# CBSE Test Paper 01
## Chapter 13 Probability

1. Five cards are drawn successively with replacement from a well – shuffled deck of 52 cards. What is the probability that only 3 cards are spades?

   a. $\frac{77}{512}$

   b. $\frac{45}{512}$

   c. $\frac{57}{512}$

   d. $\frac{41}{512}$

2. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

   a. $\frac{11}{13}$

   b. $\frac{7}{13}$

   c. $\frac{12}{13}$

   d. $\frac{9}{13}$

3. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

   a. $\frac{1}{52}$

   b. $\frac{3}{52}$

   c. $\frac{5}{52}$

   d. $\frac{1}{4}$

4. In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is

a. $\left(\frac{1}{2}\right)^5$

b. 0.1

c. $\frac{9}{10}$

d. $\left(\frac{9}{10}\right)^5$

5. There are three coins. One is a two headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads, what is the probability that it was the two headed coin?

a. $\frac{4}{9}$

b. $\frac{5}{9}$

c. $\frac{1}{9}$

d. $\frac{2}{9}$

6. The possibility of having 53 Thursdays in a non-leap year is _____.

7. A pot has 2 white, 6 black, 4 grey and 8 green balls. If one ball is picked randomly from the pot, the probability of it being black or green is _____.

8. If A and B' are independent events then P(A' $\cup$ B) = 1 - _____.

9. If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability $\frac{1}{2}$ ).

10. Two natural numbers r, s are drawn one at a time, without replacement from the set S = {1, 2, 3,...., n}. Find $P[r \leqslant p | s \leqslant p]$, where $p \in S$.

11. Evaluate $P\left(A \cap B\right)$ if 2P(A) = P (B) = $\frac{5}{13}$ and $P\left(A|B\right) = \frac{2}{5}$.

12. A die, whose faces are marked 1, 2, 3 in red and 4, 5, 6 in green, is tossed. Let A be the event "number obtained is even" and B be the event "number obtained is red". Find if A and B are independent events.

13. Prove that if E and F are independent events, then the events E and F' are also independent.

14. A factory produces bulbs. The probability that any one bulb is defective is $\frac{1}{50}$ and they are packed in boxes of 10. From a single box, find the probability that

    i.  none of the bulbs is defective
    ii. exactly two bulbs are defective
    iii. more than 8 bulbs work properly

15. There are two bags, one of which contains 3 black and 4 white balls while the other contains 4 black and 3 white balls. A die is thrown. If it shows up 1 or 3, a ball is taken from the 1st bag; but it shows up any other number, a ball is chosen from the second bag. Find the probability of choosing a black ball.

16. A black and a red die are rolled.

    a. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
    b. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

17. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of accidents is 0.01, 0.03 and 0.15 respectively. One of the insured persons meet with an accident what is the probability that he is scooter driver.

18. There are 5 cards numbered 1 to 5, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on two cards drawn. Find the mean and variance of X.

Solution

1. b. $\frac{45}{512}$

    **Explanation:** Here , probability of getting a spade from a deck of 52 cards = 13/52 = $\frac{1}{4}$ , p = $\frac{1}{4}$ , q = $\frac{3}{4}$ . let , x is the number of spades , then x has the binomial distribution with n = 5 , p = $\frac{1}{4}$ , q = $\frac{3}{4}$ .

    P( only 3 cards are spades) =P (x = 3) = $^5C_3\left(\frac{3}{4}\right)^{5-3}\left(\frac{1}{4}\right)^3 = \frac{45}{512}$

2. c. $\frac{12}{13}$

    **Explanation:** Let $E_1$ and $E_2$ are events that the student knows the answer and the student guesses respectively. $P(E_1) = \frac{3}{4}, P(E_2) = \frac{1}{4}$ .

    Let A = event that the student answers correctly.

    $P(E_1) = \frac{3}{4}, P(E_2) = \frac{1}{4}$

    $P(A/E_1) = 1$(as it is sure event)

    $P(A/E_2) = \frac{1}{4}$

    $P(E_1/A) = \dfrac{P(E_1).P(A/E_1)}{P(E_1).P(A/E_1)+P(E_2).P(A/E_2)} = \dfrac{\frac{3}{4}}{\frac{3}{4}+\frac{1}{16}} = \dfrac{12}{13}$

3. a. $\frac{1}{52}$

    **Explanation:** Let $E_1$, $E_2$ and $E_3$ are events of selection of a scooter driver, car driver and truck driver respectively.

    $\therefore P(E_1) = \frac{2000}{12000} = \frac{1}{6}, P(E_2) = \frac{4000}{12000} = \frac{1}{3}, P(E_3) = \frac{6000}{12000} = \frac{1}{2}$.

    Let A = event that the insured person meet with the accident.

    $\therefore$ P(A|$E_1$ ) = 0.01 , P(A|$E_2$) = 0.03, P(A|$E_3$ ) = 0.15

    $\Rightarrow$ P($E_1$|A) = $\dfrac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$

    $= \dfrac{\frac{1}{6}\times0.01}{\frac{1}{6}\times0.01+\frac{1}{3}\times0.03+\frac{1}{2}\times0.15} = \dfrac{1}{52}$

4. d. $\left(\frac{9}{10}\right)^5$

    **Explanation:** Probability of defective bulbs = p = $\frac{10}{100} = \frac{1}{10}$

    Probability of non defective bulb = q = $1 - \frac{1}{10} = \frac{9}{10}$

Let x be the number of defective bulbs .

Therefore , x is the binomial distribution with n = 5 , p = $\frac{1}{10}$ , q = $\frac{9}{10}$ .

Required probability = P (x = 0) = = $^5C_0\left(\frac{9}{10}\right)^5\left(\frac{1}{10}\right)^0 = \left(\frac{9}{10}\right)^5$ .

5.    a.   $\frac{4}{9}$

Explanation: Let $E_1$, $E_2$ and $E_3$ and are events of selection of a two headed coin ,

biased coin and unbiased coin respectively.

$\therefore P(E_1) = P(E_2) = P(E_2) = \frac{1}{3}$ .

Let A = event of getting head.

$P(A/E_1) = 1, P(A/E_2) = \frac{3}{4}, P(A/E_3) = \frac{1}{2}$ .

$P(E_1/A) = \dfrac{P(A/E_1).P(E_1)}{P(A/E_1).P(E_1)+P(A/E_2).P(E_2)+P(A/E_3).P(E_3)}$

$= \dfrac{\frac{1}{3}.1}{\frac{1}{3}.1+\frac{1}{3}.\frac{3}{4}+\frac{1}{3}\frac{1}{2}}$

$= \dfrac{4}{9}$

6. $\frac{1}{7}$

7. $\frac{7}{10}$

8. P(A) P(B')

9. There are four entries in a determinant of $2 \times 2$ order. Each entry may be filled up in two ways with 0 or 1.

$\therefore$ Number of determinants that can be formed = $2^4$ = 16

The value of determinants is positive in the following cases:

$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 3$

Therefore, the probability that the determinant is positive $= \frac{3}{16}$

10. $\because$ Set S = {1, 2, 3,......., n}

$\therefore P(r \leqslant p | s \leqslant p) = \dfrac{P(p \cap S)}{P(S)}$

$\therefore P\left(\dfrac{r \leqslant p}{s \leqslant p}\right) = \dfrac{P(p \cap S)}{P(S)}$

$= \dfrac{p-1}{n} \times \dfrac{n}{n-1} = \dfrac{p-1}{n-1}$

11. Given: 2 P (A) = P (B) = $\frac{5}{13}$ , $P\left(A|B\right) = \frac{2}{5}$

$\therefore$ P (A) = $\frac{5}{26}$

Now, $P(A \cap B) = P(B|A) \cdot P(B) = \frac{2}{5} \times \frac{5}{13} = \frac{2}{13}$
and $P(A \cup B)$ = P(A) + P (B) – $P(A \cap B)$ = $\frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{11}{26}$

12. When a die is thrown, the sample space is

S = {1, 2, 3, 4, 5, 6}

$\Rightarrow$ n(S) = 6

Also, A : number is even and B : number Is red

$\therefore$ A = {2, 4, 6} and B = {1, 2, 3} and A∩B = {2}

$\Rightarrow$ n(A) = 3, n (B) = 3 and n(A∩B) = 1

Now, P(A) = $\frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

P(B) = $\frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

and P(A∩B) = $\frac{n(A \cap B)}{n(S)} = \frac{1}{6}$

Now, P(A) $\times$ P(B) = $\frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4} \neq \frac{1}{6} = P(A \cap B)$

$\therefore \quad P(A \cap B) \neq P(A) \times P(B)$

Thus, A and B are not independent events.

13. We have to prove that if E and F are independent events, then the events E and F' are also independent.

Given, E and F are independent events, therefore

$\Rightarrow$ P(E∩F) = P(E ) P(F) ..... ( i )

Now, we have,

P(E∩F') + P(E ∩F ) = P (E)

$\Rightarrow$ P(E ∩F') = P(E ) - P(E∩F)

$\Rightarrow$ P(E ∩F') = P(E) - P(E )P(F) [using Eq.1] $\Rightarrow$ P (E ∩F ') = P(E ) [l - P (F )]

$\Rightarrow$ P(E ∩ F ') = P(E )P(F')

$\therefore$ E and F 'are also independent events.

14. Let X is the random variable which denotes that a bulb is defective.

Also, n = 10, $p = \frac{1}{50}$ and $q = \frac{49}{50}$ and $P(X = r) = {}^nC_r p^r q^{n-r}$

i. None of the bulbs are defective i.e., r = 0

$\therefore P(X = r) = P_{(0)} = {}^{10}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{10-0} = \left(\frac{49}{50}\right)^{10}$

ii. Exactly two bulbs are defective i.e., r = 2

$$\therefore P(X = r) = P_{(2)} = {}^{10}C_2 \left(\frac{1}{50}\right)^2 \left(\frac{49}{50}\right)^8$$

$$= \frac{10!}{8!2!} \left(\frac{1}{50}\right)^2 \cdot \left(\frac{49}{50}\right)^8 = 45 \times \left(\frac{1}{50}\right)^{10} \times (49)^8$$

iii. More than 8 bulbs work properly i.e., there is less than 2 bulbs which are defective.

So, $r < 2 \Rightarrow r = 0, 1$

$\therefore$ P(X = r) = P(r < 2) = P(0) + P(1)

$$= {}^{10}C_0 \left(\frac{1}{50}\right)^0 \left(\frac{49}{50}\right)^{10-0} + {}^{10}C_1 \left(\frac{1}{50}\right)^1 \left(\frac{49}{50}\right)^{10-1}$$

$$= \left(\frac{49}{50}\right)^{10} + \frac{10!}{1!9!} \cdot \frac{1}{50} \cdot \left(\frac{49}{50}\right)^9$$

$$= \left(\frac{49}{50}\right)^{10} + \frac{1}{5} \cdot \left(\frac{49}{50}\right)^9 = \left(\frac{49}{50}\right)^9 \left(\frac{49}{50} + \frac{1}{5}\right)$$

$$= \left(\frac{49}{50}\right)^9 \left(\frac{59}{50}\right) = \frac{59(49)^9}{(50)^{10}}$$

15. Since Bag I = {3 black, 4 white balls}, Bag II = {4 black, 3 white balls}

Let $E_1$ be the event that bag I is selected and $E_2$ be the event that bag II is selected.

Let $E_3$ be the event that black ball is chosen.

$$\therefore P(E_1) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3} \text{ and } P(E_2) = 1 - \frac{1}{3} = \frac{2}{3}$$
$$\text{And } P\left(\frac{E_3}{E_1}\right) = \frac{3}{7} \text{ and } P\left(\frac{E_3}{E_2}\right) = \frac{4}{7}$$
$$\therefore P(E_3) = P(E_1) \cdot P\left(\frac{E_3}{E_1}\right) + P(E_2) \cdot P\left(\frac{E_3}{E_2}\right)$$
$$= \frac{1}{3} \cdot \frac{3}{7} + \frac{2}{3} \cdot \frac{4}{7} = \frac{11}{21}$$

16. a. n(s) = 6 × 6 = 36

Let A represents obtaining a sum greater than 9 and B represents black die resulted in a 5.

A = (46, 64, 55, 36, 63, 45, 54, 65, 56, 66) $\Rightarrow$ n(A) = 10

P (A) = B = (51, 52, 53, 54, 55, 56) $\Rightarrow$ n(B) = 6

P (B) = $\dfrac{n(B)}{n(S)} = \dfrac{6}{36}$

$A \cap B$ = (55, 56) $\Rightarrow n(A \cap B)$ = 2

$$P(A \cap B) = \frac{2}{36}$$

$$P\left(A|B\right) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{2}{6} = \frac{1}{3}$$

b. Let A denote the sum is 8

∴ A = {(2, 6). (3, 5), (4, 4), (5, 3), (6, 2)}

B = Red die results in a number less than 4, either first or second die is red

∴ B = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)}

P (B) = $\frac{n(B)}{n(S)} = \frac{18}{36} = \frac{1}{2}$

$A \cap B$ = {(2, 6), (3, 5) $\Rightarrow n\left(A \cap B\right) = 2$

$P\left(A \cap B\right)$ = $\frac{2}{36} = \frac{1}{18}$

$$P\left(A|B\right) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1}{18}}{\frac{1}{2}} = \frac{2}{18} = \frac{1}{9}$$

17. $E_1$ : Insured person is a scooter driver

$E_2$ : Insured person is a car driver

$E_3$ : Insured person is a truck driver

$$P(E_1) = \frac{2000}{2000+4000+6000} = \frac{2000}{12000} = \frac{1}{6}$$
$$P(E_2) = \frac{1}{3}$$
$$P(E_3) = \frac{1}{2}$$

Let A insured per son meets with an accident

$$P\left(\frac{A}{E_1}\right) = 0.01 = \frac{1}{100}$$
$$P\left(\frac{A}{E_2}\right) = 0.03 = \frac{3}{100}$$
$$P\left(\frac{A}{E_3}\right) = 0.15 = \frac{15}{100}$$

$$P\left(\frac{A}{E_1}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)+P(E_3)P\left(\frac{A}{E_3}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{1}{100}}{\frac{1}{6} \times \frac{1}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{2} \times \frac{15}{100}}$$

$$= \frac{1}{52}$$

18. Here, S = {(1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2), (1, 4), (4, 1), (1, 5), (5, 1), (2, 4), (4, 2), (2, 5), (5, 2), (3, 4), (4, 3), (3, 5), (5, 3), (5, 4), (4, 5)}

$\Rightarrow n(S) = 20$

Let random variable be X which denotes the sum of the numbers on two cards drawn.

∴ X = 3, 4, 5, 6, 7, 8, 9

At X = 3, P(X) = $\frac{2}{20}$ = $\frac{1}{10}$

At X = 4, P(X) = $\frac{2}{20}$ = $\frac{1}{10}$

At X = 5, P(X) = $\frac{4}{20}$ = $\frac{1}{5}$

At X = 6, P(X) = $\frac{4}{20}$ = $\frac{1}{5}$

At X = 7, P(X) = $\frac{4}{20}$ = $\frac{1}{5}$

At X = 8, P(X) = $\frac{2}{20}$ = $\frac{1}{5}$

At X = 9, P(X), = $\frac{2}{20}$ = $\frac{1}{10}$

∴ Mean, E(X) = $\sum XP(X) = \frac{3}{10} + \frac{4}{10} + \frac{5}{5} + \frac{6}{5} + \frac{7}{5} + \frac{8}{10} + \frac{9}{10}$

$= \frac{3+4+10+12+14+8+9}{10} = 6$

Also, $\sum X^2 P(X) = \frac{9}{10} + \frac{16}{10} + \frac{25}{5} + \frac{36}{5} + \frac{49}{5} + \frac{64}{10} + \frac{81}{10}$

$= \frac{9+16+50+72+98+64+81}{10} = 39$

$\therefore Var(X) = \sum X^2 P(X) - [\sum XP(X)]^2$

= 39 - $(6)^2$ = 39 - 36 = 3