Sample Question Paper - 31
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours **Maximum Marks : 40**

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. Two numbers differ by 3 and their product is 504. Find the numbers.

OR

For what values of n, the equation $2x^2 - nx + n = 0$ has coincident roots?

2. Draw a line segment of length 8 cm and divide it internally in the ratio 4 : 5.

3. The height of a pillar is 8 m. What is the length of its shadow, when sun's altitude is 30°?

4. Find the mean of the following distribution :

Class	0-6	6-12	12-18	18-24	24-30
Frequency	7	5	10	12	2

5. A tower stands vertically on the ground. From a point on the ground which is 25 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 45°. Find the height (in meters) of the tower.

OR

The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 45°. Find the height of the tower (in metres).

6. Find the mode of the following data :

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	7	14	13	12	20	11	15	8

SECTION - B

7. A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?

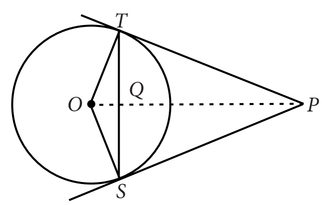
8. An electrician has to repair an electric fault on a pole of height 7 m. He needs to reach a point 2.2 m below the top of the pole to undertake the repair work. What should be the length of the ladder that he should use which, when inclined at an angle of 60° to the horizontal, would enable him to reach the required position? Also, how far from the foot of the pole should he place the foot of the ladder?

OR

A man standing on the deck of a ship, which is 10 m above water level, observes the angle of elevation of the top of a hill as 60° and angle of depression of the base of the hill as 30°. Find the horizontal distance of the hill from the ship and height of the hill.

9. A metallic cylinder has radius 3 cm and height 5 cm. To reduce its weight, a conical hole is drilled in the cylinder. The conical hole has a radius of $\frac{3}{2}$ cm and its depth is $\frac{8}{9}$ cm. Calculate the ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape.

10. In the given figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r. If $OP = 2r$, show that $\angle OTS = \angle OST = 30°$.



SECTION - C

11. Two pipes running together can fill a tank in 12 mins. If one pipe takes 10 mins less than twice the other to fill the tank. Find the time in which each pipe would fill the tank.

OR

Prove that the equation $x^2(a^2 + b^2) + 2x(ac + bd) + (c^2 + d^2) = 0$ has no real roots, if $ad \neq bc$.

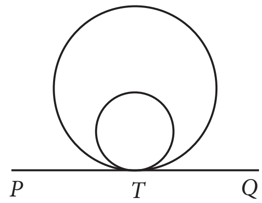
12. Kanika was given her pocket money on Jan 1st, 2008. She puts ₹ 1 on Day 1, ₹ 2 on Day 2, ₹ 3 on Day 3, and continued doing so till the end of the month, from this money into her piggy bank. She also spent ₹ 204 of her pocket money, and found that at the end of the month she still had ₹ 100 with her. How much was her pocket money for the month?

Case Study - 1

13. Following are questions of section-A in assessment test on circle that Eswar attend last month in school. He scored full marks in this section. Answer the questions and check your score if 2 marks is allotted to each question.

(i) If two tangents AB and CD drawn to a circle with centre O at P and Q respectively, are parallel to each other, then show that PQ is a diameter of a circle.

(ii) In the given figure, is PQ a tangent to both the circles?



Case Study - 2

14. An inspector in an enforcement squad of electricity department visit to a locality of 100 families and record their monthly consumption of electricity, on the basis of family members, electronic items in the house and wastage of electricity, which is summarise in the following table.

Monthly Consumption (in kwh)	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
Number of families	2	5	x	12	17	20	y	9	7	4

Based on the above information, answer the following questions.

(i) If the median of the above data is 545, then find the value of x.

(ii) Find the average monthly consumption of a family of this locality approximately.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. Let one number be x.

$\therefore$ Other number be $x + 3$.

According to question, $x(x + 3) = 504$

$\Rightarrow x^2 + 3x - 504 = 0 \Rightarrow x^2 + 24x - 21x - 504 = 0$

$\Rightarrow x(x + 24) - 21(x + 24) = 0 \Rightarrow (x + 24)(x - 21) = 0$

$\Rightarrow x + 24 = 0$ or $x - 21 = 0 \Rightarrow x = -24$ or $x = 21$

When $x = -24$, numbers are -24 and $-24 + 3 = -21$

When $x = 21$, numbers are 21 and $21 + 3 = 24$.

OR

We have, $2x^2 - nx + n = 0$

Here, $a = 2$, $b = -n$ and $c = n$.

$\therefore D = b^2 - 4ac = (-n)^2 - 4(2)(n) = n^2 - 8n = n(n - 8)$

Now, the given equation has coincident roots *i.e.,* equal roots, so $D = 0 \Rightarrow n(n - 8) = 0 \Rightarrow n = 0$ or $n = 8$

2. Steps of construction :

Step-I : Draw a line segment $AB = 8$ cm.

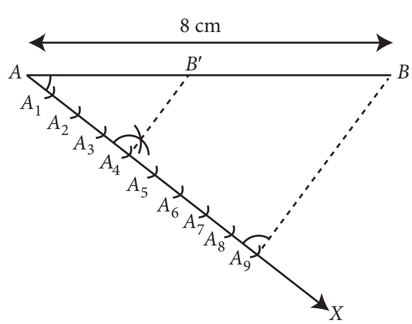
Step-II : Taking point A, draw a ray AX below the line segment AB making an acute angle $\angle BAX$.

Step-III : Mark 9 points $A_1, A_2, \ldots\ldots, A_9$ on AX such that $AA_1 = A_1A_2 = A_2A_3$ and so on.

Step-IV : Join A_9B.

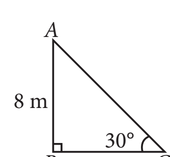
Step-V : Now, draw a parallel line, from point $A_4(A_4B' \parallel A_9B)$ which intersect AB at B'.

Thus, AB is divided internally in the ratio 4 : 5.



3. Let AB be the pillar of height 8 m and BC be the length of its shadow.

In right ΔABC,



$$\tan 30° = \frac{AB}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{8}{BC}$$

$\Rightarrow BC = 8\sqrt{3}$ m

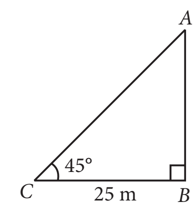
Thus, length of the shadow of the pillar is $8\sqrt{3}$ m.

4.

Class	Class marks (x_i)	Frequency (f_i)	$f_i x_i$
0-6	3	7	21
6-12	9	5	45
12-18	15	10	150
18-24	21	12	252
24-30	27	2	54
		$\Sigma f_i = 36$	$\Sigma f_i x_i = 522$

$$\therefore \text{Mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{522}{36} = 14.5$$

5. Let AB be the tower and C be the point on the ground 25 m away from the foot of the tower such that $\angle ACB = 45°$.



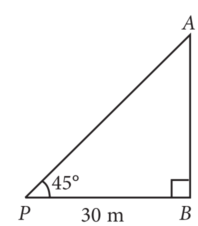
Now, In right ΔABC,

$$\tan 45° = \frac{AB}{BC} \Rightarrow 1 = \frac{AB}{25} \Rightarrow AB = 25 \text{ m}$$

Thus, the height of the tower is 25 m.

OR

Let AB be the tower and P be the point on the ground.



In ΔABP, $\frac{AB}{BP} = \tan 45°$

$$\Rightarrow \frac{AB}{30} = 1 \Rightarrow AB = 30 \text{ m}$$

6. From the given data, we observe that, highest frequency is 20, which lies in the class-interval 40-50. Here, $l = 40, f_1 = 20, f_0 = 12, f_2 = 11, h = 10$.

$$\therefore \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left(\frac{20 - 12}{40 - 12 - 11}\right) \times 10$$

$$= 40 + \frac{80}{17} = 40 + 4.7 = 44.7$$

7. Given that a heap of rice is in the form of a cone.

Height of a heap of rice (h) = 3.5 m

and diameter of a heap of rice = 24 m

$\therefore$ Radius of a heap of rice (r) = 12 m

$$\left[\because \text{Radius} = \frac{1}{2} \times \text{diameter}\right]$$

So, volume of rice $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 3.5 = 528 \text{ m}^3$

Now, canvas cloth required to just cover heap of rice = Surface area of a heap of rice = πrl

$= \frac{22}{7} \times r \times \sqrt{r^2 + h^2} = \frac{22}{7} \times 12 \times \sqrt{(12)^2 + (3.5)^2}$

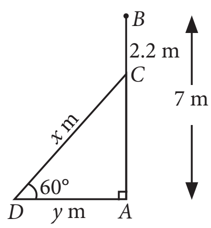
$= \frac{12 \times 22}{7} \times \sqrt{144 + 12.25} = \frac{12 \times 22}{7} \times \sqrt{156.25}$

$= \frac{12 \times 22}{7} \times 12.5 = 471.42 \text{ m}^2$

Hence, 471.42 m^2 canvas cloth is required to just cover the heap.

8. Let AB be the pole of height 7 m.

Let at point C, the electrician has to do the repair work and CD be the ladder of length x m. Let $AD = y$ m



Here, $AC = AB - CB$

$= 7 - 2.2 = 4.8$ m

In right ΔADC, $\tan 60° = \frac{AC}{AD}$

$\Rightarrow \sqrt{3} = \frac{4 \cdot 8}{y} \Rightarrow y = \frac{4 \cdot 8}{\sqrt{3}} = \frac{4.8 \times \sqrt{3}}{3} = 1.6 \times 1.732 = 2.77$

and $\operatorname{cosec} 60° = \frac{CD}{AC} \Rightarrow \frac{2}{\sqrt{3}} = \frac{x}{4.8} \Rightarrow x = \frac{9.6}{\sqrt{3}}$

$\Rightarrow x = \frac{9.6 \times \sqrt{3}}{3} = 3.2 \times 1.732 = 5.54$

$\therefore$ Length of ladder is 5.54 m and distance between foot of pole and foot of ladder is 2.77 m.

OR

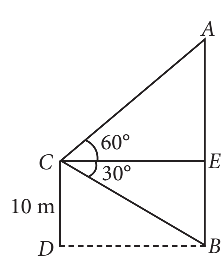
Let AB be the hill and C be the position of man on the deck of a ship which is 10 m above water level.

Then, $CD = BE = 10$ m

In right ΔBEC, $\frac{CE}{BE} = \cot 30°$

$\Rightarrow CE = BE \cdot \cot 30°$

$\Rightarrow CE = 10 \times \sqrt{3} = 17.32$ m



In right ΔAEC, $\frac{AE}{CE} = \tan 60°$

$\Rightarrow AE = CE \cdot \sqrt{3}$

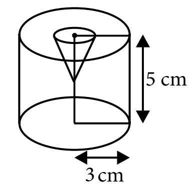
$\Rightarrow AE = 10 \times \sqrt{3} \times \sqrt{3} = 30$ m

$\therefore$ Height of the hill, $AB = AE + EB = (30 + 10)\text{m} = 40$ m

Distance of hill from the ship = $CE = 17.32$ m

9. Radius of cylinder, $r_1 = 3$ cm

Height of cylinder, $h_1 = 5$ cm



$\therefore$ Volume of cylinder

$= \pi r_1^2 h_1 = \pi(3)^2 \times 5$

$= 45\pi \text{ cm}^3$

Radius of cone, $r_2 = \frac{3}{2}$ cm;

Height of cone, $h_2 = \frac{8}{9}$ cm

Volume of cone = Volume of metal taken out

$= \frac{1}{3}\pi r_2^2 h_2$

$= \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 \times \frac{8}{9} = \frac{2}{3}\pi \text{ cm}^3$ ...(i)

Volume of metal left in the cylinder = Volume of cylinder – Volume of cone

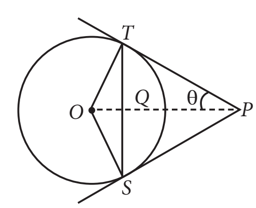
$= 45\pi - \frac{2}{3}\pi = \frac{133\pi}{3}$

$\therefore \frac{\text{Volume of metal left in cylinder}}{\text{Volume of metal taken out}} = \frac{\frac{133\pi}{3}}{\frac{2}{3}\pi}$

$= \frac{133}{2} = 133 : 2$

10. In ΔOTP, $OT = r$, $OP = 2r$ [Given]

$\angle OTP = 90°$[Radius is perpendicular to tangent at the point of contact]



Let $\angle TPO = \theta$

$\therefore \sin\theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2}$

$\Rightarrow \theta = 30°$

$\therefore$ In ΔTOP, $\angle TOP = 60°$

[By angle sum property]

$\angle TOP = \angle SOP$ [As Δ's are congruent]

$\Rightarrow \angle SOP = 60° \therefore \angle TOS = 120°$

In ΔOTS,

$OT = OS$ (Radii of same circle)

$\therefore \angle OST = \angle OTS$

Now, $\angle OTS + \angle OST + \angle SOT = 180°$

$\Rightarrow 2\angle OST + 120° = 180°$

$\therefore \angle OTS = \angle OST = 30°$

Hence Proved.

11. Let one pipe fill the tank in x mins, then the other pipe fill the tank in $(2x - 10)$ mins.

According to question,

$\frac{1}{x} + \frac{1}{2x - 10} = \frac{1}{12} \Rightarrow \frac{2x - 10 + x}{x(2x - 10)} = \frac{1}{12}$

$\Rightarrow$ $(3x - 10)\,12 = x(2x - 10) \Rightarrow 36x - 120 = 2x^2 - 10x$

$\Rightarrow$ $2x^2 - 46x + 120 = 0 \Rightarrow x^2 - 23x + 60 = 0$

Here, $a = 1$, $b = -23$ and $c = 60$

$\therefore$ $b^2 - 4ac = (-23)^2 - 4(1)(60) = 529 - 240 = 289 > 0$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-23) \pm \sqrt{289}}{2(1)} = \frac{23 \pm 17}{2}$$

$$\Rightarrow \quad x = \frac{23+17}{2} \text{ or } x = \frac{23-17}{2} \quad \Rightarrow x = \frac{40}{2} \text{ or } x = \frac{6}{2}$$

$\Rightarrow$ $x = 20$ or $x = 3$

If $x = 3$, then $2x - 10 = 2(3) - 10 = -4$, which is not possible. $\therefore$ $x = 20$

Thus, one pipe fill the tank in 20 mins and other pipe fill the tank in $2(20) - 10 = 30$ mins.

OR

We have, $x^2(a^2 + b^2) + 2x\,(ac + bd) + (c^2 + d^2) = 0$

Comparing the given equation with $Ax^2 + Bx + C = 0$, we have, $A = a^2 + b^2$, $B = 2(ac + bd)$, $C = c^2 + d^2$

$\therefore$ $D = B^2 - 4AC = [2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2)$

$= 4(a^2c^2 + b^2d^2 + 2acbd) - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2)$

$= 4[2acbd - a^2d^2 - b^2c^2] = -4[a^2d^2 + b^2c^2 - 2(ad)(bc)]$

$= -4(ad - bc)^2 < 0$ if $ad \neq bc$

Thus, given equation has no real roots if $ad \neq bc$.

12. Let her pocket money be ₹ x.

Now, she puts ₹ 1 on day 1, ₹ 2 on day 2, ₹ 3 on day 3 and so on till the end of the month, from this money into her piggy bank. *i.e.,* $1 + 2 + 3 + 4 + ... + 31$, which forms an A.P. in which number of terms are 31 and first term, $a = 1$, common difference, $d = 2 - 1 = 1$

$\therefore$ Sum of first 31 terms *i.e.*,

$$S_{31} = \frac{31}{2}[2 \times 1 + (31 - 1) \times 1] \quad \left(\because\ S_n = \frac{n}{2}[2a + (n-1)d]\right)$$

$$= \frac{31}{2}(2 + 30) = \frac{31 \times 32}{2} = 31 \times 16 = 496$$

So, Kanika takes ₹ 496 till the end of the month from this money.

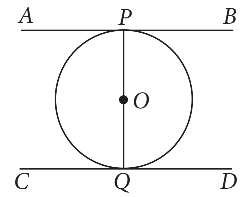
Also, she spent ₹ 204 of her pocket money and found that at the end of the month she still has ₹ 100 with her.

Now, according to the question, $(x - 496) - 204 = 100$

$\Rightarrow$ $x - 700 = 100$ $\therefore$ $x =$ ₹ 800

Hence, ₹ 800 was her pocket money for the month.

13. (i)



Two tangents of a circle are parallel only when they are drawn at ends of a diameter.

So, PQ is the diameter of the circle.

(ii) Here, the two circles have a common point of contact T and PQ is the tangent at T. So, PQ is the tangent to both the circles.

14. We have the following table :

Class interval	Frequency	Cumulative frequency
0-100	2	2
100-200	5	7
200-300	x	$7 + x$
300-400	12	$19 + x$
400-500	17	$36 + x$
500-600	20	$56 + x$
600-700	y	$56 + x + y$
700-800	9	$65 + x + y$
800-900	7	$72 + x + y$
900-1000	4	$76 + x + y$
Total	$76 + x + y$	

(i) Here, $\frac{N}{2} = \frac{100}{2} = 50$

Also, median = 545 [Given]

$\therefore$ Median class is 500-600.

Now, median $= l + \left(\frac{N/2 - c.f.}{f}\right) \times h$

$\Rightarrow$ $545 = 500 + \left(\frac{50 - (36 + x)}{20}\right) \times 100$

$\Rightarrow$ $9 = 50 - 36 - x \Rightarrow x = 5$

(ii) Since, $x + y = 24$

$\Rightarrow$ $y = 24 - 5 = 19$

Required average consumption

$$= \frac{50 \times 2 + 150 \times 5 + 250 \times 5 + 350 \times 12 + 450 \times 17 + 550 \times 20 + 650 \times 19 + 750 \times 9 + 850 \times 7 + 950 \times 4}{100}$$

$$= \frac{53800}{100} = 538 \text{ kwh}$$