Straight Lines

Case Study Based Questions

Read the following passages and answer the questions that follow:

1. If A and B are two persons standing at the positions (2,-3) and (6, -5). If C is a third person who is standing between A and B such that it divides the line AB in the ratio 1:3.

Based on the above information answer the following questions.

(A) The distance between A and B is:

(a) √5

(b) 2√5

(c) 3√5

(d) 4√5

(B) The equation of AB is:

(a) x+2y+4=0

(b) x+2y-4=0

(c) x-2y+4=0

(d) none of these

(C) Coordinates of points C are:

(a) $\left(\frac{7}{2},-3\right)$ (b) $\left(3,\frac{7}{2}\right)$

(c) (3, 3) (d) $\left(3,-\frac{7}{2}\right)$

(D) Distance between A and C is:

(a) $\sqrt{5}$ (b) $2\sqrt{5}$

(c) $\frac{\sqrt{5}}{2}$ (d) $\sqrt{\frac{5}{2}}$

(E) Distance between C and B is:

(a) $\frac{3\sqrt{5}}{2}$ (b) $3\sqrt{5}$

(c) $\frac{2\sqrt{5}}{3}$ (d) **None of these**

Ans. (A) (b) 2√5

Explanation: Given positions of person A and B are as follows:

A(2,-3) and B(6,-5)

$$d = \sqrt{(6-2)^2+(-5+3)^2}$$

[using distance formula]

$$= \sqrt{(4)^2+(-2)^2} = \sqrt{16+4}$$

$$= \sqrt{20} = 2\sqrt{5}$$

(B) (a)x+2y+4=0

Explanation: We have, A(2, -3) and B(6,-5)

$$\text{Slope, } m = \frac{-5-(-3)}{6-2}$$

$$= \frac{-5+3}{4}$$

$$= \frac{-2}{4} = -\frac{1}{2}$$

Taking point A(2, –3) = (x_1, y_1) and $m = -\frac{1}{2}$

Equation of line AB is

$$(y-(-3)) = -\frac{1}{2}(x-2)$$

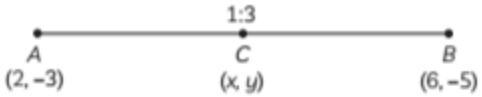
$$\Rightarrow \quad 2(y+3) = -(x-2)$$

$$\Rightarrow \quad 2y+6 = -x+2$$

$$\Rightarrow \quad x+2y+4 = 0$$

(C) *(d)* $\left(3, -\frac{7}{2}\right)$

Explanation: Let point C divides AB in the ratio m_1, and m_2.



$$\text{Then, } (x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{1\times6 + 3\times2}{1+3}, \frac{1\times(-5) + 3(-3)}{1+3}\right)$$

$$= \left(\frac{12}{4}, \frac{-14}{4}\right) = \left(3, -\frac{7}{2}\right)$$

(D)

(c) $\frac{\sqrt{5}}{2}$

Explanation: We have, A(2, -3) and

$C\left(3, -\frac{7}{2}\right)$

$$AC = \sqrt{(3-2)^2 + \left(-\frac{7}{2} + 3\right)^2}$$

$$= \sqrt{1^2 + \left(-\frac{1}{2}\right)^2}$$

$$= \sqrt{1 + \frac{1}{4}} = \frac{\sqrt{5}}{2}$$

(E)

(a) $\frac{3\sqrt{5}}{2}$

Explanation: We have, $C\left(3, -\frac{7}{2}\right)$ and B(6, –5)

$$CB = \sqrt{(6-3)^2 + \left(-5 + \frac{7}{2}\right)^2}$$

$$= \sqrt{3^2 + \left(-\frac{3}{2}\right)^2}$$

$$= \sqrt{9 + \frac{9}{4}} = \frac{\sqrt{45}}{4} = \frac{3\sqrt{5}}{2}$$

2. The corner points of a square plot are (1, 2), (2, 3), (3, 1) (0,-4). Plot is located in an industrial area to build a well known company showroom.

Based on the above information, answer the following questions:

(A) Find the distance between (1, 2) and
(3, 1) and also find the slope of (1, 2) and (3, 1).

(B) Find the distance between (2, 3) and
(0, -4) and also find the slope of (2, 3) and (0, -4).

(C) Determine <B of the triangle with vertices
A(-2, 1), B(2, 3) and C(-2, -4).

Ans. (A) Here, A = (1, 2) B = (3, 1)

$$AB = \sqrt{(3-1)^2+(1-2)^2}$$

$$= \sqrt{4+1}$$

$$= \sqrt{5}$$

Here, $x_1 = 2, x_2 = 0$

$y_1 = 3, y_2 = -4$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-4-3}{0-2}$$

$$= \frac{-7}{-2}$$

$$= \frac{7}{2}$$

(B)

Here, $A = (2, 3)$

$B = (0, -4)$

So, $AB = \sqrt{(0-2)^2 + (-4-3)^2}$

$= \sqrt{4+49}$

$= \sqrt{53}$

Here, $x_1 = 1, x_2 = 3$

$y_1 = 2, y_2 = 1$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1-2}{3-1}$$

$$= \frac{-1}{2}$$

(C)

Slope of line, AB $= \frac{3-1}{2+2} = \frac{2}{4} = \frac{1}{2} = m_1$ (say)

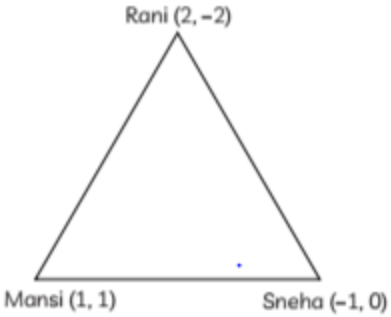
Slope of line, BC $= \frac{-4-3}{-2-2} = \frac{7}{4} = m_2$

$$\therefore \quad \tan B = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{7}{4} - \frac{1}{2}}{1 + \frac{1}{2} \cdot \frac{7}{4}}\right|$$

$$\angle B = \tan^{-1}\left(\frac{2}{3}\right)$$

3. Three girls Rani, Mansi, Sneha are talking to each other while maintaining a social distance due to covid-19. They are standing on vertices of a triangle, whose coordinates are given.



Based on the above information answer the following questions.

(A) The equation of lines formed by Rani and Mansi is:

(a) 3x - y = 4

(b) 3x + y = 4

(c) x-3y=4

(d) x + 3y = 4

(B) Slope of equation of line formed by Rani and Sneha is:

(a) $\frac{2}{3}$ (b) $\frac{-3}{2}$

(c) $\frac{-2}{3}$ (d) $\frac{1}{3}$

(C) The equation of median of lines through Rani is:

(a) 5x + 4y = 2

(b) 5x-4y = 2

(c) 4x-5y = 1

(D) The equation of

(d) none of these altitude through Mansi is:

(a) 3x-2y=1

(b) 2x + 3y = 5

(c) x+2y=3

(d) none of these

(E) The equation of line passing through the Rani and parallel to line formed by Mansi and Sneha is:

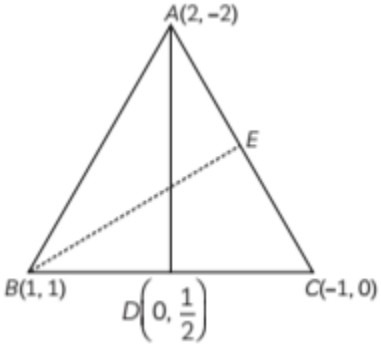
(a) x-2y= 4

(b) x + 2y = 6

(c) x-2y=6

(d) 2x + y = 4

Ans. Let the point on Rani, Mansi and Sneha stand on a vertices of triangles be A, B, C.
:- A(2,-2), B(1, 1), C(-1, 0)



(A) (b) 3x+y=4

Explanation: The equation of line AB is

$$y - 1 = \frac{-2-1}{2-1}(x-1)$$

$$\left[\because y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right]$$

$$\Rightarrow \quad y - 1 = -3x + 3$$

$$\Rightarrow \quad 3x + y = 4$$

(B)

(c) $-\frac{2}{3}$

Explanation: Slope of equation of line AC is

$$m = \frac{0+2}{-1-2} = \frac{2}{-3} = -\frac{2}{3}$$

(C) (a) 5x + 4y = 2

Explanation: Let D be the mid-point of BC.

Coordinates of D are $\left(\frac{1-1}{2}, \frac{0+1}{2}\right) = \left(0, \frac{1}{2}\right)$

$\therefore$ Equation of AD is $y + 2 = \frac{\frac{1}{2}+2}{0-2}(x-2)$

$$\Rightarrow \quad y + 2 = \frac{-5}{4}(x-2)$$

$$\Rightarrow \quad 4y + 8 = -5x + 10$$

$$\Rightarrow \quad 5x + 4y = 2$$

(D) (a) 3x-2y= 1

Explanation: Slope of AC = $\frac{-2}{3}$

$\therefore$ Slope of BE = $\frac{3}{2}$ [$\because$ BE $\perp$ AC]

Equation of altitude through B is

$$y - 1 = \frac{3}{2}(x - 1)$$

$\Rightarrow$ $3x - 2y = 1$

(E) (c) x-2y= 6

Explanation: Slope of line BC = $\frac{0-1}{-1-1} = \frac{1}{2}$

Equation of line passing through A and parallel to BC is

$$y + 2 = \frac{1}{2}(x - 2)$$

$\Rightarrow$ $2y + 4 = x - 2$

$\Rightarrow$ $x - 2y = 6$

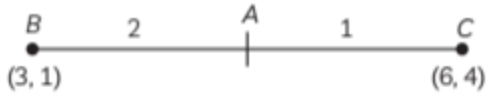
4. A triangular park has two of its vertices as B(-4, 1) and C(2, 11). The third vertex A is a point dividing the line joining the points (3, 1) and (6, 4) in the ratio 2:1.

(A) Find the coordinates of third vertex A.

(B) Find the equation of line passing through B and C.

(C) Find the equations of the sides of a triangle whose vertices are A(-1, 8), B(4,-2) and C(-5,-3).

Ans. (A)



Coordinates of

$$A = \left(\frac{2\times6+1\times3}{2+1}, \frac{2\times4+1\times1}{2+1}\right)$$

i. e., (5, 3)

(B) Equation of line through B(-4, 1)and C(2, 11) is

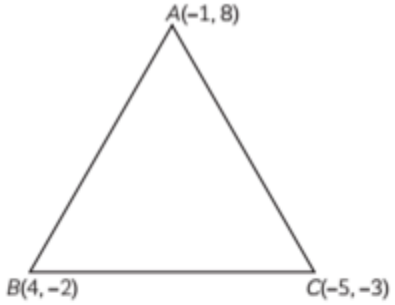
$$y-1=\frac{11-1}{2+4}(x+4)$$

$$\Rightarrow \quad y-1=\frac{5}{3}(x+4)$$

$$\Rightarrow \quad 3y-3=5x+20$$

$$\Rightarrow \quad 5x-3y+23=0$$

(C) Here, we use two points form to find the equation of sides.



Equation of AB is $y-8=\frac{-2-8}{4+1}(x+1)$

$$\Rightarrow \quad 5(y-8)+10(x+1)=0$$

$$\Rightarrow \quad 10x+5y-30=0$$

$$\Rightarrow \quad 2x+y-6=0$$

[dividing both sides by 5]

Equation of BC is

$$y+2=\frac{-3+2}{-5-4}(x-4)$$

$$\Rightarrow \quad -9(y+2)+(x-4)=0$$

$$\Rightarrow \quad x-9y-22=0$$

Equation of AC is $y-8=\frac{-3-8}{-5+1}(x+1)$

$$\Rightarrow \quad -4(y-8)+11(x+1)=0$$

$$\Rightarrow \quad 11x-4y+43=0$$