parallel to each other.

(b)

The given statement is **false**.

In three-dimensional Cartesian coordinate system, the axes *x* and *y* are perpendicular to the *z*-axis, but they are perpendicular to each other (not parallel).

(c)

The given statement is **true**.

Because normal vectors of two planes are parallel to the normal vector of the third plane, so these two normal vectors are each scalar multiple of the third vector, therefore, these two normal vectors are also scalar multiple of each other. So these two lines are also parallel to each other.

(d)

The given statement is **false**.

In three-dimensional Cartesian coordinate system, the planes *xy* and *yz* are perpendicular to *xz* plane, but not parallel to each other.

(e)

The given statement is **false**.

This is same as we discussed in part(d) , in our three coordinate system, *x* and *y* axes are parallel to the plane $z=1$, but they are not parallel to each other.

(f)

The given statement is true.

Because if two lines are perpendicular to the plane then directional vectors of these two lines would be parallel to the normal vector of the plane, therefore both directional vectors would be parallel to each other and so lines would be parallel.

(g)

The given statement is false.

Since the planes $y = 2$ and $z = 2$ are parallel to $x-$axis, but not parallel to each other.

(h)

The given statement is true.

If two planes are perpendicular to a line then normal vectors of the planes would be parallel to the directional vector of the line and so both normal vectors would be parallel to each other and hence those two planes would be parallel to each other.

(i)

The given statement is true.

Recall that if two planes are not parallel, then they intersect in a straight line and the angle between the two planes is defined as the acute angle between their normal vectors

(j)

The given statement is false.

If two lines are not parallel and not perpendicular to each other then they can be skew lines

(k)

The given statement is true.

If line and the plane are parallel then the normal vector of the plane and directional vector of the line are perpendicular to each other, otherwise these vectors meet an angle. Therefore, the line would intersect the plane.

Answer 2E.

We want to find line that goes through the point (6,-5,2) and that it parallel to the vector <1,3,-2/3>.

Here, $r_0 = <6, -5, 2> = 6\vec{i} - 5\vec{j} + 2\vec{k}$

And,

$\vec{v} = \vec{i} + 3\vec{j} - \frac{2}{3}\vec{k}$ So the vector equation becomes:

$$\vec{r} = \left(6\vec{i} - 5\vec{j} + 2\vec{k}\right) + t\left(\vec{i} + 3\vec{j} - \frac{2}{3}\vec{k}\right)$$

$$\vec{r} = \left(6 + t\right)\vec{i} + \left(-5 + 3t\right)\vec{j} + \left(2 - \frac{2}{3}t\right)\vec{k}$$

Or, the parametric equations are:

$$x = 6 + t,\ y = -5 + 3t,\ z = 2 - \frac{2}{3}t$$

Answer 3E.

Consider the line through the point $(2, 2.4, 3.5)$ and parallel to the vector $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

The objective is to find a vector equation and parametric equations for the line.

Write the formula to find the vector equation to the line passing through point $\mathbf{r}_0$ and parallel to the vector $\mathbf{v}$ is,

$$\mathbf{r} = \mathbf{r}_0 + t\,\mathbf{v}$$

Here,

$$\mathbf{r}_0 = 2\mathbf{i} + 2.4\mathbf{j} + 3.5\mathbf{k}$$
$$\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$$

Use this formula to find the vector equation to the line passing through the point $(2, 2.4, 3.5)$ and parallel to the vector $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$$
$$\mathbf{r} = 2\mathbf{i} + 2.4\mathbf{j} + 3.5\mathbf{k} + (3\mathbf{i} + 2\mathbf{j} - \mathbf{k})t$$
$$\mathbf{r} = (2 + 3t)\mathbf{i} + (2.4 + 2t)\mathbf{j} + (3.5 - t)\mathbf{k}$$

Hence, the vector equation of the required line is $\boxed{\mathbf{r} = (2 + 3t)\mathbf{i} + (2.4 + 2t)\mathbf{j} + (3.5 - t)\mathbf{k}}$.

To find the parametric equations for the line, simply look at each component of the vector equation $\mathbf{r} = (2+3t)\mathbf{i} + (2.4+2t)\mathbf{j} + (3.5-t)\mathbf{k}$.

Therefore, the parametric equations for the line is,

$$\boxed{\begin{aligned} x &= 2+3t \\ y &= 2.4+2t \\ z &= 3.5-t \end{aligned}}.$$

Answer 4E.

Vector Equation takes the form: $r = r_o + \mathrm{v}t$

r_o =the vector of the point from origin

v= the direction vector

Since the line is parallel, it would go the same direction as the line given, so simply take out the vector from the parametric equation given:

since the formula for parametric equation are:

X(t)=X+ta

Y(t)=Y+tb

Z(t)=Z+tc

we have:

v=<2,-3,9>

r_o = <0,14,-10>

So:

$$r(t) = <0,\ 14,\ -10> + <2,\ -3,\ 9> t$$

$r(t) = \ <2t, 14-3t, \ -10+9t>$ (ANSWER)

for parametric, use the formula given previously:

$x(t) = 2t$

$y(t) = 14-3t$

$z(t) = \ -10+9t$

(ANSWER)

Answer 5E.

A line which is perpendicular to the plane x + 3y+z = 5 is parallel to the normal vector $\hat{i}+3\hat{j}+\hat{k}$ of this plane.

Here $\vec{r_0} = \langle 1,0,6\rangle = \hat{i}+o\hat{j}+6\hat{k}$

And $\vec{v} = \hat{i}+3\hat{j}+\hat{k}$

Then the vector equation of line is

$$\vec{r} = \vec{r_0}+t\vec{v}$$
$$= \left(\hat{i}+o\hat{j}+6\hat{k}\right)+t\left(\hat{i}+3\hat{j}+\hat{k}\right)$$
$$= (1+t)\hat{i}+3t\hat{j}+(6+t)\hat{k}$$

The parametric equations are

$$x = 1+t, \quad y = 3t, \quad z = 6+t$$

Answer 6E.

Let P (0, 0, 0) and $Q(4, 3, -1)$ be two points on the line. We first need to find the direction vector $\mathbf{u} = \langle a, b, c\rangle$ for the line.

$$\mathbf{v} = \overline{PQ}$$
$$= \langle 4-0, 3-0, -1-0\rangle$$
$$= \langle 4, 3, -1\rangle$$

The parametric equations of a line in space parallel to a nonzero vector $\mathbf{v} = \langle a, b, c\rangle$ and passing through the point $P(x_1, y_1, z_1)$ are $x = x_1 + at$, $y = y_1 + bt$, and $z = z_1 + ct$. The numbers a, b, and c are called direction numbers.

We have $x_1 = 0$, $y_1 = 0$, and $z_1 = 0$. The direction numbers are $a = 4$, $b = 3$, and $c = -1$.

Thus, the parametric equations are $\boxed{x = 4t, y = 3t, \text{ and } z = -t}$.

The symmetric equations of the line is obtained by eliminating the parameter t and is given by $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$.

$$\frac{x-0}{4}=\frac{y-0}{3}=\frac{z-0}{-1}$$

$$\frac{x}{4}=\frac{y}{3}=-\frac{z}{1}$$

Therefore, the symmetric equations are $\boxed{\frac{x}{4}=\frac{y}{3}=-z}$.

Answer 7E.

(a) Let $P\left\langle 0, \frac{1}{2}, 1\right\rangle$ and $Q(2, 1, -3)$ be two points on the line. We first need to find the direction vector $\mathbf{u}=\langle a, b, c\rangle$ for the line.

$$\begin{aligned}\mathbf{v} &= \overrightarrow{PQ}\\ &= \left\langle 2-0, 1-\frac{1}{2}, -3-1\right\rangle\\ &= \left\langle 2, \frac{1}{2}, -4\right\rangle\end{aligned}$$

The parametric equations of a line in space parallel to a nonzero vector $\mathbf{v}=\langle a, b, c\rangle$ and passing through the point $P(x_1, y_1, z_1)$ are $x=x_1+at$, $y=y_1+bt$, and $z=z_1+ct$. The numbers a, b, and c are called direction numbers.

We have $x_1=2$, $y_1=1$, and $z_1=-3$. The direction numbers are $a=2$, $b=\frac{1}{2}$, and $c=-4$.

Thus, the parametric equations are $\boxed{x=2+2t,\ y=1+\frac{1}{2}t, \text{ and } z=-3-4t.}$

(b) The symmetric equations of the line is obtained by eliminating the parameter t and is given by $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$.

$$\frac{x-2}{2}=\frac{y-1}{\frac{1}{2}}=\frac{z+3}{-4}$$

$$\frac{x-2}{2}=2y-2=\frac{z+3}{-4}$$

Therefore, the symmetric equations are $\boxed{\frac{x-2}{2}=2y-2=\frac{z+3}{-4}}$.

Answer 8E.

(a) Let $P = (1.0, 2.4, 4.6)$ and $Q(2.6, 1.2, 0.3)$ be two points on the line. We first need to find the direction vector $\mathbf{v} = \langle a, b, c \rangle$ for the line.

$$\begin{aligned}\mathbf{v} &= \overrightarrow{PQ} \\ &= \langle 2.6 - 1.0, 1.2 - 2.4, 0.3 - 4.6 \rangle \\ &= \langle 1.6, -1.2, -4.3 \rangle\end{aligned}$$

The parametric equations of a line in space parallel to a nonzero vector $\mathbf{v} = \langle a, b, c \rangle$ and passing through the point $P(x_1, y_1, z_1)$ are $x = x_1 + at, y = y_1 + bt,$ and $z = z_1 + ct$. The numbers a, b, and c are called direction numbers.

We have $x_1 = 1.0, y_1 = 2.4,$ and $z_1 = 4.6$. The direction numbers are $a = 1.6$, $b = -1.2$, and $c = -4.3$.

Thus, the parametric equations are
$\boxed{x = 1.0 + 1.6t, , y = 2.4 - 1.2t, \text{ and } z = 4.6 - 4.3t}$.

(b) The symmetric equations of the line is obtained by eliminating the parameter t and is given by $\dfrac{x - x_1}{a} = \dfrac{y - y_1}{b} = \dfrac{z - z_1}{c}$.

$$\frac{x - 1}{1.6} = \frac{y - 2.4}{-1.2} = \frac{z - 4.6}{-4.3}$$

Therefore, the symmetric equations are $\dfrac{x - 1}{1.6} = \dfrac{y - 2.4}{-1.2} = \dfrac{z - 4.6}{-4.3}$.

Answer 9E.

(a) Let $P = (-8, 1, 4)$ and $Q(3, -2, 4)$ be two points on the line. We first need to find the direction vector $\mathbf{v} = \langle a, b, c \rangle$ for the line.

$$\begin{aligned}\mathbf{v} &= \overrightarrow{PQ} \\ &= \langle 3 + 8, -2 - 1, 4 - 4 \rangle \\ &= \langle 11, -3, 0 \rangle\end{aligned}$$

The parametric equations of a line in space parallel to a nonzero vector $\mathbf{v} = \langle a, b, c \rangle$ and passing through the point $P(x_1, y_1, z_1)$ are $x = x_1 + at, y = y_1 + bt,$ and $z = z_1 + ct$. The numbers a, b, and c are called direction numbers.

We have $x_1 = -8, y_1 = 1,$ and $z_1 = 4$. The direction numbers are $a = 11$, $b = -3$, and $c = 0$.

Thus, the parametric equations are
$\boxed{x = -8 + 11t, y = 1 - 3t, \text{ and } z = 4}$.

(b) The symmetric equations of the line is obtained by eliminating the parameter t and is given by $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$.

$$\frac{x-(-8)}{11}=\frac{y-1}{-3}, \text{ and } z=4$$

Therefore, the symmetric equations are $\boxed{\frac{x+8}{11}=\frac{y-1}{-3},\ z=4}$.

Answer 10E.

Consider the line passing through the point $(2,1,0)$ and perpendicular to both $\mathbf{i}+\mathbf{j}$ and $\mathbf{j}+\mathbf{k}$.

The objective is to find parametric and symmetric equations of the line.

Recollect that the vector equation of a line passing through the point $\mathbf{r}_o$ and parallel to the vector $\mathbf{v}$ is $\mathbf{r}=\mathbf{r}_o+t\mathbf{v}$.

Here, the point $\mathbf{r}_o$ is given by,

$$\begin{aligned}\mathbf{r}_o&=(2,1,0)\\&=2\mathbf{i}+\mathbf{j}\end{aligned}$$

First, find the cross product of vectors $\mathbf{i}+\mathbf{j}$ and $\mathbf{j}+\mathbf{k}$.

$$\begin{aligned}\mathbf{v}&=(\mathbf{i}+\mathbf{j})\times(\mathbf{j}+\mathbf{k})\\&=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&1&0\\0&1&1\end{vmatrix}\\&=\mathbf{i}(1-0)-\mathbf{j}(1-0)+\mathbf{k}(1-0)\\&=\mathbf{i}-\mathbf{j}+\mathbf{k}\end{aligned}$$

Substitute the values $\mathbf{v}=\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $\mathbf{r}_o=2\mathbf{i}+\mathbf{j}$ in $\mathbf{r}=\mathbf{r}_o+t\mathbf{v}$.

$$\begin{aligned}\mathbf{r}&=\mathbf{r}_o+t\mathbf{v}\\&=2\mathbf{i}+\mathbf{j}+t(\mathbf{i}-\mathbf{j}+\mathbf{k})\\&=\mathbf{i}(2+t)+\mathbf{j}(1-t)+t\mathbf{k}\end{aligned}$$

Therefore, the required parametric equations of the line are,

$\boxed{x=2+t,y=1-t,z=t}$.

The symmetric equations of line is,

$$\frac{x-x_o}{a}=\frac{y-y_o}{b}=\frac{z-z_o}{c},$$

Where, (x_o,y_o,z_o) is a point on the line and (a,b,c) are direction numbers of line or the components of vectors parallel to the line.

So, substitute $(x_o,y_o,z_o)=(2,1,0)$ and $(a,b,c)=(1,-1,1)$ in $\frac{x-x_o}{a}=\frac{y-y_o}{b}=\frac{z-z_o}{c}$.

$$\frac{x-2}{1}=\frac{y-1}{-1}=\frac{z-0}{1}$$

Therefore, the symmetric equations of the line is,

$$\boxed{\frac{x-2}{1}=\frac{y-1}{-1}=\frac{z-0}{1}}.$$

Answer 11E.

The required line passes through (1,-1,1) and is parallel to the line

$$\frac{x+2}{1}=\frac{y}{2}=\frac{z-3}{1}$$

The direction numbers of given line are 1,2,1, since required line is parallel to given line, then the direction numbers of required line are proportional to 1,2,1.

Then $\vec{v}=\langle 1,2,1\rangle$

Since line passes through (1,-1,1) then

$$\vec{r_0}=\langle 1,-1,1\rangle$$

Hence the required equation of the line is

$$\vec{r}=\vec{r_0}+t\vec{v}$$

i.e. $\vec{r}=\langle 1,-1,1\rangle+t\langle 1,2,1\rangle$

i.e. $\vec{r}=\langle 1+t,-1+2t,1+t\rangle$

This is the vector equation of line.

Now parametric equation is:

$$x=1+t,y=-1+2t,z=1+t$$

And the symmetric equation is:

$$\frac{x-1}{1}=\frac{y+1}{2}=\frac{z-1}{1}$$

Answer 12E.

To find the line of intersection of the planes $x+2y+3z=1$ and $x-y+z=1$, first need to find a point on the required line L.

For instance, find the point where the line intersects the *xy*-plane by setting $z=0$ in the equations of both planes.

Set z = 0 in both the planes.

$$x+2y+3z=1$$
$$x+2y+3(0)=1 \quad \text{...... (1)}$$
$$x+2y=1$$

And

$$x-y+z=1$$
$$x-y+0=1 \quad \text{...... (2)}$$
$$x-y=1$$

Solve the two equations (1) and (2).

From equation (2), $x=1+y.$

Substitute $x=1+y$ in the equation (1).

$$x+2y=1$$
$$1+y+2y=1$$
$$1+3y=1$$
$$3y=0$$

$$y=0$$

Substitute $y=0$ in the equation $x=1+y.$

$$x=1+y$$
$$=1+0$$
$$=1$$

So the solution for the two equations (1) and (2) is $x=1, y=0.$

So the point $(x,y,z)=(1,0,0)$ lies on the line L.

The normal vector for the plane $x+2y+3z=1$ is $\mathbf{n}_1 : \langle 1,2,3 \rangle$.

And the normal vector for the plane $x-y+z=1$ is $\mathbf{n}_2 : \langle 1,-1,1 \rangle$.

Since L lies on both the planes, it is perpendicular to both of the normal vectors **n**1 and **n**2. Thus, a vector **v** parallel to L is given by the cross product $\mathbf{n}_1 \times \mathbf{n}_2$.

$$\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{vmatrix}$$
$$= (2+3)\mathbf{i} - (1-3)\mathbf{j} + (-1-2)\mathbf{k}$$
$$= 5\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$$

Thus, the vector parallel to L is $\mathbf{v} = 5\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$.

The vector $\mathbf{v} = \langle 5,2,-3 \rangle$ is describes the direction of the line L so the direction numbers of L are $\langle a,b,c \rangle = \langle 5,2,-3 \rangle$.

Now substitute the known values in the symmetric equations of line L.

Symmetric equations of L:

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

Where a,b and c are called direction number of L and $P_0(x_0,y_0,z_0)$ be any point on the line L.

Taking the point $(1,0,0)$ as P_0 and direction numbers are $\langle a,b,c \rangle = \langle 5,2,-3 \rangle$, the symmetric equations are

$$\frac{x-1}{5} = \frac{y-0}{2} = \frac{z-0}{-3}$$

Or

$$\frac{x-1}{5} = \frac{y}{2} = \frac{z}{-3}$$

Hence the required line of intersection of the two given planes is $\boxed{\frac{x-1}{5} = \frac{y}{2} = \frac{z}{-3}}$.

Answer 13E.

The direction numbers of line L_1 through $(-4,-6,1)$ *and* $(-2,0-3)$ are

$$\langle -2-(-4),0-(-6),-3-1\rangle$$

That is, $\langle 2,6,-4\rangle$

And the direction numbers of line L_2 through

$(10,18,4)$ *and* $(5,3,14)$ *are*

$$\langle 5-10,3-18,14-4\rangle$$

That is, $\langle -5,-15,10\rangle$

The components of two lines L_1 *and* L_2 are proportional

$$\frac{2}{-5}=\frac{6}{-15}=\frac{-4}{10}$$

That is

$$-\frac{2}{5}=-\frac{2}{5}=-\frac{2}{5}$$

Therefore Lines L_1 *and* L_2 are parallel.

Answer 14E.

Let the points be $A(-2, 4, 0)$ and $B(1, 1, 1)$.

Find the side $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \langle (1+2), (1-4), (1-0)\rangle$$
$$= \langle 3, -3, 1\rangle$$

Now, let us represent $C(2, 3, 4)$ and $D(3, -1, -8)$.

$$\overrightarrow{CD} = \langle (3-2), (-1-3), (-8-4)\rangle$$
$$= \langle 1, -4, -12\rangle$$

We know that two vectors are perpendicular if their dot product is zero.

Now, let us represent $C(2, 3, 4)$ and $D(3, -1, -8)$.

$$\overrightarrow{CD} = \langle (3-2), (-1-3), (-8-4)\rangle$$
$$= \langle 1, -4, -12\rangle$$

We know that two vectors are perpendicular if their dot product is zero.

Answer 15E.

A)

We are given that P0=(1,-5,6), so the position vector **r0** = <1,-5,6> = 1**i** -5**j** + 6**k**

It is also given that the line L is parallel to the vector <-1,2,-3>, so let vector **v** = <-1,2,-3> = -1**i** + 2**j** - 3**k**

We can plug these points in to obtain the vector equation of L:

r = **r0** + t**v** ∴ **r** = (1**i** - 5**j** + 6**k**) + t(-1**i** + 2**j** - 3**k**) ∴

r = (1-1t)**i** + (-5+2t)**j** + (6-3t)**k**

From the vector equation, we can obtain the parametric equations:

x = x0 + at ∴ x = 1 - t

y = y0 + at ∴ y = -5 + 2t

z = z0 + at ∴ z = 6 - 3t

From the parametric equations, we can obtain the symmetric equations. By solving each parametric equation for t, we can eliminate the parameter t, and the symmetric equations we are left with are:

$$\frac{x-x_0}{a}=\frac{y-y_0}{b}=\frac{z-z_0}{c}$$

$$\therefore \frac{x-1}{-1}=\frac{y+5}{2}=\frac{z-6}{-3}$$

B)

The line intersects the x-y plane when z = 0.

To find the point of intersection, let z = 0 in the symmetric equations:

$$\frac{x-1}{-1} = \frac{y+5}{2} = \frac{z-6}{-3} \therefore \frac{x-1}{-1} = \frac{y+5}{2} = \frac{0-6}{-3}$$

$\frac{x-1}{-1} = 2,\ \frac{y+5}{2} = 2$ gives x = -1 and y = -1, so the point of intersection is (-1,-1,0)

The line intersects the x-z plane when y = 0.

To find the point of intersection, let y = 0 in the symmetric equations:

$$\frac{x-1}{-1} = \frac{y+5}{2} = \frac{z-6}{-3} \therefore \frac{x-1}{-1} = \frac{0+5}{2} = \frac{z-6}{-3}$$

$\frac{x-1}{-1} = \frac{5}{2},\ \frac{z-6}{-3} = \frac{5}{2}$ gives x = -3/2 and z = -3/2, so the point of intersection is (-3/2,0,-3/2)

The line intersects the y-z plane when x = 0.

To find the point of intersection, let x = 0 in the symmetric equations:

$$\frac{x-1}{-1} = \frac{y+5}{2} = \frac{z-6}{-3} \therefore \frac{0-1}{-1} = \frac{y+5}{2} = \frac{z-6}{-3}$$

$\frac{y+5}{2} = 1,\ \frac{z-6}{-3} = 1$ gives y = -3 and z = 3, so the point of intersection is (0,-3,3)

Answer 16E.

(a)

Consider the following data:

- A line passes through the point $(2,4,6)$.
- This line is perpendicular to the plane $x-y+3z=7$.

The objective is to find parametric equations for this line.

Use following theorem, to find parametric equations for this line:

"The parametric equations for a line through the point (x_0, y_0, z_0) and parallel to the direction vector $\mathbf{v}=\langle a,b,c\rangle$ are defined as follows:

$x=x_0+at,\ \ y=y_0+bt,\ \ z=z_0+ct$." ......(1)

To find the parametric equations for the desired line, first find the direction vector $\mathbf{v}=\langle a,b,c\rangle$ parallel to the desired line.

If a line is perpendicular to a plane, then the normal of the plane is the direction vector of the line.

Because, the normal of the plane,

$x-y+3z=7$,

is $\langle 1,-1,3\rangle$, so the direction vector of the required line is also $\langle 1,-1,3\rangle$, that is,

$\mathbf{v}=\langle a,b,c\rangle$

$=\langle 1,-1,3\rangle$. ......(i)

For the point (x_0, y_0, z_0), the position vector of a particular point on the line, take,

$(x_0, y_0, z_0)=(2,4,6)$. ....(ii)

Now, find the parametric equations for the desired line.

Use (1) with $\langle a,b,c\rangle = \langle 1,-1,3\rangle$, and $(x_0,y_0,z_0)=(2,4,6)$, the parametric equations for the desired line is,

$x = x_0 + at,\ \ y = y_0 + bt,\ \ z = z_0 + ct$, or

$x = 2 + 1t,\ \ y = 4 + (-1)t,\ \ z = 6 + 3t$, or

$x = 2 + t,\ \ y = 4 - t,\ \ z = 6 + 3t$.

Therefore, the parametric equations for the desired line is,

$\boxed{x = 2 + t,\ \ y = 4 - t,\ \ z = 6 + 3t}$.

(b)

Consider the following data:

- A line passes through the point $(2,4,6)$.
- This line is perpendicular to the plane $x - y + 3z = 7$.

The objective is to find the points in which this line intersects the coordinate planes.

By part (a), the parametric equations for this line is,

$x = 2 + t,\ \ y = 4 - t,\ \ z = 6 + 3t$. ......(2)

Find the *xy*-plane intercept:

To find the intersection with *xy*-plane, find the values of *x* and *y* when $z = 0$. Thus,

- first, set $z = 0$ into the equation of *z* in equation (2) and solve for *t*,
- then put this value of *t* back into the equations of *x* and *y* in equation (2),
- and find the values of *x* and *y*.

Now, write equation (2):

$x = 2 + t,\ \ y = 4 - t,\ \ z = 6 + 3t$.

Set $z = 0$ the equation of *z* and solve for *t*:

$z = 6 + 3t$ Write equation of *z*

$0 = 6 + 3t$ Set $z = 0$

$t = -2$. Solve for *t*

Substitute $t = -2$ back into the equations of *x* and *y* in equation (2), and solve for *x* and *y*:

$x = 2 + t,\ \ \ \ y = 4 - t$ Write equations of *x* and *y*

$x = 2 + (-2),\ \ y = 4 - (-2)$ Substitute $t = -2$

$x = 0,\ \ \ \ y = 6$. Solve for *x* and *y*

So, this line intersects the *xy*-plane at the point $\boxed{(0,6,0)}$.

Find **the *yz*-plane intercept**:

To find the intersection with *yz*-plane, find the values of *y* and *z* when $x=0$. Thus,

• first, set $x=0$ into the equation of *x* in equation (2) and solve for *t*,

• then put this value of *t* back into the equations of *y* and *z* in equation (2),

• and find the values of *y* and *z*.

Now, write equation (2):

$x=2+t,\ \ y=4-t,\ \ z=6+3t$.

Set $x=0$ the equation of *x* and solve for *t*:

$x=2+t$ Write equation of *x*

$0=2+t$ Set $x=0$

$t=-2$. Solve for *t*

Substitute $t=-2$ back into the equations of *y* and *z* in equation (2), and solve for *y* and *z*:

$y=4-t,\qquad z=6+3t$ Write equations of *x* and *y*

$y=4-(-2),\ \ z=6+3(-2)$ Substitute $t=-2$

$y=6,\qquad z=0$. Solve for *x* and *y*

So, this line intersects the *yz*-plane at the point $\boxed{(0,6,0)}$.

Find the *xz*-plane intercept:

To find the intersection with *xz*-plane, find the values of *x* and *z* when $y=0$. Thus,

• first, set $y=0$ into the equation of *y* in equation (2) and solve for *t*,

• then put this value of *t* back into the equations of *x* and *z* in equation (2),

• and find the values of *x* and *z*.

Now, write equation (2):

$x=2+t,\ \ y=4-t,\ \ z=6+3t$.

Set $z=0$ the equation of *z* and solve for *t*:

$y=4-t$ Write equation of *y*

$0=4-t$ Set $y=0$

$t=4$. Solve for *t*

Substitute $t=4$ back into the equations of *x* and *z* in equation (2), and solve for *x* and *z*:

$x=2+t,\qquad z=6+3t$ Write equations of *x* and *z*

$x=2+4,\qquad z=6+3(4)$ Substitute $t=4$

$x=6,\qquad z=18$. Solve for *x* and *z*

So, this line intersects the *xz*-plane at the point $\boxed{(6,0,18)}$.

Therefore, finally, this line intersects the *xy*- and *yz*-plane at the point $\boxed{(0,6,0)}$, and *xz*-plane at the point $\boxed{(6,0,18)}$.

Answer 17E.

The vector $\vec{v}$ with $A=(2,-1,4)$ *and*

$B(4,6,1)$ *is*

$$\vec{v}=\langle 4-2,6-(-1).1-4\rangle$$

$$=\langle 2,7,-3\rangle$$

Taking the point $(2,-1,4)$ as $\vec{r_0}$, the vector equation of line becomes

$$\vec{r}=\vec{r_0}+t\vec{v},\ 0\le t\le 1.$$

$$=\left(2\hat{i}-\hat{j}+4\hat{k}\right)+t\left(2\hat{i}+7\hat{j}-3\hat{k}\right),0\le t\le 1.$$

$$=(2+2t)\hat{i}+(-1+7t)\hat{j}+(4-3t)\hat{k},0\le t\le 1.$$

Answer 18E.

The vector along the line joining $A(10,3,1)$ and $B(5,6,-3)$ is

$$\vec{v}=\langle 5-10,6-3,-3-1\rangle$$

$$=\langle -5,3,-4\rangle$$

Taking (10, 3, 1) as the point P_0, the parametric equations of the line are:

$$\boxed{x=10-5t,y=3+3t,z=1-4t},0\le t\le 1$$

Answer 19E.

Consider the lines:

$$L_1: x=3+2t, y=4-t, z=1+3t$$
$$L_2: x=1+4s, y=3-2s, z=4+5s$$

To determine whether the given lines are parallel, skew or intersecting first need to write the symmetric form of the given lines.

The symmetric equation of a line in space is given by

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c},$$

where *a*, *b*, and *c* are the direction numbers.

Now, rewrite the given parametric equations in the symmetric form.

$$L_1 : \frac{x-3}{2} = \frac{y-4}{-1} = \frac{z-1}{3} = t$$
$$L_2 : \frac{x-1}{4} = \frac{y-3}{-2} = \frac{z-4}{5} = s$$

The direction numbers of the line L_1 are $\langle 2,-1,3 \rangle$.

And the direction numbers of the line L_2 are $\langle 4,-2,5 \rangle$.

The ratios of the direction numbers of the two lines are

$$\frac{2}{4} = \frac{-1}{-2} = \frac{3}{5}$$
$$\frac{1}{2} = \frac{1}{2} = \frac{3}{5}$$

These ratios are not equal and therefore the direction numbers are not parallel.

Hence the given lines L_1 and L_2 are not parallel because the corresponding vectors $\langle 2,-1,3 \rangle$ and $\langle 4,-2,5 \rangle$ are not parallel. (Their components are not proportional.)

Now, need to check if the lines intersect or not.

If L_1 and L_2 had a point of intersection, there would be values of t and s such that

$3 + 2t = 1 + 4s$ ...... (1)

$4 - t = 3 - 2s$ ...... (2)

$1 + 3t = 4 + 5s$ ...... (3)

From equation (1),

$$\begin{aligned} 3+2t &= 1+4s \\ 2t &= 1+4s-3 \\ t &= \frac{1+4s-3}{2} \\ &= \frac{4s-2}{2} \\ &= 2s-1 \end{aligned}$$

Substitute the value of t in the equation (2).

$$4-t = 3-2s$$
$$4-(2s-1) = 3-2s \quad \text{Not true.}$$
$$5-2s = 3-2s$$

So on solving the two equations, note that there exist no value of t and s such that

$3 + 2t = 1 + 4s$ and $4 - t = 3 - 2s$.

Therefore there are no values of t and s that satisfy the three equations, so L_1 and L_2 do not intersect.

Thus the two given lines do not intersect and are not parallel and therefore do not lie in the same plane.

Hence the given lines L_1 and L_2 are skew lines.

Answer 20E.

Consider the lines:

$$L_1: x = 5 - 12t, y = 3 + 9t, z = 1 - 3t$$
$$L_2: x = 3 + 8s, y = -6s, z = 7 + 2s$$

To determine whether the given lines are parallel, skew or intersecting first need to write the symmetric form of the given lines.

The symmetric equation of a line in space is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c},$$

where a, b, and c are the direction numbers.

Now, rewrite the given parametric equations in the symmetric form.

$$L_1: \frac{x-5}{-12} = \frac{y-3}{9} = \frac{z-1}{-3} = t$$
$$L_2: \frac{x-3}{8} = \frac{y}{-6} = \frac{z-7}{2} = s$$

The direction numbers of the line L_1 are $\langle -12, 9, -3 \rangle$.

And the direction numbers of the line L_2 are $\langle 8, -6, 2 \rangle$.

The ratios of the direction numbers of the two lines are

$$\frac{-12}{8} = \frac{9}{-6} = \frac{-3}{2}$$
$$-\frac{3}{2} = -\frac{3}{2} = -\frac{3}{2}$$

These ratios are equal and therefore the direction numbers are parallel.

Hence the given lines L_1 and L_2 are parallel because the corresponding vectors $\langle -12, 9, -3 \rangle$ and $\langle 8, -6, 2 \rangle$ are parallel. (Their components are proportional.)

Answer 21E.

The symmetric equation of a line in space is given by $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$, where a, b, and c are the direction numbers.

We then get the direction numbers of L_1 as $(1, -2, -3)$ and that of L_2 as $(1, 3, -7)$.

Since the direction numbers are not proportional, we can say that the lines are not parallel.

Now, let us check if the lines intersect.

Set $2 + t = 3 + s$, $3 - 2t = -4 + 3s$, and $1 - 3t = 2 - 7s$ and solve for the variables.
On solving the two equations, we get s as 1 and t as 2. We note that the third equation also satisfies the values for t and s. Thus, we can say that the lines intersect.

On replacing t with 2 in $x = 2 + t, y = 3 - 2t$, and $z = 1 - 3t$,
we get x as 4, y as -1, and z as -5.

Therefore, we can say that L_1 and L_2 intersects t at $\boxed{(4, -1, -5)}$.

Answer 22E.

Consider the lines:

$$L_1 : \frac{x}{1} = \frac{y-1}{-1} = \frac{z-2}{3}$$
$$L_2 : \frac{x-2}{2} = \frac{y-3}{-2} = \frac{z}{7}$$

To determine whether the given lines are parallel, skew or intersecting first need to compare the given lines with the symmetric form of the line.

The symmetric equation of a line in space is given by

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c},$$

where a, b, and c are the direction numbers.

The direction numbers of the line L_1 are $\langle 1, -1, 3 \rangle$.

And the direction numbers of the line L_2 are $\langle 2, -2, 7 \rangle$.

The ratios of the direction numbers of the two lines are

$$\frac{1}{2} = \frac{-1}{-2} = \frac{3}{7}$$
$$\frac{1}{2} = \frac{1}{2} = \frac{3}{7}$$

These ratios are not equal and therefore the direction numbers are not parallel.

Hence the given lines L_1 and L_2 are not parallel because the corresponding vectors $\langle 1, -1, 3 \rangle$ and $\langle 2, -2, 7 \rangle$ are not parallel. (Their components are not proportional.)

Now, need to check if the lines intersect or not.

First write the parametric equations of the lines.

$$L_1 : \frac{x}{1} = \frac{y-1}{-1} = \frac{z-2}{3} = t$$
$$L_2 : \frac{x-2}{2} = \frac{y-3}{-2} = \frac{z}{7} = s$$

The parametric equations are

$$L_1 : x = t, y = 1 - t, z = 2 + 3t$$
$$L_2 : x = 2 + 2s, y = 3 - 2s, z = 7s$$

If L_1 and L_2 had a point of intersection, there would be values of t and s such that

$t = 2 + 2s$ ...... (1)

$1 - t = 3 - 2s$ ...... (2)

$2 + 3t = 7s$ ...... (3)

From equation (1), $t = 2 + 2s$

Substitute the value of *t* in the equation (2).

$$1 - t = 3 - 2s$$
$$1 - (2 + 2s) = 3 - 2s$$
$$1 - 2 - 2s = 3 - 2s$$
$$-1 - 2s = 3 - 2s$$

Not true.

So on solving the two equations (1) and (2), observe that there exist no value of *t* and *s* such that $t = 2 + 2s$ and $1 - t = 3 - 2s$.

Therefore there are no values of *t* and *s* that satisfy the three equations, so L_1 and L_2 do not intersect.

Thus the two given lines do not intersect and are not parallel and therefore do not lie in the same plane.

Hence the given lines L_1 and L_2 are skew lines.

Answer 23E.

The scalar equation of the plane passing through $P_0(x_0, y_0, z_0)$ with normal vector $\mathbf{n} = \langle a, b, c \rangle$ is given by $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$.

Replace x_0 with 0, y_0 with 0, z_0 with 0, a with 1, b with -2, and c with 5.

$$1(x - 0) + (-2)(y - 0) + 5(z - 0) = 0$$
$$x - 2y + 5z = 0$$

Thus, the equation of the plane is obtained as $\boxed{x - 2y + 5z = 0}$.

Answer 24E.

The scalar equation of the plane passing through $P_0(x_0, y_0, z_0)$ with normal vector $\mathbf{n} = \langle a, b, c\rangle$ is given by $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$.

Replace x_0 with 5, y_0 with 3, z_0 with 5, a with 2, b with 1, and c with -1.

$$\begin{aligned} 2(x-5)+1(y-3)+(-1)(z-5) &= 0 \\ 2x-10+y-3-z+5 &= 0 \\ 2x+y-z-8 &= 0 \end{aligned}$$

Thus, the equation of the plane is obtained as $\boxed{2x + y - z - 8 = 0}$.

Answer 25E.

The scalar equation of the plane passing through $P_0(x_0, y_0, z_0)$ with normal vector $\mathbf{n} = \langle a, b, c\rangle$ is given by $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$.

Replace x_0 with -1, y_0 with $\frac{1}{2}$, z_0 with 3, a with 1, b with 4, and c with 1.

$$\begin{aligned} 1(x-(-1))+4\left(y-\frac{1}{2}\right)+1(z-3) &= 0 \\ x+1+4y-2+z-3 &= 0 \\ x+4y+z &= 4 \end{aligned}$$

Thus, the equation of the plane is obtained as $\boxed{x + 4y + z = 4}$.

Answer 26E.

The scalar equation of the plane passing through $P_0(x_0, y_0, z_0)$ with normal vector $\mathbf{n} = \langle a, b, c\rangle$ is given by $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$.

The normal to the line $x = 3t, y = 2 - t$, and $z = 3 + 4t$ is obtained as $\langle 3, -1, 4\rangle$.

Replace x_0 with 2, y_0 with 0, z_0 with 1, a with 3, b with -1, and c with 4.

$$\begin{aligned} 3(x-2)+(-1)(y-0)+(4)(z-1) &= 0 \\ 3x-6-y+4z-4 &= 0 \\ 3x-y+4z-10 &= 0 \end{aligned}$$

Thus, the equation of the plane is obtained as $\boxed{3x - y + 4z = 10}$.

Answer 27E.

To find an equation for the plane that passes through the point $(1,-1,-1)$ and is parallel to the plane $5x - y - z = 6$, use the scalar equation of the plane.

The scalar equation of the plane through $P_0(x_0, y_0, z_0)$ with normal vector $\mathbf{n} = \langle a,b,c\rangle$ is

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0. \quad \ldots\ldots (1)$$

Let a,b,c be direction numbers of the normal $\mathbf{n}$ to the required plane.

The normal vector of the given plane $5x-y-z=6$ is $\mathbf{n}_1:\langle 5,-1,-1\rangle$.

Since the required plane is parallel to the given plane $z=x+y$ so the normal of the required plane is parallel to the normal of the given plane.

Therefore, the normal vector of the given plane can be taken as the normal vector of the required plane. Thus

$$\begin{aligned}\mathbf{n}&=\langle a,b,c\rangle\\&=\langle 5,-1,-1\rangle\end{aligned}$$

And the required plane passing through the point $(1,-1,-1)$ and this point $(1,-1,-1)$ satisfies the equation (1).

Substitute the point $P_0(x_0,y_0,z_0)=(1,-1,-1)$ and the normal vector $\mathbf{n}$ in the equation (1), get

$$\begin{aligned}a(x-x_0)+b(y-y_0)+c(z-z_0)&=0\\5(x-1)+(-1)(y-(-1))+(-1)(z-(-1))&=0\\5(x-1)-(y+1)-1(z+1)&=0\\5x-y-z-5-1-1&=0\end{aligned}$$

$$5x-y-z-7=0$$

Hence, the required equation of the plane is $\boxed{5x-y-z=7}$.

Answer 28E.

To find an equation for the plane that passes through the point $(2,4,6)$ and is parallel to the plane $z=x+y$, use the scalar equation of the plane.

The scalar equation of the plane through $P_0(x_0,y_0,z_0)$ with normal vector $\mathbf{n}=\langle a,b,c\rangle$ is

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0. \quad \text{...... (1)}$$

Let a,b,c be direction numbers of the normal $\mathbf{n}$ to the required plane.

The normal vector of the given plane $z=x+y$ is $\mathbf{n}_1:\langle 1,1,-1\rangle$.

Since the required plane is parallel to the given plane $z=x+y$ so the normal of the required plane is parallel to the normal of the given plane.

Therefore, the normal vector of the given plane can be taken as the normal vector of the required plane. Thus

$$\mathbf{n} = \langle a, b, c \rangle$$
$$= \langle 1, 1, -1 \rangle$$

And the required plane passing through the point $(2,4,6)$ and this point $(2,4,6)$ satisfies the equation (1).

Substitute the point $P_0(x_0, y_0, z_0) = (2,4,6)$ and the normal vector $\mathbf{n}$ in the equation (1), get

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$
$$1(x-2) + (1)(y-4) + (-1)(z-6) = 0$$
$$x - 2 + y - 4 - z + 6 = 0$$
$$x + y - z = 0$$

Hence, the required equation of the plane is $\boxed{x + y - z = 0}$.

Answer 29E.

The scalar equation of the plane passing through $P_0(x_0, y_0, z_0)$ with normal vector $\mathbf{n} = \langle a, b, c \rangle$ is given by $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$.

Since the required plane is parallel to $x + y + z = 0$, the normal to the planes will be equal. Then, $\mathbf{n} = \langle -1, -1, -1 \rangle$.

Replace x_0 with 1, y_0 with $\frac{1}{2}$, z_0 with $\frac{1}{3}$, a with -1, b with -1, and c with -1.

$$(-1)(x-1) + (-1)\left(y - \frac{1}{2}\right) + (-1)\left(z - \frac{1}{3}\right) = 0$$
$$-x + 1 - y + \frac{1}{2} - z + \frac{1}{3} = 0$$
$$x + y + z = \frac{11}{6}$$
$$6x + 6y + 6z = 11$$

Thus, the equation of the plane is obtained as $\boxed{6x + 6y + 6z = 11}$.

Answer 30E.

To find an equation for the plane that contains the line $x = 1 + t$, $y = 2 - t$, and $z = 4 - 3t$ and parallel to the plane $5x + 2y + z = 1$, use the scalar equation of the plane.

The scalar equation of the plane through $P_0(x_0, y_0, z_0)$ with normal vector $\mathbf{n} = \langle a, b, c \rangle$ is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0. \quad \text{......} \ (1)$$

Since the required plane contains the line $x = 1 + t$, $y = 2 - t$, and $z = 4 - 3t$, so any point on the line lies on the plane.

Let $(1, 2, 4)$ be any point on the line, it must be lies on the plane.

So take $P_0(x_0, y_0, z_0) = (1, 2, 4)$.

The required plane is parallel to the plane $5x + 2y + z = 1$ with normal vector $\langle 5, 2, 1 \rangle$. Since the two planes are parallel, so the corresponding normal vectors of the two planes are proportional. Then, the normal vector to the required plane is

$$\begin{aligned} \mathbf{n} &= \langle a, b, c \rangle \\ &= \langle 5, 2, 1 \rangle \end{aligned}$$

Substitute the known values $P_0(x_0, y_0, z_0) = (1, 2, 4)$ and the normal vector $\mathbf{n}$ in the equation (1), get

$$\begin{aligned} a(x - x_0) + b(y - y_0) + c(z - z_0) &= 0 \\ 5(x - 1) + 2(y - 2) + 1(z - 4) &= 0 \\ 5x - 5 + 2y - 4 + z - 4 &= 0 \\ 5x + 2y + z - 13 &= 0 \end{aligned}$$

$$5x + 2y + z = 13$$

Hence the required plane equation is $\boxed{5x + 2y + z = 13}$.

Answer 31E.

Consider the points $P(0,1,1), Q(1,0,1), R(1,1,0)$

To find the plane equation using the above three points

First convert the three points into two vectors by subtracting one point from the other two.

Consider the vectors $\vec{a}$ and $\vec{b}$.

The vector $\vec{a}$ and $\vec{b}$ corresponding to $\overrightarrow{PQ}$ *and* $\overrightarrow{PR}$ are

$$\begin{aligned}\mathbf{a} &= \overrightarrow{PQ} \\ &= \langle Q-P \rangle \\ &= (1,0,1)-(0,1,1) \\ &= \langle 1,-1,0 \rangle\end{aligned}$$

And

$$\begin{aligned}\mathbf{b} &= \overrightarrow{PR} \\ &= \langle R-P \rangle \\ &= (1,1,0)-(0,1,1) \\ &= \langle 1,0,-1 \rangle\end{aligned}$$

Since both $\vec{a}$ and $\vec{b}$ lie in the plane, their cross product $\vec{a} \times \vec{b}$ is orthogonal to the plane and can be taken as the normal vector $\mathbf{n}$.

Thus the normal vector

$$\mathbf{n} = \mathbf{a} \times \mathbf{b}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix}$$

$$= (1-0)\mathbf{i} - (-1-0)\mathbf{j} + (0+1)\mathbf{k}$$

$$= \langle \mathbf{i} + \mathbf{j} + \mathbf{k} \rangle$$

It is known that

The equation of the plane through the point $P(x_0, y_0, z_0)$ with normal vector $\mathbf{n} = \langle a, b, c \rangle$ is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Now

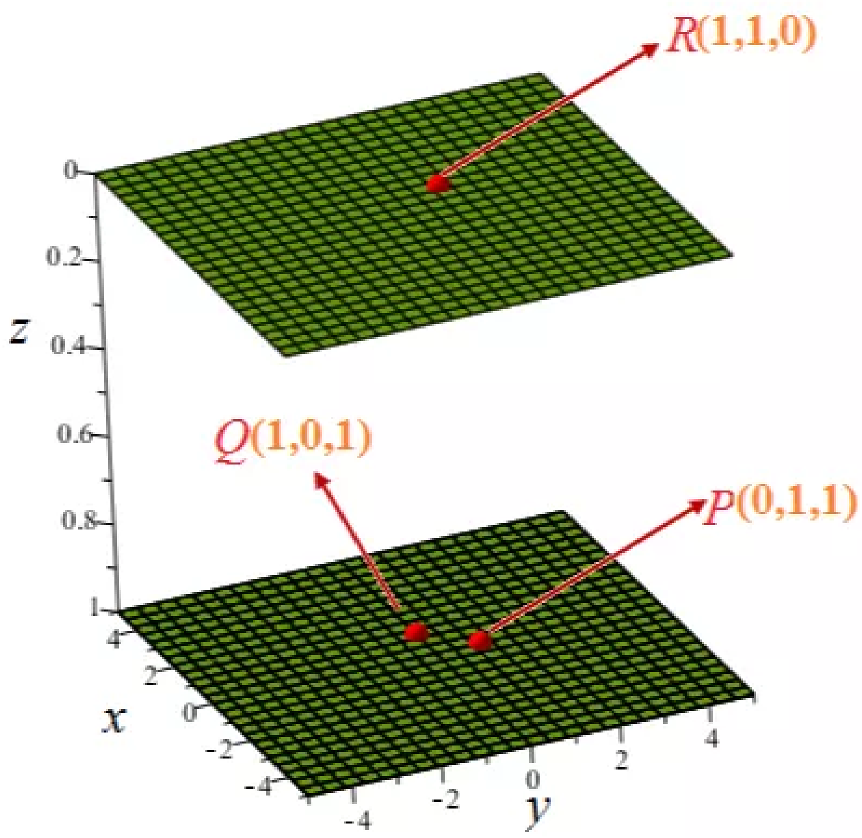
With the point $P(0,1,1)$ and the normal vector $\mathbf{n} = \langle \mathbf{i} + \mathbf{j} + \mathbf{k} \rangle$.

The equation of the plane is

$$1(x-0) + 1(y-1) + 1(z-1) = 0$$
$$x + y + z = 2$$

Thus the plane equation is $\boxed{x + y + z = 2}$

The sketch of the plane passing through the given 3 points is as below.



Answer 32E.

$O(0,0,0)$, $P(2,-4,6)$, $Q(5,1,3)$

The vector $\vec{a}$ and $\vec{b}$ correspoinding to $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are

$\vec{a} = \langle 2, -4, 6 \rangle$, $\vec{b} = \langle 5, 1, 3 \rangle$

Since both $\vec{a}$ and $\vec{b}$ lie in the plane, their cross product $\vec{a}\times\vec{b}$ is orthogonal to the plane and can be taken as the normal vector
Thus

$$\vec{n}=\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -4 & 6\\ 5 & 1 & 3\end{vmatrix}$$

$$=(-12-6)\hat{i}-(6-30)\hat{j}+(2+20)\hat{k}$$

$$=-18\hat{i}+24\hat{j}+22\hat{k}$$

With the point O(0, 0, 0) and the normal vector $\vec{n}$, an equation of the plane is

$$-18x+24y+22z=0.$$

Answer 33E.

$$P(3,-1,2,),\quad Q=(8,2,4),\quad R=(-1,-2,-3)$$

The vectors $\vec{a}$ and $\vec{b}$ corresponding to $\overrightarrow{PQ}$ *and* $\overrightarrow{PR}$ *are*

$$\vec{a}=\langle 5,3,2\rangle \textit{and } \vec{b}=\langle -4,-1,-5\rangle$$

Since both $\vec{a}$ and $\vec{b}$ lie in the plane, their cross product $\vec{a}\times\vec{b}$is orthogonal to the plane and can be taken as the normal vector.
Thus

$$\vec{n}=\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 5 & 3 & 2\\ -4 & -1 & -5\end{vmatrix}$$

$$=(-15+2)\hat{i}-(-25+8)\hat{j}+(-5+12)\hat{k}$$

$$=-13\hat{i}+17\hat{j}+7\hat{k}$$

With the point P (3,-1,2) and the normal vector is an equation of the plane is

$$-13(x-3)+17(y-(-1))+7(z-2)=0$$

or, $-13x+17y+7z=-42$

Answer 34E.

The plane contains the line

$$x = 3t,\ \ y = 1 + t, \qquad z = 2 - t$$

Then the point (0, 1, 2) lies in the plane, and the vector $\langle 3,1,-1\rangle$ lies in the plane.

The vector joining the point (1, 2, 3) and (0, 1, 2) is $\langle 1-0, 2-1, 3-2\rangle$ *i.e.* $\langle 1,1,1\rangle$

Then the normal of the plane is given by the cross product of the vectors $\langle 3,1,-1\rangle = \vec{a}\,(say)$ and $\langle 1,1,1\rangle = \vec{b}\,(say)$

$$i.e.\ \vec{n} = \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 2\hat{i} - 4\hat{j} + 2\hat{k}$$

$$= \langle 2,-4,2\rangle$$

Therefore the equation of the plane passing through (1, 2, 3) and having normal $\langle 2,-4,2\rangle$ is

$$2(x-1) - 4(y-2) + 2(z-3) = 0$$

$$i.e. \qquad 2x - 2 - 4y + 8 + 2z - 6 = 0$$

$$i.e \qquad 2x - 4y + 2z = 0$$

$$or \qquad \boxed{x - 2y + z = 0}$$

Answer 35E.

Consider the parametric equations of the line and the point is,

$x = 4 - 2t, y = 3 + 5t, z = 7 + 4t.$ and $P(6,0,-2)$

The objective is to find an equation of plane.

Since the parametric equations are,

$Q(4,3,7)$

These equations are called parametric equations of the line $\mathbf{b} = \langle -2,5,4\rangle$. passing through the point P,Q lies on the required plane so the vector $\overline{PQ} = \mathbf{a}$. and parallel to the vector

$$\overline{PQ} = \mathbf{a}$$

$$= \langle 4-6, 3-0, 7-(-2)\rangle$$

$$= \langle -2,3,9\rangle$$

Let the given point be $\mathbf{a} = \langle -2,3,9 \rangle$ is passing through the plane and the point $\langle -2,5,4 \rangle$ is also passing through the plane.

So, the plane passing through the points are, $\mathbf{a} \times \mathbf{b}$ and

$$\mathbf{n} = \mathbf{a} \times \mathbf{b}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 3 & 9 \\ -2 & 5 & 4 \end{vmatrix}$$
$$= \mathbf{i}[12-45] - \mathbf{j}[-8+18] + \mathbf{k}[-10+6]$$
$$= -33\mathbf{i} - 10\mathbf{j} - 4\mathbf{k}$$

Since the points $P(x_1, y_1, z_1) =$ and -2 lie on the plane

The corresponding vector,

$$\mathbf{n} = \langle a,b,c \rangle$$

$$a(x-x_1) + b(y-y_1) + c(z-z_1) = 0$$

Let it be $-33(x-6) - 10(y-0) - 4(z-(-2)) = 0$

$$-33(x-6) - 10(y) - 4(z+2) = 0$$

$$-33x + 198 - 10y - 4z - 8 = 0$$

So,
$$-33x + 190 - 10y - 4z = 0$$
$$33x - 190 + 10y + 4z = 0$$
$$33x + 10y + 4z = 190$$

Since the corresponding vectors are, $\boxed{33x + 10y + 4z = 190}$.

Since the both vectors **a** and **b** are lie on the plane and their cross product $\mathbf{a} \times \mathbf{b}$ is orthogonal to the plane.

So, it can be taken as the normal vector.

$$\mathbf{n} = \mathbf{a} \times \mathbf{b}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 5 & 4 \\ -2 & 3 & 9 \end{vmatrix}$$
$$= \mathbf{i}(45-12) - \mathbf{j}(-18+8) + (-6+10)\mathbf{k}$$
$$= 33\mathbf{i} + 10\mathbf{j} + 4\mathbf{k}$$

Use the result to find the equation of the plane.

The scalar equation of the plane passing through $P_0(x_0, y_0, z_0)$ with normal vector $\mathbf{n} = \langle a, b, c \rangle$

Then $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

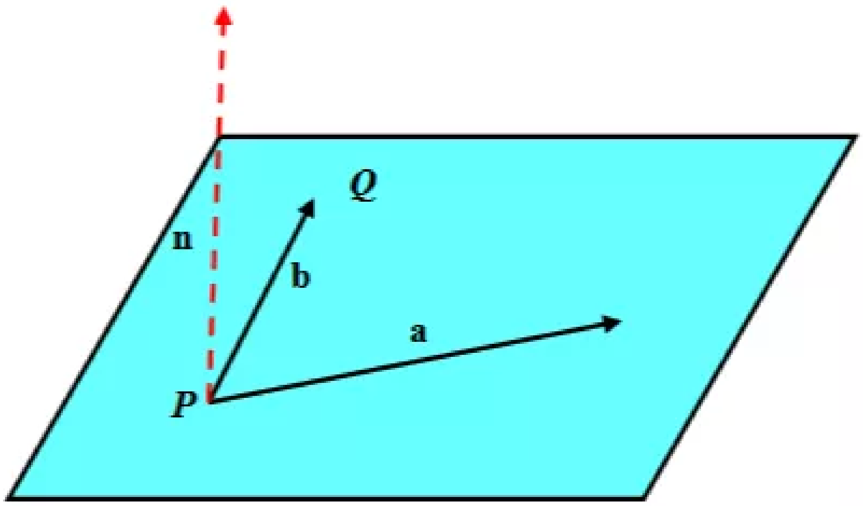
Here, the plane passing through the point $P(6, 0, -2)$ and the normal vector $\mathbf{n} = \langle 33, 10, 4 \rangle$

Now an equation of the plane is,

$$33(x-6)+10(y-0)+4(z-(-2))=0$$
$$33(x-6)+10y+4(z+2)=0$$
$$33x-198+10y+4z+8=0$$
$$33x+10y+4z-190=0$$
$$33x+10y+4z=190$$

Therefore, the equation of the plane is, $\boxed{33x+10y+4z=190}$.

Sketch the following plane:



Answer 36E.

The equation of the plane passing through $P_0(x_0, y_0, z_0)$ with normal vector $\mathbf{n} = \langle a, b, c \rangle$ is

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

Therefore, the equation of the plane passing through $(1, -1, 1)$ is

$$a(x-1)+b(y+1)+c(z-1)=0 \quad \cdots\cdots(1)$$

The plane contains the line $x = 2y = 3z$

Let $x = 2y = 3z = t$, where $t \in \mathbb{R}$

So that $x = t, \quad y = \frac{t}{2}, \quad z = \frac{t}{3}$

That is, $x = 0 + 1t, \quad y = 0 + \frac{1}{2}t, \quad z = 0 + \frac{1}{3}t$

Therefore, the plane contains the point $(0,0,0)$ and its normal vector $\langle a,b,c \rangle$ perpendicular to the vector $\left\langle 1, \frac{1}{2}, \frac{1}{3} \right\rangle$.

That is the point $(0,0,0)$ satisfies the equation (1).

So, $a(0-1) + b(0+1) + c(0-1) = 0$

$-a + b - c = 0 \quad \cdots\cdots(2)$

And its normal vector $\langle a,b,c \rangle$ perpendicular to the vector $\left\langle 1, \frac{1}{2}, \frac{1}{3} \right\rangle$.

Note that, two vectors $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ are perpendicular, then

$a_1b_1 + a_2b_2 + a_3b_3 = 0$

So, $a + \frac{b}{2} + \frac{c}{3} = 0 \quad \cdots\cdots(3)$

Adding equations (2) and (3), obtain that

$$\frac{3}{2}b-\frac{2}{3}c=0$$

$$\frac{3}{2}b=\frac{2}{3}c$$

$$b=\frac{4}{9}c \quad \cdots\cdots(4)$$

Substitute (4) in (1), obtain that

$$-a+\frac{4}{9}c-c=0$$

$$a=-\frac{5}{9}c$$

$$\frac{a}{-5}=\frac{c}{9} \quad \cdots\cdots(5)$$

Equation (4) can be written as

$$\frac{b}{4}=\frac{c}{9} \quad \cdots\cdots(6)$$

Therefore, from (5) and (6) obtain that

$$\frac{a}{-5}=\frac{b}{4}=\frac{c}{9}$$

So, the direction numbers for the required line is

$$\langle a,b,c\rangle=\langle -5,4,9\rangle. \quad \cdots\cdots(7)$$

Now substitute these values of equation (7) in (1) obtain that

$$-5(x-1)+4(y+1)+9(z-1)=0$$

$$-5x+5+4y+4+9z-9=0$$

$$-5x+4y+9z=0$$

$$5x-4y-9z=0$$

Thus, the required equation of the plane is $\boxed{5x-4y-9z=0}$.

Answer 37E.

Consider the following equation of planes

$x+y-z=2$ ...... (1)

$2x-y+3z=1$ ...... (2)

Convert the equation of the line in symmetric form.

Find a point on the line of intersection *L* of the given planes.

The normal vectors of these planes are $\mathbf{n}_1=(1,1,-1)$ and $\mathbf{n}_2=(2,-1,3)$.

The intersection of two planes gives a line equation.

Set $z=0$ in the equations of both planes (1) and (2) to find the point where the line intersects the xy plane.

Thus, the equations (1) and (2) reduces,

$x+y=2$ And $2x-y=1$

Add above two equations

$$3x=3$$
$$x=1$$

Substitute $x=1$ in $x+y=2$ to find y value.

$$1+y=2$$
$$y=1$$

The point lies on *L* is $(1,1,0)$.

Since *L* lies in both planes, it is perpendicular to both of the normal vectors.

Thus, a vector **v**, parallel to *L*, is given by the cross product.

$$\mathbf{v}=\mathbf{n}_1\times\mathbf{n}_2$$
$$=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & 1 & -1\\ 2 & -1 & 3\end{vmatrix}$$
$$=(3-1)\mathbf{i}-(3+2)\mathbf{j}+(-1-2)\mathbf{k}$$
$$=2\mathbf{i}-5\mathbf{j}-3\mathbf{k}$$

Let the point on the line be $(x_1, y_1, z_1) = (1,1,0)$.

The direction ratios of line are $(a,b,c) = (2,5,-3)$.

The symmetric equations of *L* can be written as

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Substitute the values $(x_1, y_1, z_1) = (1,1,0)$, and $(a,b,c) = (2,5,-3)$ in above equation.

$$\frac{x-1}{2} = \frac{y-1}{-5} = \frac{z}{-3}$$

Let $\frac{x-1}{2} = \frac{y-1}{-5} = \frac{z}{-3} = t$

$$x = 2t + 1, y = -5t + 1, z = -3t$$

This represents a line passing through the point $Q(1,1,0)$ and parallel to the vector $\mathbf{b} = (2,-5,-3)$.

The required plane passes through the point $P = (-1,2,1)$.

P, Q lies on the required plane so the vector $\overline{PQ} = \mathbf{a}$

$$\begin{aligned}\mathbf{a} &= \overline{\mathrm{PQ}} \\ &= \langle 1+1, 1-2, 0-1 \rangle \\ &= \langle 2, -1, -1 \rangle\end{aligned}$$

Therefore, both vectors **a** and **b** are parallel to the required plane.

Their cross product $\mathbf{a}\times\mathbf{b}$ is orthogonal to required plane.

$$\mathbf{n} = \mathbf{a}\times\mathbf{b}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -1 & -1 \\ 2 & -5 & -3 \end{vmatrix}$$
$$= -2\mathbf{i} + 4\mathbf{j} - 8\mathbf{k}$$

Let $(l,m,n) = (-2,4,-8)$.

The equation of required plane passing through $(-1,2,1)$ and having normal vector $-2\mathbf{i}+4\mathbf{j}-8\mathbf{k}$ is

$$-2(x-(-1))+4(y-2)-8(z-1)=0$$
$$-2(x+1)+4(y-2)-8(z-1)=0$$
$$-2x-2+4y-8-8z+8=0$$
$$-2x+4y-8z-2=0$$
$$x-2y+4z+1=0 \qquad \text{Divide each side by } -2.$$

Therefore, the required equation of plane is $\boxed{x-2y+4z+1=0}$.

Answer 38E.

Consider the points are $(0,-2,5)$ and $(-1,3,1)$.

The equation of the plane is as follows:

$$2z = 5x + 4y$$

Find the equation of the plane passing through the given points and perpendicular to the given plane $2z = 5x+4y$.

Rewrite the above plane equation is as follows:

$$5x + 4y - 2z = 0$$

Let $\mathbf{P_1} = \langle 0,-2,5 \rangle$ and $\mathbf{P_2} = \langle -1,3,1 \rangle$.

Find the directional vector of $\mathbf{P_1P_2}$.

$$\begin{aligned}\mathbf{P_1P_2} &= \langle \mathbf{P_2} - \mathbf{P_1} \rangle \\ &= \langle -1-0, 3+2, 1-5 \rangle \\ &= \langle -1,5,-4 \rangle\end{aligned}$$

Required plane is perpendicular to the given plane. So the normal vector $\mathbf{n}$ of the given plane is also directional vector to the required plane.

$$\mathbf{n} = \langle 5,4,-2 \rangle$$

The normal vector $\mathbf{n_1}$ of the required plane is orthogonal to the directional vectors $\mathbf{n}$ and $\mathbf{P_1P_2}$.

Now, find the cross product.

$$\begin{aligned}\mathbf{n_1} &= \mathbf{n} \times \mathbf{P_1P_2} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 4 & -2 \\ -1 & 5 & -4 \end{vmatrix}\end{aligned}$$

$$\begin{aligned}&= \mathbf{i}(-16+10) - \mathbf{j}(-20-2) + \mathbf{k}(25+4) \\ &= -6i + 22y + 29k \\ &= \langle -6,22,29 \rangle\end{aligned}$$

The normal vector of plane is orthogonal to any vector lies in the plane.

And also dot product of orthogonal vectors is zero.

Let $Q(x,y,z)$ be any point on the plane. So the vector $\mathbf{P_1Q}$ is lies on the plane.

Consider $\mathbf{n_1} \bullet \mathbf{P_1Q} = 0$

$$\begin{aligned}\mathbf{n_1} \bullet \langle \mathbf{Q} - \mathbf{P_1} \rangle &= 0 \\ \langle -6,22,29 \rangle \bullet \langle x, y+2, z-5 \rangle &= 0 \\ -6x + 22(y+2) + 29(z-5) &= 0 \\ -6x + 22y + 44 + 29z - 145 &= 0 \\ -6x + 22y + 29z - 101 &= 0 \\ 6x - 22y - 29z + 101 &= 0\end{aligned}$$

Therefore, the equation of the plane is $\boxed{6x - 22y - 29z + 101 = 0}$.

Answer 39E.

To find an equation for the plane that passes through the points $(1,5,1)$ and is perpendicular to the planes $2x+y-2z=2$ and $x+3z=4,$ use the scalar equation of the plane.

The scalar equation of the plane through $P_0(x_0,y_0,z_0)$ with normal vector $\mathbf{n}=\langle a,b,c\rangle$ is

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0. \text{ ...... (1)}$$

Let a,b,c be direction numbers of the normal $\mathbf{n}$ to the plane. (That is, this normal is the intersecting line of the given two planes.

The normal vector of the plane $2x+y-2z=2$ is $\mathbf{n}_1:\langle 2,1,-2\rangle.$

And the normal vector of the plane $x+3z=4$ is $\mathbf{n}_1:\langle 1,0,3\rangle.$

The normal of the required plane is perpendicular to the normals of the given two planes.

And their cross product $\mathbf{n}_1\times\mathbf{n}_2$ is perpendicular to the required plane and can be taken as the normal vector to the required plane.

$$\begin{aligned}\mathbf{n}&=\mathbf{n}_1\times\mathbf{n}_2\\&=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\2&1&-2\\1&0&3\end{vmatrix}\\&=\mathbf{i}(3)-\mathbf{j}(6+2)+\mathbf{k}(-1)\\&=3\mathbf{i}-8\mathbf{j}-\mathbf{k}\end{aligned}$$

Therefore the normal vector to the required plane is $\mathbf{n}\langle a,b,c\rangle=\langle 3,-8,-1\rangle.$

And the required plane passing through the point $(1,5,1)$ and this point $(1,5,1)$ satisfies the equation (1).

Substitute the point $P_0(x_0,y_0,z_0)=(1,5,1)$ and the normal vector $\mathbf{n}:\langle 3,-8,-1\rangle$ in the equation (1), get

$$\begin{aligned}a(x-x_0)+b(y-y_0)+c(z-z_0)&=0\\3(x-1)+(-8)(y-5)+(-1)(z-1)&=0\\3x-8y-z-3-5(-8)-(-1)&=0\\3x-8y-z-3+40+1&=0\\3x-8y-z+38&=0\end{aligned}$$

Hence, the required equation of the plane is $\boxed{3x-8y-z=-38}$.

Answer 40E.

To find an equation for the plane that passes through the line of intersection of the planes $x-z=1, y+2z=3$ and is perpendicular to the plane $x+y-2z=1,$ use the scalar equation of the plane.

The scalar equation of the plane through $P_0(x_0, y_0, z_0)$ with normal vector $\mathbf{n}=\langle a,b,c\rangle$ is

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0.$ ...... (1)

Since the required plane contains the line $L,$ so any point on the line lies on the plane.

To find the line of intersection of the planes $x-z=1$ and $y+2z=3,$ first need to find a point on the required line $L.$

For instance, find the point where the line intersects the *xy*-plane by setting $z=0$ in the equations of both planes.

Set z = 0 in both the planes.

$x-z=1$ ...... (1)

$y+2z=3$ ...... (2)

Put z = 0 in the equation (1)

$$\begin{aligned} x-z&=1 \\ x-0&=1 \\ x&=1 \end{aligned}$$

And put z = 0 in the equation (2)

$$\begin{aligned} y+2z&=3 \\ y+2(0)&=3 \\ y&=3 \end{aligned}$$

So the solution for the two equations (1) and (2) is $x=1, y=3.$

So the point $(x,y,z)=(1,3,0)$ is any point on the line L, it must be lies on the plane.

So take $P_0(x_0, y_0, z_0)=(1,3,0).$

Since the required plane is passes through the line *L*, so the normal vector $\mathbf{n} = \langle a, b, c \rangle$ of the required plane is perpendicular to the direction numbers of the line *L*.

So their dot product is zero.

The normal vector for the plane $x - z = 1$ is $\mathbf{n}_1 : \langle 1, 0, -1 \rangle$.

And the normal vector for the plane $y + 2z = 3$ is $\mathbf{n}_2 : \langle 0, 1, 2 \rangle$.

Since *L* lies on both the planes, it is perpendicular to both of the normal vectors **n**1 and **n**2. Thus, a vector **v** parallel to *L* is given by the cross product $\mathbf{n}_1 \times \mathbf{n}_2$.

$$\begin{aligned}
\mathbf{v} &= \mathbf{n}_1 \times \mathbf{n}_2 \\
&= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{vmatrix} \\
&= (0+1)\mathbf{i} - (2-0)\mathbf{j} + (1-0)\mathbf{k} \\
&= \mathbf{i} - 2\mathbf{j} + \mathbf{k}
\end{aligned}$$

Thus, the vector parallel to *L* is $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

The vector $\mathbf{v} = \langle 1, -2, 1 \rangle$ is describes the direction of the line *L* so the direction numbers of *L* are $\langle 1, -2, 1 \rangle$.

Since the required plane is passes through the line *L*, so the normal vector $\mathbf{n} = \langle a, b, c \rangle$ of the required plane is perpendicular to the direction numbers $\mathbf{v} = \langle 1, -2, 1 \rangle$ of the line *L*. So their dot product is zero.

$a - 2b + c = 0.$ ...... (3)

Also the required plane is perpendicular to the plane $x+y-2z=1$.

The normal vector of the plane $x+y-2z=1$ is $\langle 1,1,-2\rangle$.

So the dot product of the normal vector of the plane $x+y-2z=1$ and the normal vector $\mathbf{n}=\langle a,b,c\rangle$ of the required plane is zero. Thus

$a+b-2c=0.$ ...... (4)

Now solve the equations (3) and (4) to find the normal vector $\mathbf{n}=\langle a,b,c\rangle$ of the required plane.

Subtract equation (4) from equation (3), get

$$
\begin{aligned}
-3b+3c&=0\\
-b+c&=0\\
b&=c
\end{aligned}
$$

Choose $c=k$, then $b=k$.

Substitute $c=k,b=k$ in the equation (4), to find the value of *a*.

$$
\begin{aligned}
a+b-2c&=0\\
a+k-2k&=0\\
a-k&=0\\
a&=k
\end{aligned}
$$

The ratio of the normal vector $\mathbf{n}=\langle a,b,c\rangle$ is

$$
\begin{aligned}
a:b:c&=k:k:k\\
&=1:1:1
\end{aligned}
$$

Therefore the normal vector of the required plane is $\mathbf{n}=\langle 1,1,1\rangle$.

To find the equation of the plane substitute the known values in scalar equation of the plane.

Substitute the known values $P_0(x_0,y_0,z_0)=(1,3,0)$ and the normal vector $\mathbf{n}=\langle 1,1,1\rangle$ in the equation (1), get

$$
\begin{aligned}
a(x-x_0)+b(y-y_0)+c(z-z_0)&=0\\
1(x-1)+1(y-3)+1(z-0)&=0\\
x-1+y-3+z&=0\\
x+y+z&=4
\end{aligned}
$$

$5x+2y+z=13$

Hence the required plane equation is $\boxed{x+y+z=4}$.

Answer 41E.

Given 2x + 5y + z =10 we want to find out where this plane intersects the x, y, & z axes so we can sketch a graph of the given plane

To find out where the given plane crosses the x axis, set z and y = 0 and solve for x

$2x+5(0)+0 = 10$

$2x = 10$

$x = 5$

So the plane intersects the x axis at (5,0,0)

For intersection of y axis, set x = 0 & z = 0

$2(0)+5y+0=10$

$5y = 10$

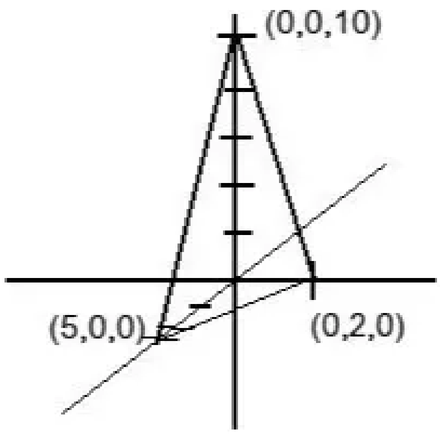
$y = 2$

So plane intersects the y-axis at (0,2,0)

For intersection of z axis: set x=0, y=0

$2(0)+5(0)+z=10$

z =10 so intersection of z-axis at (0,0,10)



Answer 42E.

We are given that an equation of the plane

$$3x + y + 2z = 6$$

Then, we can write in the standard form of the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

So,

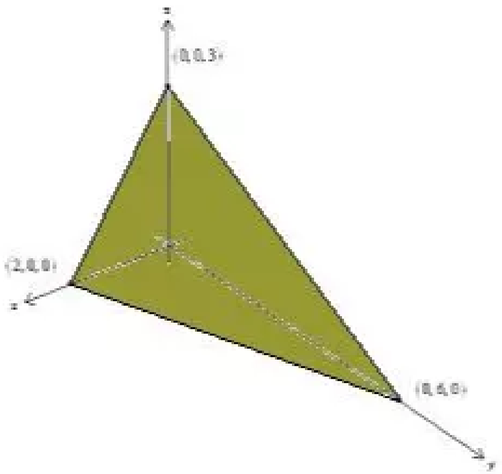
$$3x + y + 2z = 6$$

$$\Rightarrow \frac{3x}{6} + \frac{y}{6} + \frac{2z}{6} = 1$$

$$\Rightarrow \frac{x}{2} + \frac{y}{6} + \frac{z}{3} = 1$$

The x-intercept is $(2, 0, 0)$

The y-intercept is $(0, 6, 0)$ and the z-intercept is $(0, 0, 3)$



Answer 43E.

We are given that an equation of the plane

$$6x - 3y + 4z = 6$$

Then, we can write in the standard form of the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

So, $6x - 3y + 4z = 6$

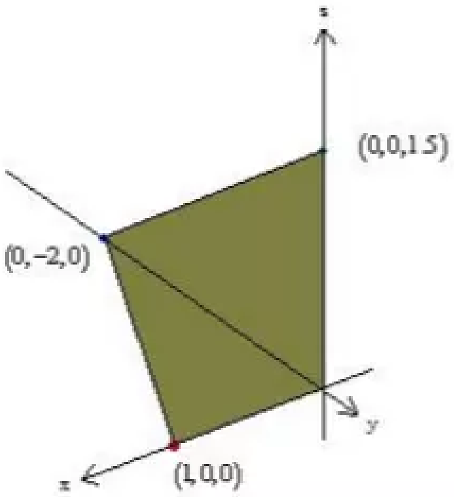
$$\Rightarrow \frac{6x}{6} - \frac{3y}{6} + \frac{4z}{6} = 1$$

$$\Rightarrow \frac{x}{1} + \frac{y}{-2} + \frac{z}{1.5} = 1$$

The x-intercept is $(1, 0, 0)$

The y-intercept is $(0, -2, 0)$

and the z-intercept is $(0, 0, 1.5)$



Answer 44E.

We are given that an equation of the plane

$$6x + 5y - 3z = 15$$

Then, we can write in the standard form of the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

So, $6x + 5y - 3z = 15$

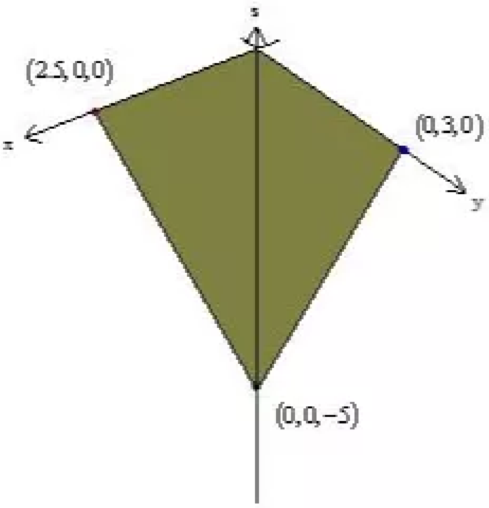
$$\Rightarrow \frac{6x}{15} + \frac{5y}{15} - \frac{3z}{15} = 1$$

$$\Rightarrow \frac{x}{2.5} + \frac{y}{3} + \frac{z}{-5} = 1$$

The x-intercept is $(2.5, 0, 0)$

The y-intercept is $(0, 3, 0)$

and the z-intercept is $(0, 0, -5)$



Answer 45E.

Consider the line π:

$x = 3 - t, y = 2 + t, z = 5t$.

And plane π: $x - y + 2z = 9$.

To determine the point of intersection at which the line and plane intersects, find the point, such that the point is on a line and the plane.

Let any point on line L be $P = (3 - t, 2 + t, 5t)$.

Substitute the point $(3 - t, 2 + t, 5t)$ in plane $x - y + 2z = 9$.

$$(3 - t) - (2 + t) + 2(5t) = 9$$
$$3 - t - 2 - t + 10t = 9$$
$$1 + 8t = 9$$
$$8t = 8$$
$$t = 1$$

To find the point of intersection of the line and plane, substitute $t=1$ in $(3-t,2+t,5t)$.

$$P=(3-(1),2+(1),5(1))$$
$$=(2,3,5)$$

Clearly, the point $(2,3,5)$ is on line for $t=1$.

To check the point $(2,3,5)$ is the point of intersection a line L and the plane π, substitute the point on $(2,3,5)$ in $x-y+2z=9$.

$$(2)-(3)+2(5)\stackrel{?}{=}9$$
$$2-3+10\stackrel{?}{=}9$$
$$9=9 \quad \text{True}$$

Therefore, the point $(2,3,5)$ satisfies the plane $x-y+2z=9$.

Hence, the point of intersection is $\boxed{(2,3,5)}$.

Answer 46E.

x = 1 + 2t

y = 4t

z = 2 - 3t

x + 2y - z + 1 = 0

Substitute the values of x,y and z in terms of t

(1 + 2t) + 2(4t) - (2 - 3t) + 1 = 0

1 + 2t + 8t - 2 + 3t +1 = 0

13t = 0

t = 0

x = 1 + 2(0) = 1

y = 4(0) = 0

z = 2 - 3(0) = 2

So the point of intersection is (1, 0 ,2)

Answer 47E.

Given the parametric equations of the line solve for t

x = y -1 = 2z ---> x =t , y = t + 1, z = t/2

this is possible because parametric equations are equal to t

next substitute these values into the equation of the plane 4x - y +3z = 8 for x, y, and z

which gives 4(t) - (t + 1) +3(t/2) = 8 --> 4t - t -1 +((3t)/2) = 8 --> 3t + ((3t)/2) = 9

--> 2(3t + ((3t)/2)) = 2(9) --> 6t + 3t = 18 --> 9t = 18 --> t = 2

next take this t value and substitute it back into the original parametric equations that were solved for x, y, and z

x = t --> x = 2

y = t+1 --> y = 2 +1 --> y =3

z= t/2 --> z=2/2 --> z=1

the point at which the line intersects the plane is (2,3,1)

Answer 48E.

The direction vector of the line through $(1,0,1)$ and $(4,-2,2)$ is

$\vec{v} = \langle 4-1, -2-0, 2-1 \rangle$

Taking $P_0 = (1,0,1)$, the parametric equations of the line are:

$$x = 1+3t, \qquad y = -2t, \qquad z = 1+t$$

Substituting for x, y, z in the equation of plane, $x+y+z=6$

$$1+3t+(-2t)+1+t=6$$
$$\Rightarrow 2t = 4$$
$$\Rightarrow t = 2$$

Then the point of intersection is :

$$x = 1+3(2), \quad y = -2(2), \quad z = 1+2$$

i.e. $(7,-4,3)$

Answer 49E.

Since line L lies in both the planes, It is perpendicular to both of the normal vector. Thus it is parallel to cross product of normal vectors

$$\vec{n_1} = \langle 1,1,1 \rangle \quad and \quad \vec{n_2} = \langle 1,0,1 \rangle$$

Now,

$$\vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= (1-0)\hat{i} - (1-1)\hat{j} + (0-1)\hat{k}$$

$$= \hat{i} - \hat{k}$$

Then direction numbers for line L are

$$\langle 1,0,-1 \rangle.$$

Answer 50E.

The normal vector perpendicular to plane $x+y+z=0$ is

$$\vec{n_1} = \langle 1,1,1 \rangle$$

The normal vector perpendicular to plane

$x+2y+3z=1$ is

$$\vec{n_2} = \langle 1,2,3 \rangle$$

The angle between two planes is the angle between their normals. Let θ be the angle between the planes.

Therefore, angle between their normals will be θ.

Now, $\vec{n}_1 \cdot \vec{n}_2 = 1\times1+1\times2+1\times3$
$= 6$

$$|\vec{n}_1| = \sqrt{(1)^2+(1)^2+(1)^2} = \sqrt{3}$$

$$|\vec{n}_2| = \sqrt{(1)^2+(2)^2+(3)^2} = \sqrt{14}$$

Therefore, $\cos\theta = \dfrac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \dfrac{6}{\sqrt{3}\sqrt{14}} = \dfrac{\sqrt{6}\times\sqrt{6}}{\sqrt{3\times2\times7}}$

$$= \frac{\sqrt{6}\times\sqrt{6}}{\sqrt{6}\sqrt{7}}$$

$$= \frac{\sqrt{6}}{\sqrt{7}} = \sqrt{\frac{6}{7}}$$

Hence,

$$\boxed{\text{Cosine of angle between the given planes} = \sqrt{\frac{6}{7}}}$$

Answer 51E.

The normal vector of given planes are

$$\vec{n}_1 = \langle 1,4,-3\rangle,\quad \vec{n}_2 = \langle -3,6,7\rangle$$

Because $\vec{n}_1 \neq \vec{n}_2$, thus two planes are not parallel.

Now, $\vec{n}_1 . \vec{n}_2 = 1(-3)+4(6)+(-3)7$
$= -3+24-21$
$= 0$

Thus $\vec{n}_1$ *and* $\vec{n}_2$ are perpendicular and their corresponding planes are also perpendicular.

Answer 52E.

The normal vectors of given planes are

$$\vec{n}_1 = \langle 1,-4,2\rangle \quad \vec{n}_2 = \langle 3,-12,6\rangle$$

Since $\vec{n}_2 = 3\vec{n}_1$, thus $\vec{n}_1$ *and* $\vec{n}_2$ are parallel and their corresponding planes are also parallel.

Answer 53E.

The normal vectors of gives planes are

$$\vec{n_1} = \langle 1,1,1 \rangle, \qquad \vec{n_2} = \langle 1,-1,1 \rangle$$

Since

$\vec{n_1} \neq \vec{n_2}$, The two planes are not parallel and

$$\begin{aligned}\vec{n_1}.\vec{n_2} &= (1)(1)+(1)(-1)+(1)(1)\\ &= 1-1+1\\ &= 1 \neq 0\end{aligned}$$

The two planes are not perpendicular.

$$\begin{aligned}\text{Thus} \quad \cos\theta &= \frac{\vec{n_1}.\vec{n_2}}{|\vec{n_1}||\vec{n_2}|}\\ &= \frac{1}{\sqrt{1^2+1^2+1^2}\sqrt{1^2+(-1)^2+1^2}}\\ &= \frac{1}{\sqrt{3}\sqrt{3}}\\ &= \frac{1}{3}\end{aligned}$$

or, $\theta = \cos^{-1}\left(\frac{1}{3}\right) = 70.5^o$

Answer 54E.

The normal vector of given planes are

$$\vec{n_1} = \langle 2,-3,4 \rangle, \quad \vec{n_2} = \langle 1,6,4 \rangle$$

Because $\vec{n_1} \neq \vec{n_2}$, thus two planes are not parallel.

$$\begin{aligned}\text{Since } \vec{n_1}.\vec{n_2} &= (2)(1)+(-3)(6)+4(4)\\ &= 2-18+16\\ &= 0\end{aligned}$$

Thus $\vec{n_1}$ and $\vec{n_2}$ are perpendicular, their corresponding planes are perpendicular.

Answer 55E.

The normal vectors of given planes are

$$\vec{n_1} = \langle 1,-4,2 \rangle, \quad \vec{n_2} = \langle 2,-8,4 \rangle$$

Since $\vec{n_2} = 2\vec{n_1}$, the vectors $\vec{n_1}$ *and* $\vec{n_2}$ are parallel, so their corresponding planes are also parallel.

Answer 56E.

The normal vectors of given planes are

$$\vec{n_1} = \langle 1, 2, 2 \rangle, \quad \vec{n_2} = \langle 2, -1, 2 \rangle$$

As $\vec{n_1} \neq \vec{n_2}$, thus planes are not parallel

and
$$\vec{n_1} \cdot \vec{n_2} = 1(2) + 2(-1) + 2(2)$$
$$= 2 - 2 + 4$$
$$= 4 \neq 0$$

The two planes are not perpendicular.

Thus
$$\cos\theta = \frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$$
$$= \frac{4}{\sqrt{1^2 + 2^2 + 2^2}\sqrt{2^2 + (-1)^2 + 2^2}}$$
$$= \frac{4}{\sqrt{9}\sqrt{9}} = \frac{4}{9}$$
$$\theta = \cos^{-1}\left(\frac{4}{9}\right)$$
$$\Rightarrow \theta = \cos^{-1}(0.444)$$
$$\Rightarrow \boxed{\theta = 64^\circ}$$

Answer 57E.

(a) Given Planes are Plane1: $x + y + z = 1$

Plane2: $x + 2y + 2z = 1$

normal vectors of the plane are:

n1 = <1, 1, 1>

n2 = <1, 2, 2>

we first need to find a point on L:

if $z = 0$ then:

$x + y = 1$

$x + 2y = 1$

$x = 1 - y$

$(1 - y) + 2y = 1$

$y + 1 = 1$

$y = 0$

then $x = 1$

Point P = (1, 0, 0) is on the line L

Since L lies in both planes, it is perpendicular to both of the normal vectors. thus a vector **v** is parallel to L:

v = **n1** x **n2** =

| **i** **j** **k** |

| 1 1 1 |

| 1 2 2 |

= (2 - 2)**i** - (2 - 1)**j** + (2 - 1)**k** = -**j** + **k**

now you have a vector (**v**) that is parallel to a point (P):

parametric equation of L:

$x = 1$

$y = -t$

$z = t$

(b)

The angle between 2 planes is:

cosθ = (**n1 . n2**) / |**n1**||**n2**|

cosθ = [(1*1) + (1*2) + (1*2)] / √(12 + 12 + 12) * √(12 + 22 + 22)

cosθ = [1 + 2 + 2] / √3 * √9

cosθ = 5/3√3

θ = arccos(5/3√3)

Answer 58E.

Consider the following planes:

$3x-2y+z=1$ and $2x+y-3z=3$

a)

The objective is to find parametric equations for line of intersection of the planes.

The planes $3x-2y+z=1$ and $2x+y-3z=3$ intersect in a line.

For instance, find the point where the line intersects the $xy-$ plane by setting $z=0$

in the equations of both planes.

This gives the equations,

$$3x-2y=1 \quad \text{......}(1)$$
$$2x+y=3 \quad \text{......}(2)$$

Solve the equations (1) and (2).

Multiply the equation (2) with 2

Then,

$$4x+2y=6 \quad \text{......}(3)$$

Subtract the equations (1) and (3) get

$$\begin{aligned} 3x-2y&=1 \\ \underline{4x+2y}&\underline{=6} \\ 7x&=7 \\ x&=1 \end{aligned}$$

Substitute the value $x=1$ in the equation (2).

$$\begin{aligned} 2(1)+y&=3 \\ y&=3-2 \\ y&=1 \end{aligned}$$

The solution is, $x=1, y=1$

The point $(1,1,0)$ lies on line L.

Hence, the line intersects the $xy-$ plane at the point $(1,1,0)$

Now observe that the line L lies on both planes and it is perpendicular to both of the normal vectors.

The two normal vectors for the planes are,

$\langle 3,-2,1\rangle$ and $\langle 2,1,-3\rangle$

The cross product of the two vectors gives a vector and it is parallel to the line of intersection.

Let $\mathbf{a}$ be the vector parallel to line.

Then the vector $\mathbf{a}$ is parallel to both two planes $x+y+z=1$ and $x+2y+2z=1$.

Let $\mathbf{n}_1,\mathbf{n}_2$ be the normal vectors to the planes.

$$\mathbf{a}=\mathbf{n}_1\times\mathbf{n}_2$$

$$\begin{aligned}\mathbf{a}&=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 3 & -2 & 1\\ 2 & 1 & -3\end{vmatrix}\\ &=\mathbf{i}(6-1)-\mathbf{j}(-9-2)+\mathbf{k}(3+4)\\ &=5\mathbf{i}+11\mathbf{j}+7\mathbf{k}\end{aligned}$$

$$\mathbf{a}=\langle 5,11,7\rangle$$

The vector equation of the line passing through point $(1,1,0)$ and parallel to vector $\mathbf{a}=\langle 5,11,7\rangle$ is given as,

$$\mathbf{r}=\langle 1,1,0\rangle+t\langle 5,11,7\rangle$$
$$\mathbf{r}=\langle 1+5t,1+11t,7t\rangle$$

The parametric equations of the line are, $x=1+5t,\ y=1+11t$, and $z=7t$

Thus, the parametric equations of line are,

$\boxed{x=1+5t,\ y=1+11t,\text{ and } z=7t}$.

b)

The objective is to find the angle between the two planes, using the result.

Let θ be the angle between the vectors $\mathbf{n}_1$ and $\mathbf{n}_2$, then

$$\cos\theta = \frac{\mathbf{n}_1 \bullet \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}$$

The two normal vectors of these planes are,

$\langle 3,-2,1\rangle$ and $\langle 2,1,-3\rangle$

Find the angle θ between two planes.

$$\cos\theta = \frac{3(2)-2(1)+1(-3)}{\sqrt{9+4+1}\sqrt{4+1+9}}$$
$$= \frac{6-2-3}{\sqrt{14}\sqrt{14}}$$
$$= \frac{1}{14}$$

$$\cos\theta = \frac{1}{14}$$
$$\theta = \cos^{-1}\left(\frac{1}{14}\right)$$
$$= 85.904°$$

Therefore, the angle between the two planes are $\boxed{\theta = 85.904°}$.

Answer 59E.

We are given that two equations of the planes

$5x-2y-2z=1$ and $4x+y+z=6$

We first need to find a point on L. For instance, we can find the point where the line intersects the xy- plane by setting $z=0$ in the equations of both planes.

This gives the equations

$5x-2y=1$ and $4x+y=6$

Let

$$5x-2y=1............(1)$$

$$4x+y=6............(2)$$

The equation (2) multiply by 2 and adding with, (1), we get

$$5x - 2y + 8x + 2y = 1 + 12$$

$$\Rightarrow 13x = 13$$

$$\Rightarrow x = 1$$

Put $x = 1$ in the equation (1), we get

$$5(1) - 2y = 1$$

$$\Rightarrow -2y = 1 - 5$$

$$\Rightarrow -2y = -4$$

$$\Rightarrow y = 2$$

So, the solution is $x = 1$, $y = 2$

So, the point $(1, 2, 0)$ lies on L

Now, we observe that since L lies in both planes, it is perperdicular to both of the normal vectors. Thus a vector v parallel to L is given by the cross product

$$v = n_1 \times n_2 = \begin{vmatrix} i & j & k \\ 5 & -2 & -2 \\ 4 & 1 & 1 \end{vmatrix}$$

$$= i(-2 + 2) + (-8 - 5)j + (5 + 8)k$$

$$= 0i - 13j + 13k$$

$$v = 0i - 13j + 13k$$

And so the symmetric equations of L can be written as

$$\frac{x-1}{0} = \frac{y-2}{-13} = \frac{z-0}{13} = g\left(\text{let}\right)$$

We get $x = 1$ and $y - 2 = -z$

So, the required symmetric equations are

$$x = 1 \text{ and } y - 2 = -z$$

Answer 60E.

We are given that two equations of the planes

$$z = 2x - y - 5 \text{ and } z = 4x + 3y - 5$$

We first need to find a point on L. For instance, we can find the point where the line intersects the xy- plane by setting $z = 0$ in the equations of both planes.

This gives the equations

$$2x - y = 5$$

and

$$4x + 3y = 5$$

Let

$$2x - y = 5 \text{.........}(1)$$

$$4x + 3y = 5 \text{.......}(2)$$

The equation (1) multiply by 3 and adding with (2), we get

$$4x + 3y + 6x - 3y = 5 + 15$$

$$\Rightarrow 10x = 20$$

$$\Rightarrow x = 2$$

Put $x = 2$ in the equation (1), we get

$$2(2) - y = 5$$

$$\Rightarrow -y = 5 - 4$$

$$\Rightarrow y = -1$$

So, the solution is $x = 2$ **and** $y = -1$
So, the point $(2, -1, 0)$ **lies on** L

Now, we observe that since L lies in both planes, it is perperdicular to both of the normal vectors. Thus a vector v parallel to L is given by the cross product

$$v = n_1 \times n_2 = \begin{vmatrix} i & j & k \\ 2 & -1 & -1 \\ 4 & 3 & -1 \end{vmatrix}$$

$$= (1+3)i + (-4+2)j + (6+4)k$$

$$= 4i - 2j + 10k$$

$$\Rightarrow v = 4i - 2j + 10k$$

And so the symmetric equations of L can be written as

$$\frac{x-2}{4} = \frac{y+1}{-2} = \frac{z-0}{10}$$

So, the required symmetric equations are $\frac{x-2}{4} = \frac{y+1}{-2} = \frac{z}{10}$

Answer 61E.

Given that the points are (3,4,0) and (1,0,-2) is

a = <3-1,4-0,0-(-2)> = <2,4,2>

Since vector will be perpendicular to our plane this will be our normal vector.

Now to find a point on a plane.

we know the plane lies halfway between the two given points so a point a plane is hafway between the 2 points

$\left(\frac{(1+3)}{2}, \frac{(0+4)}{2}, \frac{(-2+0)}{2}\right) = (2,2,-1)$

Now we write our plane equation

2(x-2)+4(y-2)+2(z+1)=0

2x-4+4y-8+2z+2=0

2x+4y+2z=10

x + 2y + z = 5

Answer 62E.

Given that a plane that is equaldistant from the points (2,5,5) & (-6,3,1) lies halfway between the points and is perpendicular to the vector formed by the points

n = <2-(-6),5-3,5-1>

= <8,2,4> This is our normal vector.

Now to find a point on the plane $\left(\frac{2+-6}{2}, \frac{5+3}{2}, \frac{5+1}{2}\right)$ = (-2,4,3)

Now write plane equation

8(x-(-2))+2(y-4)+4(z-3)=0

8x+16+2y-8+4z-12=0

8x+2y+4z=4

4x + y + 2z = 2

Answer 63E.

The plane intersects at point $P(a,0,0)$ with x-axis $Q(0,b,0)$ with y-axis, and $R(0,0,c)$ with z-axis.

The vector $\vec{a}$ and $\vec{b}$ corresponding to $\overrightarrow{PQ}$ *and* $\overrightarrow{PR}$ are

$$\vec{a} = \langle -a, b, 0 \rangle$$

$$and\ \vec{b} = \langle -a, 0, c \rangle$$

Since both a and b lie in the plane, their cross product $\vec{a} \times \vec{b}$ is orthogonal to the plane and can be taken as normal vector, then

$$\vec{n} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix}$$

$$= bc\hat{i} + ac\hat{j} + ab\hat{k}$$

With the point (a,0,0) and the normal vector $\vec{n}$ and equation of the plane is

$$bc(x-a)+ac(y-0)+ab(z-0)=0$$

$$bcx+acy+abz=abc$$

Divide by abc on both sides, we get

$$\boxed{\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1}$$

Answer 64E.

a)

Consider the following parametric form of the lines:

$$\mathbf{r}=\langle 1,1,0\rangle+t\langle 1,-1,2\rangle$$

$$\mathbf{r}=\langle 2,0,2\rangle+s\langle -1,1,0\rangle$$

The objective is to find the point of intersection of these lines.

The parametric equations of given lines are

$$x=1+t$$

$$y=1-t$$

$$z=2t$$

and

$$x=2-s$$

$$y=s$$

$$z=2$$

At intersection point, each coordinates are same.

So, equate the two coordinates of parametric lines.

$$\begin{cases} 1+t=2-s \\ s+t=1 \end{cases}$$

$$\begin{cases} 1-t=s \\ s+t=1 \end{cases}$$

$$\begin{cases} 2t=2 \\ t=1 \end{cases}$$

To find the point where the lines intersect, put $t=1, s=0$. Then,

$$x=2-0$$

$$y=0$$

$$z=2$$

Therefore, the required point of intersection is $\boxed{(2,0,2)}$.

b)

Find the equation of the plane that contains the given lines as follows:

Let $\mathbf{n}$ be vector normal to the plane.

The vectors $\langle 1,-1,2\rangle,\langle -1,1,0\rangle$ are parallel to plane.

Note that $\mathbf{n}$ is perpendicular to $\langle 1,-1,2\rangle,\langle -1,1,0\rangle$ and it is given by their cross product as follows:

$$\mathbf{n}=\langle 1,-1,2\rangle\times\langle -1,1,0\rangle$$
$$\mathbf{n}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & -1 & 2\\ -1 & 1 & 0\end{vmatrix}$$
$$\mathbf{n}=\mathbf{i}\left((-1)\cdot 0-2\cdot 1\right)-\mathbf{j}\left(1\cdot 0-(-1)\cdot 2\right)+\mathbf{k}\left(1\cdot 1-(-1)\cdot(-1)\right)$$
$$\mathbf{n}=\langle -2,-2,0\rangle$$

As the plane contain both lines, the common point also lies on the plane.

Thus, the equation of the plane passing through point $(2,0,2)$ and has normal vector $\mathbf{n}$ is

$$-2(x-2)+-2(y-0)+0(z-2)=0$$
$$-2x-2y=-4$$
$$\boxed{x+y=2}$$

Therefore, the required equation of plane is $\boxed{x+y=2}$.

Answer 65E.

The parametric equation of the required line L that passes through the point $(x_1,y_1,z_1)=(0,1,2)$ is

$$x=x_1+at,\quad y=y_1+bt,\quad z=z_1+ct$$
$$x=0+at,\quad y=1+bt,\quad z=2+ct$$

Here, $\langle a,b,c\rangle$ are direction numbers of the line L.

The normal vector to the plane $x+y+z=2$ is $\mathbf{n}=\langle 1,1,1\rangle$.

Given that the line L is parallel to the plane $x+y+z=2,$ so the line L is perpendicular to the normal vector $\langle 1,1,1\rangle$.

Then $\langle a,b,c\rangle\cdot\langle 1,1,1\rangle=0$

Thus, $a+b+c=0$ .... (1)

Also the line L is perpendicular to the following line

$x=1+t, \quad y=1-t, \quad z=2t$ ...... (2)

The direction numbers of the this line (2) is $\langle 1,-1,2\rangle$.

Then $\langle a,b,c\rangle \cdot \langle 1,-1,2\rangle = 0$

Thus, $a-b+2c=0$ .... (3)

Solve (1) and (3).

Add (1) and (3).

$$2a+3c=0$$
$$c=-\frac{2}{3}a$$

Take $a=3$, then $c=-2$.

Substitute $a=3, c=-2$ in the equation (1) and solve for b.

$$a+b+c=0$$
$$3+b-2=0$$
$$c=-1$$

Hence, the required parametric equations of the line L is $\boxed{x=3t,\ \ y=1-t,\ z=2-2t}$.

Answer 66E.

The scalar equations;

$x=x_0+at,$
$y=y_0+bt,$ ...... (1)
$z=z_0+ct$

are called parametric equations of a line L through the point $P_0(x_0,y_0,z_0)$ and parallel to the vector $\mathbf{v}=\langle a,b,c\rangle$. Each value of the parameter $t\in\mathbb{R}$ gives a point (x,y,z) on L.

Consider the line

$x = 1 + t,\ y = 1 - t,\ z = 2t.$ ...... (2)

To find the parametric equations for the line through the point $(0,1,2)$ that is perpendicular to (2), use (1) as follows;

Let the direction numbers of required line be $a, b,$ and $c.$

Since the line passes through (0, 1, 2);

Therefore, the parametric equations of the required line are;

$$x = 0 + at,$$
$$y = 1 + bt,$$
$$z = 2 + ct$$

Implies;

$$x = at,$$
$$y = 1 + bt, \quad \text{...... (3)}$$
$$z = 2 + ct$$

Since the required line is perpendicular to (2);

The direction numbers of this line are $(1, -1, 2).$

Also then the product of the direction numbers of the required line and (2) is zero.

Implies;

$$a(1) + b(-1) + c(2) = 0$$

Implies;

$$a - b + 2c = 0 \quad \text{...... (4)}$$

Let $(1+t, 1-t, 2t)$ be any point on the line (2).

Since line (2) and (3) intersect;

Therefore, the above point lies on line (3) for some *t*.

Implies;

The point $(1+t, 1-t, 2t)$ satisfies the parametric equations (3).

Implies;

$$x = at,$$
$$y = 1+bt,$$
$$z = 2+ct$$

Implies;

$$1+t = at,$$
$$1-t = 1+bt,$$
$$2t = 2+ct$$

Implies;

$$a = \frac{1+t}{t},$$
$$b = -1, \quad \ldots\ldots (5)$$
$$c = \frac{2t-2}{t}$$

Substitute (5) values in the equation (4), to obtain;

$$\frac{1+t}{t} - (-1) + 2\left(\frac{2t-2}{t}\right) = 0$$

Implies;

$$\frac{1+t}{t} + 1 + \left(\frac{4t-4}{t}\right) = 0$$

Implies;

$$\frac{1+t+t+4t-4}{t} = 0$$

Implies;

$$\frac{6t-3}{t} = 0$$

Implies;

$$6 - \frac{3}{t} = 0$$

Implies;

$$\frac{3}{t} = 6$$

Implies;

$$t = \frac{1}{2} \quad \ldots\ldots (6)$$

Substitute (6) in (5) to obtain;

$$a = \frac{1+t}{t}$$
$$= \frac{1+\frac{1}{2}}{\frac{1}{2}}$$
$$= 3$$

And;

$$c = \frac{2t-2}{t}$$
$$= \frac{2\left(\frac{1}{2}\right)-2}{\frac{1}{2}}$$
$$= -2$$

Therefore, the direction numbers for the required line are;

$$a = 3,$$
$$b = -1,$$
$$c = -2$$

Thus, the parametric equations of the required line passing through the point $(x_0, y_0, z_0) = (0,1,2)$ and parallel to the vector $\langle a,b,c\rangle = \langle 3,-1,-2\rangle$ are;

$$x = 0 + 3t,$$
$$y = 1 + (-1)t,$$
$$z = 2 + (-2)t$$

Implies;

$$x = 3t,$$
$$y = 1 - t,$$
$$z = 2 - 2t$$

Hence, the parametric equations of the required line are;

$$\boxed{x = 3t,\ y = 1 - t,\ z = 2 - 2t}.$$

Answer 67E.

Consider the planes are as follows:

$$P_1 : 3x + 6y - 3z = 6$$
$$P_2 : 4x - 12y + 8z = 5$$
$$P_3 : 9y = 1 + 3x + 6z$$
$$P_4 : z = x + 2y - 2$$

Find which of the given planes are parallel and which of them are identical.

Parallel planes: Two planes are parallel if their normal vectors are parallel.

The normal vector is a vector perpendicular to it, and is commonly denoted as $\mathbf{N}$ or $\mathbf{n}$.

The normal vectors for the following four planes are as follows:

The normal vectors for the plane P_1 are as follows:

$$P_1 : 3x + 6y - 3z = 6$$
$$x + 2y - z = 2 \qquad \text{(Dividing by 3)}$$

The normal vector for *P*1 is $\mathbf{n}_1 = \langle 1, 2, -1 \rangle$.

The normal vectors for the plane P_2 are as follows:

$$P_2 : 4x - 12y + 8z = 5$$

So, the normal vector for *P*2 is $\mathbf{n}_2 = \langle 4, -12, 8 \rangle$

The normal vectors for the plane P_3 are as follows:

$$P_3 : 9y = 1 + 3x + 6z$$
$$3x - 9y + 6z = 1 \qquad \text{(Standard form)}$$

So, the normal vector for *P*3 is $\mathbf{n}_3 = \langle 3, -9, 6 \rangle$

The normal vectors for the plane P_4 are as follows:

$$P_4 : z = x + 2y - 2$$
$$x + 2y - z = 2 \qquad \text{(Standard form)}$$

So, the normal vector for *P*4 is $\mathbf{n}_4 = \langle 1, 2, -1 \rangle$

Since $\mathbf{n}_1 = \langle 1, 2, -1 \rangle$ and also $\mathbf{n}_4 = \langle 1, 2, -1 \rangle$.

So, **n**1 and **n**4 are identical vectors.

Thus, the planes ***P*1 and *P*4 are identical**.

Also, $\mathbf{n}_2 = \langle 4, -12, 8 \rangle$

$= 4\langle 1, -3, 2 \rangle$ Common out 4.

$\frac{1}{4}\mathbf{n}_2 = \langle 1, -3, 2 \rangle$ Multiply by $\frac{1}{4}$ on both sides.

Thus, $\frac{1}{4}\mathbf{n}_2 = \langle 1, -3, 2 \rangle$ ...... (1).

And $\mathbf{n}_3 = \langle 3, -9, 6 \rangle$ Common out 3
$= 3\langle 1, -3, 2 \rangle$

$= 3\left(\frac{1}{4}\mathbf{n}_2\right)$ Since from (1)

Since the vector $\mathbf{n}_3$ is a scalar multiple of vector $\mathbf{n}_2$, $\mathbf{n}_3 = \frac{3}{4}\mathbf{n}_2$, which means that **n**2 and **n**3 are parallel vectors.

Therefore, the corresponding planes *P*2 **and** *P*3 **are parallel**.

Answer 68E.

Consider the lines,

$$L_1 : x = 1 + 6t, y = 1 - 3t, z = 12t + 5$$
$$L_2 : x = 1 + 2t, y = t, z = 1 + 4t$$
$$L_3 : 2x - 2 = 4 - 4y = z + 1$$
$$L_4 : \mathbf{r} = \langle 3, 1, 5 \rangle + t\langle 4, 2, 8 \rangle$$

The objective is to find which of the above four lines are parallel and which of them are identical.

Consider the line,

$$L_1 : x = 1 + 6t, y = 1 - 3t, z = 12t + 5$$

The symmetric equations of the line L_1 are,

$$x - 1 = 6t, y + 1 = -3t, z - 5 = 12t$$
$$\frac{x-1}{6} = \frac{y+1}{-3} = \frac{z-5}{12} = t$$

Therefore, the normal vector for line *L*1 is, $\mathbf{v}_1 = \langle 6, -3, 12 \rangle$.

Consider the line,

$$L_2 : x = 1 + 2t, y = t, z = 1 + 4t$$

The symmetric equations of line L_2 are,

$$x - 1 = 2t, y - 0 = t, z - 1 = 4t$$
$$\frac{x-1}{2} = \frac{y-0}{1} = \frac{z-1}{4} = t$$

Therefore, the normal vector for line *L*2 is, $\mathbf{v}_2 = \langle 2, 1, 4 \rangle$.

Consider the line,

$$L_3 : 2x - 2 = 4 - 4y = z + 1$$

The symmetric equations of the line L_3 are,

$$2x - 2 = 4 - 4y = z + 1 = t \text{ (say)}$$

Divide whole equation by 4.

$$\frac{2(x-1)}{4} = \frac{-4(y-1)}{4} = \frac{z+1}{4} = t$$
$$\frac{x-1}{2} = \frac{y-1}{-1} = \frac{z+1}{4} = t$$

Therefore, the normal vector for line *L*3 is, $\mathbf{v}_3 = \langle 2, -1, 4 \rangle$.

Consider the line,

$$L_4 : \mathbf{r} = \langle 3,1,5 \rangle + t\langle 4,2,8 \rangle$$

The symmetric equations of the line L_4 are,

$$\frac{x-3}{4} = \frac{y-1}{2} = \frac{z-5}{8} = t$$

Therefore, the normal vector for line *L*4 is, $\mathbf{v}_4 = \langle 4, 2, 8 \rangle$.

In this problem,

$$\mathbf{v}_1 = 3\mathbf{v}_3, \mathbf{v}_2 = \frac{1}{2}\mathbf{v}_4$$

Thus, $L_1 \parallel L_3$, and $L_2 \parallel L_4$.

Hence, the lines L_1, L_3 and L_2, L_4 are parallel.

Since the normal vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ are distinct.

Therefore, there are no identical lines.

Answer 69E.

If we put $t = 0$ in parametric equations,and Let P (4,1,-2) we get the point $Q(1, 3, 4)$ and if we put $t = 1$, we get $R(2, 1, 1)$.

The vectors corresponding to $\overrightarrow{QR}$ $\overrightarrow{QP}$ are

$$a = \langle 1, -2, -3 \rangle, b = \langle 3, -2, -6 \rangle$$

Now, we compute

$$a \times b = \begin{vmatrix} i & j & k \\ 1 & -2 & -3 \\ 3 & -2 & -6 \end{vmatrix} = 6i - 3j + 4k$$

We can use

$$d = \frac{|a \times b|}{|a|} \dots\dots\dots (1)$$

To compute distance P from the given line

$$|a| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

Putting values in (1), we get

$$d = \frac{\sqrt{(6)^2 + (3)^2 + (4)^2}}{\sqrt{14}} = \sqrt{\frac{61}{14}}$$

Answer 70E.

Given that P(0,1,3)

If we put $t = 0$ in parametric equations, we get the point $Q(0, 6, 3)$ and if we put $t = 1$, we get $R(2, 4, 4)$.

The vectors corresponding to $\overrightarrow{QR}$ and $\overrightarrow{QP}$ are

$$a = \langle 2, -2, 1\rangle, b = \langle 0, -5, 0\rangle$$

Now, we compute

$$a \times b = \begin{vmatrix} i & j & k \\ 2 & -2 & 1 \\ 0 & -5 & 0 \end{vmatrix} = 5i - 10k$$

We can use

$$d = \frac{|a \times b|}{|a|} \ldots\ldots(1)$$

to compute distance P from the given line

$$|a| = \sqrt{(2)^2 + (-2)^2 + 1} = 3$$

Putting values in (1), we get

$$d = \frac{\sqrt{(5)^2 + (-10)^2}}{3} = \frac{\sqrt{125}}{3} = \frac{5\sqrt{5}}{3}$$

Answer 71E.

We are given that a point $(1, \ -2, 4)$ and the plane $3x + 2y + 6z = 5$

We have to find the distance D from the point to the given plane .

Let

$$P_1(x_1, y_1, z_1) = P_1(1, \ -2, 4)$$

Then, the distance D given by the formula

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

where $a = 3, b = 2, c = 6 \text{and} d = \ -5$

So,

$$D = \frac{|(3)(1) + 2(-2) + 6(4) - 5|}{\sqrt{(3)^2 + (2)^2 + (6)^2}}$$

$$= \frac{|3 - 4 + 24 - 5|}{\sqrt{9 + 4 + 36}}$$

$$= \frac{18}{7}$$

$$\Rightarrow D = \frac{18}{7}$$

So, the distance from the point to the given plane is $\frac{18}{7}$

Answer 72E.

We are given that a point $(-6, 3, 5)$ and the plane $x-2y-4z=8$

We have to find the distance D from the point to the given plane.

Let $P_1(x_1, y_1, z_1) = P_1(-6, 3, 5)$

Then, the distance D given by the formula

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Where $a = 1, b = -2, c = -4 \text{ and } d = -8$

So, $D = \frac{|1(-6) + (-2)(3) + (-4)(5) - 8|}{\sqrt{(1)^2 + (-2)^2 + (-4)^2}}$

$$= \frac{|-6 - 6 - 20 - 8|}{\sqrt{1 + 4 + 16}}$$

$$= \frac{|-40|}{\sqrt{21}}$$

$$= \frac{40}{\sqrt{21}}$$

$$\Rightarrow D = \frac{40}{\sqrt{21}}$$

So, the distance from the point to the given plane is $\frac{40}{\sqrt{21}}$.

Answer 73E.

We are given that two equations of the parallel planes

$2x - 3y + z = 4$ and $4x - 6y + 2z = 3$

First we note that the planes ar parallel because their normal vectors $< 2, \ -3, 1 >$ and

$< 4, \ -6, 2 >$ are parallel.

To find the distance D between the planes, we choose any point on one plane and calculate its distance to the other plane. In particular, if we put $y = z = 0$ in the equation of the first plane,

we get $2x = 4 \Rightarrow x = 2$

So $(2, 0, 0)$ is a point in this plane.

The distance between $(2, 0, 0)$ and the plane $4x - 6y + 2z = 3$ is

$$D = \frac{|4(2) - 6(0) + 2(0) - 3|}{\sqrt{(4)^2 + (-6)^2 + (2)^2}}$$

$$= \frac{5}{\sqrt{16 + 36 + 4}}$$

$$= \frac{5}{\sqrt{56}}$$

$$= \frac{5}{2\sqrt{14}}$$

$$\Rightarrow D = \frac{5}{2\sqrt{14}}$$

So, the distance between the planes is $\frac{5}{2\sqrt{14}}$

Answer 74E.

Consider the following planes:

$$6z = 4y - 2x$$
$$9z = 1 - 3x + 6y$$

The objective is to find the distance between the given parallel planes.

Rewrite the plane equations as follows:

$$2x - 4y + 6z = 0$$
$$3x - 6y + 9z = 1$$

Note that the planes are parallel because their normal vectors $\langle 2, -4, 6 \rangle$ and $\langle 3, -6, 9 \rangle$ are parallel.

Find the distance D between the planes by choosing any point on one plane and calculate its distance to the other plane.

In particular, if $y = z = 0$ is substitute in the equation of the second plane,

That implies,

$$3x - 6(0) + 9(0) = 1$$
$$3x = 1$$
$$x = \frac{1}{3}$$

Hence $\left(\frac{1}{3}, 0, 0\right)$ is a point in this plane.

Find the distance D as follows:

The distance D from a point $P_1(x_1, y_1, z_1)$ to the plane $ax+by+cz+d$ is,

$$D=\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

The distance D from a point $P_1\left(\frac{1}{3},0,0\right)$ to the plane $2x-4y+6z=0$ is as shown below:

$$\begin{aligned}D&=\frac{|2(1/3)+(-4)(0)+6(0)|}{\sqrt{(2)^2+(-4)^2+(6)^2}}\\&=\frac{|2(1/3)|}{\sqrt{56}}\\&=\frac{2}{3\sqrt{56}}\\&=\frac{2}{3(2)\sqrt{14}}\\&=\frac{1}{3\sqrt{14}}\end{aligned}$$

Therefore, the distance between the two parallel planes is $\boxed{\frac{1}{3\sqrt{14}}}$.

Answer 75E.

The two given planes

$ax+by+cz+d_1=0$ *and* $ax+by+cz+d_2=0$ are parallel.

We choose any point on second plane by putting y =z = 0, we get $x=-\frac{d_2}{a}$. so

$\left(-\frac{d_2}{a},0,0\right)$ is a point on the second plane.

This distance of this point from the first plane is

$$\begin{aligned}&=\frac{\left|a\left(-\frac{d_2}{a}\right)+0+0+d_1\right|}{\sqrt{a^2+b^2+c^2}}\\&=\frac{|-d_2+d_1|}{\sqrt{a^2+b^2+c^2}}\\&=\boxed{\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}}\end{aligned}$$

Answer 76E.

The equation of given plane is

$$x+2y-2z=1$$

Or $\quad x+2y-2z-1=0 \qquad$ --- (1)

The equation of the plane parallel to given plane is:

$$x+2y-2z+d=0$$

It is given that the distance between these two parallel planes is 2 units.
Then using the formula of distance between two parallel planes, we have:

$$2=\frac{|-1-d|}{\sqrt{1^2+2^2+2^2}}$$

i.e. $\quad 2=\frac{|1+d|}{\sqrt{9}}$

i.e. $\quad \pm 2=\frac{1+d}{3}$

i.e. $\quad 2=\frac{1+d}{3} \quad or \quad -2=\frac{1+d}{3}$

i.e. $\quad d=5,-7$

Hence the required equations of planes are:

$$\boxed{\begin{array}{l} x+2y-2z+5=0 \\ \text{and } x+2y-2z-7=0 \end{array}}$$

Answer 77E.

The given lines are

$$x=y=z$$

And $\quad x+1=y/2=z/3$

Or $\quad \frac{x-0}{1}=\frac{y-0}{1}=\frac{z-0}{1}=t(say) \qquad ---L_1$

And $\quad \frac{x+1}{1}=\frac{y-0}{2}=\frac{z-0}{3}=s(say) \qquad ---L_2$

Now the two lines are not parallel because the associated vectors $\langle 1,1,1\rangle$ *and* $\langle 1,2,3\rangle$ are not parallel (as their components are neither proportional nor equal.)

If the above two lines had a point of intersection then for some values of t and s we must have

$t=-1+s \qquad ---(i)$

$t=2s \qquad ---(ii)$

$t=3s \qquad ---(iii)$

On solving equations (i) and (ii) we find s = -1, t = -2. But these values do not satisfy the equation (iii). Hence the given lines do not intersect and are skew.

Since two lines are skew, they lie in two different planes P_1 *and* P_2. The distance between two lines is same as the distance between planes P_1 *and* P_2. The common normal vector to both planes must be orthogonal to both $\vec{v_1} = \langle 1,1,1 \rangle$ (*the direction of* L_1) and $\vec{v_2} = \langle 1,2,3 \rangle$ (*the direction of* L_2). so a normal vector is

$$\vec{n} = \vec{v_1} \times \vec{v_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= \hat{i} - 2\hat{j} + \hat{k}$$

If we put s = 0 in equation of L_2 we get the point $(-1,0,0)$ and L_2 and so an equation for P_2 is

$$1(x+1) - 2(y-0) + 1(z-0) = 0$$

Or $\quad x - 2y + z - 1 = 0$

If we now set t = 0 in equation for L_1, we get the point $(0,0,0)$ *on* P_1. So the distance between L_1 and L_2 is the same as the distance from $(0,0,0)$ to plane $x - 2y + z - 1 = 0$

Thus $\quad D = \dfrac{|(0) - 2(0) + (0) - 1|}{\sqrt{1^2 + 2^2 + 1^2}}$

i.e. $\quad \boxed{D = \dfrac{1}{\sqrt{6}}}$

Answer 78E.

Consider the parametric equations,

$x = 1+t, y = 1+6t, z = 2t$ and $x = 1+2s, y = 5+15s, z = -2+6s$.

The object is to find the distance between the skew lines.

Rewrite the parametric equations as,

$x - 1 = t, \dfrac{y-1}{6} = t, \dfrac{z}{2} = t$ and $\dfrac{x-1}{2} = s, \dfrac{y-5}{15} = s, \dfrac{z+2}{6} = s$.

It follows that,

$\dfrac{x-1}{1} = \dfrac{y-1}{6} = \dfrac{z-0}{2} = t$ and $\dfrac{x-1}{2} = \dfrac{y-5}{15} = \dfrac{z-(-2)}{6} = s$.

Their relative parallel vectors are given as,

$\mathbf{v_1} = \langle 1,6,2 \rangle$

$\mathbf{v_2} = \langle 2,15,6 \rangle$

The points on the lines are $(1,1,0), (1,5,-2)$.

The vector perpendicular to both vectors is given by the cross product,

$$\mathbf{v}_1 \times \mathbf{v}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 6 & 2 \\ 2 & 15 & 6 \end{vmatrix}$$

$$= \mathbf{i}(36-30) - \mathbf{j}(6-4) + \mathbf{k}(15-12)$$

$$= \langle 6,-2,3 \rangle$$

The plane passing through $(1,1,0)$ and has normal vector $\langle 6,-2,3\rangle$ is given as,

$$6(x-1)-2(y-1)+3(z-0)=0$$

$$6x-6-2y+2+3z=0$$

$$6x-2y+3z-4=0$$

The plane parallel to $6x-2y+3z-4=0$ and passing through point $(1,5,-2)$ is,

$$6x-2y+3z+d=0$$

$$6(1)-2(5)+3(-2)+d=0$$

$$6-10-6+d=0$$

$$d=10$$

$$6x-2y+3z+10=0$$

These two planes contain given lines which are parallel, so given lines are skew lines.

The distance between skew lines is distance between parallel planes and the distance is calculated as,

$$D=\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$$

$$=\frac{|10-(-4)|}{\sqrt{6^2+(-2)^2+3^2}}$$

$$=\frac{14}{7}$$

$$=2$$

Hence, the distance between the two skew lines is

$$D=\boxed{2}.$$

Answer 79E.

Let L_1 be the line through the origin and the point $(2,0,-1)$

Then L_1 is given by $\frac{x}{2}=\frac{y}{0}=\frac{z}{-1}=t$

From this, we get $x=2t, y=0, z=-t$

And let L_2 be the line passing through the points $(1,-1,1)$ and $(4,1,3)$

Then L_2 is given by $\frac{x-1}{3}=\frac{y+1}{2}=\frac{z-1}{2}=s$

From this, we get $x=3s+1, y=2s-1, z=2s+1$

We have to find the distance between the lines L_1 and L_2

We have the vector representation of the lines are $\langle 2, 0, -1\rangle$ and $\langle 3, 2, 2\rangle$.

Now the cross product of the

$$A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & -1 \\ 3 & 2 & 2 \end{vmatrix}$$

$$=(0+2)\mathbf{i}-(4+3)\mathbf{j}+(4-0)\mathbf{k}$$

$$= 2\mathbf{i} - 7\mathbf{j} + 4\mathbf{k}$$

Since the cross product of the vectors is not zero, we can say that the lines are not parallel.

Now, let us check if the lines are skew or intersecting.

On equating the expressions for x, y, and z, we get $2t=3s+1,\ 0=2s-1,\ -t=2s+1$.

Solve the expressions to obtain the values of s as $\frac{1}{2}$ and t as -2.

Replace t with -2 and s with $\frac{1}{2}$ in $x=2t$ and $x=3s+1$.

$x = 2(-2)$ and $x = 3\left(\frac{1}{2}\right)+1$

$x = -4$ and $x = \frac{5}{2}$

Since x takes different values for $s=\frac{1}{2}$ and $t=-2$, we can say that the lines do not intersect. Thus, we can say that L_1 and L_2 are skew lines.

Now, we have $A = L_1 \times L_2$.

$$A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & -1 \\ 3 & 2 & 2 \end{vmatrix}$$
$$= 2\mathbf{i} - 7\mathbf{j} + 4\mathbf{k}$$

Now, if we put $t = 0$ in the equation for L_1, we get the point (0, 0, 0) on L_1 and so an equation for P_1 is $2x - 7y + 4z = 0$.

On setting $s = 0$ in L_2, we get the point $(1, -1, 1)$ on P_2.
So, the distance between L_1 and L_2 is the same as the distance from $(1, -1, 1)$ to $2x - 7y + 4z = 0$.

$$D = \frac{|2(1) - 7(-1) + 4(1)|}{\sqrt{4 + 49 + 16}}$$
$$= \frac{13}{\sqrt{69}}$$
$$\approx 1.565$$

Therefore, we get the distance as about $\boxed{1.565}$

Answer 80E.

Consider L_1 is the line through the points $A(1,2,6)$ and $B(2,4,8)$ and L_2 be the line of intersection of the planes π_1 and π_2, where π_1 is the plane $x - y + 2z + 1 = 0$ and π_2 is the plane through the points $P(3,2,-1), Q(0,0,1)$, and $R(1,2,1)$.

The objective is to find the distance between the lines L_1 and L_2.

First find parametric equations for L_1.

Calculate the vector $\mathbf{v}_1$ that gives the direction of L_1.

$$\begin{aligned}\mathbf{v}_1 &= \overrightarrow{AB} \\ &= \langle 2-1,\ 4-2,\ 8-6\rangle \\ &= \langle 1,\ 2,\ 2\rangle\end{aligned}$$

Use this to write parametric equations for L_1 with point A and normal $\mathbf{v}_1$.

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-6}{2} = t$$

Then,

$$\begin{aligned}x &= 1+t \\ y &= 2+2t \\ z &= 6+2t\end{aligned}$$

Find parametric equations for L_2.

Start by finding π_2.

Find the normal vector $\mathbf{n}_2$ by taking the cross product of two vectors **w1** and **w2**.

$$\begin{aligned}\mathbf{w}_1 &= \overrightarrow{PQ} \\ &= \langle 0-3,\ 0-2,\ 1-(-1)\rangle \\ &= \langle -3,\ -2,\ 2\rangle\end{aligned}$$

$$\begin{aligned}\mathbf{w}_2 &= \overrightarrow{QR} \\ &= \langle 1-0,\ 2-0,\ 1-1\rangle \\ &= \langle 1,\ 2,\ 0\rangle\end{aligned}$$

Calculate the cross product $\mathbf{w}_1 \times \mathbf{w}_2$.

$$\begin{aligned}\mathbf{w}_1 \times \mathbf{w}_2 &= \langle -3,\ -2,\ 2\rangle \times \langle 1,\ 2,\ 0\rangle \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2 & 2 \\ 1 & 2 & 0 \end{vmatrix} \\ &= \mathbf{i}(0-4) - \mathbf{j}(0-2) + \mathbf{k}(-6+2) \\ &= \langle -4,\ 2,\ -4\rangle\end{aligned}$$

Thus, the normal vector $\mathbf{n}_2$ to the plane π_2 is $\langle -4,\ 2,\ -4\rangle$ or $\langle 2,\ -1,\ 2\rangle$.

Therefore, the equation of plane π_2 with point Q and normal $\mathbf{n}_2$ is,

$$\begin{aligned}2x - y + 2(z-1) &= 0 \\ 2x - y + 2z - 2 &= 0 \\ 2x - y + 2z &= 2\end{aligned}$$

The line L_2 is the line of intersection of the planes π_1 and π_2.

The normal vector to the plane π_2 is $\mathbf{n}_1 = \langle 1, -1, 2 \rangle$.

Let $\mathbf{v}_2$ be the direction of *L*2.

This vector is perpendicular to both π_1 and π_2.

So it is parallel to the cross product of their normal vectors given by,

$$\begin{aligned}
\mathbf{v}_2 &= \mathbf{n}_1 \times \mathbf{n}_2 \\
&= \langle 1,\ -1,\ 2 \rangle \times \langle 2,\ -1,\ 2 \rangle \\
&= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 2 \\ 2 & -1 & 2 \end{vmatrix} \\
&= \mathbf{i}(-2+2) - \mathbf{j}(2-4) + \mathbf{k}(-1+2) \\
&= \langle 0,\ 2,\ 1 \rangle
\end{aligned}$$

Find a point on the intersection of π_1 and π_2 when $y = 0$.

$$\begin{aligned}
x - 0 + 2z &= -1 \quad x = 3 \\
2x - 0 + 2z &= 2 \quad z = -2
\end{aligned}$$

Therefore, a point of intersection is $(3, 0, -2)$.

Use this to write parametric equations for L_2:

$$x = 3$$
$$y = 2t$$
$$z = -2 + t$$

Thus, the equations for the lines are $L_1:\ x = 1 + t,\ y = 2 + 2t,\ z = 6 + 2t$ and

$L_2:\ x = 3,\ y = 2t,\ z = -2 + t$.

To find the distance between these two lines, find the equation for the plane that contains L_1 and is perpendicular to the shortest line between L_1 and L_2.

The normal vector **n** of this plane is the cross product of $\mathbf{v_1}$ and $\mathbf{v_2}$ given as,

$$\begin{aligned}\mathbf{n} &= \mathbf{v}_1 \times \mathbf{v}_2 \\ &= \langle 1,\ 2,\ 2\rangle \times \langle 0,\ 2,\ 1\rangle \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 2 \\ 0 & 2 & 1 \end{vmatrix} \\ &= \mathbf{i}(2-4) - \mathbf{j}(1-0) + \mathbf{k}(2-0) \\ &= \langle -2,\ -1,\ 2\rangle\end{aligned}$$

The point $(1,2,6)$ is on L_1.

Therefore, the equation for the plane is,

$$\begin{aligned}-2(x-1)-(y-2)+2(z-6) &= 0 \\ -2x+2-y+2+2z-12 &= 0 \\ -2x-y+2z-8 &= 0\end{aligned}$$

The distance *D* from a point $P_1(x_1, y_1, z_1)$ to the plane $ax+by+cz+d=0$ is given by,

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}$$

Use the equation for the plane, $-2x-y+2z-8=0$ and the point $(3,0,-2)$ on L_2 to find the distance *D* between L_1 and L_2 as,

$$\begin{aligned}D &= \frac{|-2(3)-0+2(-2)-8|}{\sqrt{(-2)^2+(-1)^2+(2)^2}} \\ &= \frac{|-6-4-8|}{\sqrt{4+1+4}} \\ &= \frac{18}{3} \\ &= \boxed{6}\end{aligned}$$

Answer 81E.

The given equation is $ax + by + cz + d = 0$ --- (1)

Now suppose $a \neq 0$.

Then equation (1) can be written as

$$a\left(x + \frac{d}{a}\right) + b(y - 0) + c(z - 0) = 0$$

If $b \neq 0$, equation (1) can be written as:

$$a(x - 0) + b\left(y + \frac{d}{b}\right) + c(z - 0) = 0$$

And if $c \neq 0$ equation (1) can be written as:

$$a(x - 0) + b(y - 0) + c\left(z + \frac{d}{c}\right) = 0$$

In each case we find the equation of the form $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$, the scalar equation of the plane through (x_0, y_0, z_0) with normal vector $\langle a, b, c\rangle$

Hence we say that equation (1) represents a plane and $\langle a, b, c\rangle$ is the normal vector to the plane.

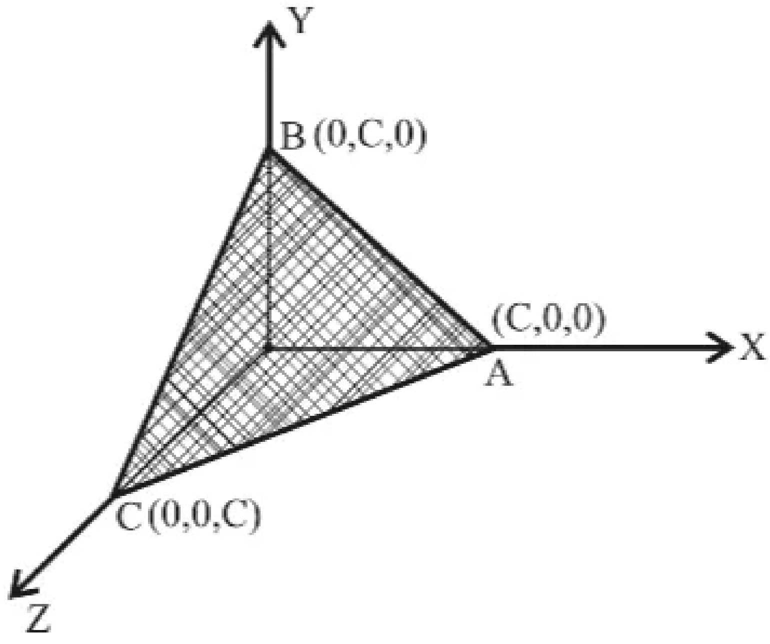
Answer 82E.

(A) The equation of given family of planes is:-

$$x + y + z = c$$

or, $$\frac{x}{c} + \frac{y}{c} + \frac{z}{c} = 1$$

Thus, the given planes cut intercepts of lengths *c, c,* and c from *x, y* and *z* axes.



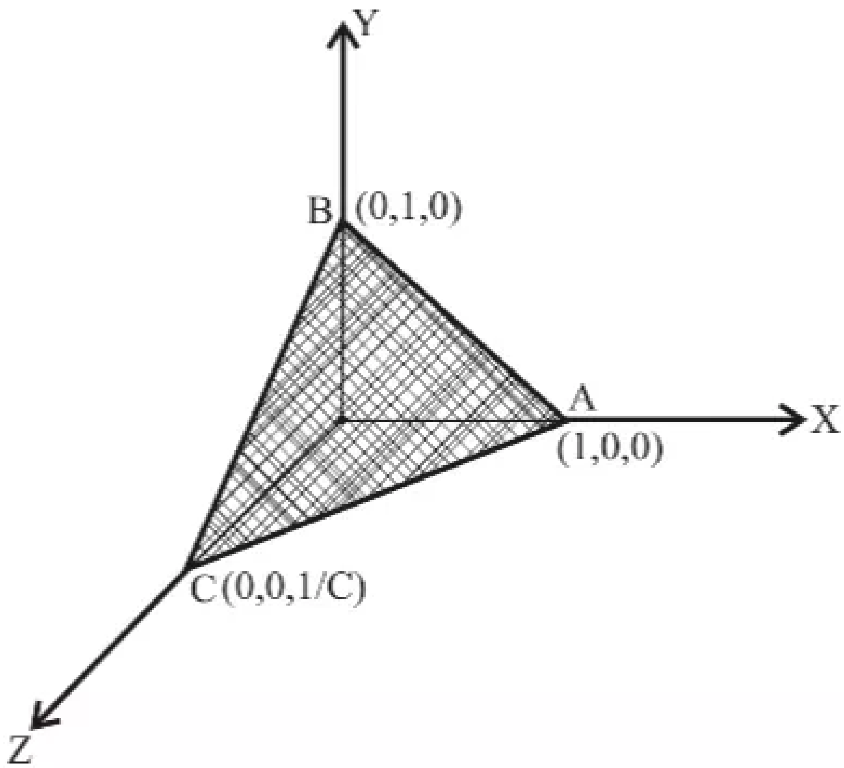
Since all the planes have normal vector $\langle 1,1,1\rangle$ then all the planes given by $x+y+z=c$ are parallel.

(B) The equation of given family of planes is:-

$$x+y+cz=1$$

or, $$\frac{x}{1}+\frac{y}{1}+\frac{z}{\frac{1}{C}}=1$$

Which cuts off intercepts of lengths $1,1,\frac{1}{c}$ from x, y, z axes respectively.



When $c=0$, the equation of the planes is $x+y=1$, which is a plane parallel to z-axis.

Since the normal vector of each plane is $\left\langle 1,1,\frac{1}{c}\right\rangle$.

Then the planes are not parallel.

Also the intercept of each plane with z-axis is 1/c, then as c increases, the plane becomes closer to xy - plane.