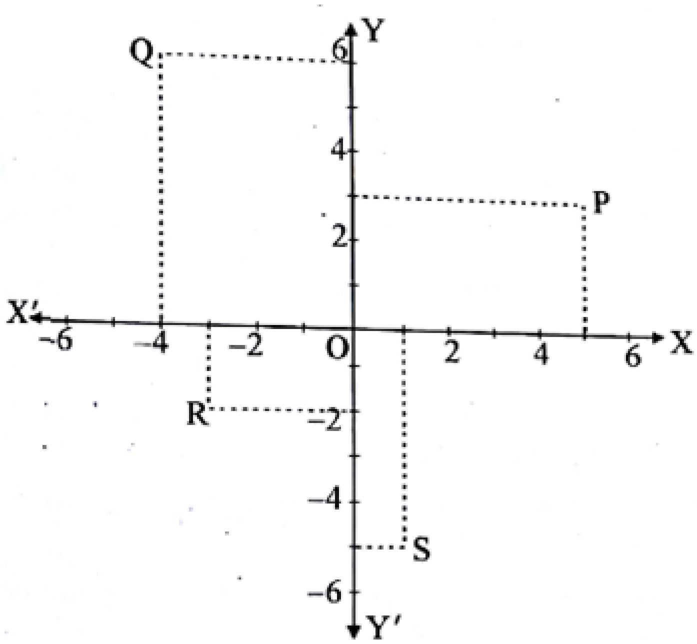
9. Co-ordinate Geometry

Exercise 9.1

1. Question

From the given figure find the coordinates of the points P, Q, R and S.



Answer

The coordinates of a point in represented as the ordered pair (abscissa, ordinate).

where abscissa is the X-coordinate or distance of that point from Y-axis

and ordinate is the Y-coordinate or distance of that point from X-axis.

Abscissa of point P = 5

Ordinate of point P = 3

⇒ Coordinates of point P = (5,3)

Abscissa of point Q = −4

Ordinate of point Q = 6

⇒ Coordinates of point Q = (−4,6)

Abscissa of point R = −3

Ordinate of point R = −2

⇒ Coordinates of point R = (–3,–2)

Abscissa of point S = –1

Ordinate of point S = –5

⇒ Coordinates of point S = (–1,–5)

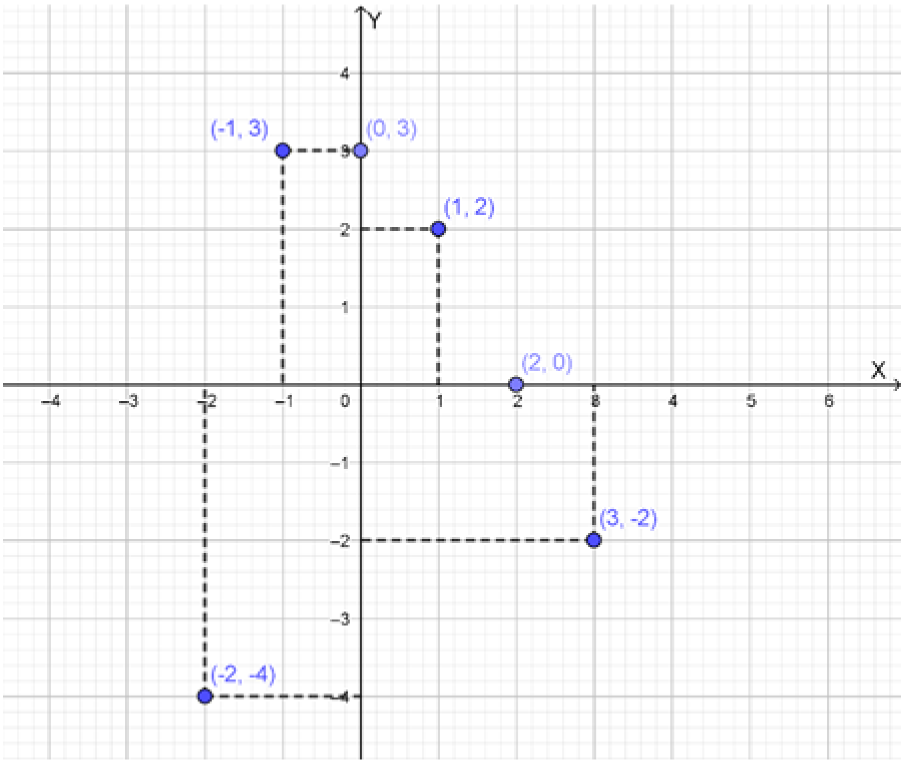
2. Question

Plot the points with following coordinates (1, 2), (–1, 3), (–2, –4), (3, –2), (2, 0), (0, 3).

Answer

Draw the rectangular coordinate axes.

The given points (x,y) are marked by measuring the distance x from Y-axis and distance y from X-axis to obtain the following:



3. Question

Taking rectangular axes plot the points O(0, 0), P(3, 0) and R(0, 4). If OPQR is a rectangle, then find the coordinates of point Q.

Answer

Mark the points O(0,0), P(3,0) and R(0,4) on the rectangular coordinate axes.

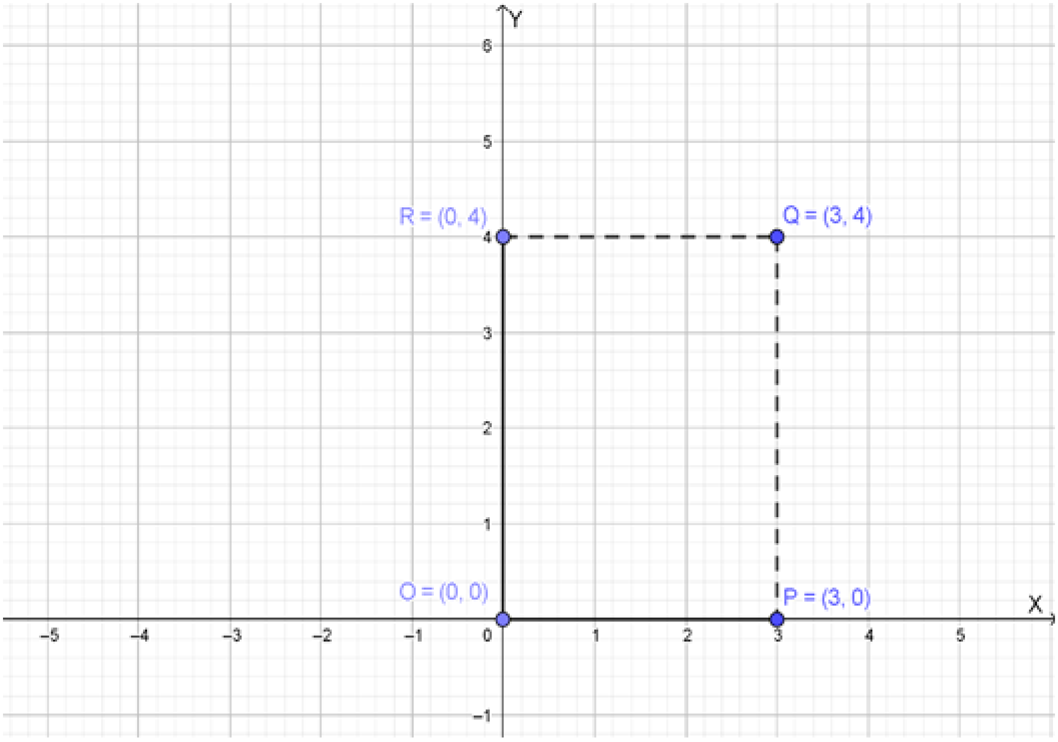
Since OPQR is a rectangle, OP = RQ = 3 units

OR = PQ = 4 units

So, abscissa of point Q = 3

ordinate of point Q = 4

The coordinates of point Q are (3,4).

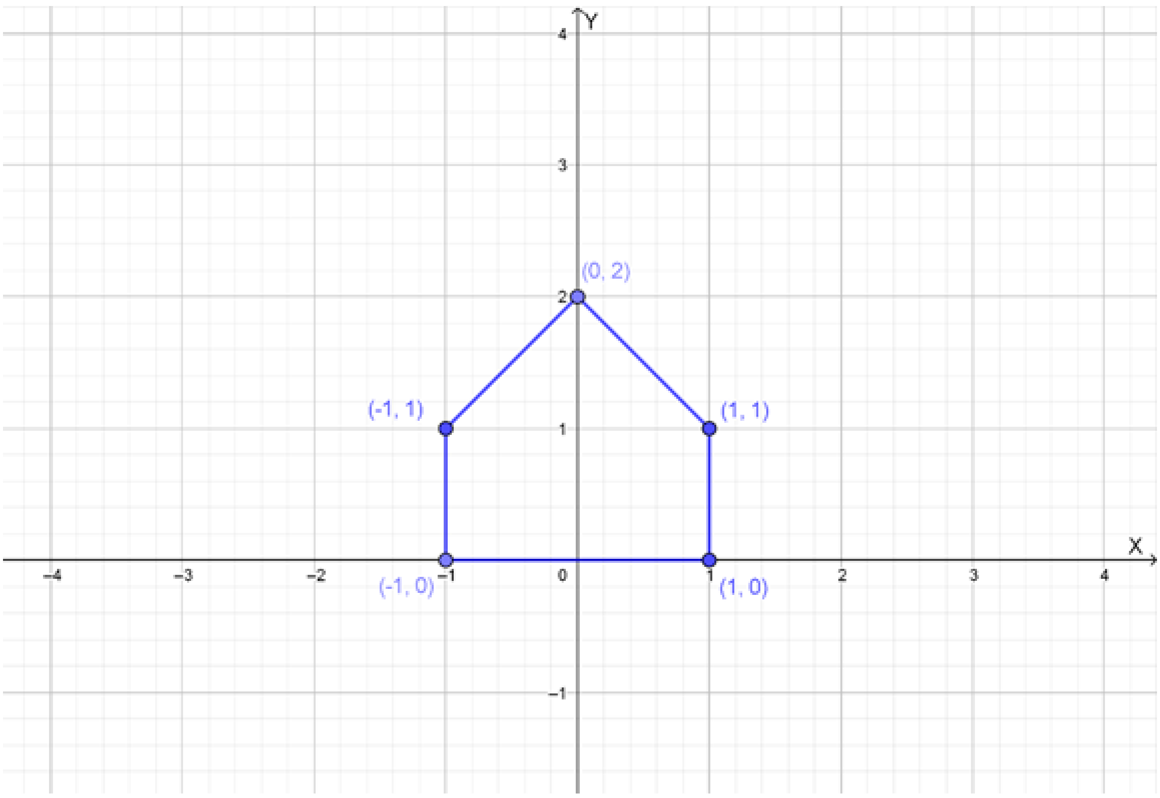


4. Question

Plot the points (−1, 0), (1, 0), (1, 1), (0, 2), (−1, 1), and which figure is obtained by joining these correctively.

Answer

On plotting the given points, we observe that a pentagon is obtained.



5. Question

Draw a quadrilateral, if its vertices are as follows:

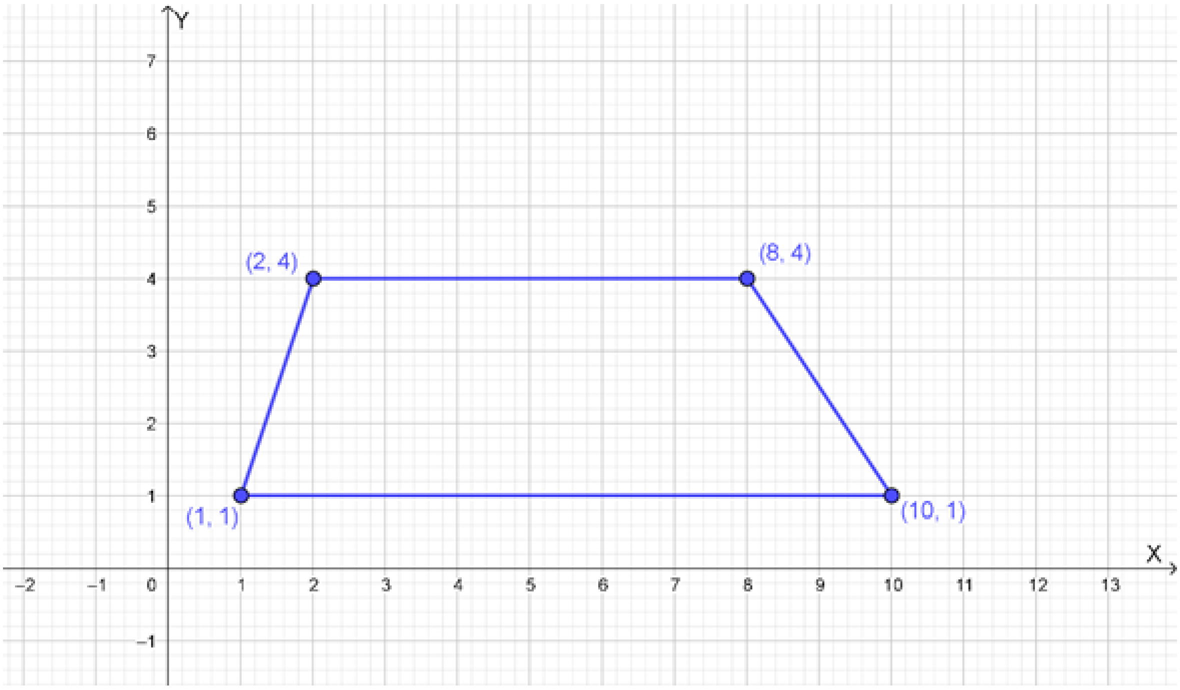
(i) (1, 1), (2, 4), (8, 4) and (10, 1)

(ii) (−2, −2), (−4, 2), (−6, −2) and (−4, −6).

In each case tell the type of the quadrilateral.

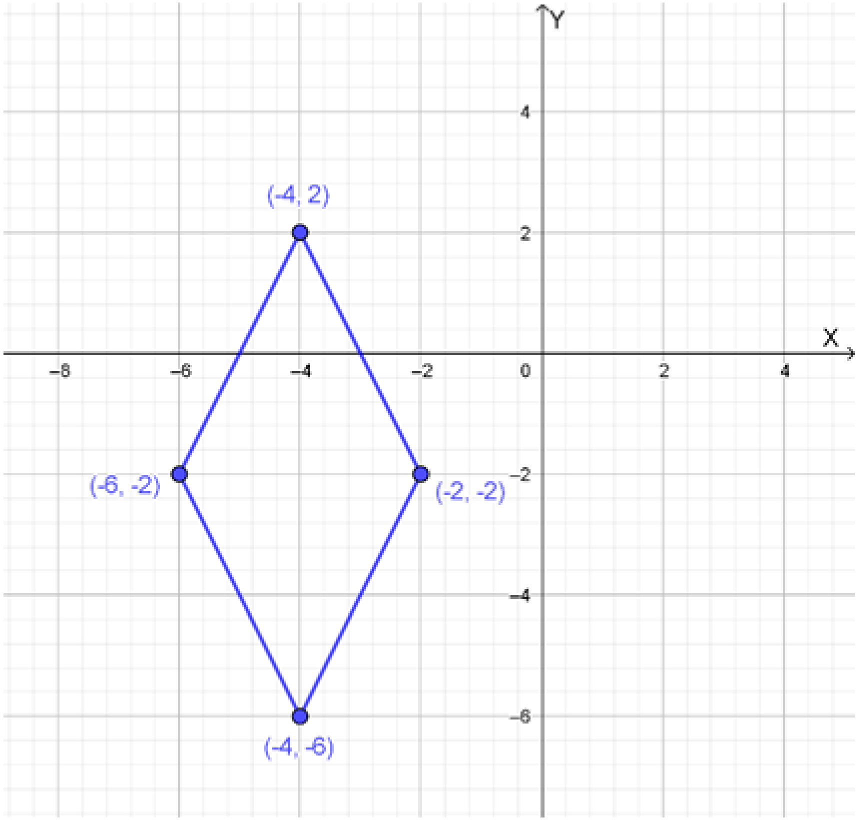
Answer

(i) The vertices are plotted on the rectangular axes.



Since only a pair of the opposite sides is parallel, the obtained quadrilateral is trapezium.

(ii) The vertices are plotted on the rectangular axes.



Since both pairs of the opposite sides are parallel and equal, the obtained quadrilateral is rhombus.

6. Question

Find the distance between the following points:

(i) (−6, 7) and (−1, −5)

(ii) (−1, −1) and (8, −2)

(iii) $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$

Answer

We know that distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(i) Let the points (−6, 7) and (−1, −5) be A and B respectively, so the distance between them

$$AB = \sqrt{(-1-(-6))^2 + (-5-7)^2}$$

$$= \sqrt{(5)^2 + (-12)^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169} = 13$$

(ii) Let the points (−1, −1) and (8, −2) be A and B respectively, so the distance between them

$$AB = \sqrt{(8-(-1))^2+(-2-(-1))^2}$$

$$= \sqrt{(9)^2+(-1)^2}$$

$$= \sqrt{81+1}$$

$$= \sqrt{82}$$

(iii) Let the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ be A and B respectively, so the distance between them

$$AB = \sqrt{(at_2{}^2 - at_1{}^2)^2 + (2at_2 - 2at_1)^2}$$

$$= \sqrt{\{a(t_2-t_1)(t_2+t_1)\}^2+\{2a(t_2-t_1)\}^2}$$

$$= \sqrt{a^2(t_2-t_1)^2(t_2+t_1)^2+4a^2(t_2-t_1)^2}$$

$$= a(t_2-t_1)(\sqrt{(t_2+t_1)^2+4^2}$$

7. Question

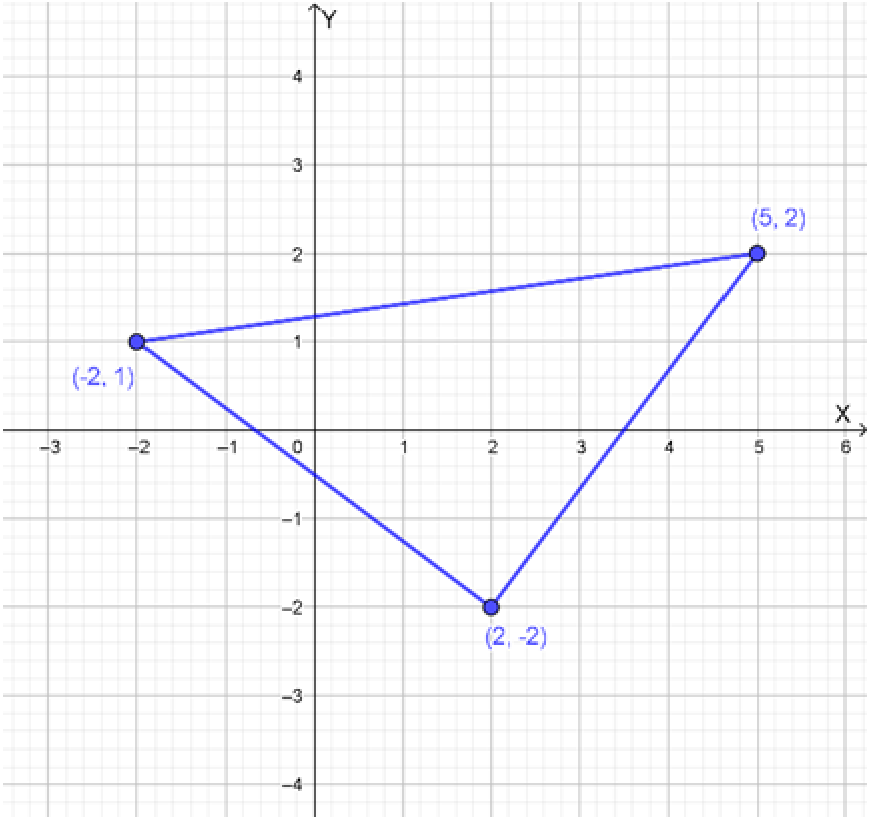
Prove that the points (2, −2), (−2, 1) and (5, 2) are the vertices of a right angled triangle.

Answer

We have A → (2, −2)

B → (−2, 1)

C → (5, 2)



Using the distance formula,

$$AB = \sqrt{(-2-2)^2 + (1-(-2))^2}$$

$$= \sqrt{(-4)^2 + (3)^2}$$

$$= \sqrt{16+9}$$

$$= \sqrt{25} = 5$$

$$BC = \sqrt{(5-(-2))^2 + (2-1)^2}$$

$$= \sqrt{(7)^2 + (1)^2}$$

$$= \sqrt{49+1}$$

$$= \sqrt{50}$$

$$CA = \sqrt{(2-5)^2 + (-2-2)^2}$$

$$= \sqrt{(-3)^2 + (-4)^2}$$

$$= \sqrt{9+16}$$

$$= \sqrt{25} = 5$$

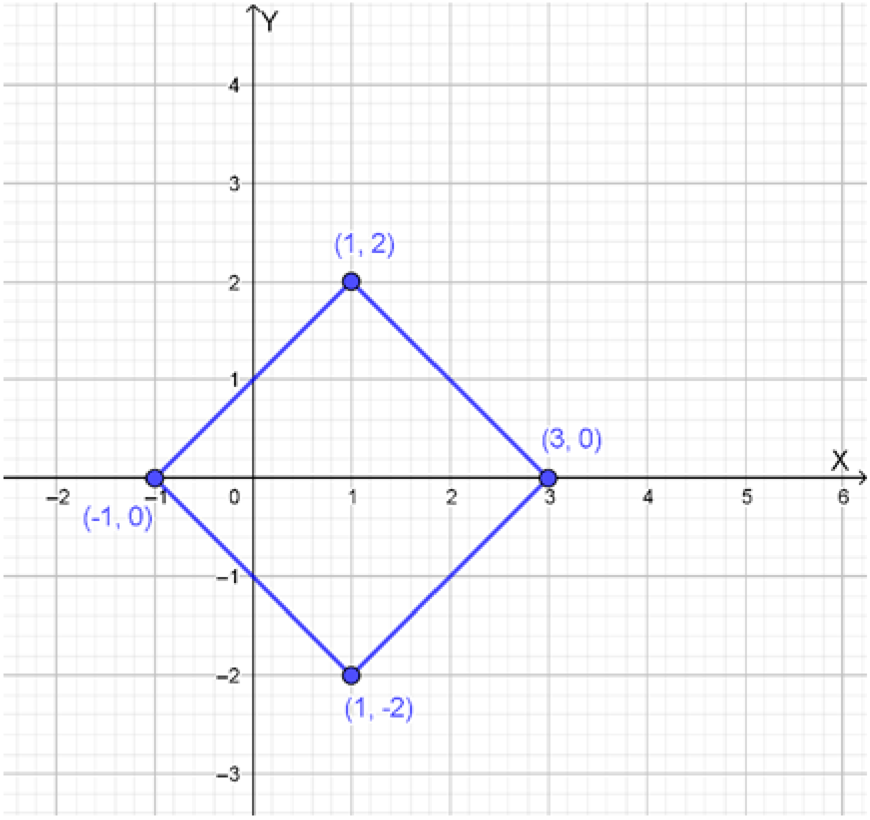
We find that $(AB)^2 + (CA)^2 = (BC)^2$, or simply $(5, 5, \sqrt{50})$ forms a pythagorean triplet and hence the points A, B and C form a right angled triangle.

8. Question

Prove that the points (1, −2), (3, 0), (1, 2) and (−1, 0) are the vertices of a square.

Answer

We have A → (1, −2)



B → (3, 0)

C → (1, 2)

D → (−1, 0)

Using the distance formula,

$$AB = \sqrt{(3-1)^2 + (0-(-2))^2}$$

$$= \sqrt{(2)^2 + (2)^2}$$

$$= \sqrt{4+4}$$

$$= \sqrt{16} = 4$$

$$BC = \sqrt{(1-3)^2 + (2-0)^2}$$

$= \sqrt{(-2)^2 + (2)^2}$

$= \sqrt{4 + 4}$

$= \sqrt{16} \; = \; 4$

$CD = \sqrt{(-1-1)^2 + (0-2)^2}$

$= \sqrt{(-2)^2 + (-2)^2}$

$= \sqrt{4 + 4}$

$= \sqrt{16} \; = \; 4$

$DA = \sqrt{(-1-1)^2 + (0-(-2))^2}$

$= \sqrt{(-2)^2 + (2)^2}$

$= \sqrt{4 + 4}$

$= \sqrt{16} \; = \; 4$

We find that AB = BC = CD = DA.

⇒ ABCD is a square.

9. Question

Prove that the points (a, a), (–a, –a) and $\left(-\sqrt{3}a, \sqrt{3}a\right)$ are the vertices of an equilateral triangle.

Answer

We have A → (a, a)

B → (–a, –a)

C → $(-\sqrt{3}a, \sqrt{3}a)$

Using the distance formula,

$AB = \sqrt{(-a-a)^2 + (-a-a)^2}$

$= \sqrt{(-2a)^2 + (-2a)^2}$

$= \sqrt{4a^2 + 4a^2}$

$= \sqrt{8a^2} = 2\sqrt{2}a$

$$BC = \sqrt{\left(-\sqrt{3}a-(-a)\right)^2+\left(\sqrt{3}a-(-a)\right)^2}$$

$$= \sqrt{\left(a-\sqrt{3}a\right)^2+\left(a+\sqrt{3}a\right)^2}$$

We know that $(a - b)^2 + (a + b)^2 = (a^2 + b^2 - 2\ ab) + (a^2 + b^2 + 2\ ab) = 2(a^2 + b^2)$

$$BC = \sqrt{2(a^2+\left(\sqrt{3}a\right)^2)}$$

$$= \sqrt{2\times 4a^2}$$

$$= \sqrt{8a^2} = 2\sqrt{2}a$$

$$CA = \sqrt{\left(a-(-\sqrt{3}a)\right)^2+\left(a-\sqrt{3}a\right)^2}$$

$$= \sqrt{\left(a+\sqrt{3}a\right)^2+\left(a-\sqrt{3}a\right)^2}$$

$$= \sqrt{2(a^2+\left(\sqrt{3}a\right)^2)}$$

$$= \sqrt{2\times 4a^2}$$

$$= \sqrt{8a^2} = 2\sqrt{2}a$$

We find that AB = BC = CA.

⇒ Points A, B and C form an equilateral triangle.

10. Question

Prove that the points (1, 1) (–2, 7) and (3, –3) are collinear.

Answer

We have A → (1, 1)

B → (–2, 7)

C → (3, –3)

Using the distance formula,

$$AB = \sqrt{(-2-1)^2+(7-1)^2}$$

$$= \sqrt{(-3)^2+(6)^2}$$

$= \sqrt{9 + 36}$

$= \sqrt{45} = 3\sqrt{5}$

$BC = \sqrt{(3 - (-2))^2 + (-3 - 7)^2}$

$= \sqrt{(5)^2 + (-10)^2}$

$= \sqrt{25 + 100}$

$= \sqrt{125} = 5\sqrt{5}$

$CA = \sqrt{(1 - 3)^2 + (1 - (-3))^2}$

$= \sqrt{(-2)^2 + (4)^2}$

$= \sqrt{4 + 16}$

$= \sqrt{20} = 2\sqrt{5}$

We find that,

AB + BC = $3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5}$ = CA

⇒ The points A, B and C are collinear.

11. Question

Find the points on x-axis which are situated at equal distances from the points (–2, –5) and (2, –3).

Answer

We have A → (–2, –5)

B → (x, 0) equidistant from A and C

(y = 0 since B lies on X-axis)

C → (2, –3)

Using the distance formula,

$AB = \sqrt{(x - (-2))^2 + (0 - (-5))^2}$

$= \sqrt{(x + 2)^2 + (5)^2}$

$= \sqrt{x^2 + 4x + 4 + 25}$

$= \sqrt{x^2 + 4x + 29}$

$BC = \sqrt{(2 - x)^2 + (-3 - 0)^2}$

$= \sqrt{4 - 4x + x^2 + 9}$

$= \sqrt{x^2 - 4x + 13}$

Since B is equidistant from A and C

∴ AB = BC

$\sqrt{x^2 + 4x + 29} = \sqrt{x^2 - 6x + 13}$

Squaring both sides

$x^2 + 4x + 29 = x^2 - 4x + 13$

8x + 16 = 0

x = –2

The required point is B (–2, 0).

12. Question

Find the points on y-axis which are situated at equal distances from the points (–5, –2) and (3, 2).

Answer

We have A → (–5, –2)

B → (0, y) equidistant from A and C

(x = 0 since B lies on Y-axis)

C → (3, 2)

Using the distance formula,

$AB = \sqrt{(0 - (-5))^2 + (y - (-2))^2}$

$= \sqrt{(5)^2 + (y + 2)^2}$

$= \sqrt{25 + y^2 + 4y + 4}$

$= \sqrt{y^2 + 4y + 29}$

$BC = \sqrt{(3 - 0)^2 + (2 - y)^2}$

$$= \sqrt{9 + 4 - 4y + y^2}$$

$$= \sqrt{y^2 - 4y + 13}$$

Since B is equidistant from A and C

∴ AB = BC

$$\sqrt{y^2 + 4y + 29} = \sqrt{y^2 - 4y + 13}$$

Squaring both sides

$$y^2 + 4y + 29 = y^2 - 4y + 13$$

8y + 16 = 0

y = –2

The required point is B (0, –2).

13. Question

If the distances of the point (0, 2) from the points (3, k) and (k, 5) are equal then find the value of k.

Answer

We have A → (3, k)

B → (0, 2)

C → (k, 5)

Using the distance formula,

$$AB = \sqrt{(0 - 3)^2 + (2 - k)^2}$$

$$= \sqrt{9 + 4 - 4k + k^2}$$

$$= \sqrt{k^2 - 4k + 13}$$

$$BC = \sqrt{(k - 0)^2 + (5 - 2)^2}$$

$$= \sqrt{k^2 + 9}$$

Since B is equidistant from A and C

∴ AB = BC

$$\sqrt{k^2 - 4k + 13} = \sqrt{k^2 + 9}$$

Squaring both sides

$$k^2 - 4k + 13 = k^2 + 9$$

4k = 4

∴ k = 1

14. Question

If the coordinates of P and Q are (a cos θ, b sin θ) and (–a sin θ, b cos θ) respectively, then prove that $OP^2 + OQ^2 = a^2 + b^2$, where O is the origin.

Answer

We have P → (a cos θ, b sin θ)

Q → (–a sin θ, b cos θ)

We know that distance of a point A (x,y) from origin O (0, 0) is given as OA = $\sqrt{x^2 + y^2}$

Using the above formula,

OP = $\sqrt{(a\cos\theta)^2 + (b\sin\theta)^2}$

= $\sqrt{a^2\cos^2\theta + b^2\sin^2\theta}$

$OP^2 = a^2\cos^2\theta + b^2\sin^2\theta$

OQ = $\sqrt{(-a\sin\theta)^2 + (b\cos\theta)^2}$

= $\sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$

$OQ^2 = a^2\sin^2\theta + b^2\cos^2\theta$

Now, $OP^2 + OQ^2 = a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta$

$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

We know that, $\cos^2\theta + \sin^2\theta = 1$

∴ $OP^2 + OQ^2 = a^2 + b^2$

15. Question

If two vertices of an equilateral triangle are (0, 0) and $(3, \sqrt{3})$ then find the third vertex.

Answer

Given two vertices of an equilateral traingle ABC

A → (0, 0)

B → (3, $\sqrt{3}$)

Let the third vertex be C → (x, y)

We know that distance of a point A (x,y) from origin O (0, 0) is given as OA = $\sqrt{x^2 + y^2}$

So,

$$AB = \sqrt{(3)^2 + \left(\sqrt{3}\right)^2}$$

$$= \sqrt{9 + 3}$$

$$= \sqrt{12}$$

$$CA = \sqrt{x^2 + y^2}$$

Using distance formula,

$$BC = \sqrt{(x - 3)^2 + \left(y - (\sqrt{3})\right)^2}$$

$$= \sqrt{x^2 - 6x + 9 + y^2 - 2\sqrt{3}y + 3}$$

$$= \sqrt{x^2 + y^2 - 6x - 2\sqrt{3y} + 12}$$

Since ΔABC is an equilateral, AB = BC = CA.

Using CA = AB,

$$\sqrt{x^2 + y^2} = \sqrt{12}$$

Squaring both sides, we get

$x^2 + y^2 = 12$ ...(i)

Now, using AB = BC

$$\sqrt{12} = \sqrt{x^2 + y^2 - 6x - 2\sqrt{3}y + 12}$$

$$\sqrt{12} = \sqrt{12 - 6x - 2\sqrt{3}y + 12}$$ (Using eq (i))

Squaring both sides, we get

$-6x - 2\sqrt{3}y + 24 = 12$

$\Rightarrow 6x + 2\sqrt{3}y = 12$

$\Rightarrow 3x + \sqrt{3}y = 6$

$$\Rightarrow \ x = \frac{6 - \sqrt{3}y}{3}$$

Substituting the value of x in eq (i), we get

$$\left(\frac{6 - \sqrt{3}y}{3}\right)^2 + y^2 = 12$$

$$\frac{36 - 12\sqrt{3}y + 3y^2}{9} + y^2 = 12$$

$$\frac{36 - 12\sqrt{3}y + 3y^2 + 9y^2}{9} = 12$$

$\Rightarrow 12y^2 - 12\sqrt{3}y + 36 = 108$

$\Rightarrow 12y^2 - 12\sqrt{3}y - 72 = 0 \Rightarrow y^2 - \sqrt{3}y - 6 = 0$

$\Rightarrow y^2 - 2\sqrt{3}y + \sqrt{3}y - 6 = 0$

$\Rightarrow y\,(y - 2\sqrt{3}) + \sqrt{3}\,(y - 2\sqrt{3}) = 0$

$\Rightarrow (y - 2\sqrt{3})\,(y + \sqrt{3}) = 0 \Rightarrow y = 2\sqrt{3}$ or $-\sqrt{3}$If $y = 2\sqrt{3}$, then $x = \frac{6-6}{3} = 0$If $y = -\sqrt{3}$, $x = \frac{6+3}{3} = 3$So, the third vertex of the equilateral triangle = $(0, 2\sqrt{3})$ or $(3, -\sqrt{3})$.

Exercise 9.2

1. Question

Find the coordinates of the point which divides the line segment joining the points (3, 5) and (7, 9) internally in the ratio 2 : 3.

Answer

We know that

Coordinates of a point P(x,y) dividing the line segment joining A (x_1, y_1) and B (x_2, y_2) in the ratio m:n internally are $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

We have A → (3, 5)

and B → (7, 9).

Coordinates of point P(x,y) dividing AB in the ratio 2:3 internally are

$$x = \frac{2 \times 7 + 3 \times 3}{2 + 3} = \frac{14 + 9}{5} = \frac{23}{5}$$

$$y = \frac{2 \times 9 + 3 \times 5}{2 + 3} = \frac{18 + 15}{5} = \frac{33}{5}$$

$P\left(\frac{23}{5}, \frac{33}{5}\right)$ is the required point.

2. Question

Find the coordinates of the point which divides the line segment joining the points (5, –2) and $\left(-1\frac{1}{2}, 4\right)$ externally in the ratio 7 : 9.

Answer

We know that

Coordinates of a point P(x,y) dividing the line segment joining A (x_1, y_1) and B (x_2, y_2) in the ratio m:n externally are $\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$

We have A → (5, –2)

and B → $(-1\frac{1}{2}, 4)$ or B $(\frac{-3}{2}, 4)$.

Coordinates of point P (x,y) dividing AB in the ratio 7:9 externally are

$$x = \frac{7 \times \frac{-3}{2} - 9 \times 5}{7 - 9} = \frac{\frac{-21}{2} - 45}{-2} = \frac{-21 - 90}{-4} = \frac{-111}{-4} = 27\frac{3}{4}$$

$$y = \frac{7 \times 4 - 9 \times (-2)}{7 - 9} = \frac{28 + 18}{-2} = -\frac{46}{2} = -23$$

P $(27\frac{3}{4}, -23)$ is the required point.

3. Question

Prove that the origin O divides the line segment joining the points, A(1, –3) and B(–3, 9) in the ratio 1 : 3 internally. Find the coordinates of the points dividing externally.

Answer

Internal divison formula:

Coordinates of a point P(x,y) dividing the line segment joining A (x_1, y_1) and B (x_2, y_2) in the ratio m:n internally are $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

Let O divide the line segment joining the given points in the ratio λ:1. Then by internal division formula,

$$0 = \frac{\lambda \times (-3) + 1 \times 1}{\lambda + 1}$$

$$= \frac{-3\lambda + 1}{\lambda + 1}$$

3λ = 1

$$\lambda = \frac{1}{3}$$

Hence proved that, the origin O divides the line segment joining the points, A(1, –3) and B(–3, 9) in the ratio 1 : 3 internally.

External divison formula:

Coordinates of a point P(x,y) dividing the line segment joining A (x_1, y_1) and B (x_2, y_2) in the ratio m:n externally are $\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$

Using external division formula,

$$x = \frac{1\times(-3)-3\times1}{1-3} = \frac{-3-3}{-2} = \frac{-6}{-2} = 3$$

$$y = \frac{1\times9-3\times(-3)}{1-3} = \frac{9+9}{-2} = -\frac{18}{2} = -9$$

P (3,–9) is the required point.

4. Question

Find the coordinates of the mid–point of the line joining the points (22, 20) and (0, 16).

Answer

We know that, coordinates of the mid–point of of the line segment joining the points A (x_1, y_1) and B (x_2, y_2) are given by $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

So, the coordinates of the mid–point of the line joining the points (22, 20) and (0, 16) are

$\left(\frac{22+0}{2}, \frac{20+16}{2}\right)$ =(11,18)

5. Question

In what ratio is the line segment joining the points (5, 3) and (–3, –2) divided by x-axis?

Answer

Let the line segment joining (5, 3) and (–3, –2) be divided by X-axis in the ratio $\lambda : 1$ by point P.

Since P lies on X-axis, so its ordinate = 0

By internal division formula,

$y = \frac{\lambda\times(-2)+1\times3}{\lambda+1}$

$$0 = \frac{-2\lambda + 3}{\lambda + 1}$$

$\Rightarrow 2\lambda = 3$

$\Rightarrow \lambda = \frac{3}{2}$

∴ Required ratio is =3:2

6. Question

In what ratio is the line segment joining the points (2, –3) and (5, 6) is divided by y-axis?

Answer

Let the line segment joining (2, –3) and (5, 6) be divided by Y-axis in the ratio $\lambda : 1$ by point P.

Since P lies on Y-axis, so its abscissa = 0

By internal division formula are

$x = \frac{\lambda\times5+1\times2}{\lambda+1}$

$$0 = \frac{5\lambda + 2}{\lambda + 1}$$

$\Rightarrow 5\lambda = -2$

The negative sign implies external division.

$\Rightarrow \lambda = \frac{2}{5}$ (externally)

∴ Required ratio is =2:5 externally.

7. Question

In what ratio does the point (11, 15) divide the line segment joining the points (15, 5) and (9, 20)?

Answer

Let the line segment joining (15, 5) and (9, 20) be divided by P(11, 15) in the ratio λ : 1.

By internal division formula,

$11 = \frac{\lambda \times 9 + 1 \times 15}{\lambda + 1}$

$\Rightarrow 11\lambda + 11 = 9\lambda + 15$

$\Rightarrow 2\lambda = 4$

$\Rightarrow \lambda = 2$

∴ Required ratio is =2:1

Note: Alternatively, the procedure could have been carried out by y-coordinate.

8. Question

If the point P(3, 5) divides the line segment joining the points A(−2, 3) and B in the ratio 4 : 7 then find the coordinates of B.

Answer

Let the coordinates of B be (x, y).

Using internal division formula, we have

$$3 = \frac{4x + 7 \times (-2)}{4 + 7}$$

$\Rightarrow 3 = \frac{4x - 14}{11}$

$\Rightarrow 4x - 14 = 33$

$\Rightarrow 4x = 47$

$\Rightarrow x = \frac{47}{4}$

and $5 = \frac{4 \times y + 7 \times 3}{4 + 7}$

$\Rightarrow 5 = \frac{4y + 21}{11}$

$\Rightarrow 4y + 21 = 55$

$\Rightarrow$ 4y = 34

$\Rightarrow y = \frac{17}{2}$

$B\left(\frac{47}{4}, \frac{17}{2}\right)$ is the required point.

9. Question

Find the points of trisection of the line joining the points (11, 9) and (1, 2).

Answer

Let the line segment joining the points A (1, 2) and B (11, 9) be trisected at points P (x_1, y_1) and Q (x_2, y_2).

Clearly, P divides the line segment AB internally in the ratio1:2.

∴ By internal division formula,

$x_1 = \frac{1\times11+2\times1}{1+2} = \frac{11+2}{3} = \frac{13}{3}$

$y_1 = \frac{1\times9+2\times2}{1+2} = \frac{9+4}{3} = \frac{13}{3}$

Again, Q divides the line segment AB internally in the ratio 2:1.

∴ By internal division formula,

$x_2 = \frac{2\times11+1\times1}{2+1} = \frac{22+1}{3} = \frac{23}{3}$

$y_2 = \frac{2\times9+1\times2}{2+1} = \frac{18+2}{3} = \frac{20}{3}$

∴ The required points are $P\left(\frac{13}{3}, \frac{13}{3}\right)$ and $Q\left(\frac{23}{3}, \frac{20}{3}\right)$.

10. Question

Find the coordinates of points dividing the line segment joining the points (–4, 0) and (0, 6) into 4 equal parts.

Answer

Let the line segment joining the points A (–4, 0) and B (0, 6) be divided into 4 parts by points P (x_1, y_1), Q (x_2, y_2) and R(x_3, y_3).

Clearly, P divides the line segment AB internally in the ratio1:3.

∴ By internal division formula,

$x_1 = \frac{1 \times 0 + 3 \times (-4)}{1+3} = \frac{-12}{4} = -3$

$y_1 = \frac{1 \times 6 + 3 \times 0}{1+3} = \frac{6}{4} = \frac{3}{2}$

Again, Q divides the line segment AB internally in the ratio 1:1.

∴ By mid–point formula,

$x_2 = \frac{-4+0}{2} = -2$

$y_2 = \frac{0+6}{2} = 3$

Also, R divides the line segment AB internally in the ratio3:1.

∴ By internal division formula,

$x_3 = \frac{3 \times 0 + 1 \times (-4)}{3+1} = \frac{-4}{4} = -1$

$y_3 = \frac{3 \times 6 + 1 \times 0}{3+1} = \frac{18}{4} = \frac{9}{2}$

∴ The required points are P$\left(-3, \frac{3}{2}\right)$, Q (–2, 3)and R $\left(-1, \frac{9}{2}\right)$.

11. Question

Find in what ratio does the line 3x + y = 9 divide the line segment joining the points (1, 3) and (2, 7)?

Answer

Let the line segment joining (1, 3) and (2, 7) be divided by the given line in the ratio λ : 1 at point P (x, y).

By internal division formula

$x = \frac{\lambda \times 2 + 1 \times 1}{\lambda+1} = \frac{2\lambda+1}{\lambda+1}$

$y = \frac{\lambda \times 7 + 1 \times 3}{\lambda+1} = \frac{7\lambda+3}{\lambda+1}$

Since P lies on the line 3x + y = 9, it must satisfy its equation.

Hence, $3 \times \frac{2\lambda+1}{\lambda+1} + \frac{7\lambda+3}{\lambda+1} = 9$

$6\lambda + 3 + 7\lambda + 3 = 9(\lambda + 1)$

$13\lambda + 6 = 9\lambda + 9$

$4\lambda = 3$

$\Rightarrow \lambda = \frac{3}{4}$

∴ Required ratio is =3:4.

12. Question

Find the ratio in which the point (–3, p) divides the join of points (–5, –4) and (–2, 3) internally. Also find the value of p.

Answer

Let the line segment joining (–5, –4) and (–2, 3) be divided by the point (–3, p) in the ratio λ : 1.

By internal division formula

$$-3 = \frac{\lambda \times (-2) + 1 \times (-5)}{\lambda + 1} = \frac{-2\lambda - 5}{\lambda + 1}$$

$\Rightarrow -2\lambda - 5 = -3\lambda - 3$

$\Rightarrow \lambda = 2$

∴ Required ratio is =2:1.

Now, $p = \frac{2 \times 3 + 1 \times (-4)}{2+1} = \frac{6-4}{3}$

∴ $p = \frac{2}{3}$

Miscellaneous Exercise 9

1. Question

The distance of point (3, 4) from y-axis will be:

A. 1

B. 4

C. 2

D. 3

Answer

Distance of point (3,4) from Y-axis = abscissa of the point = 3

∴The correct option is **D**.

2. Question

The distance of point (5, –2) from x-axis will be:

A. 5

B. 2

C. 3

D. 4

Answer

Distance of point (5, –2) from X-axis = ordinate of the point = |–2| = 2

∴The correct option is **B.**

3. Question

The distance between the points (0, 3) and (–2, 0) will be:

A. $\sqrt{14}$

B. $\sqrt{15}$

C. $\sqrt{13}$

D. $\sqrt{5}$

Answer

We have A → (0, 3)

B → (–2, 0)

Using the distance formula,

AB = $\sqrt{(-2-0)^2+(0-3)^2}$

= $\sqrt{(-2)^2+(-3)^2}$

= $\sqrt{4+9} = \sqrt{13}$

∴ The correct option is **C.**

4. Question

The triangle with vertices (–2, 1), (2, –2) and (5, 2) is :

A. right angled

B. equilateral

C. isosceles

D. none of these

Answer

We have A → (−2, 1)

B → (2, −2)

C → (5, 2)

Using the distance formula,

AB = $\sqrt{(2-(-2))^2+(-2-1)^2}$

= $\sqrt{(4)^2+(-3)^2}$

= $\sqrt{16+9}$

= $\sqrt{25}$ = 5

BC = $\sqrt{(5-2)^2+(2-(-2))^2}$

= $\sqrt{(3)^2+(4)^2}$

= $\sqrt{9+16}$

= $\sqrt{25} = 5$

CA = $\sqrt{(-2-5)^2+(1-2)^2}$

= $\sqrt{(-7)^2+(-1)^2}$

= $\sqrt{49+1}$

= $\sqrt{50}$

We find that $(AB)^2 + (CA)^2 = (BC)^2$, or simply (5, 5, √50) forms a pythagorean triplet and hence the points A, B and C form a right angled triangle.

∴ The correct option is **A**.

5. Question

The quadrilateral formed by points (−1, 1), (0, −3), (5, 2) and (4, 6) will be:

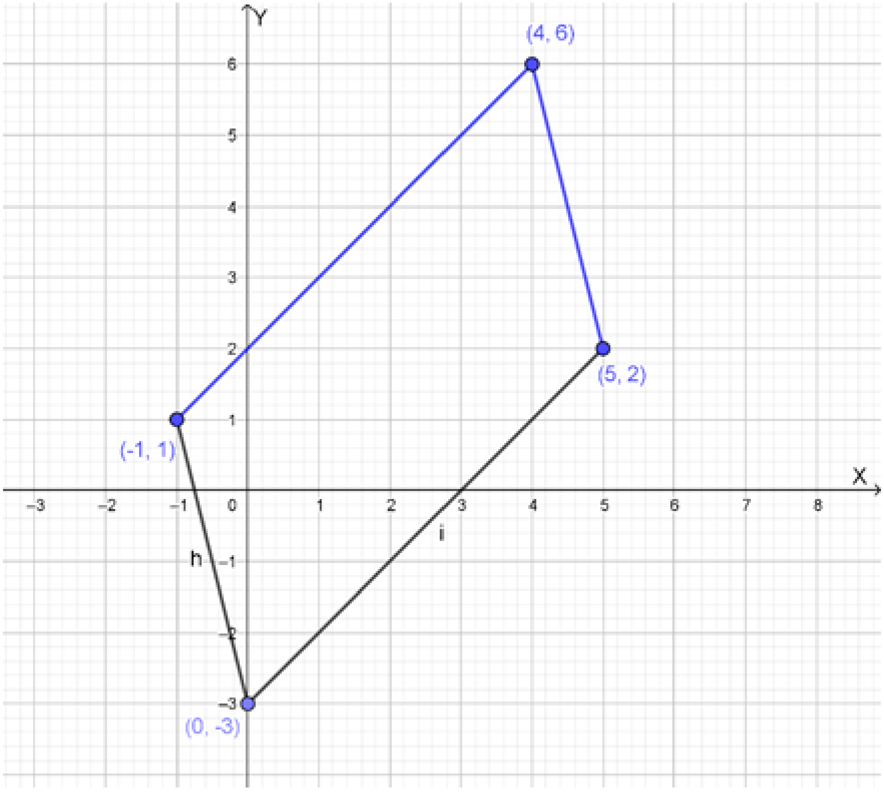
A. square

B. rectangular

C. rhombus

D. parallelogram

Answer



We have A → (−1, 1)

B → (0, −3)

C → (5, 2)

D → (4, 6)

Using the distance formula,

AB = $\sqrt{(0-(-1))^2+(-3-1)^2}$

= $\sqrt{(1)^2+(-4)^2}$

= $\sqrt{1+16}$

= $\sqrt{17}$

BC = $\sqrt{(5-0)^2+(2-(-3))^2}$

= $\sqrt{(5)^2+(5)^2}$

= $\sqrt{25+25}$

$= \sqrt{50}$

CD= $\sqrt{(4-5)^2+(6-2)^2}$

$= \sqrt{(-1)^2+(4)^2}$

$= \sqrt{1+16}$

$= \sqrt{17}$

DA = $\sqrt{(-1-4)^2+(1-6)^2}$

$= \sqrt{(-5)^2+(-5)^2}$

$= \sqrt{25+25}$

$= \sqrt{50}$s

We find that AB = CDand BC = DA.

The oppposite pairs of sides are equal for the given quadrilateral.

Consider the diagonals, AC and BD.

AC = $\sqrt{(5-(-1))^2+(2-1)^2}$

$= \sqrt{(6)^2+(1)^2}$

$= \sqrt{36+1}$

$= \sqrt{37}$

BD = $\sqrt{(4-0)^2+(6-(-3))^2}$

$= \sqrt{(4)^2+(9)^2}$

$= \sqrt{16+81}$

$= \sqrt{97}$

Since the diagonals are not equal, the given quadrilateral is not a rectangle.

(This is also visible from the figure since angles are not 90°.)

⇒ ABCD is a parallelogram.

∴ The correct option is **D**.

6. Question

The point equidistant from points (0, 0), (2, 0) and (0, 2) is:

A. (1, 2)

B. (2, 1)

C. (2, 2)

D. (1, 1)

Answer

We have A → (0, 0)

B → (2, 0)

C → (0, 2)

Let D (x,y) be equidistant from A, B and C.

We know that distance of a point A (x,y) from origin O (0, 0) is given as OA = $\sqrt{x^2 + y^2}$

∴ AD = $\sqrt{x^2 + y^2}$ ...(i)

Using the distance formula,

BD = $\sqrt{(x-2)^2 + (y-0)^2}$

= $\sqrt{x^2 - 4x + 4 + y^2}$

= $\sqrt{x^2 + y^2 - 4x + 4}$ ---(ii)

CD = $\sqrt{(x-0)^2 + (y-2)^2}$

= $\sqrt{x^2 + y^2 - 4y + 4}$ ---(iii)

Since D is equidistant from A, B and C

Equating eq (i) and (ii)

$\sqrt{x^2 + y^2} = \sqrt{x^2 + y^2 - 4y + 4}$

Squaring both sides

$x^2 + y^2 = x^2 + y^2 - 4y + 4$

4y – 4 = 0

y = 1

Equating eq (i) and (iii)

$$\sqrt{x^2+y^2} = \sqrt{x^2+y^2-4x+4}$$

Squaring both sides

$$x^2+y^2 = x^2+y^2-4x+4$$

4x – 4 = 0

x = 1

The required point is D (1, 1).

∴ The correct option is **D**.

7. Question

The point P divides the line segment joining the points (5, 0) and (0, 4) in the ratio 2 : 3 internally. The coordinates of P are:

A. $\left(3, \frac{8}{5}\right)$

B. $\left(1, \frac{4}{5}\right)$

C. $\left(\frac{5}{2}, \frac{3}{4}\right)$

D. $\left(2, \frac{12}{5}\right)$

Answer

Coordinates of point P (x,y) dividing the line segment joining the points (5, 0) and (0, 4) in the ratio 2 : 3 internally are:

x = $\frac{2\times0+3\times5}{2+3} = \frac{15}{5}$

∴ x = 3

And y = $\frac{2\times4+3\times0}{2+3} = \frac{8}{5}$

P $\left(3, -\frac{8}{5}\right)$ is the required point.

∴ The correct option is **A**.

8. Question

If points (1, 2), (–1, x) and (2, 3) are collinear, then the value of x will be:

A. 2

B. 0

C. –1

D. 1

Answer

Given, points A (1, 2), B (–1, x) and C (2, 3) are collinear.

⇒ Slope of AB = Slope of AC

$$\frac{x-2}{-1-1} = \frac{3-2}{2-1}$$

∴ x – 2 = –2 (1)

∴ x = 0.

∴ The correct option is **B.**

9. Question

If the distance between the points (3, a) and (4, 1) be $\sqrt{10}$ then the value of a will be:

A. 3, –1

B. 2, –2

C. 4, –2

D. 5, –3

Answer

Using the distance formula, the distance between the points (3, a) and (4, 1)

$\sqrt{10} = \sqrt{(4-3)^2 + (1-a)^2}$

$= \sqrt{(1)^2 + 1 - 2a + a^2}$

$= \sqrt{1 + 1 - 2a + a^2}$

$= \sqrt{a^2 - 2a + 2}$

Squaring both sides, we get

$a^2 - 2a + 2 = 10$

$\Rightarrow a^2 - 2a - 8 = 0$

$\Rightarrow a^2 - 4a + 2a - 8 = 0$

$\Rightarrow a(a - 4) + 2(a - 4) = 0$

$\Rightarrow (a - 4)(a + 2) = 0$

∴ Either $a = 4$ or $a = -2$.

∴ The correct option is **C**.

10. Question

If point (x, y) is at equal distance from points (2, 1) and (1, −2), then the true statement out of the following:

A. $x + 3y = 0$

B. $3x + y = 0$

C. $x + 2y = 0$

D. $2y + 3x = 0$

Answer

We have $A \rightarrow (2, 1)$

$B \rightarrow (1, -2)$

$P \rightarrow (x, y)$

Using the distance formula,

$AP = \sqrt{(x-2)^2 + (y-1)^2}$

$= \sqrt{x^2 - 4x + 4 + y^2 - 2y + 1}$

$= \sqrt{x^2 + y^2 - 4x - 2y + 5}$

$BP = \sqrt{(x-1)^2 + (y-(-2))^2}$

$= \sqrt{(x-1)^2 + (y+2)^2}$

$= \sqrt{x^2 - 2x + 1 + y^2 + 4y + 4}$

$= \sqrt{x^2 + y^2 - 2x + 4y + 5}$

Since P is equidistant from A and B

∴ AP = BP

$\sqrt{x^2+y^2-4x-2y+5} = \sqrt{x^2+y^2-2x+4y+5}$

Squaring both sides, we get

$x^2 + y^2 - 4x - 2y + 5 = x^2 + y^2 - 2x + 4y + 5$

$2x + 6y = 0$

or $x + 3y = 0$

∴ The correct option is **A**.

11. Question

If the vertices of a quadrilateral are (1, 4), (−5, 4), (−5, −3) and (1, −3), then find the type of quadrilateral.

Answer

We have A → (1, 4)

B → (−5, 4)

C → (−5, −3)

D → (1, −3)

Using the distance formula,

AB = $\sqrt{(-5-1)^2+(4-4)^2}$

= $\sqrt{(-6)^2+(0)^2}$

= $\sqrt{36}$

=6

BC = $\sqrt{(-5-(-5))^2+(-3-4)^2}$

= $\sqrt{(-5+5)^2+(-7)^2}$

= $\sqrt{0+49}$

= 7

CD = $\sqrt{(1-(-5))^2+(-3-(-3))^2}$

= $\sqrt{(1+5)^2+(-3+3)^2}$

= $\sqrt{(6)^2+(0)^2}$

$= \sqrt{36}$

= 6

DA = $\sqrt{(1-1)^2 + (4-(-3))^2}$

$= \sqrt{(0)^2 + (4+3)^2}$

$= \sqrt{0+49}$

= 7

The opposite sides are equal in length.

Consider the diagonals, AC and BD.

AC = $\sqrt{(-5-1)^2 + (-3-4)^2}$

$= \sqrt{(-6)^2 + (-7)^2}$

$= \sqrt{36+49}$

$= \sqrt{85}$

BD = $\sqrt{(1-(-5))^2 + (-3-4)^2}$

$= \sqrt{(6)^2 + (-7)^2}$

$= \sqrt{36+49}$

$= \sqrt{85}$

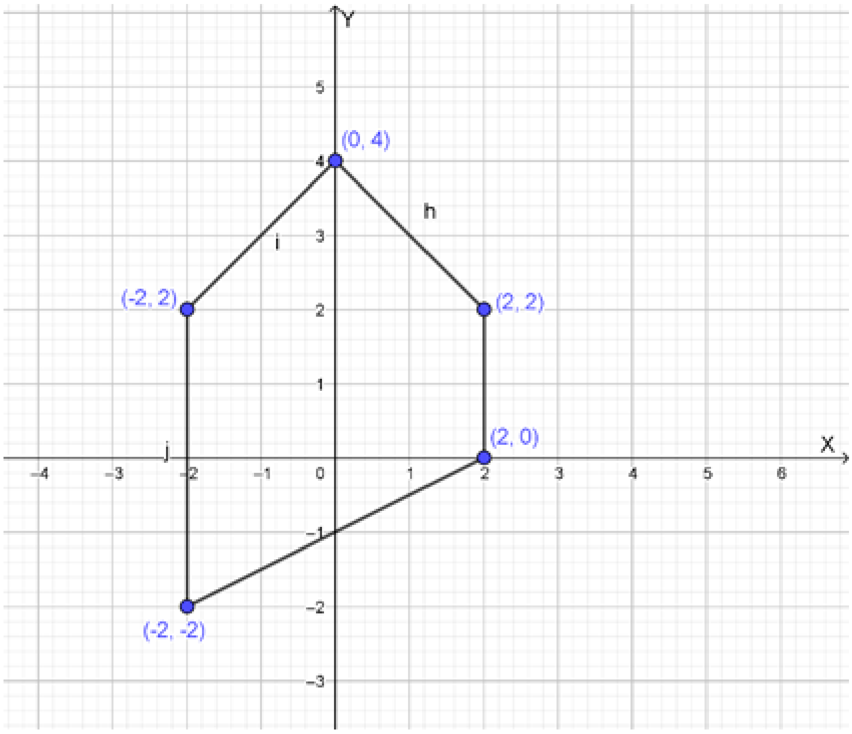
Since the diagonals are also equal, the given quadrilateral is a rectangle.

12. Question

Which figure will be obtained by joining the points (–2, 0), (2, 0), (2, 2), (0, 4), (–2, 2) in order?

Answer

On plotting and joining the points (–2, 0), (2, 0), (2, 2), (0, 4), (–2, 2) in order on the rectangular axes, we obtain the following figure:



which corresponds to a pentagon.

13. Question

In what ratio does the point (3, 4) divide the line segment joining the points (1, 2) and (6, 7)?

Answer

Let the line segment joining (1, 2) and (6, 7) be divided by the point (3, 4) in the ratio λ : 1.

By internal division formula,

$3 = \frac{\lambda \times 6 + 1 \times 1}{\lambda + 1}$

$3\lambda + 3 = 6\lambda + 1$

$\Rightarrow 3\lambda = 2$

$\Rightarrow \lambda = \frac{2}{3}$

∴ Required ratio is =2:3.

14. Question

The opposite vertices of a square are (5, −4) and (−3, 2) find the length of its diagonal.

Answer

Using the distance formula, length of the diagonal

$= \sqrt{(-3-5)^2 + (2-(-4))^2}$

$= \sqrt{(-8)^2 + (6)^2}$

$= \sqrt{64 + 36}$

$= \sqrt{100} = 10$

∴ The length of the diagonal of given square = 10 units.

15. Question

One end of a line segment is (4, 0) and mid–point is (4, 1) then find the coordinates of the other end of the line segment.

Answer

Let the coordinates of the other end of the line segment be (x, y).

Using the mid–point formula, we get

$4 = \frac{4+x}{2}$

$\Rightarrow 4 + x = 8$

$\Rightarrow x = 4$

and

$1 = \frac{0+y}{2}$

$\Rightarrow y = 2$

∴ The coordinates of the other end of the line segment are (4, 2).

16. Question

Find the distance of the point (1, 2) from the mid–point of the line segment joining the points (6, 8) and (2, 4).

Answer

Using mid–point formula:

Coordinates of the mid–point of the line segment joining the points (6, 8) and (2, 4) are

$\left(\frac{6+2}{2}, \frac{8+4}{2}\right) = (4, 6)$

Using the distance formula,

Distance between the points (1, 2) and (4, 6) = $\sqrt{(4-1)^2+(6-2)^2}$

= $\sqrt{(3)^2+(4)^2}$

= $\sqrt{9+16} = \sqrt{25}$

= 5

17. Question

There are four points P(2, −1), Q(3, 4), R(−2, 3) and S(−3, −2) in a plane. Then prove that PQRS in not a square, rather it is a rhombus.

Answer

We have P → (2,–1)

Q → (3,4)

R → (–2,3)

S → (–3,–2)

Using the distance formula,

PQ = $\sqrt{(3-2)^2+(4-(-1))^2}$

= $\sqrt{(1)^2+(5)^2}$

= $\sqrt{1+25}$

= $\sqrt{26}$

QR = $\sqrt{(-2-3)^2+(3-4)^2}$

= $\sqrt{(-5)^2+(-1)^2}$

= $\sqrt{25+1}$

= $\sqrt{26}$

RS = $\sqrt{(-3-(-2))^2+(-2-3)^2}$

= $\sqrt{(-3+2)^2+(-5)^2}$

= $\sqrt{(-1)^2+25}$

= $\sqrt{26}$

SP = $\sqrt{(2-(-3))^2+(-1-(-2))^2}$

= $\sqrt{(2+3)^2+(-1+2)^2}$

= $\sqrt{(5)^2+(1)^2}$

= $\sqrt{26}$

All four sides of quadrilateral PQRS are equal.

Consider the diagonals, AC and BD.

PR = $\sqrt{(-2-2)^2+(3-(-1))^2}$

= $\sqrt{(-4)^2+(3+1)^2}$

= $\sqrt{16+16}$

= $\sqrt{32}$

QS = $\sqrt{(-3-3)^2+(-2-4)^2}$

= $\sqrt{(-6)^2+(-6)^2}$

= $\sqrt{36+36}$

= $\sqrt{72}$

Since the diagonals are not equal, the given quadrilateral is not a square.

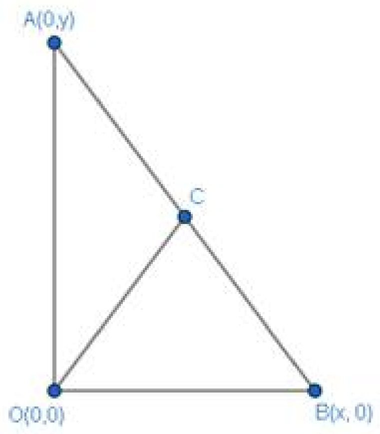
⇒ PQRS is a rhombus.

18. Question

Prove that the mid–point C of the hypotenuse in a right angled triangle AOB is situated at equal distances from the vertices O, A and B of the triangle.

Answer

Consider a right angled ΔAOB, such that C is the mid–point of hypotenuse AB.



We have O → (0, 0)

Since A lies on y-axis, A→ (0, y)

and B lies on x-axis, B→ (x, 0)

Using mid–point formula, coordinates of C are $\left(\frac{0+x}{2}, \frac{y+0}{2}\right) = \left(\frac{x}{2}, \frac{y}{2}\right)$

Using distance formula,

AC = $\sqrt{\left(\frac{x}{2} - 0\right)^2 + \left(\frac{y}{2} - y\right)^2}$

= $\sqrt{\left(\frac{x}{2}\right)^2 + \left(\frac{y-2y}{2}\right)^2}$

= $\sqrt{\frac{x^2}{4} + \left(\frac{-y}{2}\right)^2}$

= $\sqrt{\frac{x^2}{4} + \frac{y^2}{4}}$

= $\frac{\sqrt{x^2+y^2}}{2}$

BC = $\sqrt{\left(\frac{x}{2} - x\right)^2 + \left(\frac{y}{2} - 0\right)^2}$

= $\sqrt{\left(\frac{x-2x}{2}\right)^2 + \left(\frac{y}{2}\right)^2}$

= $\sqrt{\left(\frac{-x}{2}\right)^2 + \frac{y^2}{4}}$

= $\sqrt{\frac{x^2}{4} + \frac{y^2}{4}}$

$= \frac{\sqrt{x^2+y^2}}{2}$

We know that distance of a point P (x,y) from origin O (0, 0) is given as OP = $\sqrt{x^2+y^2}$

$\therefore$ OC = $\sqrt{\left(\frac{x}{2}\right)^2 + \frac{y^2}{4}}$

$= \sqrt{\frac{x^2}{4} + \frac{y^2}{4}}$

$= \frac{\sqrt{x^2+y^2}}{2}$

We can observe that OA = OB = OC.

$\therefore$ C (mid–point of hypotenuse AB) is equidistant from all the three vertices of the right angled ΔAOB.

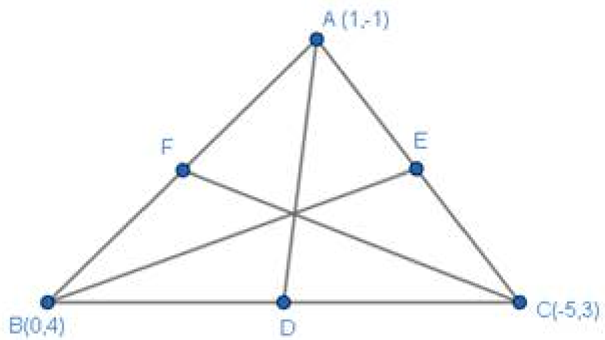
19. Question

Find the length of medians of the triangle whose vertices are (1, −1), (0, 4) and (−5, 3).

Answer

We have the vertices of ΔABC, A (1, −1), B (0, 4) and C (−5, 3).

Let D, E and F be the mid points of the sides BC, CA and AB respectively.



Using the mid–point formula,

Coordinates of D are $\left(\frac{0+(-5)}{2}, \frac{4+3}{2}\right) = \left(\frac{-5}{2}, \frac{7}{2}\right)$

Coordinates of E are $\left(\frac{1+(-5)}{2}, \frac{-1+3}{2}\right)$ = (−2, 1)

Coordinates of F are $\left(\frac{0+1}{2}, \frac{-1+4}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$

Using distance formula,

Length of median AD = $\sqrt{\left(\frac{-5}{2}-1\right)^2+\left(\frac{7}{2}-(-1)\right)^2}$

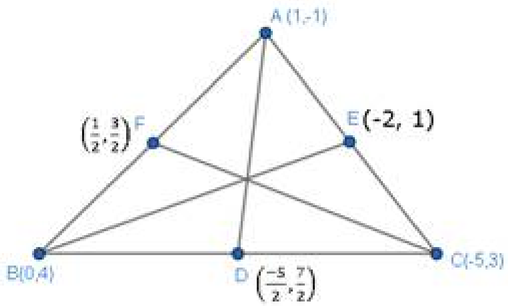
= $\sqrt{\left(\frac{-5-2}{2}\right)^2+\left(\frac{7+2}{2}\right)^2}$

= $\sqrt{\left(\frac{-7}{2}\right)^2+\left(\frac{9}{2}\right)^2}$

= $\sqrt{\frac{49}{4}+\frac{81}{4}}$

= $\sqrt{\frac{130}{4}}$

= $\frac{\sqrt{130}}{2}$



Length of median BE = $\sqrt{(-2-0)^2+(1-4)^2}$

= $\sqrt{(-2)^2+(-3)^2}$

= $\sqrt{4+9}$

= $\sqrt{13}$

Length of median CF = $\sqrt{\left(\frac{1}{2}-(-5)\right)^2+\left(\frac{3}{2}-3\right)^2}$

= $\sqrt{\left(\frac{1+10}{2}\right)^2+\left(\frac{3-6}{2}\right)^2}$

= $\sqrt{\left(\frac{11}{2}\right)^2+\left(\frac{-3}{2}\right)^2}$

= $\sqrt{\frac{121}{4}+\frac{9}{4}}$

= $\sqrt{\frac{130}{4}}$

$=\frac{\sqrt{130}}{2}$

∴ The lengths of medians of the triangle whose vertices are (1, −1), (0, 4) and (−5, 3) are $\frac{\sqrt{130}}{2}$, $\frac{\sqrt{130}}{2}$ and $\sqrt{13}$.

20. Question

Prove that the mid–point of the line segment joining the points (5, 7) and (3, 9) is the same as the mid–point the line segment joining the points (8, 6) and (0, 10).

Answer

Using mid–point formula:

Coordinates of the mid–point of the line segment joining the points (5, 7) and (3, 9) are

$\left(\frac{5+3}{2}, \frac{7+9}{2}\right)$ = (4, 8)

and

Coordinates of the mid–point of the line segment joining the points (8, 6) and (0, 10) are

$\left(\frac{8+0}{2}, \frac{6+10}{2}\right)$ = (4, 8)

Hence proved that the mid–point of the line segment joining the points (5, 7) and (3, 9) is the same as the mid–point the line segment joining the points (8, 6) and (0, 10).

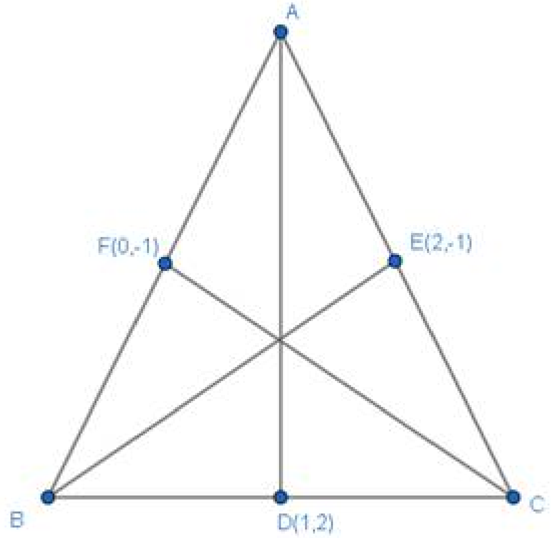
21. Question

If the mid–points of the sides of a triangle are (1, 2), (0, −1) and (2, −1), then find the coordinates of the vertices of the triangle.

Answer

Let A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) be the vertices of the given triangle

and D (1, 2), E (2, −1) and F (0, −1) be the mid points of the sides BC, CA and AB respectively.



Using the mid–point formula,

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (0, -1)$

$\Rightarrow x_1 + x_2 = 0$ and $y_1 + y_2 = -2$ ……(i)

$\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right) = (2, -1)$

$\Rightarrow x_1 + x_3 = 4$ and $y_1 + y_3 = -2$ ……(ii)

$\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right) = (1, 2)$

$\Rightarrow x_2 + x_3 = 2$ and $y_2 + y_3 = 4$ ……(iii)

Adding all the equations obtained for x coordinates and y coordinates respectively, we get

$2(x_1 + x_2 + x_3) = 6$

and $2(y_1 + y_2 + y_3) = 0$

$\Rightarrow x_1 + x_2 + x_3 = 3$ and $y_1 + y_2 + y_3 = 0$ –––(iv)

From eq (i) and (iv), we get $x_3 = 3$, $y_3 = 2$, i.e. C (3, 2)

From eq (ii) and (iv), we get $x_2 = -1$, $y_2 = 2$, i.e. B (–1, 2)

From eq (iii) and (iv), we get $x_1 = 1$, $y_3 = -4$, i.e. A (1, –4)

∴ The coordinates of the vertices of the triangle are A (1, –4), B (–1, 2) and C (3, 2).