Sample Question Paper - 3
CLASS: XII
Session: 2021-22
Mathematics (Code-041)
Term - 1

Time Allowed: 1 hour and 30 minutes **Maximum Marks: 40**

General Instructions:

1. This question paper contains three sections – A, B and C. Each part is compulsory.
2. Section - A has 20 MCQs, attempt any 16 out of 20. 3
3. . Section - B has 20 MCQs, attempt any 16 out of 20
4. Section - C has 10 MCQs, attempt any 8 out of 10.
5. There is no negative marking.
6. All questions carry equal marks.

SECTION – A

Attempt any 16 questions

1. Let A = { 2 , 3 , 6 }. Which of the following relations on A are reflexive? **[1]**

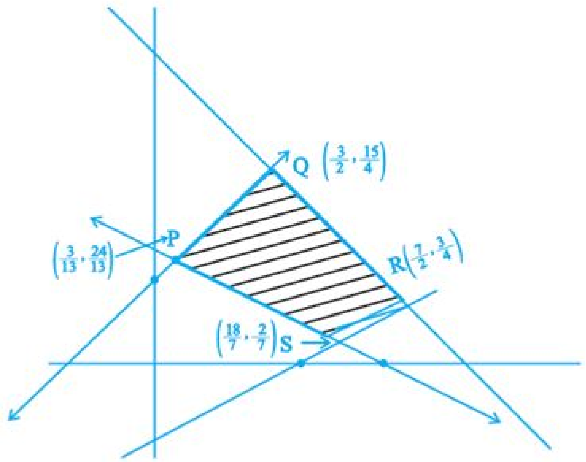
a) None of these

b) R_1 = {(2,2) , (3,3) , (6,6)}

c) R_2 = {(2,2) , (3,3) , (3,6) , (6,3)}

d) R_3 = {(2,2) , (3,6) , (2,6)}

2. In Figure, the feasible region (shaded) for a LPP is shown. Determine the maximum and minimum value of Z = x + 2y **[1]**



a) Maximum = 10, minimum = $3\frac{1}{4}$

b) Maximum = 8, minimum = $3\frac{1}{6}$

c) Maximum = 7, minimum = $3\frac{1}{9}$

d) Maximum = 9, minimum = $3\frac{1}{7}$

3. If x = $a\cos^3 t$, y = $a\sin^3 t$, then $\frac{dy}{dx}$ is equal to **[1]**

a) – tan t

b) cosec t

c) cos t

d) cot t

[1]

4. The function f(x) = $\cot^{-1} x + x$ increases in the interval

a) 0 , ∞ b) -∞ ,∞

c) (1, ∞) d) -1, ∞

5. The point at which the maximum value of x + y, subject to the constraints x + 2y $\leq$ 70, 2x + y $\leq$ 95, x, y $\geq$ 0 is obtained, is **[1]**

a) (20, 35) b) (30, 25)

c) (35, 20) d) (40,15)

6. The system of equations, x + y = 2 and 2x + 2y = 3 has **[1]**

a) a unique solution b) finitely many solutions

c) no solution d) infinitely many solutions

7. If y = $x^{x\sin x}$ then $\frac{dy}{dx} = ?$ **[1]**

a) $x^{\sin x}\left\{\frac{\sin x + x\log x \cdot \sin x}{x}\right\}$ b) $(\sin x\cos x)\cdot x^{(\sin x-1)}$

c) None of these d) $(\sin x)\cdot x^{(\sin x-1)}$

8. If A and B are matrices of same order, then (AB' – BA') is a **[1]**

a) null matrix b) unit matrix

c) symmetric matrix d) skew-symmetric matrix

9. Maximize Z = 3x + 4y, subject to the constraints : x + y ≤ 1, x ≥ 0, y ≥ 0. **[1]**

a) 4 b) 5

c) 6 d) 3

10. If $y = \sqrt{\frac{1+\tan x}{1-\tan x}}$ then $\frac{dy}{dx} = ?$ **[1]**

a) $\frac{\sec^2\left(\frac{x}{4}\right)}{\sqrt{\tan\left(x+\frac{\pi}{4}\right)}}$ b) $\frac{1}{2}\sec^2 x \cdot \tan\left(x+\frac{\pi}{4}\right)$

c) $\frac{\sec^2\left(x+\frac{\pi}{4}\right)}{2\sqrt{\tan\left(x+\frac{\pi}{4}\right)}}$ d) none of these

11. Let $f(x) = \begin{cases} \frac{1}{|x|} & \text{for } |x| \geq 1 \\ ax^2 + b & \text{for } |x| < 1 \end{cases}$ If f(x) is continuous and differentiable at any point, then **[1]**

a) a = 1, b = –1 b) $a = \frac{1}{2}, b = -\frac{3}{2}$

c) $a = \frac{1}{2}, b = \frac{3}{2}$ d) none of these

12. Minimize Z = 5x + 10 y subject to x + 2y ≤ 120, x + y ≥ 60, x – 2y ≥ 0, x, y ≥ 0 **[1]**

a) Minimum Z = 310 at (60, 0) b) Minimum Z = 320 at (60, 0)

c) Minimum Z = 330 at (60, 0) d) Minimum Z = 300 at (60, 0)

13. The normal to the curve x = a $(\cos\theta + \theta\sin\theta)$,y = a $(\sin\theta - \theta\cos\theta)$ at any point θ is such that **[1]**

a) it is at a constant distance from the origin

b) it passes through the origin

c) it makes a constant angle with X – axis

d) none of these

14. The function f(x) = $\sin^{-1}(\cos x)$ is **[1]**

a) None of these

b) differentiable at x = 0

c) discontinuous at x = 0

d) continuous at x = 0

15. The equation of the tangent to the curve y = x log x is parallel to the chord joining the points (1, 0) and (e, e), the value of x is: **[1]**

a) $e^{1/1-e}$

b) $e^{(e-1)(2e-1)}$

c) $e^{\frac{2e-1}{e-1}}$

d) $\frac{e-1}{e}$

16. Assume X, Y, Z, W, and P are matrices of order $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ and $p \times k$, respectively. **[1]**

The restriction on n, k and p so that PY + WY will be defined are

a) p is arbitrary, k = 3

b) k is arbitrary, p = 2

c) k = 2, p = 3

d) k = 3, p = n

17. At what points the slope of the tangent to the curve $x^2 + y^2 - 2x - 3 = 0$ is zero **[1]**

a) (3, 0), (1, 2)

b) (-1, 0), (1, 2)

c) (3, 0), (-1, 0)

d) (1, 2), (1, -2)

18. If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then x = **[1]**

a) $\frac{1}{2}$

b) None of these

c) $\frac{\sqrt{3}}{2}$

d) $-\frac{1}{2}$

19. If $f(x) = \sqrt{x^2 + 6x + 9}$, then f'(x) is equal to **[1]**

a) 1 for all $x \in R$

b) none of these

c) 1 for x < -3

d) -1 for x < -3

20. If A is a square matrix, then AA is a **[1]**

a) none of these

b) skew-symmetric matrix

c) symmetric matrix

d) diagonal matrix

SECTION – B

Attempt any 16 questions

21. Let $f : R \to R$ be defined by f (x) = $\frac{1}{x}$, $\forall x \in R$. Then f is **[1]**

a) one – one

b) Bijective

c) f is not defined

d) Onto

22. The minimum value of $f(x) = 3x^4 - 8x^3 - 48x + 25$ on [0, 3] is **[1]**

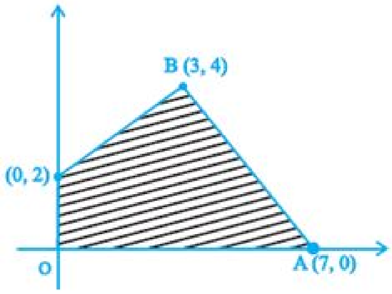
a) 25

b) 16

c) -39

d) None of these

23. Feasible region (shaded) for a LPP is shown in Figure. Maximize Z = 5x + 7y. **[1]**



a) 45

b) 49

c) 47

d) 43

24. If y = $\cos^2 x^3$ then $\frac{dy}{dx} = ?$ **[1]**

a) $-3x^2 \sin^2 x^3$

b) none of these

c) $-3x^2 \cos^2 (2x^3)$

d) $-3x^2 \sin (2x^3)$

25. If y = $ax^2 + bx + c$, then $y^3 \frac{d^2y}{dx^2}$ is **[1]**

a) a constant

b) a function of x only

c) a function of y only

d) a function of x and y

26. $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to **[1]**

a) $\frac{1}{4}$

b) $\frac{1}{3}$

c) 1

d) $\frac{1}{2}$

27. R is a relation on the set Z of integers and it is given by (x, y) $\in$ R $\Leftrightarrow$ $|x - y| \leq 1$. Then, R is **[1]**

a) an equivalence relation

b) symmetric and transitive

c) reflexive and symmetric

d) reflective and transitive

28. $\sin^{-1}(1 - x) - 2\sin^{-1} x = \frac{\pi}{2}$ then x is equal to **[1]**

a) $\frac{1}{2}$

b) $(0, \frac{1}{2})$

c) $(1, \frac{1}{2})$

d) 0

29. If A + B = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and A – 2 B = $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$, then A = ? **[1]**

a) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

b) none of these

c) $\frac{1}{3}\begin{bmatrix} 1 & 1 \\ 3 & 1 \end{bmatrix}$

d) $\frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

30. If y = $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$ then $\frac{dy}{dx} = ?$ **[1]**

a) 1

b) $\frac{1}{2}$

c) -1

d) $\frac{-1}{2}$

31. If f(x) = $|\log_e x|$, then **[1]**

a) f'(1) = –1

b) f'(1) = 1

c) $f'(1^-) = -1$

d) $f'(1^+) = 1$

32. If $y = \log\sqrt{\tan x}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{4}$ is given by [1]

a) 0

b) ∞

c) $\frac{1}{2}$

d) 1

33. The function f(x) = $x^9 + 3x^7 + 64$ is increasing on [1]

a) $(-\infty, 0)$

b) R_0

c) $(0, \infty)$

d) R

34. If $\cos^{-1}x > \sin^{-1}x$, then [1]

a) $0 \leq x < \frac{1}{\sqrt{2}}$

b) $\frac{1}{\sqrt{2}} < x \leq 1$

c) $-1 \leq x < \frac{1}{\sqrt{2}}$

d) x > 0

35. Any tangent to the curve y = $2x^7 + 3x + 5$ [1]

a) is parallel to x-axis

b) is parallel to y-axis

c) makes an acute angle with x-axis

d) makes an obtuse angle with x-axis

36. In a LPP, the linear inequalities or restrictions on the variables are called [1]

a) Limits

b) Inequalities

c) Linear constraints

d) Constraints

37. If A is an invertible square matrix and k is a non-negative real number then$(kA)^{-1}$ = ? [1]

a) $\frac{1}{k} \cdot A^{-1}$

b) $-k.A^{-1}$

c) $k.A^{-1}$

d) None of these

38. If a matrix A is symmetric as well as skew symmetric, then A is a [1]

a) none of these

b) null matrix

c) unit matrix

d) diagonal matrix

39. If $\sqrt{x} + \sqrt{y} = \sqrt{a},$ then $\left(\frac{d^2y}{dx^2}\right)_{x=a}$ is equal to [1]

a) $\frac{1}{2a}$

b) a

c) None of these

d) $\frac{1}{a}$

40. Let f : R $\rightarrow$ R be a function defined by $f(x) = \frac{x^2-8}{x^2+2}$. Then, f is [1]

a) one-one and onto

b) one-one but not onto

c) onto but not one-one

d) neither one-one nor onto

SECTION – C

Attempt any 8 questions

41. If $\alpha = \tan^{-1}\left(\tan\frac{5\pi}{4}\right)$ and $\beta = \tan^{-1}\left(-\tan\frac{2\pi}{3}\right)$, then [1]

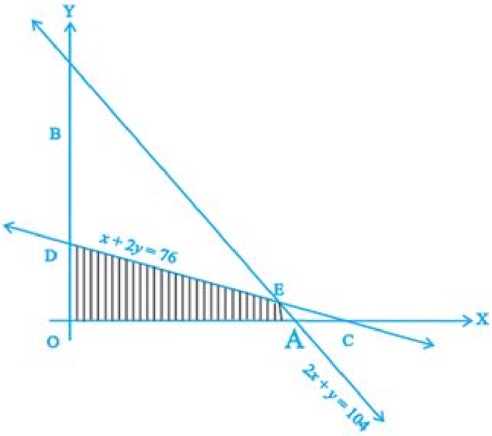
a) none of these

b) $\alpha - \beta = \frac{7\pi}{12}$

c) $3a = 4\beta$

d) $4a = 3\beta$

42. Determine the minimum value of Z = 3x + 4y if the feasible region (shaded) for a LPP is shown in Figure above. **[1]**



a) 154

b) 196

c) 112

d) 132

43. If $y = ae^{mx} + be^{-mx}$, then y_2 is equal to **[1]**

a) my_1

b) $-m^2y$

c) m^2y

d) None of these

44. Let $f(x) = 2x^3 - 3x^2 - 12x + 5$ on [-2, 4]. The relative maximum occurs at x = **[1]**

a) 2

b) -1

c) 4

d) -2

45. S is a relation over the set R of all real numbers and its is given by $(a, b) \in S \Leftrightarrow ab \geq 0$. Then, S is **[1]**

a) an equivalence relation

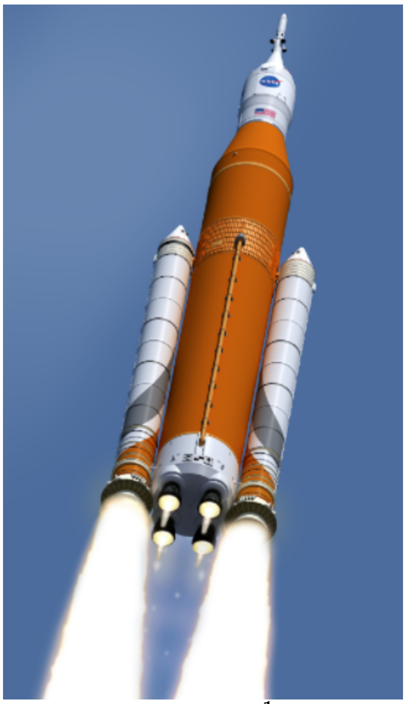
b) reflexive and symmetric only

c) symmetric and transitive only

d) antisymmetric relation

Question No. 46 to 50 are based on the given text. Read the text carefully and answer the questions:

The upward speed v(t) of a rocket at time t is approximated by $v(t) = at^2 + bt + c$, $0 \leq t \leq 100$, where a, b and c are constants. It has been found that the speed at times t = 3, t = 6 and t = 9 seconds are respectively 64, 133 and 208 miles per second.



If $\begin{pmatrix} 9 & 3 & 1 \\ 36 & 6 & 1 \\ 81 & 9 & 1 \end{pmatrix}^{-1} = \frac{1}{18}\begin{pmatrix} 1 & -2 & 1 \\ -15 & 24 & -9 \\ 54 & -54 & 18 \end{pmatrix}$,

46. v(t) is given by **[1]**

a) $t^2 + \frac{1}{3}t + 20$

b) $t^2 + 20t + 1$

c) $t^2 + t + 1$

d) $\frac{1}{3}t^2 + 20t + 1$

47. The speed at time t = 15 seconds is **[1]**

a) 366 miles/sec

b) 346 miles/sec

c) 376 miles/sec

d) 356 miles/sec

48. The time at which the speed of rocket is 784 miles/sec is **[1]**

a) 20 seconds

b) 25 seconds

c) 30 seconds

d) 27 seconds

49. The value of b + c is **[1]**

a) 21

b) $\frac{3}{4}$

c) $\frac{4}{3}$

d) 20

50. The value of a + c is **[1]**

a) 1

b) none of these

c) $\frac{4}{3}$

d) 20

Solution

SECTION – A

1. **(b)** R_1 = {(2,2) , (3,3) , (6,6)}

   Explanation: R_1 is a reflexive on A, because (a,a) $\in R_1$ for each a $\in$ A

2. **(d)** Maximum = 9, minimum = $3\frac{1}{7}$

   Explanation:

Corner points	Z = x +2 y
P(3/13,24/13)	51/13
Q(3/2,15/4)	9......................(Max.)
R(7/2,3/4)	5
S(18/7,2/7)	22/7......................(Min.)

   Hence the maximum value is 9 and the minimum value is $3\frac{1}{7}$

3. **(a)** – tan t

   Explanation: We have to find: $\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{3a\sin^2 t\cos t}{3a\cos^2 t(-\sin t)}=-\tan t$

4. **(b)** -∞ ,∞

   Explanation: $(-\infty,\infty)$

   $f(x)=\cot^{-1}x+x$

   $f'(x)=\frac{-1}{1+x^2}+1$

   $=\frac{-1+1+x^2}{1+x^2}$

   $=\frac{x^2}{1+x^2}\geq 0,\forall x\in R$

   So, f (x) is increasing on $(-\infty,\infty)$

5. **(d)** (40,15)

   Explanation: We need to maximize the function z = x + y Converting the given inequations into equations, we obtain

   x + 2y = 70, 2x + y = 95, x = 0 and y = 0

   Region represented by x + 2y $\leq$ 70 :

   The line x + 2y = 70 meets the coordinate axes at A(70, 0) and B(0, 35) respectively. By joining these points we obtain the line x + 2y = 70. Clearly (0, 0) satisfies the inequation x + 2y $\leq$ 70 . So, the region containing the origin represents the solution set of the inequation x + 2y $\leq$ 70.

   Region represented by 2x + y $\leq$ 95 :

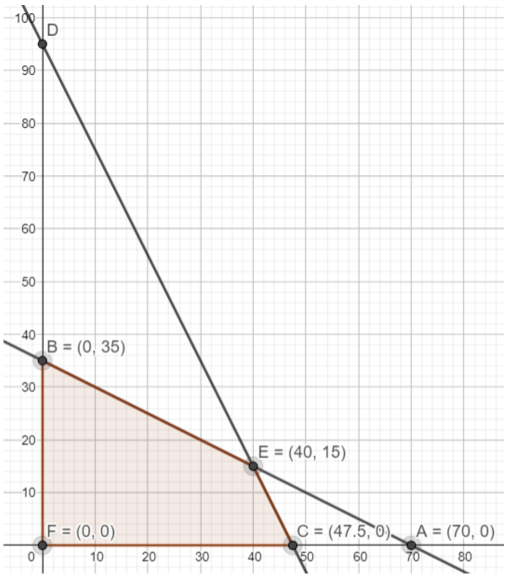
   The line 2x + y = 95 meets the coordinate axes at $C\left(\frac{95}{2},0\right)$ respectively. By joining these points we obtain the line 2x + y = 95

   Clearly (0, 0) satisfies the inequation 2x + y $\leq$ 95 . So, the region containing the origin represents the solution set of the inequation 2x + y $\leq$ 95

   Region represented by x $\geq$ 0 and y $\geq$ 0 :

   since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0, and y $\geq$ 0

   The feasible region determined by the system of constraints x + 2y $\leq$ 70, 2x + y $\leq$ 95, x $\geq$ 0, and y $\geq$ 0 are as follows.



The corner points of the feasible region are O(0, 0), C($\frac{95}{2}$, 0) E(40, 15) and B(0, 35).

The value fo Z at these corner points are as follows.

Corner point : z = x + y

O(0, 0) : 0 + 0 = 0

$C\left(\frac{95}{2},0\right) : \frac{95}{2}+0=\frac{95}{2}$

E(40, 15) : 40 + 15 = 55

B(0, 35) : 0 + 35 = 35

We see that maximum value of the objective function Z is 55 which is at (40, 15).

6. **(c)** no solution

Explanation: For No solution, $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$, for given system of equations we have:$\frac{1}{2}=\frac{1}{2}\neq\frac{2}{3}$.

7. **(a)** $x^{\sin x}\left\{\frac{\sin x+x\log x\cdot\sin x}{x}\right\}$

Explanation: Let y = f(x) = $x^{\sin x}$

Taking log both sides, we obtain

$\log_e y = \sin x \log_e x$ -(1) (Since $\log_a b^c = c \log_a b$)

Differentiating (i) with respect to x, we obtain

$\frac{1}{y}\frac{dy}{dx}=\sin x\times\frac{1}{x}+\log_e x\times\cos x$

$\Rightarrow\frac{dy}{dx}=y\times\left(\frac{\sin x}{x}+\log_e x\cos x\right)$

$\Rightarrow\frac{dy}{dx}=f'(x)=x^{sinx}\left(\frac{\sin x+x\log x\sin x}{x}\right)$.

Which is the required solution.

8. **(d)** skew-symmetric matrix

Explanation: We have matrices A and B of same order.

Let P = (AB' − BA')

Then, P' = (AB' − BA')'

= (AB')' − (BA')'

= (B')' (A)' − (A')'B' = BA' − AB' = −(AB' − BA') = −P

Therefore, the given matrix (AB - BA') is a skew-symmetric matrix.

9. **(a)** 4

Explanation: According to the question, maximize , Z = 3x + 4y, subject to the constraints: x + y ≤ 1, x ≥ 0, y ≥ 0.

Corner points	Z = 3x +4 y
C(0, 0)	0
B (1,0)	3
D(0,1)	4

Hence the maximum value is 4

10. **(c)** $\frac{\sec^2\left(x+\frac{\pi}{4}\right)}{2\sqrt{\tan\left(x+\frac{\pi}{4}\right)}}$

Explanation: Given that $y=\sqrt{\frac{1+\tan x}{1-\tan x}}$

Using $\tan\left(\frac{\pi}{4}+x\right)=\frac{1+\tan x}{1-\tan x}$, we obtain

$y=\sqrt{\tan\left(\frac{\pi}{4}+x\right)}$

Differentiating with respect to x, we obtain

$\frac{dy}{dx}=\frac{1}{2\sqrt{\tan\left(\frac{\pi}{4}+x\right)}}\times\sec^2\left(\frac{\pi}{4}+x\right)\times 1$

Hence, $\frac{dy}{dx}=\frac{\sec^2\left(\frac{\pi}{4}+x\right)}{2\sqrt{\tan\left(\frac{\pi}{4}+x\right)}}$

11. **(d)** none of these

Explanation: Given that $f(x)=\begin{Bmatrix}\frac{-1}{x}, x\le -1\\ ax^2+b, -1<x<1\\ \frac{1}{x}, x\ge 1\end{Bmatrix}$

∵ f(x) is continuous and differentiable at any point, consider x =1.

$\lim\limits_{x\to 1}\frac{1}{x}=\lim_{x\to 1}ax^2+b$

⇒a + b = 1

Also,

$\Rightarrow\lim\limits_{x\to 1^-}\frac{f(x)-f(1)}{x-1}=\lim\limits_{x\to 1^+}\frac{f(x)-f(1)}{x-1}$

$\Rightarrow\lim\limits_{x\to 1}\frac{ax^2-a}{x-1}=\lim\frac{1-x}{x(x-1)}$

$\Rightarrow\lim\limits_{x\to 1}a(x+1)=\lim\limits_{x\to 1}(-x)$

$\Rightarrow a=\frac{-1}{2}$

Putting above value in a + b = 1,we get

$\text{b}=\frac{3}{2}$.

Which is the required value of a and b.

12. **(d)** Minimum Z = 300 at (60, 0)

Explanation: Objective function is Z = 5x + 10 y ....(1).

The given constraints are : x + 2y ≤ 120, x + y ≥ 60, x – 2y ≥ 0, x, y ≥ 0 .

The corner points are obtained by drawing the lines x+2y =120, x+y = 60 and x-2y = 0. The points so obtained are (60,30),(120,0), (60,0) and (40,20)

Corner points	Z = 5x + 10y
D(60 ,30)	600
A(120,0)	600
B(60,0)	300.........................(Min.)
C(40,20)	400

Here , Z = 300 is minimum at (60, 0).

13. **(a)** it is at a constant distance from the origin

Explanation: Equation of normal at $\theta\ \ is\ x\cos\theta+y\sin\theta-a=0$. So,normal is at a fixed distance a from the origin.

14. **(d)** continuous at x = 0

Explanation: Given f(x) = $\sin^{-1}(\cos x)$,

Checking differentiability and continuity,

LHL at x = 0,

$$\lim_{\mathbf{x}\to 0^-} \mathbf{f}(\mathbf{x}) = \lim_{\mathbf{h}\to 0} \mathbf{f}(0-\mathbf{h}) = \lim_{\mathbf{h}\to 0} \sin^{-1}(\cos(0-\mathbf{h})) = \lim_{\mathbf{h}\to 0} \sin^{-1}(\cos(-\mathbf{h})) = \sin^{-1} 1 = \frac{\pi}{2}$$

RHL at x =0,

$$\lim_{\mathbf{x}\to 0^+} \mathbf{f}(\mathbf{x}) = \lim_{\mathbf{h}\to 0} \mathbf{f}(0+\mathbf{h}) = \lim_{\mathbf{h}\to 0} \sin^{-1}(\cos(0+\mathbf{h})) = \lim_{\mathbf{h}\to 0} \sin^{-1}(\cos(\mathbf{h})) = \sin^{-1} 1 = \frac{\pi}{2}$$

And $f(0) = \frac{\pi}{2}$

Hence, f(x) is continuous at x =0.

LHD at x = 0,

$$\lim_{x\to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{h\to 0} \frac{f(0-h)-f(0)}{0-h-0}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}(\cos(0-h))-\left(\frac{\pi}{2}\right)}{-h} = 1$$

RHD at x =0,

$$\lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{h\to 0} \frac{f(0+h)-f(0)}{0+h-0}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}(\cos(0+h))-\left(\frac{\pi}{2}\right)}{h} = -1$$

$\because$ LHD $\neq$ RHD

$\therefore$ f(x) is not differentiable at x =0.

15. **(a)** $e^{1/1-e}$

Explanation: y = x log x

Differentiating the function with respect to 'x',

$$\frac{dy}{dx} = 1 + \log x$$

Slope of tangent to the curve = 1 + log x

And, slope of the chord joining the points, (1, 0) & (e, e)

$$\mathrm{m} = \frac{\mathrm{e}}{\mathrm{e}-1}$$

The tangent to the curve is parallel to the chord joining the points, (1, 0) & (e, e)

$\therefore$ m = 1 + log x

$$\frac{e}{e-1} = 1 + \log x$$

$$\log x = \frac{e}{e-1} - 1$$

$$\log x = \frac{e-e+1}{e-1}$$

$$\log x = \frac{1}{e-1}$$

$$x = e^{\frac{1}{1-e}}$$

16. **(d)** k = 3, p = n

Explanation: Matrices P and Y are of the orders p $\times$ k and 3 $\times$ k respectively.

Therefore, matrix PY will be defined if k = 3.

Then, PY will be of the order p $\times$ k = p$\times$ 3.

Matrices W and Y are of the orders n $\times$ 3 and 3 $\times$ k = 3 $\times$ 3 respectively.

As, the number of columns in W is equal to the number of rows in Y, Matrix WY is well defined and is of the order n $\times$ 3.

Matrices PY and WY can be added only when their orders are the same.

Therefore, PY is of the order p $\times$ 3 and WY is of the order n $\times$ 3.

Thus, we must have p = n.

Therefore, k = 3 and p = n are the restrictions on n, k and p so that
PY + WY will be defined.

17. **(d)** (1, 2), (1, -2)

Explanation: $x^2 + y^2 - 2x - 3 = 0$

Differentiating with respect to x,

$2x+2y\frac{dy}{dx}-2=0$
$\Rightarrow \frac{dy}{dx}=\frac{2-2x}{2y}$
Given that slope of tangent $=\frac{dy}{dx}=0$
$\Rightarrow \frac{2-2x}{2y}=0$
x = 1
$x^2 + y^2 - 2x - 3 = 0$
$\Rightarrow y^2 = 2x + 3 - x^2$
x = 1
$\Rightarrow y=\pm 2$
Point are (1, 2) and (1, -2)

18. **(c)** $\frac{\sqrt{3}}{2}$
Explanation: $\sin^{-1}$ - $\cos^{-1}$ x $=\frac{\pi}{6}$
$\frac{\pi}{2}-\cos^{-1}x-\cos^{-1}x=\frac{\pi}{6}$
$\frac{\pi}{2}-2\cos^{-1}x=\frac{\pi}{6}$
$\frac{\pi}{2}-\frac{\pi}{6}=2\cos^{-1}x$
$\frac{2\pi}{6}=2\cos^{-1}x$
$\frac{\pi}{3}\times\frac{1}{2}=\cos^{-1}x$
$\frac{\pi}{6}=\cos^{-1}x$
$x=\cos\frac{\pi}{6}$
$x=\frac{\sqrt{3}}{2}$

19. **(d)** -1 for x < -3
Explanation: We have, $f(x)=\sqrt{x^2+6x+9}$
$=\sqrt{(x+3)^2}$
= |x + 3|
$f(x)=\begin{cases}x+3 & x\geq -3\\ -x-3 & x<-3\end{cases}$
$\Rightarrow f'(x)=\begin{cases}1 & x\geq -3\\ -1 & x<-3\end{cases}$
$\therefore$ f'(x) = -1 for x < -3.
Which is the required solution.

20. **(a)** none of these
Explanation: If A is a square matrix,
Let A = $\begin{bmatrix}1 & 2\\ 1 & 0\end{bmatrix}$
AA = $\begin{bmatrix}1 & 2\\ 1 & 0\end{bmatrix}\times\begin{bmatrix}1 & 2\\ 1 & 0\end{bmatrix}=\begin{bmatrix}3 & 2\\ 1 & 2\end{bmatrix}$
then AA is neither of the matrices given in the options of the question.

SECTION – B

21. **(c)** f is not defined
Explanation: Because , $\frac{1}{x}$ $is\ not\ defined\ for\ x\ =0, as\ 0\in R, \therefore f\ is\ not\ defined.$

22. **(c)** -39
Explanation: Given function,
$f(x) = 3x^4 - 8x^3 - 48x + 25$
$F'(x) = 12x^3 - 24x^2 - 48 = 0$
$F'(x) = 12(x^3 - 2x^2 - 4) = 0$
Differentiating again, we obtain

$F''(x) = 3x^2 - 4x = 0$
$x(3x - 4) = 0$
$x = 0$ or $x = \frac{4}{3}$
Putting the value in equation, we obtain
f(x) = -39

23. **(d)** 43
Explanation:

Corner points	Z = 5x + 7y
O(0,0)	0
B (3,4)	43
A(7,0)	35
C(0,2)	14

Hence the maximum value is 43

24. **(d)** $-3x^2 \sin(2x^3)$
Explanation: Given, $y = \cos^2 x^3 = (\cos(x^3))^2$
$\frac{dy}{dx} = (2\cos x^3)(-\sin(x^3)) \times 3x^2$
Using 2 sin A cos A = sin 2A
$\frac{dy}{dx} = -3x^2 \sin(2x^3)$

25. **(c)** a function of y only
Explanation: $y = ax^2 + bx + c$
$\frac{dy}{dx} = 2ax + b$
$\frac{d^2y}{dx^2} = 2a$
$y^3 \frac{d^2y}{dx^2} = 2ay^3$ = A function of y only

26. **(c)** 1
Explanation: $\sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right)$, as $sin^{-1}(-x) = -sin^{-1}x$
We all know that the principle branch of $sin^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $-\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$
Now, $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\left(\frac{3\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1$
Therefore, the required value of $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right) = 1$

27. **(c)** reflexive and symmetric
Explanation: According to the condition,
$(x,y) \in R \implies |x - y| \leq 1$
Reflexive: let $(x,x) \in R \implies |x-x| = 0 < 1$
$\Rightarrow$R is Reflexive
Symmetric:
If $(x,y) \in R \implies |x-y| \leq 1$
and $(y,x) \in R \implies |y-x| \leq 1$ [Since $|x-y| = |y-x|$]
$\Rightarrow$R is Symmetric
Transitive:
If $(x,y) \in R \Rightarrow |x-y| \leq 1$
and $(y,z) \in R \Rightarrow |y-z| \leq 1$
$\Rightarrow |x-y| = |x-y+y-z|$
$\leq |x-y| + |y-z| \leq 1+1=2$
$\Rightarrow |x-z| \leq 2$
$\therefore$R is not transitive

28. **(d)** 0

Explanation: $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$

Now, we will put x = sin y in the given equation, and we get

$\sin^{-1}(1-\sin y)-2\sin^{-1}\sin y=\frac{\pi}{2}$

$\Rightarrow \sin^{-1}(1-\sin y)-2y=\frac{\pi}{2}$

$\Rightarrow \sin^{-1}(1-\sin y)=\frac{\pi}{2}+2y$

$\Rightarrow 1-\sin y=\sin\left(\frac{\pi}{2}+2y\right)$

$\Rightarrow 1-\sin y=\cos 2y(as \sin\left(\frac{\pi}{2}+x\right)=\cos x)$

$\Rightarrow 1-cos2y=siny$

$\Rightarrow 2\ sin2y=siny$

$\Rightarrow siny.(2siny-1)=0$

$\Rightarrow siny=0\ \ or\ \ siny=\frac{1}{2}$

$\therefore\ \ x=0\ \ or\ \ x=\frac{1}{2}$

Now, if we put x = $\frac{1}{2}$, then we will see that,

L.H.S. = $\sin^{-1}\left(1-\frac{1}{2}\right)-2\sin^{-1}\frac{1}{2}$

= $\sin^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\frac{1}{2}$

= $-\sin^{-1}\frac{1}{2}$

= $-\frac{\pi}{6}\neq\frac{\pi}{2}\neq$ R.H.S

Hence, x = $\frac{1}{2}$ is not the solution of the given equation.

Thus, x = 0

29. **(c)** $\frac{1}{3}\begin{bmatrix}1 & 1\\ 3 & 1\end{bmatrix}$

Explanation: A + B = $\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$ ...(i)

A - 2B = $\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$ ...(ii)

adding 2 $\times$ (i) and (ii), we get

2A + 2B = $\begin{bmatrix}2 & 0\\ 2 & 2\end{bmatrix}$ ...(iii)

A - 2B = $\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$ ...(iv)

adding (iii) and (iv), we get

$\Rightarrow$ 3A = $\begin{bmatrix}2 & 0\\ 2 & 2\end{bmatrix}+\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$

$\Rightarrow$ A = $\frac{1}{3}\begin{bmatrix}1 & 1\\ 3 & 1\end{bmatrix}$

30. **(b)** $\frac{1}{2}$

Explanation: Given that y = $\tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$

Using 1 - cos x = 2sin$^2\frac{x}{2}$ and Using sin x = 2 sin x $\frac{x}{2}$cos $\frac{x}{2}$, we obtain

y = $\tan^{-1}\left(\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$ or y = $\tan^{-1}\tan\frac{x}{2}$

$y=\frac{x}{2}$

Differentiating with respect to x, we obtain

$\frac{dy}{dx}=\frac{1}{2}$

31. **(c)** f'(1⁻) = –1

Explanation: Given that $f(x)=\left\{\begin{array}{c}-\log_e x, 0<x<1\\ \log_e x, x\geq 1\end{array}\right\}$

Differentiability at x =1,

LHD at x =1,

$$\lim_{x\to1^-}\frac{f(x)-f(1)}{x-1}=\lim_{h\to0}\frac{f(1-h)-f(1)}{1-h-1}$$

$$=\lim_{h\to0}\frac{\log 1-h}{-h}=-1$$

RHD at x =1,

$$\lim_{x\to1^+}\frac{f(x)-f(1)}{x-1}=\lim_{h\to0}\frac{f(1+h)-f(1)}{1+h-1}$$

$$=\lim_{h\to0}\frac{\log(1+h)}{h}=1$$

So, $f'(1^+) = 1$ and $f'(1^-) = -1$

32. **(d)** 1

Explanation: $y=\log\sqrt{\tan x}$

$$\frac{dy}{dx}=\frac{1}{\sqrt{\tan x}}\times\frac{1}{2\sqrt{\tan x}}\sec^2 x$$

$$\frac{dy}{dx}=\frac{\sec^2 x}{2\tan x}$$

$$\left|\frac{dy}{dx}\right|_{x=\frac{\pi}{4}}=\frac{\sec^2\frac{\pi}{4}}{\sqrt{\tan\frac{\pi}{4}}}=\frac{2}{2\times1}=1$$

33. **(d)** R

Explanation: R

34. **(c)** $-1\leq x<\frac{1}{\sqrt{2}}$

Explanation: We have $\cos^{-1}x > \sin^{-1}x$

$\Rightarrow\frac{\pi}{2}-\sin^{-1}x>\sin^{-1}x$

$\Rightarrow\frac{\pi}{2}>2\sin^{-1}x$

$\Rightarrow\sin^{-1}x<\frac{\pi}{4}$ ... (i)

But $-\frac{\pi}{2}\leq\sin^{-1}x\leq\frac{\pi}{2}$ .... (ii)

From (i) and (ii), $-\frac{\pi}{2}\leq\sin^{-1}x<\frac{\pi}{4}$

$\Rightarrow\sin\left(-\frac{\pi}{2}\right)\leq x<\sin\frac{\pi}{4}$

$\Rightarrow-1\leq x<\frac{1}{\sqrt{2}}$

35. **(c)** makes an acute angle with x-axis

Explanation: $y = 2x^7 + 3x + 5$

$\Rightarrow\frac{dy}{dx} = 14x^6 + 3$

Even power is always positive.

Hence, $\frac{dy}{dx}>0$

$\tan\theta>0$

Hence, tangent makes an acute angle with x-axis to the curve.

36. **(c)** Linear constraints

Explanation: In a LPP, the linear inequalities or restrictions on the variables are called Linear constraints.

37. **(a)** $\frac{1}{k}\cdot A^{-1}$

Explanation: by the property of inverse

$(AB)^{-1} = B^{-1}A^{-1}$

$(KA)^{-1} = A^{-1}K^{-1}$

$=\frac{1}{K}A^{-1}$

38. **(b)** null matrix

Explanation: Only a null matrix can be symmetric as well as skew symmetric.

In Symmetric Matrix $A^T = A$,

Skew Symmetric Matrix $A^T = -A$,

Given that the matrix is satisfying both the properties.
Therefore, Equating the RHS we get A = -A i.e, 2A = 0.
Therefore A = 0, which is a null matrix.

39. **(a)** $\frac{1}{2a}$

Explanation: $\sqrt{x}+\sqrt{y}=\sqrt{a}\ldots\ldots.(1)$

$\Rightarrow \frac{1}{2\sqrt{x}}+\frac{1}{2\sqrt{y}}\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=-\frac{\sqrt{y}}{\sqrt{x}}\ldots\ldots(2)$

$\Rightarrow \frac{d^2y}{dx^2}=-\frac{\sqrt{x}\frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx}-\sqrt{y}\frac{1}{2}x^{-\frac{1}{2}}}{x}$

$=\frac{-\left(\frac{\sqrt{x}}{2\sqrt{y}}\left(-\frac{\sqrt{y}}{\sqrt{x}}\right)-\frac{\sqrt{y}}{2\sqrt{x}}\right)}{x}$

$=\frac{\sqrt{x}+\sqrt{y}}{2x\sqrt{x}}=\frac{\sqrt{a}}{2x\sqrt{x}.}=\frac{\sqrt{a}}{2a\sqrt{a}}=\frac{1}{2a}$

40. **(d)** neither one-one nor onto

Explanation: Given that f : R $\rightarrow$ R be a function where

$f(x)=\frac{x^2-8}{x^2+2}$

Here, we can see that for negative as well as positive x we will get same value.
So, it is not one-one.
y = f(x)

$\Rightarrow \mathrm{y}=\frac{\mathrm{x}^2-8}{\mathrm{x}^2+2}$

$\Rightarrow$ y(x^2 + 2) = (x^2 - 8)

$\Rightarrow$ x^2(y - 1) = -2y - 8

$\Rightarrow \mathrm{x}=\sqrt{\frac{2\mathrm{y}+8}{1-\mathrm{y}}}$

For y = 1 , no x is defined.
So, f is not onto.

SECTION – C

41. **(d)** $4a=3\beta$

Explanation: $\alpha=\tan^{-1}\left(\tan\frac{5\pi}{4}\right)$

$\Rightarrow \alpha=\tan^{-1}\left(\tan\left(\pi+\frac{\pi}{4}\right)\right)$

$\Rightarrow \alpha=\tan^{-1}\left(\tan\left(\frac{\pi}{4}\right)\right)$

$\Rightarrow \alpha=\frac{\pi}{4}$

and

$\beta=\tan^{-1}\left(\tan\left(\pi-\frac{2\pi}{3}\right)\right)$

$\beta=\tan^{-1}\left(\tan\left(\frac{\pi}{3}\right)\right)$

$\beta=\frac{\pi}{3}$

$4\alpha=4\times\frac{\pi}{4}=\pi$ ...(i)

$3\beta=3\times\frac{\pi}{3}=\pi$ ...(ii)

From (i) and (ii)

$4\alpha=3\beta$.

Which is the required solution.

42. **(d)** 132

Explanation: Here , minimize Z = 3x + 4y ,

Corner points	**Z = 3x + 4y**
C(0 ,38)	132.......................(Min.)
B (52 ,0)	156
D(44 , 16)	196

The minimum value is 132

43. **(c)** m^2y
Explanation: $y = ae^{mx} + be^{-mx} \Rightarrow y_1 = ame^{mx} + (-m)be^{-mx} \Rightarrow y_2 = am^2e^{mx} + (m^2)be^{-mx}$
$\Rightarrow y_2 = m^2(ae^{mx} + be^{-mx}) \Rightarrow y_2 = m^2y$

44. **(b)** -1
Explanation: $f(x) = 2x^3 - 3x^2 - 12x + 5$
$\Rightarrow f'(x) = 6x^2 - 6x - 12$
For local maxima or minima we have
f'(x) = 0
$6x^2 - 6x - 12 = 0$
$\Rightarrow x^2 - x - 2 = 0$
$\Rightarrow$ x = 2 or x = -1
f''(x) = 12x - 6
f''(2) = 18 > 0
function has local minima at x = 2.
f''(-1) = -18 < 0
function has local maxima at x = -1.

45. **(a)** an equivalence relation
Explanation: an equivalence relation
Reflexivity: Let a $\in$ R
Then, $\mathrm{aa} = \mathrm{a}^2 > 0$
$\Rightarrow (a, a) \in R \forall a \in R$
So, S is reflexive on R.
Symmetry: Let $(a, b) \in S$
Then,
(a, b) $\in$ S
$\Rightarrow ab \geq 0$
$\Rightarrow \mathrm{ba} \geq 0$
$\Rightarrow (b, a) \in S \forall a, b \in R$
So, S is symmetric on R.
Transitive:
If $(a, b), (b, c) \in S$
$\Rightarrow$ ac $\geq$ 0 $[\because b2 \geq 0]$
$\Rightarrow (a, c) \in S$ for all a, b, c $\in$ set R
Hence,. S is an equivalence relation on R

46. **(d)** $\frac{1}{3}t^2 + 20t + 1$
Explanation: $\frac{1}{3}t^2 + 20t + 1$

47. **(c)** 376 miles/sec
Explanation: 376 miles/sec

48. **(d)** 27 seconds
Explanation: 27 seconds

49. **(a)** 21
Explanation: 21

50. **(c)** $\frac{4}{3}$
Explanation: $\frac{4}{3}$