KVPY QUESTION PAPER-2014 (STREAM SX)

Part – I

One - Mark Questions

Date : 02 / 11 / 2014

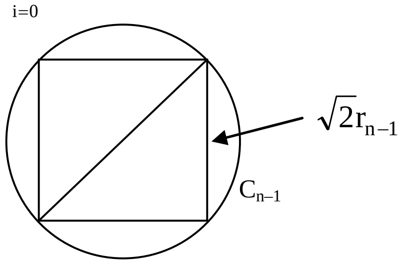
MATHEMATICS

1. Let C_0 be a circle of radius 1. For $n \geq 1$, let C_n be a circle whose area equals the area of a square inscribed in C_{n-1}. Then $\sum_{i=0}^{\infty}$ Area (C_i) equals

(A) π^2 (B) $\frac{\pi - 2}{\pi^2}$ (C) $\frac{1}{\pi^2}$ (D) $\frac{\pi^2}{\pi - 2}$

Ans. **[D]**

Sol. $\sum_{i=0}^{\infty} \text{Area}(C_i) = \pi r_0^2 + \pi r_1^2 + \pi r_2^2 + \pi r_3^2 + \ldots.. \infty$



Area of $C_n = \pi r_n^2 = (\sqrt{2}\, r_{n-1})^2$

$r_n^2 = \frac{2}{\pi} r_{n-1}^2$

so $r_1^2 = \frac{2}{\pi} r_0^2$, $r_2^2 = \frac{2}{\pi} r_1^2$

$= \frac{2}{\pi}\left(\frac{2}{\pi} r_0^2\right)$

$r_3^2 = \frac{2}{\pi}\left(r_2^2\right) = \frac{2}{\pi}\left(\frac{2}{\pi}\frac{2}{\pi} r_0^2\right)$

So $\sum_{i=0}^{\infty} \text{Area}(C_i) = \pi \left[r_0^2 + \frac{2}{\pi} r_0^2 + \frac{2}{\pi}.\frac{2}{\pi} r_0^2 + \ldots.\infty \right]$

$= \frac{\pi r_0^2}{1 - \frac{2}{\pi}} = \frac{\pi^2 r_0^2}{\pi - 2} \quad \forall\ r_0 = 1$

$= \frac{\pi^2}{\pi - 2}$

2. For a real number r we denote by [r] the largest integer less than or equal to r. If x, y are real numbers with x, y ≥ 1 then which of the following statements is always true?

(A) $[x + y] \le [x] + [y]$ (B) $[xy] \le [x]\,[y]$

(C) $[2^x] \le 2^{[x]}$ (D) $\left[\frac{x}{y}\right] \le \frac{[x]}{[y]}$

Ans. **[D]**

Sol. (A) $[x + y] \le [x] + [y]$

let x = 0.1

y = 0.9

$[0.1 + 0.9] \le [0.1] + [0.9]$

$1 \le 0 + 0$ wrong

(B) $[xy] \le [x\ ]\,[y]$

$x = 2;\ y = \frac{1}{2}$

$\left[2.\frac{1}{2}\right] \le [2]\left[\frac{1}{2}\right]$

$\Rightarrow 1 \le 0$ wrong

(C) $[2^x] \le 2^{[x]}$

$x = 0.99\quad [2^{0.99}] \le 2^{[0.99]}$

$[2^{0.99}] \le 2^o = 1$ wrong

(D) $\left[\frac{x}{y}\right] \le \frac{[x]}{[y]}$

given x, y ≥ 1

if $x < y$ $\left[\frac{x}{y}\right] = 0$ $0 \le \frac{[x]}{[y]}$ true

if $x \ge y$ $\left[\frac{x}{y}\right] \le \frac{[x]}{[y]}$ always true

3. For each positive integer n, let $A_n = \max\left\{\binom{n}{r} \middle| 0 \le r \le n\right\}$. Then the number of elements n in {1,2,…,20} for which $1.9 \le \frac{A_n}{A_{n-1}} \le 2$ is

(A) 9 (B) 10 (C) 11 (D) 12

Ans. **[C]**

Sol. Case (1) n = even

$$\frac{A_n}{A_{n-1}} = \frac{{}^{n}C_{n/2}}{{}^{n-1}C_{\frac{n-1-1}{2}}} = 2$$

so for all n even given relation is true.

Case (2) n = odd

$$\frac{A_n}{A_{n-1}} = \frac{{}^{n}C_{\frac{n-1}{2}}}{{}^{n-1}C_{\frac{n-1}{2}}} = \frac{2n}{n+1}$$

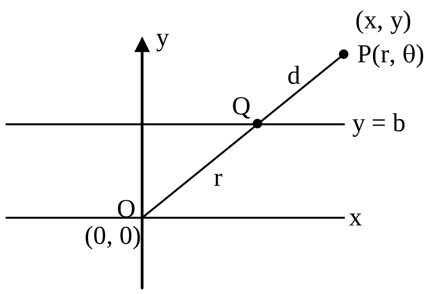
which satisfies only for n = 19

4. Let b, d > 0. The locus of all points P(r, θ) for which the line OP (where O is the origin) cuts the line r sin θ = b in Q such that PQ = d is

(A) (r – d) sin θ = b
(B) (r ± d) sin θ = b
(C) (r – d) cos θ = b
(D) (r ± d) cos θ = b

Ans. **[B]**

Sol.



equation of OP

y = x tan θ

point Q is (b cot θ, b)

∴ point P is y = b ± d sin θ

r sin θ = b ± d sin θ

(r ∓ d) sin θ = b

5. Let C be the circle $x^2 + y^2 = 1$ in the xy-plane. For each t ≥ 0, let L_t be the line passing through (0, 1) and (t, 0). Note that L_t intersects C in two points, one of which is (0,1). Let Q_t be the other point. As t varies between 1 and $1 + \sqrt{2}$, the collection of points Q_t sweeps out an arc on C. The angle subtended by this arc at (0, 0) is

(A) $\frac{\pi}{8}$
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$
(D) $\frac{3\pi}{8}$

Ans. **[B]**

Sol. $x^2 + y^2 = 1$

$L_t \quad \frac{x}{t} + \frac{y}{1} = 1$

$y = 1 - \frac{x}{t}$

$x^2 + 1 + \frac{x^2}{t^2} - \frac{2x}{t} = 1$

$x^2\left(1+\frac{1}{t^2}\right) - \frac{2x}{t} = 0$

$x = 0$, $y = 1$, $(0, 1)$

$x\left(1+\frac{1}{t^2}\right) = \frac{2}{t}$

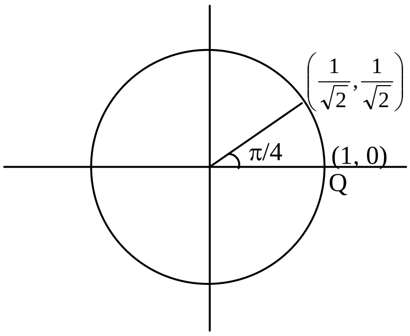
$x = \frac{2t}{t^2+1}$; $y = 1 - \frac{2}{t^2+1}$

$y = \frac{t^2-1}{t^2+1}$

$Q_t\left(\frac{2t}{1+t^2}, \frac{t^2-1}{t^2+1}\right)$

$1 \le t \le 1 + \sqrt{2}$ $\quad$ $t = \tan\theta$ $\quad$ $Q_t(\sin 2\theta, -\cos 2\theta)$

$\theta \in \left(45^\circ, 67\frac{1^\circ}{2}\right)$ $\quad$ lies on circle C



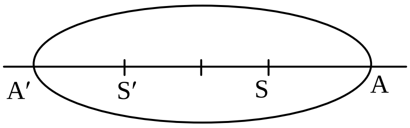
so angle at centre $= \frac{\pi}{4}$

6. In an ellipse, its foci and the ends of its major axis are equally spaced. If the length of its semi-minor axis is $2\sqrt{2}$, then the length of its semi-major axis is

(A) 4 (B) $2\sqrt{3}$ (C) $\sqrt{10}$ (D) 3

Ans. **[D]**

Sol.



$A'S' = SS' = SA$

$2ae = a - ae$

$3ae = a$

$e = 1/3$

$$1 - \frac{b^2}{a^2} = \frac{1}{9} \Rightarrow \frac{b^2}{a^2} = \frac{8}{9}$$

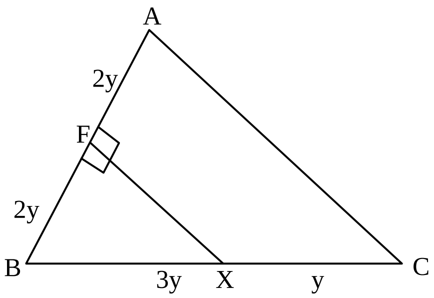
$$\Rightarrow \frac{8}{a^2} = \frac{8}{9} \Rightarrow a = 3$$

7. Let ABC be a triangle such that AB = BC. Let F be the midpoint of AB and X be a point on BC such that FX is perpendicular to AB. If BX = 3XC then the ratio BC/AC equals

(A) $\sqrt{3}$ (B) $\sqrt{2}$ (C) $\sqrt{\frac{3}{2}}$ (D) 1

Ans. **[C]**

Sol.



$\because \cos B = \frac{2}{3}$

$$\text{Also } \cos B = \frac{16y^2 + 16y^2 - AC^2}{2.4y.4y}$$

$$\frac{2}{3} = \frac{32y^2 - AC^2}{32y^2}$$

$64y^2 = 96y^2 - 3AC^2$

$3AC^2 = 32y^2$

$$AC = \frac{4\sqrt{2}}{\sqrt{3}}y$$

$$\therefore \frac{BC}{AC} = \frac{4y}{\left(\frac{4\sqrt{2}y}{\sqrt{3}}\right)} = \frac{\sqrt{3}}{\sqrt{2}}$$

8. The number of solutions to the equation $\cos^4 x + \frac{1}{\cos^2 x} = \sin^4 x + \frac{1}{\sin^2 x}$ in the interval $[0, 2\pi]$ is

(A) 6 (B) 4

(C) 2 (D) 0

Ans. **[B]**

Sol. $\cos^4 x - \sin^4 x = \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x}$

$(\cos^2 x - \sin^2 x) = \frac{(\cos^2 x - \sin^2 x)}{\sin^2 x \cos^2 x}$

$\cos 2x = \frac{4\cos 2x}{\sin^2 2x}$

$\cos 2x\,(1 - 4\,\text{cosec}^2 2x) = 0$

$\cos 2x = 0$

$2x = 2n\pi \pm \frac{\pi}{2}$

$x = n\pi \pm \frac{\pi}{4}$

At $n = 0$, $x = \frac{\pi}{4}$

$n = 1$; $x = \frac{5\pi}{4}, \frac{3\pi}{4}$

$n = 2$, $x = \frac{7\pi}{4}$

9. Consider the function $f(x) = \begin{cases} \frac{x+5}{x-2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$. Then $f(f(x))$ is discontinuous

(A) at all real numbers (B) at exactly two values of x

(C) at exactly one value of x (D) at exactly three values of x

Ans. **[B]**

Sol. discontinuous at $x = 2$

$f(f(x)) = f\left(\frac{x+5}{x-2}\right)$

$$= \frac{\left(\frac{x+5}{x-2}+5\right)}{\left(\frac{x+5}{x-2}-2\right)} = \frac{6x-5}{-x+9}$$

$$= \frac{6x-5}{9-x}$$

At x = 9 it is discontinuous

10. For a real number x let [x] denote the largest number less than or equal to x. For $x \in R$ let $f(x) = [x] \sin \pi x$. Then

(A) f is differentiable on R.

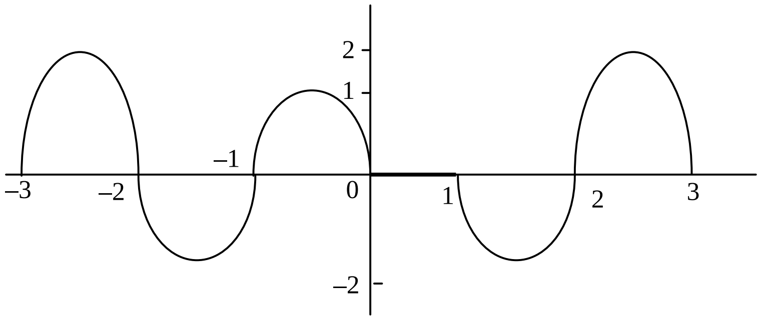
(B) f is symmetric about the line x = 0.

(C) $\int_{-3}^{3} f(x)dx = 0$.

(D) For each real α, the equation $f(x) - \alpha = 0$ has infinitely many roots.

Ans. **[D]**

Sol. $f(x) = [x] \sin \pi x$



Not diff for $\forall\ x \in R$.

Not sym about x = 0.

$$\int_{-3}^{3} f(x)dx \neq 0$$

$f(x) = \alpha$ will have ∞ sol^n

11. Let $f : [0, \pi] \rightarrow R$ be defined as

$$f(x) = \begin{cases} \sin x, & \text{if } x \text{ is irrational and } x \in [0, \pi] \\ \tan^2 x, & \text{if } x \text{ is rational and } x \in [0, \pi] \end{cases}.$$

The number of points in $[0, \pi]$ at which the function f is continuous is

(A) 6 (B) 4 (C) 2 (D) 0

Ans. **[B]**

Sol. $f(x) = \begin{cases} \sin x & x \notin Q \\ \tan^2 x & x \in Q \end{cases}$

if is continuous at x = 0, π

so 2 points

$\sin x = \tan^2 x \Rightarrow \sin x(\cos^2 x - \sin x) = 0$

$\sin x = 0$

$x = 0, \pi$

$\sin^2 x + \sin x - 1 = 0$

$\sin x = \dfrac{-1 \pm \sqrt{5}}{2}$

$\sin x = \dfrac{\sqrt{5} - 1}{2}$ 2 values

Total 4 points

12. Let $f : [0, 1] \to [0, \infty]$ be a continuous function such that $\int_0^1 f(x)dx = 10$. Which of the following statements is **NOT** necessarily true?

(A) $\int_0^1 e^{-x} f(x)dx \le 10$

(B) $\int_0^1 \dfrac{f(x)}{(1+x)^2} dx \le 10$

(C) $-10 \le \int_0^1 \sin(100x) f(x) dx \le 10$

(D) $\int_0^1 f(x)^2 dx \le 100$

Ans. **[D]**

Sol. $\because f(x) \ge 0$

$\int_0^1 f(x)^2 dx \le 100$ not necessarily true.

because $(f(x))^2$ can take very high values then area bounded by $(f(x))^2$, x-axis & x = 0 to 1 may cross 100.

13. A continuous function $f : R \to R$ satisfies the equation $f(x) = x + \int_0^x f(t) dt$. Which of the following options is true?

(A) $f(x + y) = f(x) + f(y)$

(B) $f(x + y) = f(x) f(y)$

(C) $f(x + y) = f(x) + f(y) + f(x) f(y)$

(D) $f(x + y) = f(xy)$

Ans. **[C]**

Sol. $f(x) = x + \int_0^x f(t)dt$

$f'(x) = 1 + f(x) \quad \Rightarrow f'(x) - f(x) = 1$

$\Rightarrow e^{-x} f'(x) - f(x)\, e^{-x} = e^{-x}$

$\Rightarrow \frac{d}{dx}(f(x)\, e^{-x}) = e^{-x}$

$\Rightarrow f(x)\, e^{-x} = \frac{e^{-x}}{-1} + c$

$\Rightarrow f(x) = -1 + ce^{x}$

$f(0) = 0 = -1 + ce^0 \Rightarrow c = 1$

$f(x) = e^x - 1$

$f(x) + f(y) + f(x)\, f(y) = e^x - 1 + e^y - 1 + (e^x - 1)(e^y - 1)$

$= e^x - 1 + e^y - 1 + e^x.e^y - e^y - e^x + 1$

$= e^x.e^y - 1 = e^{x+y} - 1$

$= f(x + y)$

14. For a real number x let [x] denote the largest integer less than or equal to x and {x} = x – [x]. Let n be a positive integer. Then $\int_0^n \cos(2\pi[x]\{x\})\, dx$ is equal to

(A) 0 (B) 1 (C) n (D) 2n – 1

Ans. **[B]**

Sol. $\int_0^n \cos(2\pi[x]\{x\})\, dx$

$= \int_0^1 \cos(0)\, dx + \int_1^2 \cos(2\pi(x-1))\, dx + \int_2^3 \cos(4\pi(x-2))\, dx + \ldots\ldots\ldots + \int_{n-1}^n \cos(2\pi(n-1)(x-(n-1)))\, dx$

$= (1 - 0) + \int_1^2 \cos 2\pi x\, dx + \int_2^3 \cos 4\pi x\, dx + \ldots\ldots + \int_{n-1}^n \cos(2\pi(n-1)x)\, dx$

$= 1 + \left.\frac{\sin 2\pi x}{2\pi}\right|_1^2 + \left.\frac{\sin 4\pi x}{4\pi}\right|_2^3 + \ldots\ldots + \left.\frac{\sin 2\pi(n-1)x}{2\pi(n-1)}\right|_{n-1}^n$

$= 1 + 0 = 1$

15. Two persons A and B throw a (fair) die (six-faced cube with faces numbered from 1 to 6) alternately, starting with A. The first person to get an outcome different from the previous one by the opponent wins. The probability that B wins is

(A) $\frac{5}{6}$ (B) $\frac{6}{7}$ (C) $\frac{7}{8}$ (D) $\frac{8}{9}$

Ans. **[B]**

Sol. $P = \frac{6}{6}.\frac{5}{6} + \frac{6}{6}.\frac{1}{6}.\frac{1}{6}.\frac{5}{6} + \frac{6}{6}.\frac{1}{6}.\frac{1}{6}.\frac{1}{6}.\frac{1}{6}.\frac{5}{6} + \ldots\ldots \infty$

$= \frac{5}{6} + \frac{5}{6^3} + \frac{5}{6^5} + \ldots\ldots$

$= \frac{5/6}{1-\frac{1}{36}} = \frac{30}{35} = \frac{6}{7}$

16. Let $n \geq 3$. A list of numbers $x_1, x_2, \ldots, x_n$ has mean μ and standard deviation σ. A new list of numbers $y_1, y_2, \ldots, y_n$ is made as follows: $y_1 = \frac{x_1 + x_2}{2}$, $y_2 = \frac{x_1 + x_2}{2}$ and $y_j = x_j$ for $j = 3,4,\ldots, n$. The mean and the standard deviation of the new list are $\hat{\mu}$ and $\hat{\sigma}$. Then which of the following is necessarily true?

(A) $\mu = \hat{\mu}$ and $\sigma \leq \hat{\sigma}$ (B) $\mu = \hat{\mu}$ and $\sigma \geq \hat{\sigma}$

(C) $\sigma = \hat{\sigma}$ (D) $\mu \neq \hat{\mu}$

Ans. **[B]**

Sol. $\mu = \frac{x_1 + x_2 + \ldots\ldots + x_n}{n}$

$\hat{\mu} = \frac{y_1 + y_2 + \ldots\ldots + y_n}{n} = \frac{\frac{x_1 + x_2}{2} + \frac{x_1 + x_2}{2} + x_3 + \ldots. + x_n}{n}$

$\hat{\mu} = \frac{x_1 + x_2 + x_3 + \ldots.. + x_n}{n} = \mu \Rightarrow \boxed{\hat{\mu} = \mu}$

$\sigma^2 = \frac{\sum x_i^2}{n} - \mu^2$

$\sigma^2 = \frac{x_1^2 + x_2^2 + \ldots. + x_n^2}{n} - \mu^2$ .... (1)

$\hat{\sigma}^2 = \frac{y_1^2 + y_2^2 + \ldots. + y_n^2}{n} - \mu^2 = \frac{\left(\frac{x_1 + x_2}{2}\right)^2 + \left(\frac{x_1 + x_2}{2}\right)^2 + x_3^2 + \ldots.. x_n^2}{n} - \mu^2$

$\hat{\sigma}^2 = \frac{\frac{x_1^2 + x_2^2}{2} + x_1 x_2 + x_3^2 + \ldots + x_n^2}{n} - \mu^2$ ...(2)

$\sigma^2 - \hat{\sigma}^2 = \frac{x_1^2 + x_2^2}{n} - \left(\frac{x_1^2 + x_2^2 + 2x_1 x_2}{2n}\right) = \frac{x_1^2 + x_2^2 - 2x_1 x_2}{2n}$

$= \frac{(x_1 - x_2)^2}{2n} \geq 0 \Rightarrow \sigma \geq \hat{\sigma}$ & $\mu = \hat{\mu}$

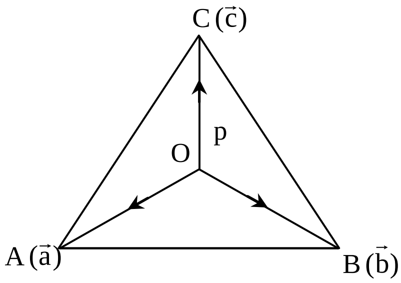
17. What is the angle subtended by an edge of a regular tetrahedron at its center?

(A) $\cos^{-1}\left(\frac{-1}{2}\right)$ (B) $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

(C) $\cos^{-1}\left(\frac{-1}{3}\right)$ (D) $\cos^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

Ans. **[C]**

Sol.



$\vec{a}, \vec{b}, \vec{c}$ are unit vectors

$$\vec{a} \wedge \vec{b} = \vec{b} \wedge \vec{c} = \vec{c} \wedge \vec{a} = \frac{\pi}{3}$$

centre p $\left(\frac{\vec{o} + \vec{a} + \vec{b} + \vec{c}}{4}\right)$

Now angle between $\overrightarrow{AP}$ & $\overrightarrow{BP}$

$$\cos\theta = \frac{\overrightarrow{AP} \cdot \overrightarrow{BP}}{|\overrightarrow{AP}||\overrightarrow{BP}|} = \frac{\left(\frac{\vec{a}+\vec{b}+\vec{c}}{4} - \vec{a}\right) \cdot \left(\frac{\vec{a}+\vec{b}+\vec{c}}{4} - \vec{b}\right)}{\left|\frac{\vec{a}+\vec{b}+\vec{c}}{4} - \vec{a}\right| \left|\frac{\vec{a}+\vec{b}+\vec{c}}{4} - \vec{b}\right|}$$

$$= \frac{(\vec{b}+\vec{c}-3\vec{a}) \cdot (\vec{a}+\vec{c}-3\vec{b})}{|\vec{b}+\vec{c}-3\vec{a}| \cdot |\vec{a}+\vec{c}-3\vec{b}|}$$

$$= \frac{\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} - 3\vec{b}^2 + \vec{a}\cdot\vec{c} + \vec{c}^2 - 3\vec{b}\cdot\vec{c} - 3\vec{a}^2 - 3\vec{a}\cdot\vec{c} + 9\vec{a}\cdot\vec{b}}{(\vec{b}^2 + \vec{c}^2 + 9\vec{a}^2 + 2\vec{b}\cdot\vec{c} - 6\vec{a}\cdot\vec{c} - 6\vec{a}\cdot\vec{b})}$$

$$= \frac{\frac{1}{2} + \frac{1}{2} - 3 + \frac{1}{2} + 1 - \frac{3}{2} - 3 - \frac{3}{2} + \frac{9}{2}}{1+1+9+1-3-3}$$

$$= \frac{-5+3}{6} = -\frac{1}{3}$$

$$\theta = \cos^{-1}\left(-\frac{1}{3}\right)$$

18. Let S = {(a, b) : a, b Z, 0 a, b ≤ 18}. The number of elements (x, y) in S such that 3x + 4y + 5 is divisible by 19 is

(A) 38 (B) 19 (C) 18 (D) 1

Ans. **[B]**

Sol. $3x + 4y + 5 = 19\,I$

$5 \le (3x + 4y + 5) \le 131$

$5 \le 19\,I \le 131$

Case (i) $3x + 4y + 5 = 19$ $3x + 4y = 14$ $y = \dfrac{14 - 3x}{4}$ $x = 2$	Case (ii) $3x + 4y + 5 = 38$ $3x + 4y = 33$ $4y = 33 - 3x$ $y = \dfrac{33 - 3x}{4}$ $x = 3, 7, 11$	Case (iii) $3x + 4y = 52$ $x = \dfrac{52 - 3x}{4}$ $x = 0, 4, 8, 12, 16$
Case (iv) $3x + 4y = 71$ $4y = 71 - 3x$ $y = \dfrac{71 - 3x}{4}$ $x = 1, 5, 9, 13, 17$	Case (v) $3x + 4y = 90$ $y = \dfrac{90 - 3x}{4}$ $x = 6, 10, 14, 18$	Case (vi) $3x + 4y = 109$ $y = \dfrac{109 - 3x}{4}$ $x = 15$ is only possibility.

Total Solution = 19

19. For a real number r let [r] denote the largest integer less than or equal to r. Let a > 1 be a real number which is not an integer and let k be the smallest positive integer such that $[a^k] > [a]^k$. Then which of the following statements is always true?

(A) $k \le 2([a] + 1)^2$ (B) $k \le ([a] + 1)^4$

(C) $k \le 2^{[a]+1}$ (D) $k \le \dfrac{1}{a - [a]} + 1$

Ans. **[B]**

Sol. By taking different values of a & k.
option (B) is possible.

20. Let X be a set of 5 elements. The number d of ordered pairs (A, B) of subsets of X such that $A \ne \phi$, $B \ne \phi$, $A \cap B = \phi$ satisfies

(A) $50 \le d \le 100$ (B) $101 \le d \le 150$

(C) $151 \le d \le 200$ (D) $201 \le d$

Ans. **[C]**

Sol. $${}^5C_2 \,.\, 2! + {}^5C_3\left(\frac{3!}{1!\,2!} \times 2!\right) + {}^5C_4\left[\frac{4!}{1!\,3!} \times 2! + \frac{4!}{2!\,2!} \times \frac{2!}{2!}\right] + {}^5C_5\left[\frac{5!}{1!\,4!} \times 2! + \frac{5!}{2!\,3!} \times 2!\right]$$

$= 10(2) + 10(6) + 5(8 + 6) + (10 + 20)$

$= 20 + 60 + 70 + 30 = 180$

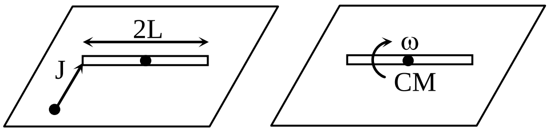
PHYSICS

21. A uniform thin rod of length 2L and mass m lies on a horizontal table. A horizontal impulse J is given to the rod at one red. There is no friction. The total kinetic energy of the rod just after the impulse will be

(A) $\frac{J^2}{2m}$ (B) $\frac{J^2}{m}$ (C) $\frac{2J^2}{m}$ (D) $\frac{6J^2}{m}$

Ans. **[C]**

Sol.



$J = mv$ .... (1)

where v is the velocity of centre of mass.

After impulse rod get angular velocity ω

Angular impulse = Iω

$$J \times L = \frac{m(2L)^2}{12} \times \omega \quad ....(2)$$

$$J = \frac{mL\omega}{3}$$

$$\boxed{\omega = \frac{3J}{mL}}$$

from equation (1); $v = \frac{J}{m}$

Kinetic energy = KE = $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$\Rightarrow \frac{1}{2}m\frac{J^2}{m^2} + \frac{1}{2} \times \frac{m \times 4L^2}{12} \times \frac{9J^2}{m^2L^2}$$

$$\Rightarrow \frac{J^2}{2m} + \frac{36J^2}{24m}$$

$$\Rightarrow \frac{48J^2}{24m} \Rightarrow \frac{2J^2}{m}$$

22. A solid cylinder P rolls without slipping from rest down an inclined plane attaining a speed v_P at the bottom. Another smooth solid cylinder Q of same mass and dimensions slides without friction from rest down the inclined plane attaining a speed v_Q at the bottom. The ratio of the speeds $\left(\frac{v_Q}{v_P}\right)$ is -

(A) $\sqrt{3/4}$ (B) $\sqrt{3/2}$ (C) $\sqrt{2/3}$ (D) $\sqrt{4/3}$

Ans. **[B]**

Sol. If perfect rolling (solid cylinder P)

According to energy conservation law

$$mgh = \frac{1}{2}mv_P^2 + \frac{1}{2}I\left(\frac{v_P}{R}\right)^2$$

Here,

$I \rightarrow$ moment of inertia, $R \rightarrow$ Radius

$$I = \frac{mR^2}{2}$$

$$\omega = \frac{v_P}{R}$$

$$mgh = \frac{1}{2}mv_P^2 + \frac{1}{2}\frac{mR^2}{2}\frac{v_P^2}{R^2}$$

$$mgh = \frac{1}{2}mv_P^2\left[1+\frac{1}{2}\right] = \frac{1}{2}mv_P^2 \times \frac{3}{2}$$

$$mgh = \frac{3}{4}mv_P^2$$

$$v_P^2 = \frac{4}{3}gh \qquad \text{.... (1)}$$

If sliding without friction

(solid cylinder Q)

According to energy conservation law

$$mgh = \frac{1}{2}mv_Q^2$$

$$\Rightarrow \quad v_Q^2 = 2gh \qquad \text{.... (2)}$$

from equation (1) and (2)

$$\frac{v_Q^2}{v_P^2} = \frac{2gh}{\left(\frac{4}{3}gh\right)} = \frac{3}{2}$$

$$\frac{v_Q}{v_P} = \sqrt{\frac{3}{2}}$$

23. A body moves in a circular orbit of radius R under the action of a central force. Potential due to the central force is given by V(r) = kr (k is a positive constant). Period of revolution of the body is proportional to-

(A) $R^{1/2}$ (B) $R^{-1/2}$ (C) $R^{3/2}$ (D) $R^{-5/2}$

Ans. **[A]**

Sol. $F = -\frac{dU}{dr} = \frac{-d}{dr}[qV]$ $\qquad q \rightarrow$ constant

$$F = -q\left[\frac{dV}{dr}\right]$$

$F = -qk \qquad \leftarrow \begin{pmatrix} v = kr \\ \dfrac{dV}{dr} = k \end{pmatrix}$

$m\omega^2 R = -qk$

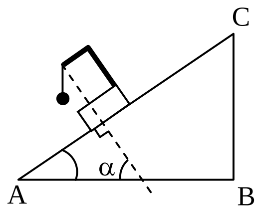
$m\left(\dfrac{2\pi}{T}\right)^2 R = -qk$

$\dfrac{m(4\pi^2)R}{T^2} = -qk$

$\Rightarrow \quad T^2 \propto R$

$\Rightarrow \quad T \propto R^{1/2}$

24. A simple pendulum is attached to the block which slides without friction down an inclined plane (ABC) having an angle of inclination α as shown.



While the block is sliding down the pendulum oscillates in such a way that its mean position the direction of the string is-

(A) at angle α to the perpendicular to the inclined plane AC .

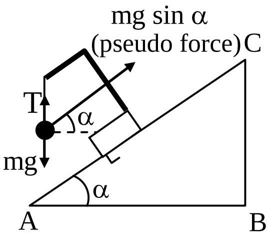
(B) parallel to the inclined plane AC.

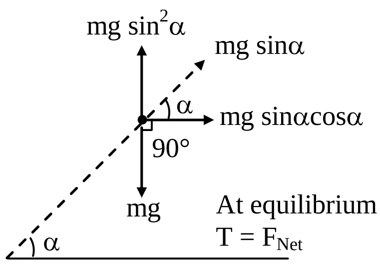
(C) vertically downwards

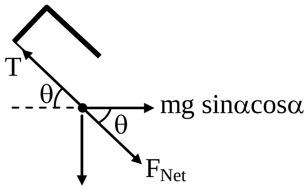
(D) perpendicular to the inclined plane AC.

Ans. **[D]**

Sol. Block slides downward along the inclined plane with acceleration g sin α.







$mg[1 - \sin^2 \alpha] \Rightarrow mg\cos^2 \alpha$

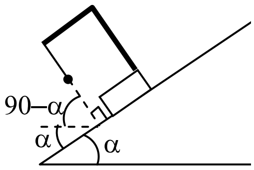
$\tan \theta = \dfrac{mg\cos^2\alpha}{mg\sin\alpha\cos\alpha}$

$\tan \theta = \cot \alpha$

$\tan \theta = \tan (90 - \alpha)$

$\theta = (90 - \alpha)$

string is perpendicular to inclined plane .



25. Water containing air bubbles flows without turbulence through a horizontal pipe which has a region of narrow cross-section. In this region the bubbles

(A) move with greater speed and are smaller than in the rest of the pipe

(B) move with greater speed and are larger in size than in the rest of the pipe

(C) move with lesser speed and are smaller than in the rest of the pipe

(D) move with lesser speed and are of the same size as in the rest of the pipe

Ans. **[B]**

Sol. According to Bernoulli theorem

In the region of narrow cross section of pipe, KE of fluid will be greater and pressure energy will be lesser.

$\Rightarrow$ less pressure results into larger in size of air bubble and greater KE results its greater speed.

26. A solid expands upon heating because-

(A) the potential energy of interaction between atoms in the solid is asymmetric about the equilibrium positions of atoms.

(B) the frequency of vibration of the atoms increases.

(C) the heating generates a thermal gradient between opposite sides.

(D) a fluid called the caloric flows into the interatomic spacing of the solid during heating thereby expanding it.

Ans. **[A]**

27. Consider two thermometers T_1 and T_2 of equal length which can be used to measure temperature over the range θ_1 to θ_2. T_1 contains mercury as the thermometric liquid while T_2 contains bromine. The volumes of the two liquids are the same at the temperature θ_1. The volumetric coefficients of expansion of mercury and bromine are $18 \times 10^{-5}\ K^{-1}$ and $108 \times 10^{-5}\ K^{-1}$, respectively . The increase in length of each liquid is the same for the same increase in temperature. If the diameters of the capillary tubes of the two thermometers are d_1 and d_2 respectively, then the ratio $d_1 : d_2$ would be closest to

(A) 6.0 (B) 2.5 (C) 0.6 (D) 0.4

Ans. **[D]**

Sol. Increase in length of each liquid is same

$$\Delta\ell = \Delta\ell$$

$$\frac{\Delta V_{Hg}}{\pi d_1^2} = \frac{\Delta V_{Bromine}}{\pi d_2^2}$$

$$\frac{(V)\gamma_{Hg}\,\Delta\theta}{\pi d_1^2} = \frac{V\,\gamma_{Bromine}\,\Delta\theta}{\pi d_2^2}$$

$$\left(\frac{d_1}{d_2}\right)^2 = \frac{\gamma_{Hg}}{\gamma_{Bromine}} = \frac{18\times10^{-5}}{108\times10^{-5}}$$

$$\frac{d_1}{d_2} = \sqrt{\frac{1}{6}} \simeq 0.4$$

28. An ideal gas follows a process described by $PV^2 = C$ from (P_1, V_1, T_1) to (P_2, V_2, T_2) (C is a constant). Then

(A) if $P_1 > P_2$ then $T_2 > T_1$ (B) if $V_2 > V_1$ then $T_2 < T_1$

(C) if $V_2 > V_1$ then $T_2 > T_1$ (D) if $P_1 > P_2$ then $V_1 > V_2$

Ans. **[B]**

Sol. $PV^2 = C$

$$\Rightarrow \left(\frac{nRT}{V}\right)V^2 = C$$

$\Rightarrow TV = C$ $\Rightarrow T_1V_1 = T_2V_2$

$\Rightarrow$ If temperature increases, volume decreases and vice versa

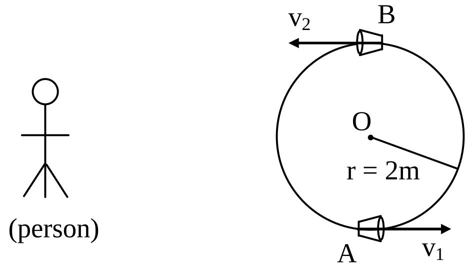
$\Rightarrow V_2 > V_1$ then $T_2 < T_1$.

29. A whistle emitting a loud sound of frequency 540 Hz is whirled in a horizontal circle of radius 2 m and at a constant angular speed of 15 rad/s. The speed of sound is 330 m/s. The ratio of the highest to the lowest frequency heard by a listener standing at rest at a large distance from the center of the circle is

(A) 1.0 (B) 1.1 (C) 1.2 (D) 1.4

Ans. **[C]**

Sol.



v_1 and v_2 are speed of whistle

$| v_1 | = | v_2 | = \omega r$

$= 15 \times 2$

$= 30$ m/s

Maximum frequency heard $\rightarrow$

Here,

$f \rightarrow$ original frequency (540 Hz)

$v \rightarrow$ speed of sound

$v_s \rightarrow$ speed of whistle

$$f_{max} = f\left[\frac{v}{v - v_s}\right]$$

Minimum frequency heard $\rightarrow$

$$f_{min} = f\left[\frac{v}{v + v_s}\right]$$

$$\frac{f_{max}}{f_{min}} = \frac{v + v_s}{v - v_s} = \frac{330 + 30}{330 - 30} = \frac{360}{300}$$

$$\frac{f_{max}}{f_{min}} = \frac{6}{5} = 1.2$$

30. Monochromatic light passes through a prism. Compared to that in air, inside the prism the light's

(A) speed and wavelength are different but frequency remains same.

(B) speed and frequency are different but wavelength remains same.

(C) wavelength and frequency are different, but speed remains same.

(D) speed, wavelength and frequency are all different.

Ans. **[A]**

Sol. On refraction of light, frequency remain unchanged. However speed and wavelength get change.

31. The flat face of a plano-convex lens of focal length 10 cm is silvered. A point source placed 30 cm in front of the curved surface will produce a

(A) real image 15 cm away from the lens (B) real image 6 cm away from the lens

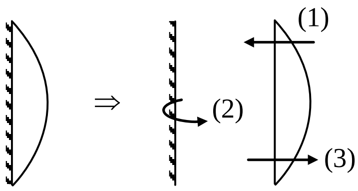
(C) virtual image 15 cm away from the lens (D) virtual image 6 cm away from the lens

Ans. **[B]**

Sol.

f = 10 cm

After silvering of flat face lens behave as mirror of focal length f_{eq}.

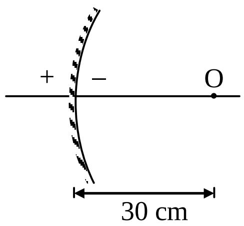


$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

$$\frac{1}{f_{eq}} = \frac{2}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f_{eq}} = \frac{2}{10} + \frac{1}{\infty}$$

$f_{eq} = 5$



mirror formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$$\frac{1}{-5} = \frac{1}{-30} + \frac{1}{v}$$

v = – 6 cm

Image is real and 6 cm away from silvered lens.

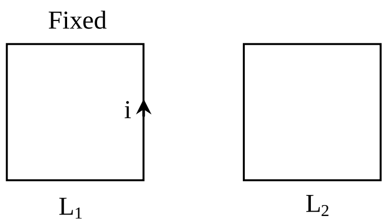
32. Two identical metallic square loops L_1 and L_2 are placed next to each other with their sides parallel on a smooth horizontal table. Loop L_1 is fixed and a current which increases as a function of time is passed through it. Then loop L_2

(A) rotates about its center of mass (B) moves towards L_1.

(C) remains stationary (D) moves away from L_1

Ans. **[D]**

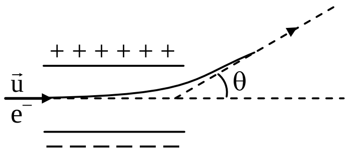
Sol.



When current through L_1 increases then flux linked through L_2 will increase.

$\therefore$ According to lenz law L_2 will move away.

33. An electron enters a parallel plate capacitor with horizontal speed u and is fond to deflect by angle θ on leaving the capacitor as shown. It is found that tanθ = 0.4 and gravity is negligible

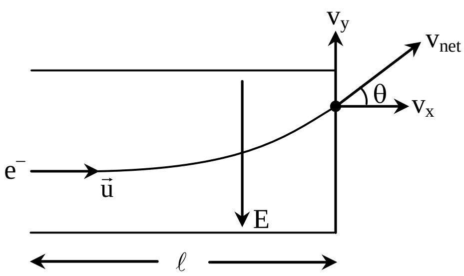


If the initial horizontal speed is doubled, then tan θ will be

(A) 0.1 (B) 0.2 (C) 0.8 (D) 1.6

Ans. **[A]**

Sol.



Horizontal displacement = ℓ

$$t = \frac{\ell}{u}$$

$$v_y = u_y + at$$

$$= 0 + \frac{eE}{m} \times \frac{\ell}{u}$$

$v_y = \frac{eE}{m} \times \frac{\ell}{u}$

v_x remain same and it is equal to u

$\tan\theta = \frac{v_y}{v_x} = \frac{eE}{m}\frac{\ell}{u} \times \frac{1}{u} = \frac{eE\ell}{mu^2}$

$\tan\theta \propto \frac{1}{u^2}$

When speed u is doubled then tan θ will become $\frac{1}{4}$ th.

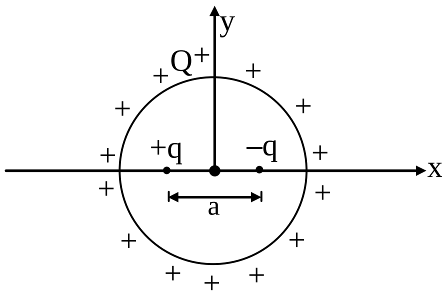
$\therefore \tan\theta = \frac{0.4}{4} = 0.1$

34. Consider a spherical shell of radius R with a total charge +Q uniformly spread on its surface (center of the shell lies at the origin x= 0). Two point charge, +q and – q are brought, one after the other, from far away and placed at x = – a/2 and x = + a/2 (a < R), respectively. Magnitude of the work done in this process is

(A) $(Q + q)^2 /4\pi\varepsilon_0 a$ (B) zero (C) $q^2 /4\pi\varepsilon_0 a$ (D) $Qq /4\pi\varepsilon_0 a$

Ans. **[C]**

Sol.



PE_i = Initial energy of system = $\frac{Q^2}{8\pi\varepsilon_0 R}$ (self energy of shell)

PE_f = Final energy of system = $\frac{Q^2}{8\pi\varepsilon_0 R} + \frac{q\times(-q)}{4\pi\varepsilon_0 a} + \frac{kQ\times q}{R} + \frac{kQ(-q)}{R} \Rightarrow \frac{Q^2}{8\pi\varepsilon_0 R} - \frac{q^2}{4\pi\varepsilon_0 a}$

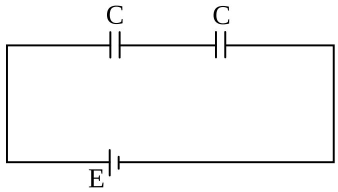
(self energy of shell) (Interaction energy between various charges)

Work done = $PE_f - PE_i$

$= \frac{-q^2}{4\pi\varepsilon_0 a}$

Magnitude of work done = $\frac{q^2}{4\pi\varepsilon_0 a}$

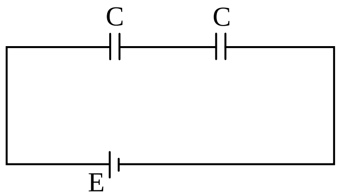
35. Two identical parallel plate capacitors of capacitance C each are connected in series with a battery of emf, E as shown. If one of the capacitors is now filled with a dielectric of dielectric constant k, the amount of charge which will flow through the battery is (neglect internal resistance of the battery)



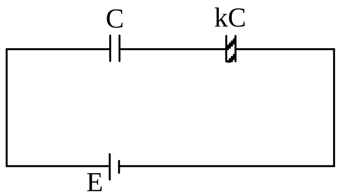
(A) $\frac{k+1}{2(k-1)}CE$ (B) $\frac{k-1}{2(k+1)}CE$ (C) $\frac{k-2}{k+2}CE$ (D) $\frac{k+2}{k-2}CE$

Ans. **[B]**

Sol.



Initial charge on both C = $\frac{CE}{2}$



New charge on each C = $\left(\frac{kC}{k+1}\right)E$

Change in charge on C is supplied by battery

$\therefore$ Charge supply by battery = $\left(\frac{kC}{k+1}\right)E - \frac{CE}{2}$

$\Rightarrow \; CE\left[\frac{k}{k+1} - \frac{1}{2}\right]$

$\Rightarrow \; CE\left[\frac{k-1}{2(k+1)}\right]$

Charge passes through battery is change supply by battery

$\therefore$ Ans. $CE\left[\frac{k-1}{2(k+1)}\right]$

36. A certain p-n junction, having a depletion region of width 20 μm, was found to have a breakdown voltage of 100 V. If the width of the depletion region is reduced to 1 μm during its production, then it can be used as a Zener diode for voltage regulation of -

(A) 5 V (B) 10 V (C) 7.5 V (D) 2000 V

Ans. **[A]**

Sol. Break down voltage is proportional to width of depletion region.

$\therefore$ When width reduce to 1 μm thus become $\frac{1}{20}$ times then break down voltage also become $\frac{1}{20}$ times thus it become 5 volt. So Zener diode can used for voltage regulation of 5 volt.

37. The half life of a particle of mass 1.6×10^{-26} kg is 6.9 s and a stream of such particles is travelling with the kinetic energy of a particle being 0.05 eV. The fraction of particles which will decay when they travel a distance of 1 m is -

(A) 0.1 (B) 0.01 (C) 0.001 (D) 0.0001

Ans. **[D]**

Sol. $KE = \frac{1}{2}mv^2$

$0.05 \times 1.6 \times 10^{-19} = \frac{1}{2} \times 1.6 \times 10^{-26} \times v^2$

$0.05 \times 2 \times 10^7 = v^2$

$10^6 = v^2$

v = 1000 m/sec

time taken to travel a distance of 1 m is $\frac{1}{v} \Rightarrow \frac{1}{1000} = 0.001$ sec

Half life of radioactive material = 6.9 sec

$T_{1/2} = \frac{0.693}{\lambda}$

$\lambda = \frac{0.693}{6.9} \Rightarrow 0.1$

fraction of particle decay in 0.001 sec or $\frac{1}{1000}$ sec $= 1 - e^{-\lambda t}$

$\Rightarrow 1 - e^{-0.1 \times \frac{1}{1000}}$

$\Rightarrow 1 - e^{-\frac{1}{10000}}$

$\Rightarrow 0.0001$

38. A 160 watt light source is radiating light of wavelength 6200 Å uniformly in all directions. The photon flux at a distance of 1.8 m is of the order of (Planck's constant 6.63×10^{-34} J-s)

(A) $10^2\ m^{-2}\ s^{-1}$ (B) $10^{12}\ m^{-2}\ s^{-1}$ (C) $10^{19}\ m^{-2}\ s^{-1}$ (D) $10^{25}\ m^{-2}\ s^{-1}$

Ans. **[C]**

Sol. Intensity of light at 1.8 m = $\frac{P}{4\pi(1.8)^2}$

$I \Rightarrow \frac{160}{4\times\pi\times(1.8)^2}$

Photon flux = Number of photon per unit area.

$\Rightarrow \frac{I}{\frac{hc}{\lambda}}$

$\Rightarrow \frac{I\lambda}{hc}$

$\Rightarrow \frac{160\times 6200\times 10^{-10}}{4\times\pi\times(1.8)^2\times 6.63\times 10^{-34}\times 3\times 10^{8}}$

$\Rightarrow 1.22 \times 10^{19}$

39. The wavelength of the first Balmer line caused by a transition from the n = 3 level to the n = 2 level in hydrogen is λ_1. The wavelength of the line caused by an electronic transition from n = 5 to n = 3 is -

(A) $\frac{375}{128}\lambda_1$ (B) $\frac{125}{64}\lambda_1$ (C) $\frac{64}{125}\lambda_1$ (D) $\frac{128}{375}\lambda_1$

Ans. **[B]**

Sol. $\frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$

$\frac{1}{\lambda_1} = R\left[\frac{1}{4} - \frac{1}{9}\right]$

$\frac{1}{\lambda_1} = R\left[\frac{5}{36}\right]$ ....(1)

$\frac{1}{\lambda_2} = R\left[\frac{1}{9} - \frac{1}{25}\right]$

$\frac{1}{\lambda_2} = R\left[\frac{16}{9\times 25}\right]$ ....(2)

From (1) & (2)

$\frac{\lambda_2}{\lambda_1} = \frac{5}{36} \div \frac{16}{9\times 25}$

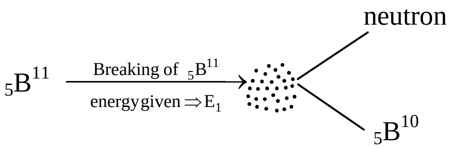
$\lambda_2 = \frac{5}{36} \times \frac{9\times 25}{16} = \frac{125}{64}\lambda_1$

40. The binding energy per nucleon of ${}_5B^{10}$ is 8.0 MeV and that of ${}_5B^{11}$ is 7.5 MeV. The energy required to remove a neutron from ${}_5B^{11}$ is (mass of electron and proton are 9.11 × 10^{31} kg and 1.67 × 10^{27} kg, respectively) -

(A) 2.5 MeV (B) 8.0 MeV (C) 0.5 MeV (D) 7.5 MeV

Ans. **[A]**

Sol.



formation of ${}_5B^{10}$ will release energy $\Rightarrow E_2$

E_1 = Binding energy of ${}_5B^{11} \Rightarrow 7.5 \times 11$ MeV

= 82.5 MeV

E_2 = Binding energy of ${}_5B^{10} = 8.0 \times 10$

= 80 MeV

Energy given = $E_1 - E_2$

= 82.5 – 80

= 2.5 MeV

CHEMISTRY

41. When 1.88 g of AgBr(s) is added to a 10^{-3} M aqueous solution of KBr, the concentration of Ag is 5×10^{-10} M. If the same amount of AgBr(s) is added to a 10^{-2} M aqueous solution of $AgNO_3$, the concentration of Br^- is

(A) 9.4×10^{-9} M (B) 5×10^{-10} M (C) 1×10^{-11} M (D) 5×10^{-11} M

Ans. **[D]**

Sol. $K_{sp(AgBr)} = [Ag^+][Br^-]$

$= (5 \times 10^{-10})(10^{-3})$

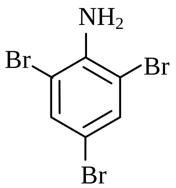
$= 5 \times 10^{-13}$

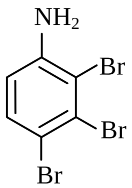
Now

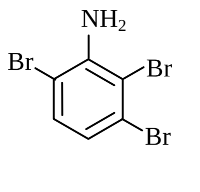
$5 \times 10^{-13} = (10^{-2})[Br^-]$

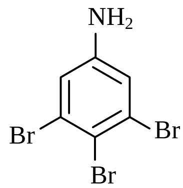
$[Br^-] = 5 \times 10^{-11}$ M

42. Aniline reacts with excess Br_2/H_2O to give the major product

(A) 

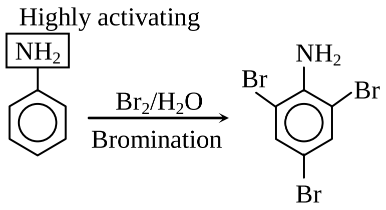
(B) 

(C) 

(D) 

Ans. **[A]**

Sol.



2,4,6-Tribromoaniline

43. The metal with the highest oxidation state present in K_2CrO_4, $NbCl_5$ and MnO_2 is -

(A) Nb (B) Mn (C) K (D) Cr

Ans. **[D]**

Sol. $K_2CrO_4 \Rightarrow Cr^{+6}$ (highest oxidation state)

$NbCl_5 \Rightarrow Nb^{+5}$

$MnO_2 \Rightarrow Mn^{+4}$

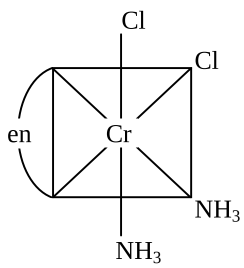
44. The number of geometrical isomers of $[CrCl_2(en)(NH_3)_2]$, where en = ethylenediamine, is -

(A) 2 (B) 3 (C) 4 (D) 1

Ans. **[B]**

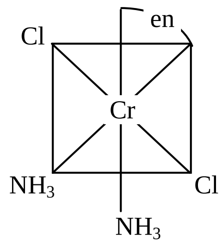
Sol. Total 3 geometrical isomers are possible -

(1)



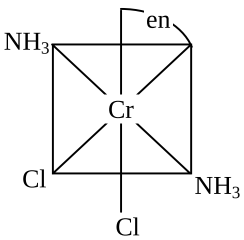
Cis

(2)



Trans

(3)



Trans

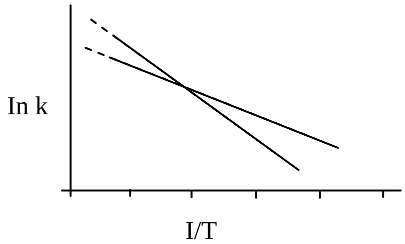
45. The element that combines with oxygen to give an amphoteric oxide is -

(A) N (B) P (C) Al (D) Na

Ans. **[C]**

Sol. Aluminium form amphoteric oxide with oxygen (Al_2O_3)

46. The Arrhenius plots of two reactions, I and II are shown graphically -



The graph suggests that -

(A) $E_I > E_{II}$ and $A_I > A_{II}$
(B) $E_{II} > E_I$ and $A_{II} > A_I$
(C) $E_I > E_{II}$ and $A_{II} > A_I$
(D) $E_{II} > E_I$ and $A_I > A_{II}$

Ans. **[A]**

Sol. For plot between In ***k*** v/s 1/T y-intercept is ln A

& slope is $\frac{-E_a}{R}$

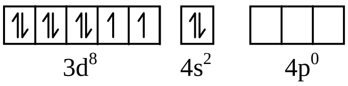
therefore; $E_{II} < E_I$ and $A_I > A_{II}$

47. $Ni(CO)_4$ is

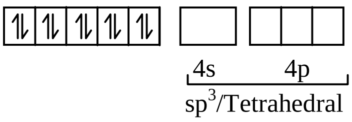
(A) tetrahedral and paramagnetic
(B) square planar and diamagnetic
(C) tetrahedral and diamagnetic
(D) square planar and paramagnetic

Ans. **[C]**

Sol. Ni exist in zero oxidation state so its configuration is -
its configuration is -
$_{28}Ni = [Ar]\ 3d^8\ 4s^2$

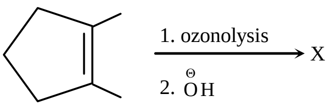


– CO is strong ligand so pairing of electron possible and configuration will be

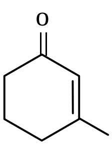
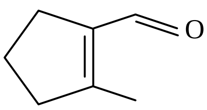
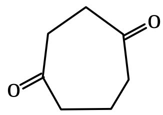
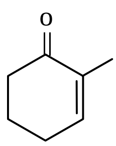


– Number of unpaired electron are zero hence it is diamagnetic in nature.

48. In the following reaction -

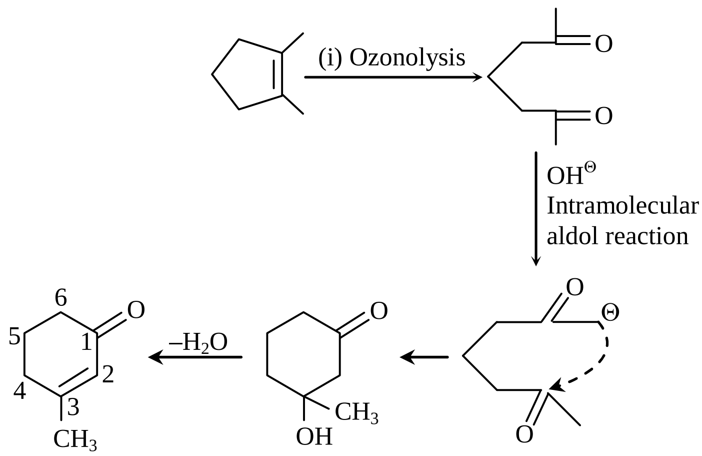


the major product X is-

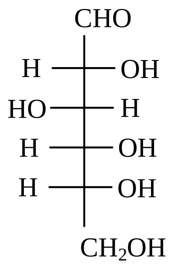
(A) 
(B) 
(C) 
(D) 

Ans. **[A]**

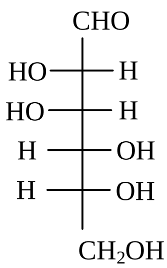
Sol.

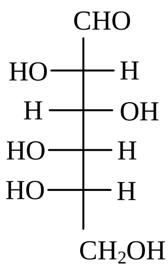


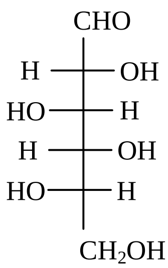
49. Given the structure of D-(+)-glucose as

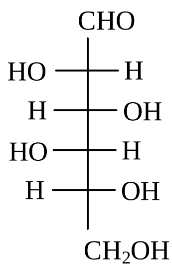


The structure of L-(–)-glucose is

(A)


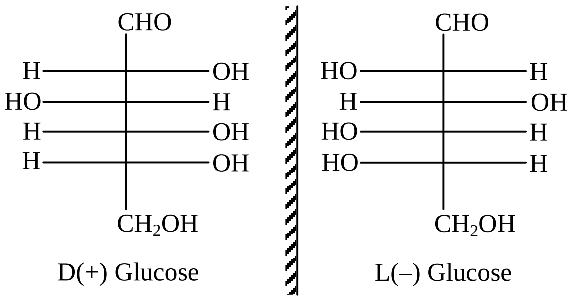
(B)


(C)


(D)


Ans. **[B]**

Sol.



D(+) Glucose L(–) Glucose

Enantiomer

50. In a cubic close packed structure, fractional contributions of an atom at the corner and at the face in the unit cell are, respectively -

(A) 1/8 and 1/2 (B) 1/2 and 1/4 (C) 1/4 and 1/2 (D) 1/4 and 1/8

Ans. **[A]**

Sol. Corner $\Rightarrow \dfrac{1}{8}$

Face $\Rightarrow \dfrac{1}{2}$

51. The equilibrium constant K_c of the reaction, $2A \rightleftharpoons B+C$ is 0.5 at 25°C and 1 atm. The reaction will proceed in the backward direction when concentrations [A], [B] and [C] are, respectively -

(A) 10^{-3}, 10^{-2} and 10^{-2} M (B) 10^{-1}, 10^{-2} and 10^{-2} M

(C) 10^{-2}, 10^{-2} and 10^{-3} M (D) 10^{-2}, 10^{-3} and 10^{-3} M

Ans. **[A]**

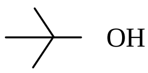
Sol. $Q = \dfrac{[B][C]}{[A^2]}$ & $K_C = 0.5$

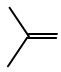
For option (A)

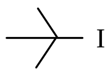
$$Q = \frac{(10^{-2})\times(10^{-2})}{[10^{-3}]^2} = 100$$

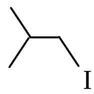
& $Q > K_C$ i.e. reaction proceed in backward direction.

52. Major products formed in the reaction of t-butyl methyl ether with HI are -

(A) $H_3C - I$ and 

(B)  and $H_3C - OH$

(C) $H_3C - OH$ and 

(D) 

and $H_3C - OH$

Ans. **[C]**

Sol.

$$CH_3 - \underset{CH_3}{\overset{CH_3}{\underset{|}{\overset{|}{C}}}} - O - CH_3 \xrightarrow[SN_1]{HI} CH_3 - \underset{CH_3}{\overset{CH_3}{\underset{|}{\overset{|}{C}}}} - I + CH_3OH$$

If one of the alkyl group is 3º. Then mechanism is SN_1 and nucleophile attach to the carbon where carbocation more stable.

53. If the molar conductivities (in S cm^2 mol^{-1}) of NaCl, KCl and NaOH at infinite dilution are 126, 150 and 250 respectively, the molar conductivity of KOH (in S cm^2 mol^{-1}) is -

(A) 526 (B) 226 (C) 26 (D) 274

Ans. **[D]**

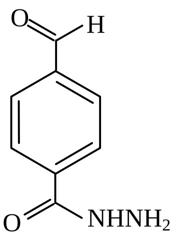
Sol.

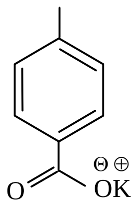
$$126 = \lambda^{\infty}_{Na^+} + \lambda^{\infty}_{Cl^-}$$
$$150 = \lambda^{\infty}_{K^+} + \lambda^{\infty}_{Cl^-}$$
$$250 = \lambda^{\infty}_{Na^+} + \lambda^{\infty}_{OH^-}$$

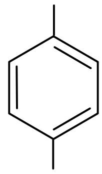
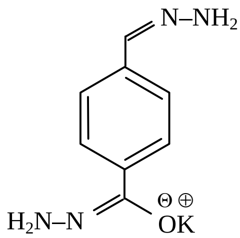
$$(150 + 250 - 126) = \lambda^{\infty}_{K^+} + \lambda^{\infty}_{OH^-}$$

or $\lambda^{\infty}_{KOH} = 274$

54. 4-Formylbenzoic acid on treatment with one equivalent of hydrazine followed by heating with alcoholic KOH gives the major product -

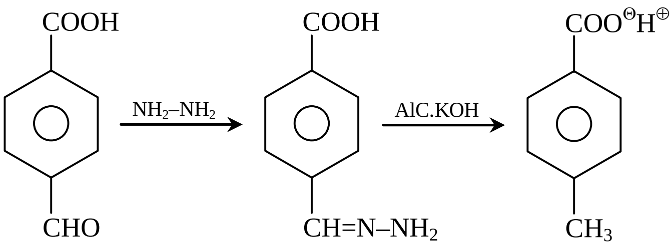
(A) 

(B) 

(C) 
(D) 

Ans. **[B]**

Sol.



This is example of wolfkishner reduction which converts.

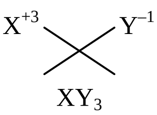
$>C = O$ in $>CH_2$ Group But do not reduce –COOH group.

55. Two elements, X and Y, have atomic numbers 33 and 17, respectively. The molecular formula of a stable compound formed between them is -

(A) XY (B) XY_2 (C) XY_3 (D) XY_4

Ans. **[C]**

Sol. Atomic no. 33 and 17 belongs to 15th & 17th group respectively therefore co-valent bond form between both elements



$\left.\begin{array}{l}\text{Atomic no. 33 = As}\\ \text{Atomic no. 17 = Cl}\end{array}\right] \longrightarrow AsCl_3$

56. The number of moles of $KMnO_4$ required to oxidize one equivalent of KI in the presence of sulfuric acid is -

(A) 5 (B) 2 (C) 1/2 (D) 1/5

Ans. **[D]**

Sol. $KMnO_4 + KI + H_2SO_4 \longrightarrow MnSO_4 + I_2 + K_2SO_4 + H_2O$

v.f = 5 v.f = 1

$\therefore\ (eq)_{KMnO_4} = (eq)_{KI} = 1$

Eq. = V.F. × mole

1 = 5 × mole

Mole = 1/5

57. Three successive measurements in an experiment gave the values 10.9, 11.4042 and 11.42. The correct way of reporting the average value is -

(A) 11.2080 (B) 11.21 (C) 11.2 (D) 11

Ans. **[C]**

Sol. The correct way of reporting the average value should have exactly the same number of digit after decimal which has least digit after decimal among the data given.

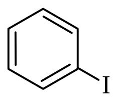
58. The latent heat of melting of ice at 0 °C is 6 kJ mol^{-1}. The entropy change during the melting in J K^{-1} mol^{-1} is closest to -

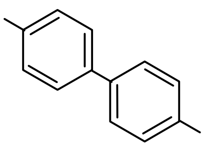
(A) 22 (B) 11 (C) –11 (D) –22

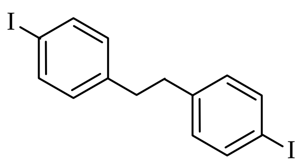
Ans. **[A]**

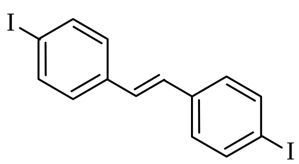
Sol. $\Delta S = \dfrac{\Delta H_{Melting}}{T_{F.P}} = \dfrac{6\times1000}{273}\ \dfrac{J}{K}$

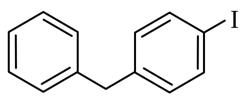
$= 21.978 \approx 22 J/k$

59. The major product of the following reaction  $\xrightarrow{Cu,\ \Delta}$ is

(A) 

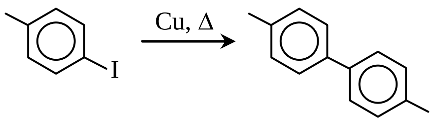
(B) 

(C) 

(D) 

Ans. **[A]**

Sol.



This is example of Ulman reaction which gives product like Wurtz reaction.

60. The energies of d_{xy} and d_z^2 orbitals in octahedral and tetrahedral transition metal complexes are such that -

(A) $E(d_{xy}) > E(d_z^2)$ in both tetrahedral and octahedral complexes

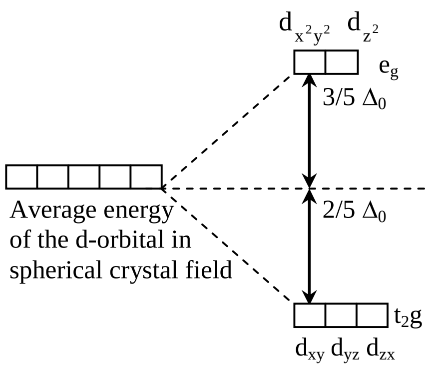
(B) $E(d_{xy}) < E(d_z^2)$ in both tetrahedral and octahedral complexes

(C) $E(d_{xy}) > E(d_z^2)$ in tetrahedral but $E(d_{xy}) < E(d_z^2)$ in octahedral complexes

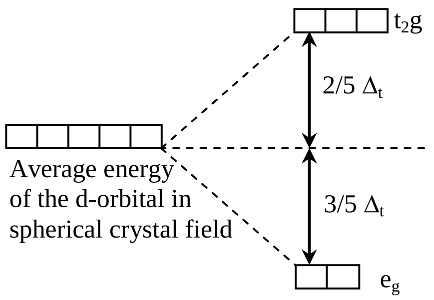
(D) $E(d_{xy}) < E(d_z^2)$ in tetrahedral but $E(d_{xy}) > E(d_z^2)$ in octahedral complexes

Ans. **[C]**

Sol. Energy of d_{z^2} is greater than d_{xy} in case of octahedral crystal field while energy of d_{z^2} is less than d_{xy} in case of tetrahedral splitting



[Spliting in octahedral crystal field]



[Spliting in tetrahedral crystal field]

BIOLOGY

61. In which of the following types of glands is the secretion collected inside the cell and discharged by disintegration of the entire gland ?

(A) Apocrine (B) Merocrine (C) Holocrine (D) Epicrine

Ans. **[C]**

Sol. Holocrine gland distingrate completely for discharge of secretion.

62. Which one of the following interactions doest **NOT** promote coevolution ?

(A) Commensalism (B) Mutualism (C) Parasitism (D) Interspecific competition

Ans. **[D]**

Sol. Interspecific competition doesn't lead to co-evolution

63. Stratification is more common in which of the following ?

(A) Deciduous forest (B) Tropical rain forest (C) Temperate forest (D) Tropical savannah

Ans. **[B]**

Sol. Tropical rain forest shows vertical zonation i.e. stratification.

64. Where is the third ventricle of the brain located ?

(A) Cerebrum (B) Cerebellum (C) Pons varoli (D) Diencephalon

Ans. **[D]**

Sol. Cavity of Diancephelon is called as diocoel or third ventricle.

65. Which of the following is the final product of a gene ?

(A) a polypeptide only (B) an RNA only

(C) either polypeptide or RNA (D) a nucleotide only

Ans. **[C]**

Sol. Gene is a segment of genetic material which produces either polypeptide or a RNA [rRNA or tRNA]

66. Forelimbs of whales, bats, humans and cheetah are examples of which of the following processes ?

(A) Divergent evolution (B) Convergent evolution

(C) Adaptation (D) Saltation

Ans. **[A]**

Sol. Fore limbs of whale, bat, human & cheetah are homologous organ which represents divergent evolution.

67. Which of the following results from conjugation in *Paramecium* ?

(A) Cell death (B) Cell division (C) Budding (D) Recombination

Ans. **[D]**

Sol. Conjugation in paramecium results in recombination

68. In an experiment investigating photoperiodic response, the leaves of a plant are removed. What is the most likely outcome ?
(A) Photoperiodism is not affected
(B) Photoperiodic response does not occur
(C) The plant starts flowering
(D) The plant starts to grow taller

Ans. **[B]**

Sol. Leaves are the site for photoperiodic perception.

69. Testosterone is secreted by which endocrine part of testis ?
(A) Leydig cells (B) Seminiferous tubules (C) Tunica albugenia (D) Sertoli cells

Ans. **[A]**

Sol. Leydig cells or Interstitial cells of testes secretes testosteron hormone.

70. The mutation of a purine to a pyrimidine is known as
(A) transition (B) frame shift (C) nonsense (D) transversion

Ans. **[D]**

Sol. Mutation of purine to purimidine is known as transversion. 8 possible transversion can occurs.

71. Which of the following is secreted at the ends of an axon ?
(A) Ascorbic acid (B) Acetic acid (C) Acetyl choline (D) Acetyl CoA

Ans. **[C]**

Sol. Synaptic bulbs of axon have vesicles which are filled with acetylcholine.

72. A bacterial colony is produced from
(A) a single bacterium by its repetitive division
(B) multiple bacterium without replication
(C) clumping of two to three bacteria
(D) a single bacterium without cell division

Ans. **[A]**

Sol. A bacterial colony on culture media is formed on repetitive division of bacterium.

73. Rhinoviruses are the causative agents of
(A) Diarrhoea (B) AIDS (C) Dengue (D) Common cold

Ans. **[D]**

Sol. Rhinovirus are the primary cause of common cold.

74. What is the genetic material of Ebola virus ?
(A) Single-stranded DNA
(B) Double-stranded RNA
(C) Single-stranded RNA
(D) Double-stranded DNA

Ans. **[C]**

Sol. Ebola virus consist of ss RNA.

75. Name the terminal acceptor of electrons in the mitochondrial electron transport chain
(A) Nitrate (B) Fumarate (C) Succinate (D) Oxygen

Ans. **[D]**

Sol. Oxygen is the terminal acceptor of electrons in mitochondrial ETS.

76. Two tubes labelled 'P' and 'Q' contain food stuff. Tube 'P' gave positive test with Benedict's solution while tube 'Q' gave positive test with Nitric acid. Which of the following is correct ?
(A) Tube 'P' contains sugar; tube 'Q' contains protein
(B) Tube 'P' contains protein; tube 'Q' contains sugar
(C) Both, tube 'P' and tube 'Q' contain sugar
(D) Both, tube 'P' and tube 'Q' contain protein

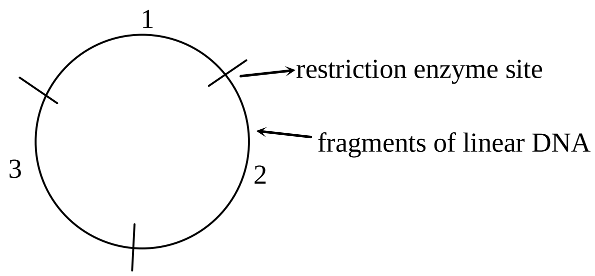
Ans. **[A]**

Sol. → Nitric acid reacts with proteins to form yellow nitrated products.
→ Benedict test is used to test the presence of monosaccharide and reducing sugar.

77. How many linear DNA fragments will be produced when a circular plasmid is digested with a restriction enzyme having 3 sites ?
(A) 4 (B) 5 (C) 3 (D) 2

Ans. **[C]**

Sol.



Plasmid

78. If the humidity of the atmosphere suddenly increases substantially, the water flow in the xylem will -
(A) increase
(B) decrease
(C) remain unaltered
(D) increase sharply and then reduce slowly to the preexisting level

Ans. **[B]**

Sol. Humidity of atmosphere is inversely proportional to transpiration (water flow). Increase in humidity will decrease the water flow in the xylem.

79. Which one of the following is the complementary sequence for the DNA with 5′-CGTACTA-3′
(A) 5′-TAGTACG-3′ (B) 5′-ATCATGC-3′ (C) 5′-UTCUTGC-3′ (D) 5′-GCUAGCA-3′

Ans. **[A]**

Sol. Double stranded DNA has antiparalllel strands and complementary N-bases
so, 5′ CGTACTA 3′
3′ GCATGAT 5′
$\longleftarrow$
thus, answer is 5′TAGTACG3′

80. A diploid plant has 14 chromosomes, but its egg cell has 6 chromosomes. which one of the following is the most likely explanation of this ?
(A) Non-disjunction in meiosis I and II (B) Non-disjunction in meiosis I
(C) Non-disjunction in mitosis (D) Normal meiosis

Ans. **[B]**

Sol. Egg cell is formed by meiosis. Less number of chromosome indicates non-disjunction in meiosis I.

Part – II

Two - Mark Questions

MATHEMATICS

81. Let $n \geq 3$ be an integer. For a permutation $\sigma = (a_1, a_2, \ldots\ldots, a_n)$ of $(1, 2, \ldots\ldots, n)$ we let $f_\sigma(x) = a_n x^{n-1} + a_{n-1}x^{n-2} + \ldots.. + a_2 x + a_1$. Let S_σ be the sum of the roots of $f_\sigma(x) = 0$ and let S denote the sum over all permutations σ of $(1, 2, \ldots.., n)$ of the numbers S_σ. Then -

(A) $S < -n!$ (B) $-n! < S < 0$ (C) $0 < S < n!$ (D) $n! < S$

Ans. **[B]**

Sol. $$S = -\left[\frac{\lambda - a_n}{a_n} + \frac{\lambda - a_{n-1}}{a_{n-1}} + \ldots\ldots. + \frac{\lambda - a_1}{a_1}\right]$$

$\forall\ \lambda = a_1 + a_2 + \ldots\ldots. + a_n$

$$S = -\left[(a_1 + a_2 + \ldots. + a_n)\left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots. + \frac{1}{a_n}\right) - n\right]$$

$$S = n - (a_1 + a_2 + \ldots + a_n)\left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots. + \frac{1}{a_n}\right)$$

from A.M. $\geq$ H.M.

$$(a_1 + a_2 + \ldots. + a_n)\left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots. + \frac{1}{a_n}\right) \geq n^2$$

$$\boxed{S \leq -n(n-1)}$$

82. If n is a positive integer and $\omega \neq 1$ is a cube root of unity, the number of possible values of

$$\left| e^{\sum_{k=0}^{n}\binom{n}{k}\omega^k} \right|$$

(A) 2 (B) 3 (C) 4 (D) 6

Ans. **[C]**

Sol. $$\sum_{k=0}^{n} {}^nC_k\omega^k = {}^nC_0 + {}^nC_1\omega + \ldots..+ {}^nC_n\omega^n$$

$= (1 + \omega)^n = (-\omega^2)^n$

$= (-1)^n\ \omega^{2n}$

$$\therefore\ \left|e^{(-1)^n\omega^{2n}}\right| = \left|e^{(-\omega^2)^n}\right|$$

$$= \left|e^{\left(-\cos\frac{4\pi}{3} - i\sin\frac{4\pi}{3}\right)^n}\right|$$

$= \left| e^{\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3}} \right|$

$= \left| e^{\cos\frac{n\pi}{3}} \right|$ can have values

$= \{e^{1}, e^{1/2}, e^{-1/2}, e^{-1}\}$

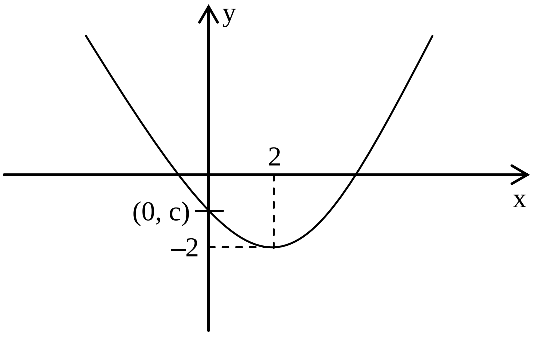
Four values.

83. Suppose a parabola $y = ax^2 + bx + c$ has two x intercepts, one positive and one negative, and its vertex is (2, –2). Then which of the following is true?

(A) $ab > 0$ (B) $bc > 0$ (C) $ca > 0$ (D) $a + b + c > 0$

Ans. **[B]**

Sol. The graph according to the question is



Clearly it can be observed

$c < 0$

$a > 0$

$\frac{-b}{a} > 0 \Rightarrow -b > 0 \Rightarrow b < 0$

$f(1) < 0 \Rightarrow a + b + c < 0$

$ab < 0$

$ac < 0$

$bc > 0$

84. Let $n \geq 3$ and let $C_1, C_2, \ldots, C_n$, be circles with radii $r_1, r_2, \ldots, r_n$, respectively. Assume that C_i and C_{i+1} touch externally for $1 \leq i \leq n - 1$. It is also given that the x-axis and the line $y = 2\sqrt{2}x + 10$ are tangential to each of the circles. Then $r_1, r_2, \ldots, r_n$ are in -

(A) an arithmetic progression with common difference $3 + \sqrt{2}$

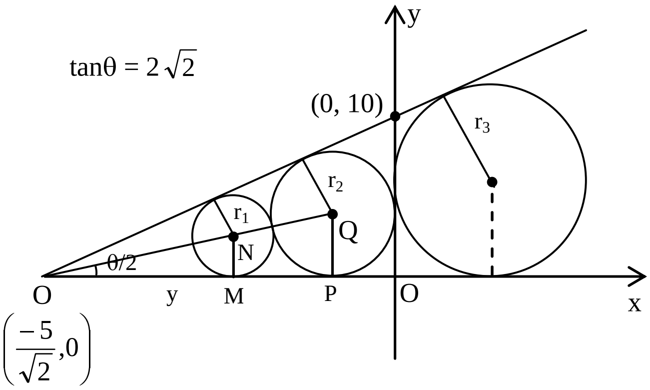
(B) a geometric progression with common ratio $3 + \sqrt{2}$

(C) an arithmetic progression with common difference $2 + \sqrt{3}$

(D) a geometric progression with common ratio $2 + \sqrt{3}$

Ans. **[D]**

Sol.



$\tan\theta = \dfrac{2\tan\theta/2}{1-\tan^2\theta/2}$

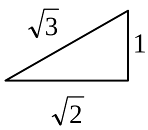
$2\sqrt{2} = \dfrac{2\tan\theta/2}{1-\tan^2\theta/2}$

$\sqrt{2}\tan^2\theta/2 + \tan\theta - \sqrt{2} = 0$

$\tan\theta/2 = \dfrac{-1\pm\sqrt{1+8}}{2\sqrt{2}}$

$= \dfrac{-1\pm 3}{2\sqrt{2}} = \dfrac{1}{\sqrt{2}}$ or $-\sqrt{2}$

$\therefore \quad \tan\theta/2 = \dfrac{1}{\sqrt{2}}$



In Δ OMN $\sin\theta/2 = \dfrac{r_1}{ON}$ $\quad \sin\theta/2 = \dfrac{1}{\sqrt{3}}$

$ON = \sqrt{3}r_1$

In Δ OPQ $\sin\theta/2 = \dfrac{r_2}{ON+r_1+r_2} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{r_2}{\sqrt{3}r_1+r_1+r_2}$

$\sqrt{3}r_1 + r_1 + r_2 = \sqrt{3}r_2$

$r_1(\sqrt{3}+1) = r_2(\sqrt{3}-1)$

$\dfrac{r_2}{r_1} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

$= \dfrac{(\sqrt{3}+1)^2}{2} = 2+\sqrt{3}$

85. The number of integers n for which $3x^3 - 25x + n = 0$ has three real roots is -

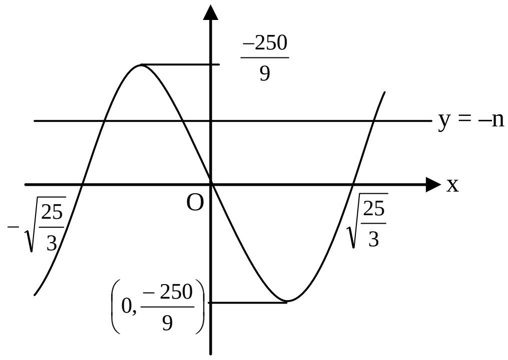
(A) 1 (B) 25 (C) 55 (D) infinite

Ans. **[C]**

Sol. $x(3x^2 - 25) = -n$

$$3x\left(x^2 - \frac{25}{3}\right) = -n$$

$$3x\left(x - \sqrt{\frac{25}{3}}\right)\left(x + \sqrt{\frac{25}{3}}\right) = -n$$



$$\therefore\ n \in \left(-\frac{250}{9}, \frac{250}{9}\right)$$

$\therefore$ But $n \in I$

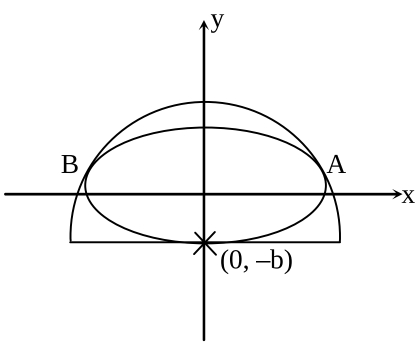
Total integer = 55

86. An ellipse inscribed in a semi-circle touches the circular arc at two distinct points and also touches the bounding diameter. Its major axis is parallel to the bounding diameter. When the ellipse has the maximum possible area, its eccentricity is -

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{\frac{2}{3}}$

Ans. **[D]**

Sol.



Let ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

and circle $x^2 + (y + b)^2 = r^2$ {let radius = r}

put $x^2 = a^2 - \frac{a^2y^2}{b^2}$

in circle $a^2 - \frac{a^2y^2}{b^2} + (y + b)^2 = r^2$

$\Rightarrow \left(1 - \frac{a^2}{b^2}\right)y^2 + 2by + (a^2 + b^2 - r^2) = 0$

$D = 0 \Rightarrow r^2 = \frac{a^4}{a^2 - b^2}$

$\Rightarrow b = a\sqrt{1 - \frac{a^2}{r^2}}$

Area $= \Delta = \pi ab = \pi a^2\sqrt{1 - \frac{a^2}{r^2}}$

$\frac{d\Delta}{da} = 0 \Rightarrow a^2 = \frac{2r^2}{3} \Rightarrow a = \sqrt{\frac{2}{3}}\,r$

$\therefore\ b = a\sqrt{1 - \frac{2}{3}} = \frac{a}{\sqrt{3}} \Rightarrow e = \sqrt{\frac{2}{3}}$

87. Let $I_n = \int_0^{\pi/2} x^n \cos x\, dx$, where n is a non-negative integer.

Then $\sum_{n=2}^{\infty}\left(\frac{I_n}{n!} + \frac{I_{n-2}}{(n-2)!}\right)$ equals -

(A) $e^{\pi/2} - 1 - \frac{\pi}{2}$ (B) $e^{\pi/2} - 1$ (C) $e^{\pi/2} - \frac{\pi}{2}$ (D) $e^{\pi/2}$

Ans. **[A]**

Sol. $I_n = \int_0^{\pi/2} \underset{\mathrm{I}}{x^n}\, \underset{\mathrm{II}}{\cos x}\, dx$

$= x^n \sin x\Big|_0^{\pi/2} - \int_0^{\pi/2} n\,x^{n-1} \sin x\, dx$

$= \left(\frac{\pi}{2}\right)^n - 0 - \left(n\,x^{n-1}\ (-\cos x)\Big|_0^{\pi/2} - \int_0^{\pi/2} n\,(n-1)x^{n-2}\,(-\cos x)\,dx\right.$

$= \left(\frac{\pi}{2}\right)^n - 0 - n(n-1)\int_0^{\pi/2} x^{n-2} \cos x\, dx$

$I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)I_{n-2}$

$$\sum_{n=2}^{\infty}\left(\frac{I_n}{n!}+\frac{I_{n-2}}{(n-2)!}\right)=\sum_{n=2}^{\infty}\left(\frac{\left(\frac{\pi}{2}\right)^n-n(n-1)I_{n-2}}{n!}+\frac{I_{n-2}}{(n-2)!}\right)$$

$$=\sum_{n=2}^{\infty}\left(\left(\frac{\pi}{2}\right)^n\frac{1}{n!}\right)$$

$$=\left(\frac{\pi}{2}\right)^2\frac{1}{2!}+\left(\frac{\pi}{2}\right)^3\frac{1}{3!}+\left(\frac{\pi}{2}\right)^4\frac{1}{4!}+\ldots.$$

$$=e^{\pi/2}-1-\left(\frac{\pi}{2}\right)$$

88. For a real number x let [x] denote the largest integer less than or equal to x. The smallest positive integer n for which the integral $\int_1^n [x][\sqrt{x}]dx$ exceeds 60 is -

(A) 8 (B) 9 (C) 10 (D) $[60^{2/3}]$

Ans. **[B]**

Sol. Let $I = \int_1^n [x][\sqrt{x}]dx$

$1 \le x < 4$ $\quad\left[\sqrt{x}\right]=1$

$4 \le x < 9$ $\quad\left[\sqrt{x}\right]=2$

$9 \le x < 16$ $\quad\left[\sqrt{x}\right]=3$

Now,

$$I=\int_1^2 dx+\int_2^3 2dx+\int_3^4 3dx+\int_4^5 8dx+\int_5^6 10dx+\int_6^7 12dx+\int_7^8 14dx+\int_8^9 16dx+\int_9^{10} 27dx+\int_{10}^{11} 30dx\ \ldots..$$

I = 1 + 2 + 3 + 8 + 10 + 12 + 14 + 16 = 66

So n = 9

89. Choose a number n uniformly at random from the set {1, 2, ….., 100}. Choose one of the first seven days of the year 2014 at random and consider n consecutive days starting from the chosen day. What is the probability that among the chosen n days, the number of Sundays is different from the number of Mondays?

(A) $\frac{1}{2}$ (B) $\frac{2}{7}$ (C) $\frac{12}{49}$ (D) $\frac{43}{175}$

Ans. **[*]**

Sol.

90. Let S = {(a, b)|a, b $\in$ Z, $0 \le a, b \le 18$}. The number of lines in R^2 passing through (0, 0) and exactly one other point in S is -

(A) 16 (B) 22 (C) 28 (D) 32

Ans. **[*]**

PHYSICS

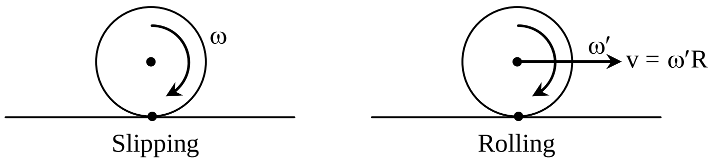
91. A solid sphere spinning about a horizontal axis with an angular velocity ω is placed on a horizontal surface. Subsequently it rolls without slipping with an angular velocity of -

(A) $2\omega/5$ (B) $7\omega/5$ (C) $2\omega/7$ (D) ω

Ans. **[C]**

Sol. Initial sphere is slipping and finally it start rolling. During its motion τ about point of contact is zero.

$\therefore$ Angular momentum of sphere about point of contact remain conserved.

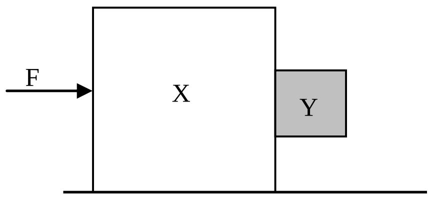


$$I\omega = (I + MR^2)\,\omega'$$

$$\frac{2}{5}MR^2\omega = \left(\frac{2}{5}MR^2 + MR^2\right)\omega'$$

$$\omega' = \frac{2\omega}{7}$$

92. Consider the system shown below.

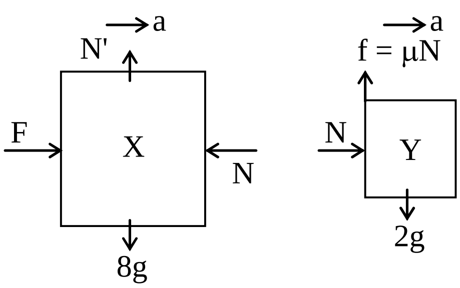


A horizontal force F is applied to a block X of mass 8 kg such that the block Y of mass 2 kg adjacent to it does not slip downwards under gravity. There is no friction between the horizontal plane and the base of the block X. The coefficient of friction between the surfaces of blocks X and Y is 0.5. Take acceleration due to gravity to be 10 ms^{-2}. The minimum value of F is

(A) 200 N (B) 160 N (C) 40 N (D) 240 N

Ans. **[A]**

Sol. According to free body diagram



$F - N = 8a$ ....(1)

$N = 2a$ ....(2)

$f = 2g = 20$ ....(3)

$\Rightarrow \mu N = 20$

$\Rightarrow N = \frac{20}{\mu} = \frac{20}{0.5} = 40$ ....(4)

$\Rightarrow N = 2a$

$\Rightarrow a = \frac{N}{2} = \frac{40}{2} = 20\ m/s^2$ ....(5)

$F = N + 8a = 10\ a$ [from equation (1)]

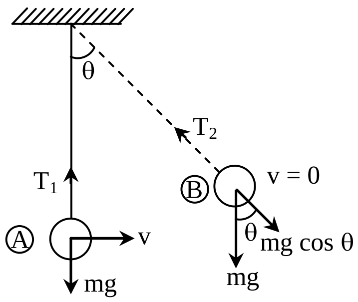
$F = 10 \times 20 = 200$ newton

93. The maximum value attained by the tension in the string of a swinging pendulum is four times the minimum value it attains. There is no slack in the string. The angular amplitude of the pendulum is

(A) 90° (B) 60° (C) 45° (D) 30°

Ans. **[B]**

Sol.



Centripetal force at point A :

$T_1 - mg = \frac{mv^2}{\ell}$ ....(1)

At point B :

$T_2 = mg \cos \theta$ ....(2)

According to question

$T_1 = 4T_2$ ....(3)

$\Rightarrow mg + \frac{mv^2}{\ell} = 4\ mg \cos \theta$ [from equation (1) & (2)]

$\Rightarrow$ mg (4 cos θ – 1) = $\frac{mv^2}{\ell}$ ….(4)

According to conservation of energy between point A and B

Also $\frac{1}{2}mv^2 + 0 = 0 + mg\ell\,(1 - \cos\theta)$

$mv^2 = 2\,mg\ell\,(1 - \cos\theta)$

$\frac{mv^2}{\ell} = 2\,mg\,(1 - \cos\theta)$ ….(5)

From equation (4) & (5)

$mg\,(4\cos\theta - 1) = 2\,mg\,(1 - \cos\theta)$

$\Rightarrow$ $4\cos\theta - 1 = 2 - 2\cos\theta$

$\Rightarrow$ $6\cos\theta = 3$

$\Rightarrow$ $\cos\theta = \frac{1}{2}$

$\Rightarrow$ $\theta = 60^o$

94. One mole of a monoatomic ideal gas is expanded by a process described by $PV^3 = C$ where C is a constant. The heat capacity of the gas during the process is given by (R is the gas constant).

(A) 2 R (B) $\frac{5}{2}R$ (C) $\frac{3}{2}R$ (D) R

Ans. **[D]**

Sol. Monoatomic gas

$\Rightarrow$ $\gamma = \frac{5}{3}$

$n = 1$

$PV^3 = C$ on comparing with $PV^\alpha = C$

Here $\alpha = 3$

Heat capacity

$C = \frac{R}{\gamma - 1} - \frac{R}{\alpha - 1}$

$C = \frac{R}{\left(\frac{2}{3}\right)} - \frac{R}{(2)}$

$C = R\left[\frac{3}{2} - \frac{1}{2}\right]$

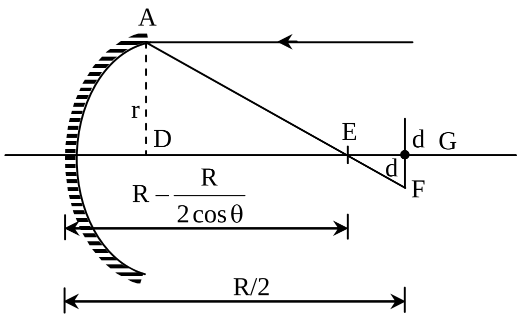
$C = R$

95. A concave mirror of radius of curvature R has a circular outline of radius r. A circular disc is to be placed normal to the axis at the focus so that it collects all the light that is reflected from the mirror from a beam parallel to the axis. For r << R, the area of this disc has to be at least

(A) $\frac{\pi r^6}{4R^4}$ (B) $\frac{\pi r^4}{4R^2}$ (C) $\frac{\pi r^5}{4R^3}$ (D) $\frac{\pi r^4}{R^2}$

Ans. **[A]**

Sol.



d = radius of disc

$A = \pi d^2$

From similar triangle

$$\frac{r}{d} = \frac{R - \frac{R}{2\cos\theta}}{\frac{R}{2} - \left(R - \frac{R}{2\cos\theta}\right)}$$

$$\frac{r}{d} = \frac{2\cos\theta - 1}{-\cos\theta + 1}$$

$$d = \left(\frac{-\cos\theta + 1}{2\cos\theta - 1}\right).r$$

$$\sin\theta = \frac{r}{R} \quad \therefore \cos\theta = \frac{\sqrt{R^2 - r^2}}{R}$$

$$\cos\theta = \left(1 - \frac{r^2}{R^2}\right)^{1/2}$$

$$\cos\theta = 1 - \frac{1}{2}\frac{r^2}{R^2}$$

$$1 - \cos\theta = \frac{r^2}{2R^2}$$

$$d = \frac{r^2 \times r}{2R^2\left[2\left(1 - \frac{r^2}{2R^2}\right) - 1\right]}$$

$$d = \frac{r^3}{2R^2\left[1 - \frac{r^2}{2R^2}\right]} = \frac{r^3}{2R^2}$$

$$A = \pi d^2 = \frac{\pi r^6}{4R^4}$$

96. The angles of incidence and refraction of a monochromatic ray of light of wavelength λ at an air-glass interface are i and r, respectively. A parallel beam of light with a small spread $\delta\lambda$ in wavelength about a mean wavelength λ is refracted at the same air-glass interface. The refractive index μ of glass depends on the wavelength λ as $\mu(\lambda) = a + b/\lambda^2$ where a and b are constants. Then the angular spread in the angle of refraction of the beam is

(A) $\left|\frac{\sin i}{\lambda^3 \cos r}\delta\lambda\right|$ (B) $\left|\frac{2b}{\lambda^3}\delta\lambda\right|$ (C) $\left|\frac{2b\tan r}{a\lambda^3 + b\lambda}\delta\lambda\right|$ (D) $\left|\frac{2b(a + b/\lambda^2)\sin i}{\lambda^3}\delta\lambda\right|$

Ans. **[C]**

Sol. Snell law

$\sin i = \mu \sin r$

$\sin i = \left(a + \frac{b}{\lambda^2}\right)\sin r$

Differentiating with respect to λ

$0 = \cos r\, dr\left(a + \frac{b}{\lambda^2}\right) + \sin r\left(\frac{b}{\lambda^3}(-2)\right)d\lambda$

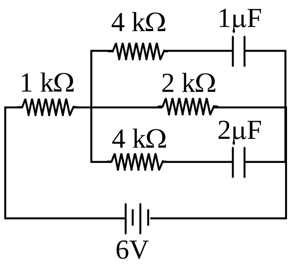
$0 = \cos r\, dr\left(\frac{a\lambda^2 + b}{\lambda^2}\right) + \sin r\left(\frac{-2b}{\lambda^3}\right)d\lambda$

$\frac{d\lambda\ 2b\ \sin r}{\lambda} = \cos r\ dr\ (a\lambda^2 + b)$

$dr = \frac{2b\,d\lambda}{\lambda}\ \frac{\tan r}{(a\lambda^2 + b)}$

$\delta r = \frac{(2b\tan r)\delta\lambda}{(a\lambda^3 + b\lambda)}$

97. What are the charges stored in the 1 μF and 2 μF capacitors in the circuit below, once the currents become steady ?

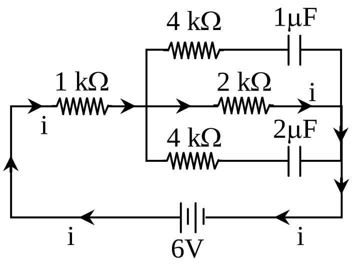


(A) 8 μC and 4 μC respectively (B) 4 μC and 8 μC respectively

(C) 3 μC and 6 μC respectively (D) 6 μC and 3 μC respectively

Ans. **[B]**

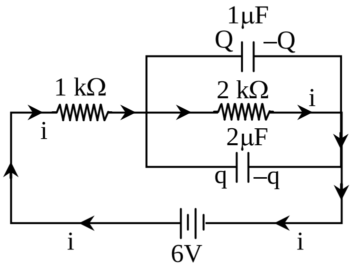
Sol.



At steady state current does not flow in the branch of capacitor.

$\therefore$ we can replace all resistor connected in branch of capacitor with wire

new circuit is



$$i = \frac{6}{(2+1)\times 10^3}$$

$\Rightarrow$ 2 mA

Potential drop across 2kΩ is same as potential drop across 1 μF & 2 μF.

Potential drop across 2 kΩ = $i \times 2 \times 10^3 = 2 \times 10^{-3} \times 2 \times 10^3 = 4$ volt.

Charge on 1μF = Q = $1 \times 4 \times 10^{-6} = 4$ μC

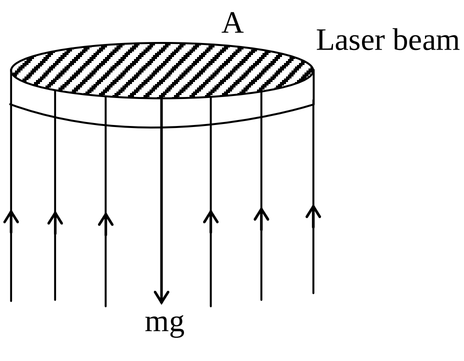
Charge on 2μF = q = $2 \times 4 \times 10^{-6} = 8$ μC

98. A 1.5 kW (kilo-watt) laser beam of wavelength 6400 Å is used to levitate a thin aluminium disc of same area as the cross section of the beam. The laser light is reflected by the aluminium disc without any absorption. The mass of the foil is close to

(A) 10^{-9} kg (B) 10^{-3} kg (C) 10^{-4} kg (D) 10^{-6} kg

Ans. **[D]**

Sol. Power of light = P



Force acting on Al disc = $\frac{2P}{C}$

$= \dfrac{2\times1.5\times10^3}{3.0\times10^8}$

$= 10^{-5}$

Force acting on Al disc = mg

$10^{-5} = m \times 10$

$m = 10^{-6}$ kg

99. When ultraviolet radiation of a certain frequency falls on a potassium target, the photoelectrons released can be stopped completely by a retarding potential of 0.6 V. If the frequency of the radiation is increased by 10%, this stopping potential rises to 0.9 V. The work function of potassium is

(A) 2.0 eV (B) 2.4 eV (C) 3.0 eV (D) 2.8 eV

Ans. **[B]**

Sol. $KE_{max} = e \times V_{retarding}$

$= e \times 0.6$

$= 0.6$ eV

Photon energy = hf = E

When frequency increase by 10% energy of photon also increases by 10%

New energy = E' = 1.1 E

New KE_{max}. $= e \times V_{retarding} = e \times 0.9 = 0.9$ eV

Einstein photoelectric equation

$hf = KE_{max} + \phi$

$E = 0.6 + \phi$ …(1)

$1.1\,E = 0.9 + \phi$ …(2)

$1.1 = \dfrac{0.9+\phi}{0.6+\phi}$

$1.1\,\phi + 0.66 = 0.9 + \phi$

$0.1\,\phi = 0.24$

$\phi = 2.4$ eV

100. The dimensions of Stefan-Boltzmann constant σ can be written in terms of Planck's constant h, Boltzmann constant k_B and the speed of light c as $\sigma = h^{\alpha}\, k_B^{\beta}\, c^{\gamma}$. Here

(A) $\alpha = 3$, $\beta = 4$ and $\gamma = -3$

(B) $\alpha = 3$, $\beta = -4$ and $\gamma = 2$

(C) $\alpha = -3$, $\beta = 4$ and $\gamma = -2$

(D) $\alpha = 2$, $\beta = -3$ and $\gamma = -1$

Ans. **[C]**

Sol. $\sigma = \overline{h^{\alpha} k_B^{\beta} c^{\gamma}}$

$\sigma \rightarrow$ Steffan boltzmann constant

$h \rightarrow$ Planck's constant

$k_B \rightarrow$ Boltzmann constant

$c \rightarrow$ speed of sight

According to stefan's law

$$\frac{Q}{At} = \sigma T^4$$

$$\sigma = \frac{Q}{At} \times \frac{1}{T^4}$$

$$[\sigma] = \frac{[M^1L^2T^{-2}]}{[L^2][T][K^4]} = [M^1T^{-3}K^{-4}]$$

& $E = h\nu$

$$\Rightarrow h = \frac{E}{\nu} \Rightarrow [h] = \frac{[M^1L^2T^{-2}]}{[T^{-1}]}$$

$[h] = [M^1L^2T^{-1}]$

$[c] = [LT^{-1}]$

$$E = \frac{3}{2} k_B T$$

$$\Rightarrow [k_B] = \frac{[M^1L^2T^{-2}]}{[K]}$$

$[k_B] = [M^1L^2T^{-2}K^{-1}]$

According to homogeneity principle of dimension

$[M^1T^{-3}K^{-4}] = [M^1L^2T^{-1}]^{\alpha}\ [M^1L^2T^{-2}K^{-1}]^{\beta}\ [L^1T^{-1}]^{\gamma}$

on comparing powers00000000 of M, L, T, K on both sides

$\Rightarrow \alpha + \beta = 1$ ….(1)

$2\alpha + 2\beta + \gamma = 0$ ….(2)

$-\alpha - 2\beta - \gamma = -3$ ….(3)

$-\beta = -4$ ….(4)

$\Rightarrow \beta = 4$

$\alpha = -3$

Putting values is equation (2)

$2(-3) + 2(4) + \gamma = 0$

$\gamma = -2$

Ans :

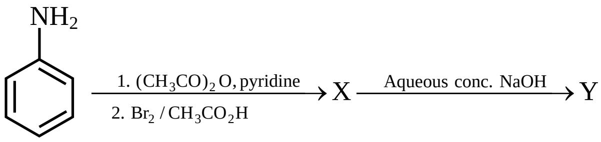
$\alpha = -3$

$\beta = 4$

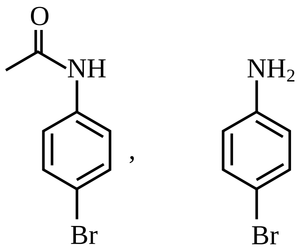
$\gamma = -2$

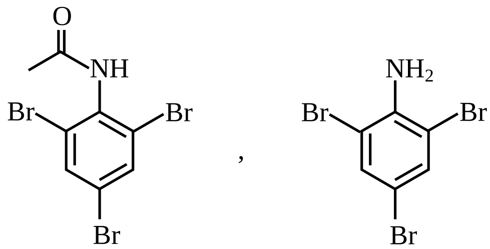
CHEMISTRY

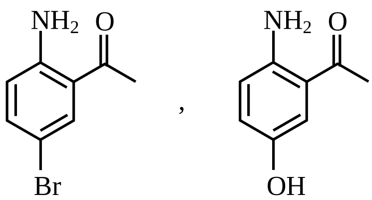
101. In the reaction sequence

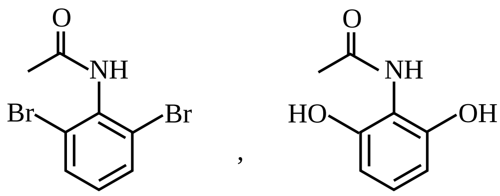


X and Y are, respectively,

(A) 

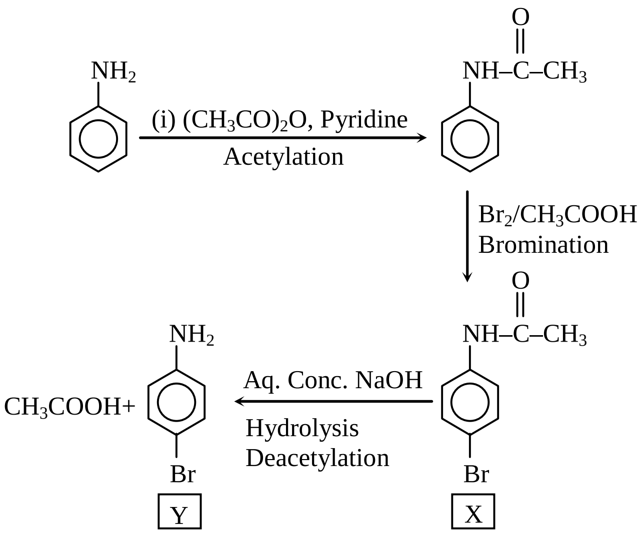
(B) 

(C) 

(D) 

Ans. **[A]**

Sol.



102. The density of acetic acid vapour at 300 K and 1 atm is 5 mg cm^{-3}. The number of acetic acid molecules in the cluster that is formed in the gas phase is closest to

(A) 5 (B) 2 (C) 3 (D) 4

Ans. **[B]**

Sol. $PM = \rho RT$

$$M = \left(\frac{5\times 0.0821\times 300}{1}\right) = 123.15$$

$\therefore$ Number of acetic acid molecule $= \frac{123.15}{60} \approx 2$

103. The molar enthalpy change for $H_2O(l) \rightleftharpoons H_2O(g)$ at 373 K and 1 atm is 41 kJ/mol. Assuming ideal behaviour, the internal energy change for vaporization of 1 mol of water at 373 K and 1 atm in kJ mol^{-1} is :

(A) 30.2 (B) 41.0 (C) 48.1 (D) 37.9

Ans. **[D]**

Sol. $W = -nRT = -(1 \times 8.314 \times 10^{-3} \times 373)kJ$

$= -3.10$ kJ

$q = \Delta H = 41$ kJ

& $\Delta E = q + w = (41 - 3.1) \cong 37.9$ kJ

104. The equilibrium constant (K_c) of two reactions $H_2 + I_2 \rightleftharpoons 2HI$ and $N_2 + 3H_2 \rightleftharpoons 2NH_3$ are 50 and 1000, respectively. The equilibrium constant of the reaction $N_2 + 6HI \rightleftharpoons 2NH_3 + 3I_2$ is closest to :

(A) 50000 (B) 20 (C) 0.008 (D) 0.005

Ans. **[C]**

Sol. $H_2 + I_2 \rightleftharpoons 2\,HI$; $K_c = 50$

$N_2 + 3H_2 \rightleftharpoons 2\,NH_3$; $K_c = 1000$

$N_2 + 6HI \rightleftharpoons 2NH_3 + 3I_2$; $K_c = \frac{1000}{(50)^3}$

$K_c = 0.008$

105. Given that the bond energies of : N≡N is 946 kJ mol^{-1}, H–H is 435 kJ mol^{-1}, N–N is 159 kJ mol^{-1}, and N–H is 389 kJ mol^{-1}, the heat of formation of hydrazine in the gas phase in kJ mol^{-1} is :

(A) 833 (B) 101 (C) 334 (D) 1268

Ans. **[B]**

Sol. $N_2 + 2H_2 \rightarrow 1\,N_2H_4$; ΔH_f

$\Delta H_f = 1 \times E_{N\equiv N} + 2\,E_{H-H} - 4\,E_{N-H} - 1\,E_{N-N}$

$= [(1 \times 946) + (2 \times 435) - 4 \times (389) - 1 \times (159)]$ kJ

$= 101$ kJ/mol

106. The radius of K^+ is 133 pm and that of Cl is 181 pm. The volume of the unit cell of KCl expressed in 10^{-22} cm^3 is :

(A) 0.31 (B) 1.21 (C) 2.48 (D) 6.28

Ans. **[C]**

Sol. $r_{K^+} + r_{Cl^-} = \frac{a}{2}$

$133 + 181 = \frac{a}{2}$

a = 2 (133 + 181)

a = 628 pm

or $a = 628 \times 10^{-10}$ cm

and

volume = $a^3 = (6.28 \times 10^{-8})^3\ cm^3$

$= 2.4767 \times 10^{-22}\ cm^3$

$\approx 2.48 \times 10^{-22}\ cm^3$

107. The reaction, $K_2Cr_2O_7$ + m $FeSO_4$ + n $H_2SO_4 \rightarrow Cr_2(SO_4)_3$ + p $Fe_2(SO_4)_3$ + K_2SO_4 + q H_2O when balanced, m, n, p and q are, respectively :

(A) 6, 14, 3, 14 (B) 6, 7, 3, 7 (C) 3, 7, 2, 7 (D) 4, 14, 2, 14

Ans. **[B]**

Sol. $1\,K_2\overset{(+6)}{Cr_2}O_7 + 6\,\overset{(+2)}{Fe}SO_4 + 7H_2SO_4 \rightarrow 1\,\overset{(+3)}{Cr_2}(SO_4)_3 + 3\,\overset{(+3)}{Fe_2}(SO_4)_3 + 1K_2SO_4 + 7H_2O$

(3 × 2) = 6 (1 × 1) = 1

i.e. answer is m = 6, n = 7, p = 3, q = 7

108. The standard free energy change (in J) for the reaction $3Fe^{2+}$ (aq) + 2Cr(s) = $2Cr^{3+}$(aq) + 3Fe(s) given $E^0_{Fe^{2+}/Fe} = -0.44$ V and $E^0_{Cr^{3+}/Cr} = -0.74$ V is (F = 96500 C)

(A) 57,900 (B) –57,900 (C) 173,700 (D) 173, 700

Ans. **[C]**

Sol. $3\,F^{2+} + 2\,Cr \longrightarrow 2Cr^{3+} + 3Fe$

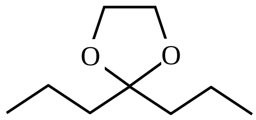
$E^o_{cell} = (-0.44) + (0.74) = 0.3$ volt

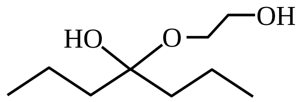
n = 6

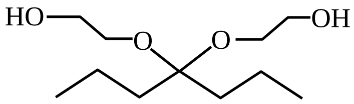
$\Delta G^o = -nFE^o = -(6 \times 96500 \times 0.3)$ J

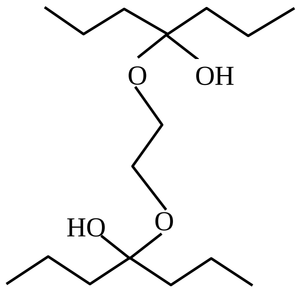
$= -173,700$ J

109. Calcium butanoate on heating followed by treatment with 1,2-ethanediol in the presence of catalytic amount of an acid, produces a major product which is :

(A) 

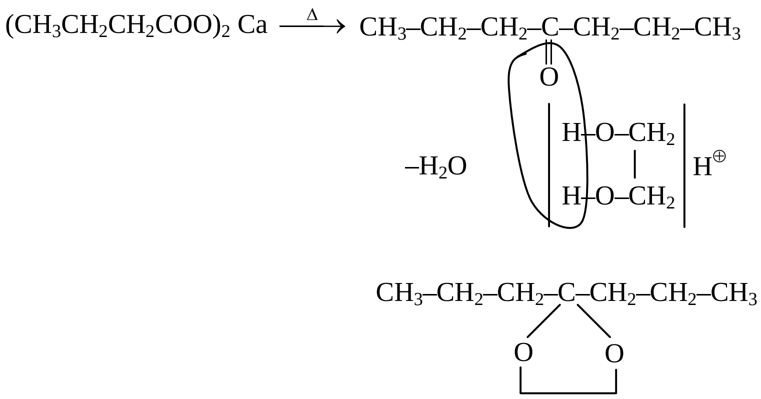
(B) 

(C) 

(D) 

Ans. **[A]**

Sol.



110. XeF_6 on complete hydrolysis yields 'X'. The molecular formula of X and its geometry, respectively, are :

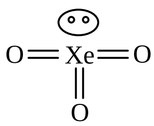
(A) XeO_2 and linear

(B) XeO_3 and trigonal planar

(C) XeO_3 and pyramidal

(D) XeO_4 and tetrahedral

Ans. **[C]**

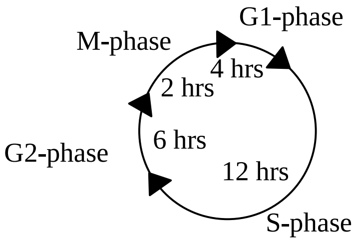


sp^3/Pyramidal geometry

$XeF_6 + 3H\text{–}OH \longrightarrow 6HF + XeO_3$

BIOLOGY

111. Following the cell cycle scheme given below, what is the probability that a cell would be in M-phase at any given time ?



(A) 1/24 (B) 1/12 (C) 1/6 (D) 1/2

Ans. **[B]**

Sol. Total time for cell cycle = 24 hrs.

Time for M-phase = 2 hrs

So, probability of cell in M-phase at any given time is $\frac{2}{24} \Rightarrow \frac{1}{12}$

112. A flower with Tt genotype is cross-pollinated by TT pollens. What will the genotypes of the resulting endosperm and embryo, respectively, be ?

(A) TTT, (TT + Tt) (B) (TTT + TTt), TT (C) TTt, Tt (D) TTt, (TT + Tt)

Ans. **[A]**

Sol. Tt × TT (Pollen means male plant)

Endosperm $\rightarrow \underbrace{\text{2 polar nuclei}}_{\text{female}}$ (should be same) + 1 male nuclei

So, TTT

Embryo $\rightarrow \underbrace{\text{Egg cell}}_{\text{female}}$ + 1 male nuclei

So, either TT or Tt

113. A new life form discovered on a distant planet has a genetic code consisting of five unique nucleotides and only one stop codon. If each codon has four bases, what is the maximum number of unique amino acids this life form can use ?

(A) 624 (B) 20 (C) 124 (D) 3124

Ans. **[A]**

Sol. No. of unique nucleotide = 5

No. of bases in codon = 4

so, total combinations $\Rightarrow 5^4 \Rightarrow 625$

No. of stop codons = 1

so, unique amino acids (maximum) = $625 - 1 \Rightarrow 624$

114. A spontaneous mutation results in a couple having only female progeny. When the daughter marries and has children, none of them are males. However, in the third generation there are few male offspring. What is the most likely explanation of this observation -

(A) The mutation reverses spontaneously in the third generation

(B) The mutation occurs on the X chromosome and is both recessive and lethal

(C) The mutation occurs on the X chromosome and is both recessive and dominant

(D) The mutation occurs on an autosome and is dominant

Ans. **[B]**

Sol. The mutation that occurred is on x-chromosome and is both recessive and lethal since, female is carrier it survives but due to hemizygous condition male is unable to survive.

$XX^m \rightarrow$ carrier (survives)

$X^mY \rightarrow$ dies

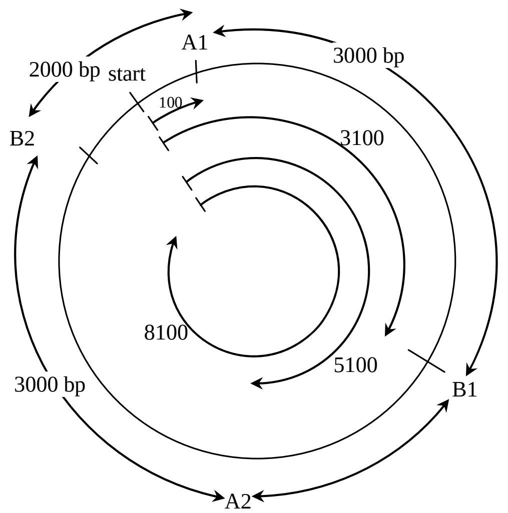
While in third generation, the possibility of male child occurs.

115. A circular plasmid of 10,000 base pairs (bp) is digested with two restriction enzymes, A and B, to produce a 3000 bp and a 2000 bp bands when visualized on an agarose gel. When digested with one enzyme at a time, only one band is visible at 5000 bp. If the first site for enzyme A (A1) is present at the 100th base, the order in which the remaining sites (A2, B1 and B2) are present is -

(A) 3100, 5100, 8100 (B) 8100, 3100, 5100 (C) 5100, 3100, 8100 (D) 8100, 5100,. 3100

Ans. **[C]**

Sol.



Question asks the position in A2 $\rightarrow$ B1 $\rightarrow$ B2

so, 5100, 3100, 8100

116. After meiosis-II, daughter cells differ from the parent cells and each other in their genotypes. This can occur because of which one of the following mechanism(s) ?

(A) Only synaptic crossing over

(B) Only crossing over and independent assortment of chromosomes

(C) Only crossing over and chromosomal segregation

(D) Crossing over, independent assortment and segregation of chromosomes

Ans. **[D]**

Sol. After meiosis-II, daughter cells differ from the parent cells and each other in their genotypes due to Crossing over, independent assortment and segregation of chromosomes

117. A desert lizard (an ectotherm) and a mouse (an endotherm) are placed inside a chamber at 15 ºC and their body temperature [T(L) for the lizard and T(M) for the mouse] and metabolic rates [M(L) for the lizard and M(M) for the mouse] are monitored. Which one of the following is correct -

(A)T(L) and M(L) will fall while T(M) and M(M) will increase

(B) T(L) and M(L) will increase while T(M) and M(M) will fall

(C) T(L) and M(L) will fall, T(M) will remain same and M(M) will increase

(D) T(L) and M(L) will remain same and T(M) and M(M) will decrease

Ans. **[C]**

Sol. Desert lizard is an ectotherm (poikilotherm) whose body temperature varies according to environmental temperature.

So at 15ºC T(L) will fall due to fall in T(M)

While mouse is an endotherm (Homeotherm) whose body temperature remains constant always due to variation is metabolic rate

In homeotherms metabolic rate is inversely proportional to environmental temperature.

So at 15ºC T(M) remain same and M((M) will increased.

118. In Griffith's experiments mice died when injected with -

(A) heat killed S-strain
(B) heat killed S-strain combined with R-strain
(C) heat killed R-strain
(D) live R-strain

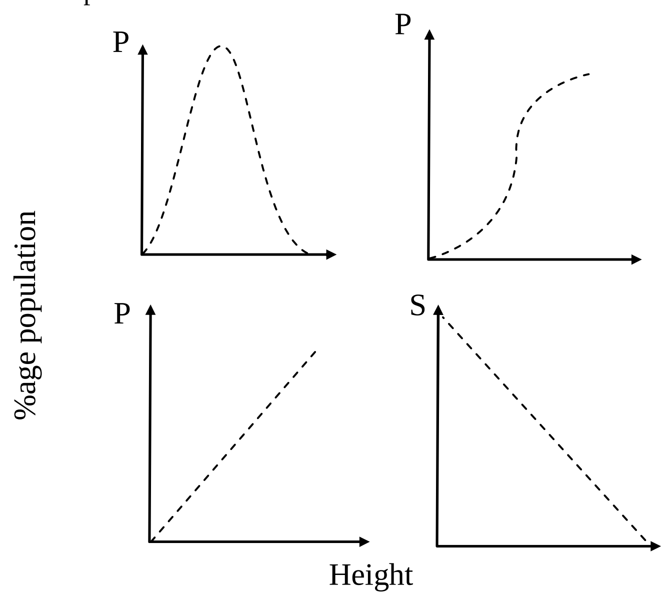
Ans. **[B]**

Sol. In Griffith experiment, mice died when injected with combination

heat killed s-strain + Live R strain

which resulted in transformation of R II into S III form.

119. Human height is a multigenic character. If the heights of all the individuals living in a metropolis are measured and the percentages of the population belonging to a specific heat are plotted as shown below, which of the plots would represent the most realistic distribution -



(A) P
(B) Q
(C) R
(D) S

Ans. **[A]**

Sol. P → plot

→ Height of human is multigenic character and shows bell shape curve as the occurance of extreme height will be low but medium height will be maximum.

→ Maximum % of population will have average medium height

120. If mitochondria isolated from a cell are first placed without carbon source in a buffer at pH 8.0 and then transferred to a buffer at pH4 , it will lead to -

(A) an increase in intra-mitochondrial acidity

(B) a decrease in intra-mitochondrial acidity

(C) blockage of ATP synthesis

(D) synthesis of ATP

Ans. **[D]**

Sol. Mitochondria synthesizes ATP based on chemiosmosis when mitochondria is transferred from a buffer pH 8.0 to buffer pH 4.0