# Geometric Progression

## Exercise 11(A)

### Solution 1(i).

Given sequence: 8, 24, 72, 216.........

Now,

$\frac{24}{8} = 3, \ \frac{72}{24} = 3, \ \frac{216}{72} = 3$

Since $\frac{24}{8} = \frac{72}{24} = \frac{216}{72} = ...... = 3$, the given sequence is a G.P.

with common ratio 3.

### Solution 1(ii).

Given sequence: $\frac{1}{8}, \frac{1}{24}, \frac{1}{72}, \frac{1}{216}$.........

Now,

$\frac{1/24}{1/8} = \frac{1}{3}, \ \frac{1/72}{1/24} = \frac{1}{3}, \ \frac{1/216}{1/72} = \frac{1}{3}$

Since $\frac{1/24}{1/8} = \frac{1/72}{1/24} = \frac{1/216}{1/72} = ....... = \frac{1}{3}$, the given sequence is a G.P.

with common ratio $\frac{1}{3}$.

### Solution 1(iii).

Given sequence: 9, 12, 16, 24.........

Now,

$\frac{12}{9} = \frac{4}{3}, \ \frac{16}{12} = \frac{4}{3}, \ \frac{24}{16} = \frac{3}{2}$

Since $\frac{24}{8} = \frac{72}{24} \neq \frac{216}{72}$, the given sequence is not a G.P.

## Solution 2.

Given sequence: 1, 4, 16, 64.........

Now,

$\frac{4}{1} = 4, \quad \frac{16}{4} = 4, \quad \frac{64}{16} = 4$

Since $\frac{4}{1} = \frac{16}{4} = \frac{64}{16} = \ldots\ldots = 4$, the given sequence is a G.P.

with first term, a = 1 and common ratio, r = 4.

Now, $t_n = ar^{n-1}$

$\Rightarrow t_9 = 1 \times 4^8 = 65536$

## Solution 3.

Given G.P.: 1, $\sqrt{3}$, 3, $3\sqrt{3}$, ....

Here,

First term, a = 1

Common ration, $r = \frac{\sqrt{3}}{1} = \sqrt{3}$

Now, $t_n = ar^{n-1}$

$\Rightarrow t_7 = 1 \times \left(\sqrt{3}\right)^6 = 27$

## Solution 4.

Given sequence: $\frac{3}{4}, 1\frac{1}{2}, 3, \ldots\ldots$

i.e. $\frac{3}{4}, \frac{3}{2}, 3, \ldots\ldots$

Now,

$\frac{3/2}{3/4} = 2, \quad \frac{3}{3/2} = 2,$

Since $\frac{3/2}{3/4} = \frac{3}{3/2} = \ldots\ldots = 2$, the given sequence is a G.P.

with first term, $a = \frac{3}{4}$ and common ratio, r = 2.

Now, $t_n = ar^{n-1}$

$\Rightarrow t_8 = \frac{3}{4} \times 2^7 = \frac{3}{4} \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 3 \times 2^5 = 96$

## Solution 5.

Given G.P.: 12, 4, $1\frac{1}{3}$,.......

Here,

First term, $a = 12$

Common ration, $r = \frac{4}{12} = \frac{1}{3}$

Now, $t_n = ar^{n-1}$

$\Rightarrow t_{10} = 12 \times \left(\frac{1}{3}\right)^9 = 12 \times \frac{1}{19683} = \frac{4}{6561}$

## Solution 6.

Given series: 1, 2, 4, 8, .....

Now,

$\frac{2}{1} = 2, \ \frac{4}{2} = 2, \ \frac{8}{4} = 2$

Since $\frac{2}{1} = \frac{4}{2} = \frac{8}{4} = ...... = 2$, the given sequence is a G.P.

with first term, $a = 1$ and common ratio, $r = 2$.

Now, $t_n = ar^{n-1}$

$\Rightarrow t_n = 1 \times 2^{n-1} = 2^{n-1}$

## Solution 7.

Given sequence: $\sqrt{5}$, 5, $5\sqrt{5}$,........

Now,

$\frac{5}{\sqrt{5}} = \sqrt{5}, \ \frac{5\sqrt{5}}{5} = \sqrt{5}$

Since $\frac{5}{\sqrt{5}} = \frac{5\sqrt{5}}{52} = ...... = \sqrt{5}$, the given sequence is a G.P.

with first term, $a = \sqrt{5}$ and common ratio, $r = \sqrt{5}$.

Now, $t_n = ar^{n-1}$

$\therefore$ Next three terms:

$4^{th}$ term $= \sqrt{5} \times \left(\sqrt{5}\right)^3 = \sqrt{5} \times 5\sqrt{5} = 25$

$5^{th}$ term $= \sqrt{5} \times \left(\sqrt{5}\right)^4 = \sqrt{5} \times 25 = 25\sqrt{5}$

$6^{th}$ term $= \sqrt{5} \times \left(\sqrt{5}\right)^5 = \sqrt{5} \times 25\sqrt{5} = 125$

## Solution 8.

Given sequence: $2^2$, $2^3$, $2^4$,.....

Now,

$$\frac{2^3}{2^2} = 2, \quad \frac{2^4}{2^3} = 2$$

Since $\frac{2^3}{2^2} = \frac{2^4}{2^3} = ...... = 2$, the given sequence is a G.P.

with first term, $a = 2^2 = 4$ and common ratio, $r = 2$.

Now, $t_n = ar^{n-1}$

$\therefore\ t_6 = 4 \times (2)^5 = 4 \times 32 = 128$

## Solution 9.

Given G.P.: $\sqrt{3} + 1, 1, \frac{\sqrt{3} - 1}{2}, ......$

Here,

First term, $a = \sqrt{3} + 1$

Common ration, $r = \frac{1}{\sqrt{3} + 1}$

Now, $t_n = ar^{n-1}$

$$\Rightarrow t_7 = \left(\sqrt{3} + 1\right) \times \left(\frac{1}{\sqrt{3} + 1}\right)^6$$

$$= \left(\frac{1}{\sqrt{3} + 1}\right)^5$$

$$= \left(\frac{1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}\right)^5$$

$$= \left(\frac{\sqrt{3} - 1}{2}\right)^5$$

$$= \frac{1}{32}\left(\sqrt{3} - 1\right)^5$$

## Solution 10.

First term, $a = 64$

Second term, $t_2 = 32$

$\Rightarrow ar = 32$

$\Rightarrow 64 \times r = 32$

$\Rightarrow r = \frac{32}{64} = \frac{1}{2}$

$\therefore$ Required G.P. $= a, ar, ar^{n-1}, ar^{n-2}, \ldots\ldots$

$$= 64, 32, 64 \times \left(\frac{1}{2}\right)^2, 64 \times \left(\frac{1}{2}\right)^3, \ldots\ldots$$

$$= 64, 32, 16, 8, \ldots\ldots$$

## Solution 11.

Given sequence: $\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \ldots\ldots$

Now,

$$\frac{2/9}{2/27} = 3, \quad \frac{2/3}{2/9} = 3$$

Since $\frac{2/9}{2/27} = \frac{2/3}{2/9} = \ldots\ldots = 3$, the given sequence is a G.P.

with first term, $a = \frac{2}{27}$ and common ratio, $r = 3$.

Now, $t_n = ar^{n-1}$

$\therefore$ Next three terms:

$4^{th}$ term $= \frac{2}{27} \times (3)^3 = \frac{2}{27} \times 27 = 2$

$5^{th}$ term $= \frac{2}{27} \times (3)^4 = \frac{2}{27} \times 27 \times 3 = 6$

$6^{th}$ term $= \frac{2}{27} \times (3)^5 = \frac{2}{27} \times 27 \times 9 = 18$

## Solution 12.

Given series: $2 - 6 + 18 - 54 \ldots\ldots$

Now,

$$\frac{-6}{2} = -3, \quad \frac{18}{-6} = -3, \quad \frac{-54}{18} = -3$$

Since $\frac{-6}{2} = \frac{18}{-6} = \frac{-54}{18} = \ldots\ldots = -3$, the given sequence is a G.P.

with first term, $a = 2$ and common ratio, $r = -3$.

Now, $t_n = ar^{n-1}$

$\therefore$ Next two terms:

$5^{th}$ term $= 2 \times (-3)^4 = 2 \times 81 = 162$

$6^{th}$ term $= 2 \times (-3)^5 = 2 \times (-243) = -486$

# Exercise 11(B)

## Solution 1.

For the given G.P.:

First term, $a = -10$

$$\text{Common ratio, } r = \frac{5/\sqrt{3}}{-10} = -\frac{1}{2\sqrt{3}}$$

If $-\frac{5}{72}$ is the $n^{th}$ term of the given G.P., then

$$-\frac{5}{72} = ar^{n-1}$$

$$\Rightarrow -\frac{5}{72} = -10 \times \left(\frac{1}{2\sqrt{3}}\right)^{n-1}$$

$$\Rightarrow \frac{1}{144} = \left(\frac{1}{2\sqrt{3}}\right)^{n-1}$$

$$\Rightarrow \frac{1}{2 \times 2 \times 2 \times 2 \times \sqrt{3} \times \sqrt{3} \times \sqrt{3} \times \sqrt{3}} = \left(\frac{1}{2\sqrt{3}}\right)^{n-1}$$

$$\Rightarrow \left(\frac{1}{2\sqrt{3}}\right)^4 = \left(\frac{1}{2\sqrt{3}}\right)^{n-1}$$

$\Rightarrow n - 1 = 4$

$\Rightarrow n = 5$

## Solution 2.

Let the first term of the G.P. be a and its common ratio be r.

$5^{th}$ term = 81 $\Rightarrow ar^4 = 81$

$2^{nd}$ term = 24 $\Rightarrow ar = 24$

Now, $\frac{ar^4}{ar} = \frac{81}{24}$

$\Rightarrow r^3 = \frac{27}{8}$

$\Rightarrow r = \frac{3}{2}$

$ar = 24$

$\Rightarrow a \times \frac{3}{2} = 24$

$\Rightarrow a = 16$

$\therefore$ G.P. = a, ar, $ar^2$, $ar^3$, ......

$= 16, 24, 16 \times \left(\frac{3}{2}\right)^2, 16 \times \left(\frac{3}{2}\right)^3, .....$

$= 16, 24, 36, 54, .....$

## Solution 3.

Let the first term of the G.P. be a and its common ratio be r.

$4^{th}$ term = $\frac{1}{18} \Rightarrow ar^3 = \frac{1}{18}$

$7^{th}$ term = $-\frac{1}{486} \Rightarrow ar^6 = -\frac{1}{486}$

Now, $\frac{ar^6}{ar^3} = \frac{-1/486}{1/18}$

$\Rightarrow r^3 = -\frac{1}{27}$

$\Rightarrow r = -\frac{1}{3}$

$ar^3 = \frac{1}{18}$

$\Rightarrow a \times \left(-\frac{1}{3}\right)^3 = \frac{1}{18}$

$\Rightarrow a = -\frac{27}{18} = -\frac{3}{2}$

$\therefore$ G.P. = a, ar, $ar^2$, $ar^3$, ......

$= -\frac{3}{2}, -\frac{3}{2} \times \left(\frac{-1}{3}\right), -\frac{3}{2} \times \left(-\frac{1}{3}\right)^2, \frac{1}{18}, .....$

$= -\frac{3}{2}, \frac{1}{2}, -\frac{1}{6}, \frac{1}{18}, .....$

## Solution 4.

Let the first term of the G.P. be a and its common ratio be r.

$\therefore$ $1^{st}$ term = a = 2

And, $3^{rd}$ term = 8 $\Rightarrow ar^2 = 8$

Now, $\frac{ar^2}{a} = \frac{8}{2}$

$\Rightarrow r^2 = 4$

$\Rightarrow r = \pm 2$

When a = 2 and r = 2

$2^{nd}$ term = ar = 2 x 2 = 4

When a = 2 and r = −2

$2^{nd}$ term = ar = 2 x (−2) = −4

## Solution 5.

Let the first term of the G.P. be a and its common ratio be r.

Now,

$t_3 \times t_8 = 243$

$\Rightarrow ar^2 \times ar^7 = 243$

$\Rightarrow a^2r^9 = 243$ ....(i)

Also,

$t_4 = 3$

$\Rightarrow ar^3 = 3$

$\Rightarrow a = \frac{3}{r^3}$

Substituting the value of a in (i), we get

$\left(\frac{3}{r^3}\right)^2 \times r^9 = 243$

$\Rightarrow \frac{9}{r^6} \times r^9 = 243$

$\Rightarrow r^3 = 27$

$\Rightarrow r = 3$

$\Rightarrow a = \frac{3}{3^3} = \frac{3}{27} = \frac{1}{9}$

$\therefore$ $7^{th}$ term = $t_7 = ar^6 = \frac{1}{9} \times (3)^6 = 81$

## Solution 6.

Let the first term of the G.P. be a and its common ratio be r.

$4^{th}$ term = 54 $\Rightarrow ar^3 = 54$

$7^{th}$ term = 1458 $\Rightarrow ar^6 = 1458$

Now, $\frac{ar^6}{ar^3} = \frac{1458}{54}$

$\Rightarrow r^3 = 27$

$\Rightarrow r = 3$

$ar^3 = 54$

$\Rightarrow a \times (3)^3 = 54$

$\Rightarrow a = \frac{54}{27} = 2$

$\therefore$ G.P. = $a, ar, ar^2, ar^3, \ldots\ldots$

$= 2, 2 \times 3, 2 \times (3)^2, 54, \ldots\ldots$

$= 2, 6, 18, 54, \ldots\ldots$

## Solution 7.

Let the first term of the G.P. be a and its common ratio be r.

Now, $2^{nd}$ term = $t_2 = 6 \Rightarrow ar = 6$

Also, $t_5 = 9 \times t_3$

$\Rightarrow ar^4 = 9 \times ar^2$

$\Rightarrow r^2 = 9$

$\Rightarrow r = \pm 3$

Since, each term of a G.P. is positive, we have r = 3

$ar = 6$

$\Rightarrow a \times 3 = 6 \Rightarrow a = 2$

$\therefore$ G.P. = $a, ar, ar^2, ar^3, \ldots\ldots$

$= 2, 6, 2 \times (3)^2, 2 \times (3)^3, \ldots\ldots$

$= 2, 6, 18, 54, \ldots\ldots$

## Solution 8.

Let the first term of the G.P. be a and its common ratio be r.

Now,

$4^{th}$ term = $t_4 = 10 \Rightarrow ar^3 = 10$

$7^{th}$ term = $t_7 = 80 \Rightarrow ar^6 = 80$

$$\frac{ar^6}{ar^3} = \frac{80}{10}$$

$\Rightarrow r^3 = 8$

$\Rightarrow r = 2$

$ar^3 = 10$

$\Rightarrow a \times (2)^3 = 10$

$\Rightarrow a = \frac{10}{8} = \frac{5}{4}$

Last term = l = 2560

Let there be n terms in given G.P.

$\Rightarrow t_n = 2560$

$\Rightarrow ar^{n-1} = 2560$

$\Rightarrow \frac{5}{4} \times (2)^{n-1} = 2560$

$\Rightarrow (2)^{n-1} = 2048$

$\Rightarrow (2)^{n-1} = (2)^{11}$

$\Rightarrow n - 1 - 11$

$\Rightarrow n = 12$

Thus, we have

First term = $\frac{5}{4}$, Common ratio = 2 and Number of terms = 12

## Solution 9.

Let the first term of the G.P. be a and its common ratio be r.

Now,

$4^{th}$ term = $t_4 = 54 \Rightarrow ar^3 = 54$

$9^{th}$ term = $t_9 = 13122 \Rightarrow ar^8 = 13122$

$$\frac{ar^8}{ar^3} = \frac{13122}{54}$$

$\Rightarrow r^5 = 243$

$\Rightarrow r = 3$

$ar^3 = 54$

$\Rightarrow a \times (3)^3 = 54$

$\Rightarrow a = \frac{54}{27} = 2$

$\therefore$ Required G.P. = $a,\ ar,\ ar^2,\ ar^3, \ldots\ldots$

$= 2, 2\times 3,\ 2\times(3)^2,\ 54$

$= 2,\ 6,\ 18,\ 54$

General term = $t_n = ar^{n-1} = 2\times(3)^{n-1}$

## Solution 10.

Let the first term of the G.P. be a and its common ratio be r.

$5^{th}$ term = $t_5 = p$

$\Rightarrow ar^4 = p$

$8^{th}$ term = $t_8 = q$

$\Rightarrow ar^7 = q$

$11^{th}$ term = $t_{11} = r$

$\Rightarrow ar^{10} = r$

Now,

$pr = ar^4 \times ar^{10} = a^2 \times r^{14} = (a\times r^7)^2 = q^2$

$\Rightarrow q^2 = pr$

## Exercise 11(C)

### Solution 1.

Given series: $\sqrt{2}, 2, 2\sqrt{2}, \ldots, 32$

Now, $\frac{2}{\sqrt{2}} = \sqrt{2}, \ \frac{2\sqrt{2}}{2} = \sqrt{2}$

So, the given series is a G.P. with common ratio, $r = \sqrt{2}$

Here, last term, $l = 32$

$\therefore$ $7^{th}$ term from an end $= \frac{l}{r^6} = \frac{32}{\left(\sqrt{2}\right)^6} = \frac{32}{8} = 4$

### Solution 2.

Given G.P.: $\frac{2}{27}, \frac{2}{9}, \frac{2}{3}, \ldots\ldots, 162$

Here,

Common ratio, $r = \frac{2/9}{2/27} = 3$

Last term, $l = 162$

$\therefore$ $3^{rd}$ term from an end $= \frac{l}{r^2} = \frac{162}{(3)^2} = \frac{162}{9} = 18$

### Solution 3.

Given G.P.: $\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, \ldots\ldots, 81$

Here,

Common ratio, $r = \frac{1/9}{1/27} = 3$

First term, $a = \frac{1}{27}$ and Last term, $l = 81$

$\therefore$ $4^{th}$ term from the beginning $= ar^3 = \frac{1}{27} \times (3)^3 = \frac{1}{27} \times 27 = 1$

And, $4^{th}$ term from an end $= \frac{l}{r^3} = \frac{81}{(3)^3} = \frac{81}{27} = 3$

Thus, required product $= 1 \times 3 = 3$

## Solution 4.

Let the first term of the G.P. be A and its common ratio be R.

Then,

$p^{th}$ term = a $\Rightarrow AR^{p-1} = a$

$q^{th}$ term = b $\Rightarrow AR^{q-1} = b$

$r^{th}$ term = c $\Rightarrow AR^{r-1} = c$

Now,

$$a^{q-r} \times b^{r-p} \times c^{p-q} = (AR^{p-1})^{q-r} \times (AR^{q-1})^{r-p} \times (AR^{r-1})^{p-q}$$
$$= A^{q-r}.R^{(p-1)(q-r)} \times A^{r-p}.R^{(q-1)(r-p)} \times A^{p-q}.R^{(r-1)(p-q)}$$
$$= A^{q-r+r-p+p-q} \times R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$$
$$= A^0 \times R^0$$
$$= 1$$

Taking log on both the sides, we get

$$\log\left(a^{q-r} \times b^{r-p} \times c^{p-q}\right) = \log 1$$

$$\Rightarrow (q-r)\log a + (r-p)\log b + (p-q)\log c = 0 \qquad \ldots.(\text{proved})$$

## Solution 5.

Let the first term of the G.P. be a and its common ratio be r.

Then,

$(p+q)^{th}$ term = m $\Rightarrow ar^{p+q-1} = m$

$(p-q)^{th}$ term = n $\Rightarrow ar^{p-q-1} = n$

Now,

$$m \times n = ar^{p+q-1} \times ar^{p-q-1}$$
$$= a^2 \times r^{p+q-1+p-q-1}$$
$$= a^2 \times r^{2p-2}$$
$$= a^2 \times r^{2(p-1)}$$
$$= (a \times r^{p-1})^2$$

$$\Rightarrow ar^{p-1} = \sqrt{mn}$$

$$\Rightarrow p^{th} \text{ term} = t_p = \sqrt{mn}$$

## Solution 6.

For a G.P.,

First term = a

Let the common ratio = r

$n^{th}$ term = b

$\Rightarrow ar^{n-1} = b$

P = Product of first n numbers of the given G.P.

$\Rightarrow P = a \times ar \times ar^2 \times ar^3 \times \ldots\ldots\ldots \times ar^{n-1}$

$\Rightarrow P = a \times ar \times ar^2 \times ar^3 \times \ldots\ldots\ldots \times b$

$\Rightarrow P = a \times ar \times ar^2 \times ar^3 \times \ldots\ldots\ldots \times \frac{b}{r^2} \times \frac{b}{r} \times b$

$\Rightarrow P = (ab) \times \left(ar \times \frac{b}{r}\right) \times \left(ar^2 \times \frac{b}{r^2}\right) \times \ldots\ldots\ldots \frac{n}{2}$ terms

$\Rightarrow P = (ab) \times (ab) \times (ab) \times \ldots\ldots\ldots \frac{n}{2}$ terms

$\Rightarrow P = (ab)^{\frac{n}{2}}$

$\Rightarrow P = \sqrt{ab^n}$

$\Rightarrow p^2 = ab^n$

## Solution 7.

Let r be the common ratio of this G.P.

Given: a, b, c, d are in G.P.

$\Rightarrow 1^{st} = a,$

$2^{nd}$ term = b = ar,

$3^{rd}$ term = c = $ar^2$

$4^{th}$ term = d = $ar^3$

Now, $\left(b^2 + c^2\right)^2 = \left[(ar)^2 + (ar^2)^2\right]^2$

$= \left[a^2r^2 + a^2r^4\right]^2$

$= \left[a^2r^2\left(1 + r^2\right)\right]^2$

$= a^4r^4\left(1 + r^2\right)^2$

And, $\left(a^2 + b^2\right) \times \left(c^2 + d^2\right) = \left[a^2 + (ar)^2\right] \times \left[(ar^2)^2 + (ar^3)^2\right]$

$= \left[a^2 + a^2r^2\right] \times \left[a^2r^4 + a^2r^6\right]$

$= a^2\left(1 + r^2\right) \times a^2r^4\left(1 + r^2\right)$

$= a^4r^4\left(1 + r^2\right)^2$

$\Rightarrow \left(b^2 + c^2\right)^2 = \left(a^2 + b^2\right) \times \left(c^2 + d^2\right)$

i.e. $\frac{b^2 + c^2}{a^2 + b^2} = \frac{c^2 + d^2}{b^2 + c^2}$

Hence, $\left(a^2 + b^2\right)$, $\left(b^2 + c^2\right)$ and $\left(c^2 + d^2\right)$ are in G.P.

## Solution 8.

Let r be the common ratio of this G.P.

Given : a, b, c, d are in G.P.

$\Rightarrow 1^{st} = a,$

$2^{nd}$ term = b = ar,

$3^{rd}$ term = c = $ar^2$

$4^{th}$ term = d = $ar^3$

$$\text{Now, } \left(\frac{1}{b^2+c^2}\right)^2 = \left[\frac{1}{(ar)^2+(ar^2)^2}\right]^2$$

$$= \left[\frac{1}{a^2r^2+a^2r^4}\right]^2$$

$$= \frac{1}{a^4r^4}\left[\frac{1}{1+r^2}\right]^2$$

$$= \frac{1}{a^4r^4} \times \frac{1}{\left(1+r^2\right)^2}$$

$$\text{And, } \left(\frac{1}{a^2+b^2}\right) \times \left(\frac{1}{c^2+d^2}\right) = \left[\frac{1}{a^2+(ar)^2}\right] \times \left[\frac{1}{(ar^2)^2+(ar^3)^2}\right]$$

$$= \left[\frac{1}{a^2+a^2r^2}\right] \times \left[\frac{1}{a^2r^4+a^2r^6}\right]$$

$$= \frac{1}{a^2}\left(\frac{1}{1+r^2}\right) \times \frac{1}{a^2r^4}\left(\frac{1}{1+r^2}\right)$$

$$= \frac{1}{a^4r^4} \times \frac{1}{\left(1+r^2\right)^2}$$

$$\Rightarrow \left(\frac{1}{b^2+c^2}\right)^2 = \left(\frac{1}{a^2+b^2}\right) \times \left(\frac{1}{c^2+d^2}\right)$$

Hence, $\frac{1}{a^2+b^2}, \frac{1}{b^2+c^2}$ and $\frac{1}{c^2+d^2}$ are in G.P.

## Exercise 11(D)

### Solution 1(i).

Here, a, b, c are in G.P.

$\Rightarrow b^2 = ac$

Taking log on both sides, we get

$\log(b^2) = \log(ac)$

$\Rightarrow 2\log b = \log a + \log c$

$\Rightarrow \log b + \log b = \log a + \log c$

$\Rightarrow \log b - \log a = \log c - \log b$

$\Rightarrow \log a, \log b$ and $\log c$ are in A.P.

### Solution 1(ii).

Here, a, b, c are in G.P.

$\Rightarrow b^2 = ac$

$\Rightarrow (b^2)^n = (ac)^n$

$\Rightarrow b^{2n} = a^n c^n$

Taking log on both sides, we get

$\log(b^{2n}) = \log(a^n c^n)$

$\Rightarrow \log(b^n)^2 = \log a^n + \log c^n$

$\Rightarrow 2\log b^n = \log a^n + \log c^n$

$\Rightarrow \log b^n + \log b^n = \log a^n + \log c^n$

$\Rightarrow \log b^n - \log a^n = \log c^n - \log b^n$

$\Rightarrow \log a^n, \log b^n$ and $\log c^n$ are in A.P.

### Solution 2.

Let $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}} = k$

$\Rightarrow a^{\frac{1}{x}} = k, \quad b^{\frac{1}{y}} = k$ and $c^{\frac{1}{z}} = k$

$\Rightarrow a = k^x, \quad b = k^y$ and $c = k^z$

Given that a, b, c are in G.P.

$\Rightarrow b^2 = ac$

$\Rightarrow (k^y)^2 = k^x \times k^z$

$\Rightarrow k^{2y} = k^{x+z}$

$\Rightarrow 2y = x + z$

$\Rightarrow x, y, z$ are in A.P.

## Solution 3.

Let $a_1, a_2, a_3, \ldots\ldots, a_n, \ldots\ldots$ be a G.P. with common ratio r.

$\Rightarrow \frac{a_{n+1}}{a_n} = r$ for all $n \in N$

If each term of a G.P. is raised to the power x, we get the sequence

$a_1^x, a_2^x, a_3^x, \ldots\ldots, a_n^x, \ldots\ldots$

Now, $\frac{(a_{n+1})^x}{(a_n)^x} = \left(\frac{a_{n+1}}{a_n}\right)^x = r^x$ for all $n \in N$

Hence, $a_1^x, a_2^x, a_3^x, \ldots\ldots, a_n^x, \ldots\ldots$ is also a G.P.

## Solution 4.

a, b and c are in A.P.

$\Rightarrow 2b = a + c$

a, x and b are in G.P.

$\Rightarrow x^2 = ab$

b, y and c are in G.P.

$\Rightarrow y^2 = bc$

Now,

$$\begin{aligned} x^2 + y^2 &= ab + bc \\ &= b(a + c) \\ &= b \times 2b \\ &= 2b^2 \end{aligned}$$

$\Rightarrow x^2, b^2$ and $y^2$ are in A.P.

## Solution 5(i).

a, b and c are in G.P.

$\Rightarrow b^2 = ac$

a, x, b, y and c are in A.P.

$\Rightarrow 2x = a + b \Rightarrow x = \dfrac{a+b}{2}$

$2b = x + y \Rightarrow b = \dfrac{x+y}{2}$

$2y = b + c \Rightarrow y = \dfrac{b+c}{2}$

Now,

$$\frac{1}{x} + \frac{1}{y} = \frac{2}{a+b} + \frac{2}{b+c}$$

$$= \frac{2b + 2c + 2a + 2b}{ab + ac + b^2 + bc}$$

$$= \frac{2a + 2c + 4b}{ab + b^2 + b^2 + bc}$$

$$= \frac{2a + 2c + 4b}{ab + 2b^2 + bc}$$

$$= \frac{2(a + c + 2b)}{b(a + 2b + c)}$$

$$= \frac{2}{b}$$

## Solution 5(ii).

a, b and c are in G.P.

$\Rightarrow b^2 = ac$

a, x, b, y and c are in A.P.

$$\Rightarrow 2x = a + b \Rightarrow x = \frac{a+b}{2}$$

$$2b = x + y \Rightarrow b = \frac{x+y}{2}$$

$$2y = b + c \Rightarrow y = \frac{b+c}{2}$$

Now,

$$\frac{a}{x} + \frac{c}{y} = \frac{2a}{a+b} + \frac{2c}{b+c}$$

$$= \frac{2a(b+c) + 2c(a+b)}{(a+b)(b+c)}$$

$$= \frac{2ab + 2ac + 2ac + 2bc}{ab + ac + b^2 + bc}$$

$$= \frac{2ab + 4ac + 2bc}{ab + b^2 + b^2 + bc}$$

$$= \frac{2(ab + 2ac + bc)}{ab + 2b^2 + bc}$$

$$= \frac{2(ab + 2ac + bc)}{ab + 2ac + bc}$$

$$= 2$$

## Solution 6.

$\log_x a^n, \log_x b^n$ and $\log_x c^n$ are in A.P.

$\Rightarrow \log_x b^n - \log_x a^n = \log c_x{}^n - \log b_x{}^n$

$\Rightarrow \log_x b^n + \log_x b^n = \log_x a^n + \log_x c^n$

$\Rightarrow 2\log_x b^n = \log_x a^n + \log_x c^n$

$\Rightarrow \log_x (b^n)^2 = \log_x a^n + \log_x c^n$

$\Rightarrow (b^2)^n = (ac)^n$

$\Rightarrow b^2 = ac$

Hence, a, b and c are in G.P.

## Solution 7.

a, b and c are in A.P.

$\Rightarrow 2b = a + c$

$\Rightarrow b = \dfrac{a+c}{2}$

a, b and c are also in G.P.

$\Rightarrow b^2 = ac$

$\Rightarrow \left(\dfrac{a+c}{2}\right)^2 = ac$

$\Rightarrow \dfrac{a^2 + c^2 + 2ac}{4} = ac$

$\Rightarrow a^2 + c^2 + 2ac = 4ac$

$\Rightarrow a^2 + c^2 - 2ac = 0$

$\Rightarrow (a - c)^2 = 0$

$\Rightarrow a - c = 0$

$\Rightarrow a = c$

Now, $2b = a + c$

$\Rightarrow 2b = a + a$

$\Rightarrow 2b = 2a$

$\Rightarrow b = a$

Thus, we have $a = b = c$

## Exercise 11(E)

## Solution 1(i).

Given G.P. : $1 + 3 + 9 + 27 + \ldots.$

Here,

first term, $a = 1$

common ratio, $r = \dfrac{3}{1} = 3\ (r > 1)$

number of terms to be added, $n = 12$

$\therefore\ S_n = \dfrac{a(r^n - 1)}{r - 1}$

$\Rightarrow S_{12} = \dfrac{1\left(3^{12} - 1\right)}{3 - 1} = \dfrac{3^{12} - 1}{2} = \dfrac{531441 - 1}{2} = \dfrac{531440}{2} = 265720$

**Solution 1(ii).**

Given G.P. : 0.3 + 0.03 + 0.003 + 0.003 + ....

Here,

first term, a = 0.3

common ratio, $r = \frac{0.03}{0.3} = 0.1\ (r < 1)$

number of terms to be added, n = 8

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_8 = \frac{0.3\left(1-(0.1)^8\right)}{1-0.1} = \frac{0.3\left(1-(0.1)^8\right)}{0.9} = \frac{1-(0.1)^8}{3} = \frac{1}{3}\left(1-\frac{1}{10^8}\right)$$

## Solution 1(iii).

Given G.P. : $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+......$

Here,

first term, $a = 1$

common ratio, $r = \frac{-\frac{1}{2}}{1} = -\frac{1}{2}\ (r < 1)$

number of terms to be added, $n = 9$

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_8 = \frac{1\left(1-\left(-\frac{1}{2}\right)^9\right)}{1-\left(-\frac{1}{2}\right)}$$

$$= \frac{1-\left(-\frac{1}{2}\right)^9}{1+\frac{1}{2}}$$

$$= \frac{1+\frac{1}{2^9}}{\frac{3}{2}}$$

$$= \frac{2}{3}\left(1+\frac{1}{2^9}\right)$$

$$= \frac{2}{3}\left(1+\frac{1}{512}\right)$$

$$= \frac{2}{3}\times\frac{513}{512}$$

$$= \frac{171}{256}$$

## Solution 1(iv).

Given G.P.: $1-\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+.....$ upto n terms

Here,

first term, $a = 1$

common ratio, $r = \frac{-1/3}{1} = -\frac{1}{3}\ (r < 1)$

number of terms to be added = n

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_n = \frac{1\left(1-\left(-\frac{1}{3}\right)^n\right)}{1-\left(-\frac{1}{3}\right)}$$

$$= \frac{1\left(1-\left(-\frac{1}{3}\right)^n\right)}{1+\frac{1}{3}}$$

$$= \frac{\left[1-\left(-\frac{1}{3}\right)^n\right]}{\frac{4}{3}}$$

$$= \frac{3}{4}\left[1-\left(-\frac{1}{3}\right)^n\right]$$

## Solution 1(v).

Given G.P.: $\frac{x+y}{x-y} + 1 + \frac{x-y}{x+y} + \ldots\ldots$ upto n terms

Here,

first term, $a = \frac{x+y}{x-y}$

common ratio, $r = \frac{1}{\frac{x+y}{x-y}} = \frac{x-y}{x+y}$ $(r < 1)$

number of terms to be added = n

$$\therefore S_n = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_n = \frac{\frac{x+y}{x-y}\left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{1-\left(\frac{x-y}{x+y}\right)}$$

$$= \frac{\frac{x+y}{x-y}\left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{\frac{x+y-x+y}{x+y}}$$

$$= \frac{\frac{x+y}{x-y}\left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{\frac{2y}{x+y}}$$

$$= \frac{(x+y)^2\left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{2y(x-y)}$$

## Solution 1(vi).

Given G.P.: $\sqrt{3}+\frac{1}{\sqrt{3}}+\frac{1}{3\sqrt{3}}+\ldots\ldots$ upto n terms

Here,

first term, $a=\sqrt{3}$

common ratio, $r=\frac{\frac{1}{\sqrt{3}}}{\sqrt{3}}=\frac{1}{3}\ \ (r<1)$

number of terms to be added = n

$$\therefore S_n=\frac{a(1-r^n)}{1-r}$$

$$\Rightarrow S_n=\frac{\sqrt{3}\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}}$$

$$=\frac{\sqrt{3}\left(1-\frac{1}{3^n}\right)}{\frac{2}{3}}$$

$$=\frac{3\sqrt{3}}{2}\left(1-\frac{1}{3^n}\right)$$

**Solution 2.**

Given G.P.: 1 + 4 + 16 + 64 + ......

Here,

first term, a = 1

common ratio, $r = \frac{4}{1} = 4 \ (r > 1)$

Let the number of terms to be added = n

Then, $S_n = 5461$

$\Rightarrow \frac{a(r^n - 1)}{r - 1} = 5461$

$\Rightarrow \frac{1\left(4^n - 1\right)}{4 - 1} = 5461$

$\Rightarrow \frac{4^n - 1}{3} = 5461$

$\Rightarrow 4^n - 1 = 16383$

$\Rightarrow 4^n = 16384$

$\Rightarrow 4^n = 4^7$

$\Rightarrow n = 7$

Hence, required number of terms = 7

## Solution 3.

Given,

First term, $a = 27$

$8^{th}$ term $= ar^7 = \frac{1}{81}$

$n = 10$

Now,

$$\frac{ar^7}{a} = \frac{\frac{1}{81}}{27}$$

$$\Rightarrow r^7 = \frac{1}{2187}$$

$$\Rightarrow r^7 = \left(\frac{1}{3}\right)^7$$

$$\Rightarrow r = \frac{1}{3} \ (r < 1)$$

$$\therefore S_n = \frac{a(1 - r^n)}{1 - r}$$

$$\Rightarrow S_{10} = \frac{27\left(1 - \left(\frac{1}{3}\right)^{10}\right)}{1 - \frac{1}{3}}$$

$$= \frac{27\left(1 - \frac{1}{3^{10}}\right)}{\frac{2}{3}}$$

$$\frac{81}{2}\left(1 - \frac{1}{3^{10}}\right)$$

## Solution 4.

Amount spent on $1^{st}$ day = Rs. 10

Amount spent on $2^{nd}$ day = Rs. 20

Amount spent on $3^{rd}$ day = Rs. 40 and so on

Now, $\frac{20}{10} = 2$, $\frac{40}{20} = 2$,

Thus, 10, 20, 40, ..... is a G.P. with first term, $a = 10$

and common ratio, $r = 2 \ (r > 1)$

$\therefore$ Total amount spent in 12 days = $S_{12}$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$\Rightarrow S_{12} = \frac{10(2^{12} - 1)}{2 - 1} = 10(2^{12} - 1) = 10(4096 - 1) = 10 \times 4095 = 40950$$

Hence, the total amount spent in 12 days is Rs. 40950.

## Solution 5.

For a G.P.,

$4^{th}$ term $= ar^3 = \frac{1}{27}$

$7^{th}$ term $= ar^6 = \frac{1}{729}$

Now, $\frac{ar^6}{ar^3} = \frac{1/729}{1/27}$

$\Rightarrow r^3 = \frac{1}{27} = \left(\frac{1}{3}\right)^3$

$\Rightarrow r = \frac{1}{3}\ (r < 1)$

$\Rightarrow a \times \frac{1}{27} = \frac{1}{27}$

$\Rightarrow a = 1$

$S_n = \frac{a(1-r^n)}{1-r}$

$\Rightarrow S_n = \frac{1\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}}$

$= \frac{\left(1-\frac{1}{3^n}\right)}{\frac{2}{3}}$

$= \frac{3}{2}\left(1-\frac{1}{3^n}\right)$

## Solution 6.

For a G.P.,

Common ratio, $r = 3\ \ (r > 1)$

Last term, $l = 486$

$S = 728$

$\Rightarrow \frac{lr - a}{r - 1} = 728$

$\Rightarrow \frac{486 \times 3 - a}{3 - 1} = 728$

$\Rightarrow \frac{1458 - a}{2} = 728$

$\Rightarrow 1458 - a = 1456$

Hence, the first term is 2.

## Solution 7.

Given G.P. : 3, 6, 12, ........, 1536

Here,

First term, $a = 3$

Common ratio, $r = \frac{6}{3} = 2 \ (r > 1)$

Last term, $l = 1536$

$\therefore$ Required sum $= \frac{lr - a}{r - 1}$

$= \frac{1536 \times 2 - 3}{2 - 1}$

$= 3072 - 3$

$= 3069$

## Solution 8.

Given series: $2 + 6 + 18 + \ldots\ldots$

Now, $\frac{6}{2} = 3, \ \frac{18}{6} = 3$

Thus, given series is a G.P. with first term, $a = 2$

and common ratio, $r = 3 \ (r > 1)$

Let the number of terms to be added = n

Then, $S_n = 728$

$\Rightarrow \frac{a(r^n - 1)}{r - 1} = 728$

$\Rightarrow \frac{2(3^n - 1)}{3 - 1} = 728$

$\Rightarrow 3^n - 1 = 728$

$\Rightarrow 3^n = 729$

$\Rightarrow 3^n = 3^6$

$\Rightarrow n = 6$

Hence, required number of terms = 6

## Solution 9.

Let a be the first term and r be the common ratio of given G.P.

Now, sum of first three terms = $S_3 = \frac{a(r^3 - 1)}{r - 1}$

Now, sum of first six terms = $S_6 = \frac{a(r^6 - 1)}{r - 1}$

It is given that

$$\frac{\frac{a(r^3 - 1)}{r - 1}}{\frac{a(r^6 - 1)}{r - 1}} = \frac{125}{152}$$

$$\Rightarrow \frac{r^3 - 1}{r^6 - 1} = \frac{125}{152}$$

$$\Rightarrow \frac{r^3 - 1}{\left(r^3\right)^2 - \left(1\right)^2} = \frac{125}{152}$$

$$\Rightarrow \frac{r^3 - 1}{\left(r^3 - 1\right)\left(r^3 + 1\right)} = \frac{125}{152}$$

$$\Rightarrow \frac{1}{r^3 + 1} = \frac{125}{152}$$

$$\Rightarrow r^3 + 1 = \frac{152}{125}$$

$$\Rightarrow r^3 = \frac{152}{125} - 1 = \frac{152 - 125}{125} = \frac{27}{125}$$

$$\Rightarrow r^3 = \left(\frac{3}{5}\right)^3$$

$$\Rightarrow r = \frac{3}{5}$$

Hence, the common ratio is $\frac{3}{5}$.

**Solution 10(i).**

$$\text{Required sum} = 4 + 44 + 444 + \ldots\ldots\ldots \text{ upto n terms}$$
$$= 4(1 + 11 + 111 + \ldots\ldots\ldots \text{ upto n terms})$$
$$= \frac{4}{9}(9 + 99 + 999 + \ldots\ldots\ldots \text{upto n terms})$$
$$= \frac{4}{9}[(10 - 1) + (100 - 1) + (1000 - 1) + \ldots\ldots\ldots \text{ upto n terms}]$$
$$= \frac{4}{9}\left[\begin{array}{l}(10 + 10^2 + 10^3 + \ldots\ldots\ldots \text{upto n terms}) \\ \qquad - (1 + 1 + 1 + \ldots\ldots \text{upto n terms})\end{array}\right]$$
$$= \frac{4}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$$
$$= \frac{4}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$$

**Solution 10(ii).**

$$\text{Required sum} = 0.8 + 0.88 + 0.888 + \ldots\ldots\ldots \text{ upto n terms}$$
$$= 8(0.1 + 0.11 + 0.111 + \ldots\ldots\ldots \text{ upto n terms})$$
$$= \frac{8}{9}(0.9 + 0.99 + 0.999 + \ldots\ldots\ldots \text{upto n terms})$$
$$= \frac{8}{9}[(1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \ldots\ldots\ldots \text{ upto n terms}]$$
$$= \frac{8}{9}\left[\begin{array}{l}(1 + 1 + 1 + \ldots\ldots\ldots \text{upto n terms}) \\ \qquad - (0.1 + 0.01 + 0.001 + \ldots\ldots \text{upto n terms})\end{array}\right]$$
$$= \frac{8}{9}\left[\begin{array}{l}(1 + 1 + 1 + \ldots\ldots\ldots \text{upto n terms}) \\ \qquad - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \ldots\ldots \text{upto n terms}\right)\end{array}\right]$$
$$= \frac{8}{9}\left[n - \frac{\frac{1}{10}\left(1 - \frac{1}{10^n}\right)}{1 - \frac{1}{10}}\right] \qquad \left[\because r = \frac{1}{10} < 1\right]$$
$$= \frac{8}{9}\left[n - \frac{10}{9} \times \frac{1}{10}\left(1 - \frac{1}{10^n}\right)\right]$$
$$= \frac{8}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$$

## Solution 10(iii).

Required sum $= 2 + 22 + 222 + \dots\dots\dots$ upto n terms

$$= 2(1 + 11 + 111 + \dots\dots\dots \text{ upto n terms})$$

$$= \frac{2}{9}(9 + 99 + 999 + \dots\dots\dots \text{upto n terms})$$

$$= \frac{2}{9}\left[(10 - 1) + (100 - 1) + (1000 - 1) + \dots\dots\dots \text{ upto n terms}\right]$$

$$= \frac{2}{9}\left[\begin{array}{l}(10 + 10^2 + 10^3 + \dots\dots\dots \text{upto n terms}) \\ \qquad - (1 + 1 + 1 + \dots\dots \text{upto n terms})\end{array}\right]$$

$$= \frac{2}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$$

$$= \frac{2}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$$

## Solution 10(iv).

Required sum $= 0.5 + 0.55 + 0.555 + \dots\dots\dots$ upto n terms

$$= 5(0.1 + 0.11 + 0.111 + \dots\dots\dots \text{ upto n terms})$$

$$= \frac{5}{9}(0.9 + 0.99 + 0.999 + \dots\dots\dots \text{upto n terms})$$

$$= \frac{5}{9}\left[(1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \dots\dots\dots \text{ upto n terms}\right]$$

$$= \frac{5}{9}\left[\begin{array}{l}(1 + 1 + 1 + \dots\dots\dots \text{upto n terms}) \\ \qquad - (0.1 + 0.01 + 0.001 + \dots\dots \text{upto n terms})\end{array}\right]$$

$$= \frac{5}{9}\left[\begin{array}{l}(1 + 1 + 1 + \dots\dots\dots \text{upto n terms}) \\ \qquad - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \dots\dots \text{upto n terms}\right)\end{array}\right]$$

$$= \frac{5}{9}\left[n - \frac{\frac{1}{10}\left(1 - \frac{1}{10^n}\right)}{1 - \frac{1}{10}}\right] \qquad \left[\because r = \frac{1}{10} < 1\right]$$

$$= \frac{5}{9}\left[n - \frac{10}{9} \times \frac{1}{10}\left(1 - \frac{1}{10^n}\right)\right]$$

$$= \frac{5}{9}\left[n - \frac{1}{9}\left(1 - \frac{1}{10^n}\right)\right]$$

**Solution 11.**

Given G.P.: $\frac{2}{9} - \frac{1}{3} + \frac{1}{2}$........

Here,

First term, $a = \frac{2}{9}$

Common ratio, $r = \frac{-1/3}{2/9} = -\frac{3}{2} < 1$

Let required number of terms be n.

$\Rightarrow S_n = \frac{55}{72}$

$\Rightarrow \frac{a(1-r^n)}{1-r} = \frac{55}{72}$

$\Rightarrow \frac{\frac{2}{9}\left(1-\left(\frac{-3}{2}\right)^n\right)}{1-\left(-\frac{3}{2}\right)} = \frac{55}{72}$

$\Rightarrow \frac{\frac{2}{9}\left(1-\left(\frac{-3}{2}\right)^n\right)}{\frac{5}{2}} = \frac{55}{72}$

$\Rightarrow \frac{2}{9}\left(1-\left(\frac{-3}{2}\right)^n\right) = \frac{55}{72} \times \frac{5}{2}$

$\Rightarrow 1-\left(\frac{-3}{2}\right)^n = \frac{55}{72} \times \frac{5}{2} \times \frac{9}{2}$

$\Rightarrow 1-\left(\frac{-3}{2}\right)^n = \frac{275}{32}$

$\Rightarrow 1-\frac{275}{32} = \left(\frac{-3}{2}\right)^n$

$\Rightarrow -\frac{243}{32} = \left(\frac{-3}{2}\right)^n$

$\Rightarrow \left(-\frac{3}{2}\right)^5 = \left(\frac{-3}{2}\right)^n$

$\Rightarrow n = 5$

$\therefore$ Required number of terms = 5

## Solution 12.

Required series: $1+2+2^2+\dots\dots\dots+2^{n-1}$

Now, $\frac{2}{1}=2,\ \frac{2^2}{2}=2$

Thus, given series is a G.P. with

first term, $a=1$

common ratio, $r=2\ (r>1)$

Last term, $l=2^{n-1}$

Let there be n terms in the series.

Then, $S_n=255$

$\Rightarrow \frac{lr-a}{r-1}=255$

$\Rightarrow \frac{2^{n-1}\times 2-1}{2-1}=255$

$\Rightarrow 2^{n-1}\times 2-1=255$

$\Rightarrow 2^{n-1}\times 2=256$

$\Rightarrow 2^{n-1}=128$

$\Rightarrow 2^{n-1}=2^7$

$\Rightarrow n-1=7$

$\Rightarrow n=8$

## Exercise 11(F)

## Solution 1(i).

Given G.P. : $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\dots\dots\dots$

Here,

First term, $a=1$

Common ratio, $r=\frac{1/3}{1}=\frac{1}{3}\ \left(|r|=\left|\frac{1}{3}\right|=\frac{1}{3}<1\right)$

$\therefore$ Required sum $=\frac{a}{1-r}=\frac{1}{1-\frac{1}{3}}=\frac{3}{2}$

**Solution 1(ii).**

Given G.P. : $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots\ldots.$

Here,

First term, $a=1$

Common ratio, $r=\frac{-1/2}{1}=-\frac{1}{2}\quad\left(|r|=\left|-\frac{1}{2}\right|=\frac{1}{2}<1\right)$

$\therefore$ Required sum $=\frac{a}{1-r}=\frac{1}{1-\left(-\frac{1}{2}\right)}=\frac{1}{1+\frac{1}{2}}=\frac{2}{3}$

**Solution 1(iii).**

Given G.P. : $\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\ldots.$

Here,

First term, $a=\frac{1}{3}$

Common ratio, $r=\frac{1/3^2}{1/3}=\frac{1}{3}\quad\left(|r|=\left|\frac{1}{3}\right|=\frac{1}{3}<1\right)$

$\therefore$ Required sum $=\frac{a}{1-r}=\frac{\frac{1}{3}}{1-\frac{1}{3}}=\frac{\frac{1}{3}}{\frac{2}{3}}=\frac{1}{2}$

**Solution 1(iv).**

Given G.P. : $\sqrt{2}-\frac{1}{\sqrt{2}}+\frac{1}{2\sqrt{2}}-\frac{1}{4\sqrt{2}}+\ldots\ldots$

Here,

First term, $a=\sqrt{2}$

Common ratio, $r=\frac{-1/\sqrt{2}}{\sqrt{2}}=-\frac{1}{2}\quad\left(|r|=\left|-\frac{1}{2}\right|=\frac{1}{2}<1\right)$

$\therefore$ Required sum $=\frac{a}{1-r}=\frac{\sqrt{2}}{1-\left(-\frac{1}{2}\right)}=\frac{\sqrt{2}}{1+\frac{1}{2}}=\frac{2\sqrt{2}}{3}$

## Solution 1(v).

Given G.P. : $\sqrt{3}+\frac{1}{\sqrt{3}}+\frac{1}{3\sqrt{3}}-\frac{1}{9\sqrt{3}}+\ldots\ldots$

Here,

First term, $a=\sqrt{3}$

Common ratio, $r=\frac{1/\sqrt{3}}{\sqrt{3}}=\frac{1}{3}\left(|r|=\left|\frac{1}{3}\right|=\frac{1}{3}<1\right)$

$\therefore$ Required sum $=\frac{a}{1-r}=\frac{\sqrt{3}}{1-\frac{1}{3}}=\frac{\sqrt{3}}{\frac{2}{3}}=\frac{3\sqrt{3}}{2}$

## Solution 2.

Let a be the first term and r be the common ratio of a G.P.

$2^{nd}$ term, $t_2=ar=9$

$\Rightarrow r=\frac{9}{a}$

Sum of its infinite terms, S = 48

$\Rightarrow \frac{a}{1-r}=48$

$\Rightarrow \frac{a}{1-\frac{9}{a}}=48$

$\Rightarrow \frac{a^2}{a-9}=48$

$\Rightarrow a^2=48a-432$

$\Rightarrow a^2-48a+432=0$

$\Rightarrow a^2-36a-12a+432=0$

$\Rightarrow a(a-36)-12(a-36)=0$

$\Rightarrow (a-36)(a-12)=0$

$\Rightarrow a=36$ or $a=12$

When $a=36$, $r=\frac{9}{36}=\frac{1}{4}$

$\Rightarrow 1^{st}$ term = 36,

$2^{nd}$ term $=ar=36\times\frac{1}{4}=9$

$3^{rd}$ term $=ar^2=36\times\frac{1}{16}=\frac{9}{4}$

When $a=12$, $r=\frac{9}{12}=\frac{3}{4}$

$\Rightarrow 1^{st}$ term = 12,

$2^{nd}$ term $=ar=12\times\frac{3}{4}=9$

$3^{rd}$ term $=ar^2=12\times\frac{9}{16}=\frac{27}{4}$

**Solution 3(i).**

$$0.\dot{2} = 0.22222222222\ldots\ldots\ldots \text{ upto infinity}$$

$$= 0.2 + 0.02 + 0.002 + \ldots\ldots\ldots\ldots \text{ upto infinity}$$

$$= \frac{2}{10} + \frac{2}{100} + \frac{2}{1000} + \ldots\ldots\ldots \text{ upto infinity}$$

$$= \frac{2/10}{1 - 1/10} \qquad \ldots\left[\frac{2/100}{2/10} = \frac{1}{10}\right]$$

$$= \frac{2/10}{9/10}$$

$$= \frac{2}{9}$$

**Solution 3(ii).**

$$0.5\dot{2} = 0.522222222222\ldots\ldots\ldots \text{ upto infinity}$$

$$= 0.5 + 0.02 + 0.002 + 0.0002 + \ldots\ldots\ldots\ldots \text{ upto infinity}$$

$$= \frac{5}{10} + \frac{2}{100} + \frac{2}{1000} + \frac{2}{10000} + \ldots\ldots\ldots \text{ upto infinity}$$

$$= \frac{5}{10} + \frac{2}{100}\left[1 + \frac{1}{10} + \frac{1}{100} + \ldots\ldots\ldots \text{ upto infinity}\right]$$

$$= \frac{5}{10} + \frac{2}{100}\left[\frac{1}{1 - 1/10}\right] \qquad \ldots\left[\frac{1/10}{1} = \frac{1}{10}\right]$$

$$= \frac{5}{10} + \frac{2}{100} \times \frac{10}{9}$$

$$= \frac{5}{10} + \frac{1}{45}$$

$$= \frac{45 + 2}{90}$$

$$= \frac{47}{90}$$

## Solution 3(iii).

$0.4\overline{23} = 0.4232323232323\ldots\ldots$ upto infinity

$= 0.4 + 0.023 + 0.00023 + 0.0000023 + \ldots\ldots\ldots$ upto infinity

$= \frac{4}{10} + \frac{23}{1000} + \frac{23}{100000} + \frac{23}{10000000} + \ldots\ldots$ upto infinity

$= \frac{4}{10} + \frac{23}{1000}\left[1 + \frac{1}{100} + \frac{1}{10000} + \ldots\ldots \text{ upto infinity}\right]$

$= \frac{4}{10} + \frac{23}{1000}\left[\frac{1}{1 - \frac{1}{100}}\right] \qquad \ldots\left[\frac{\frac{1}{100}}{1} = \frac{1}{100}\right]$

$= \frac{4}{10} + \frac{23}{1000} \times \frac{100}{99}$

$= \frac{4}{10} + \frac{23}{990}$

$= \frac{396 + 23}{990}$

$= \frac{419}{990}$

## Solution 3(iv).

$0.7\dot{6}\dot{2} = 0.76262626262\ldots\ldots$ upto infinity

$= 0.7 + 0.062 + 0.00062 + 0.0000062 + \ldots\ldots\ldots$ upto infinity

$= \frac{7}{10} + \frac{62}{1000} + \frac{62}{100000} + \frac{62}{10000000} + \ldots\ldots$ upto infinity

$= \frac{7}{10} + \frac{62}{1000}\left[1 + \frac{1}{100} + \frac{1}{10000} + \ldots\ldots \text{ upto infinity}\right]$

$= \frac{7}{10} + \frac{62}{1000}\left[\frac{1}{1 - \frac{1}{100}}\right] \qquad \ldots\left[\frac{\frac{1}{100}}{1} = \frac{1}{100}\right]$

$= \frac{7}{10} + \frac{62}{1000} \times \frac{100}{99}$

$= \frac{7}{10} + \frac{62}{990}$

$= \frac{693 + 62}{990}$

$= \frac{755}{990}$

$= \frac{151}{198}$

## Solution 3(v).

$$0.02\dot{7} = 0.0277777777 \ldots\ldots\ldots \text{ upto infinity}$$
$$= 0.02 + 0.007 + 0.0007 + 0.00007 + \ldots\ldots\ldots\ldots \text{ upto infinity}$$
$$= \frac{2}{100} + \frac{7}{1000} + \frac{7}{10000} + \frac{7}{100000} + \ldots\ldots\ldots \text{ upto infinity}$$
$$= \frac{2}{100} + \frac{7}{1000}\left[1 + \frac{1}{10} + \frac{1}{100} + \ldots\ldots\ldots \text{ upto infinity}\right]$$
$$= \frac{2}{100} + \frac{7}{1000}\left[\frac{1}{1 - 1/10}\right] \qquad \ldots\left[\frac{1/10}{1} = \frac{1}{100}\right]$$
$$= \frac{2}{100} + \frac{7}{1000} \times \frac{10}{9}$$
$$= \frac{2}{100} + \frac{7}{900}$$
$$= \frac{18 + 7}{900}$$
$$= \frac{25}{900}$$
$$= \frac{1}{36}$$

## Solution 4.

$$\text{Required sum} = \frac{2}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \frac{2}{5^5} + \frac{3}{5^6} + \ldots\ldots\ldots\ldots$$
$$= \left(\frac{2}{5} + \frac{2}{5^3} + \frac{2}{5^5} + \ldots\ldots\right) + \left(\frac{3}{5^2} + \frac{3}{5^4} + \frac{3}{5^6} + \ldots\ldots\ldots\right)$$
$$= \frac{2}{5}\left(1 + \frac{1}{5^2} + \frac{1}{5^4} + \ldots\ldots\ldots\right) + \frac{3}{5^2}\left(1 + \frac{1}{5^2} + \frac{1}{5^4} + \ldots\ldots\ldots\ldots\right)$$
$$= \frac{2}{5}\left[\frac{1}{1 - 1/25}\right] + \frac{3}{25}\left[\frac{1}{1 - 1/25}\right]$$
$$= \frac{2}{5} \times \frac{25}{24} + \frac{3}{25} \times \frac{25}{24}$$
$$= \frac{5}{12} + \frac{1}{8}$$
$$= \frac{10 + 3}{24}$$
$$= \frac{13}{24}$$

## Exercise 11(G)

### Solution 1(i).

Geometric mean between $\frac{4}{9}$ and $\frac{9}{4} = \sqrt{\frac{4}{9} \times \frac{9}{4}} = \sqrt{1} = 1$

### Solution 1(ii).

Geometric mean between 14 and $\frac{7}{32} = \sqrt{14 \times \frac{7}{32}} = \sqrt{\frac{49}{16}} = \frac{7}{4} = 1\frac{3}{4}$

### Solution 1(iii).

Geometric mean between 2a and $8a^3 = \sqrt{2a \times 8a^3} = \sqrt{16 \times a^4} = 4a^2$

### Solution 2.

Let $G_1$, $G_2$, $G_3$ be three geometric means between $a = \frac{1}{3}$ and $b = 432$.

Then, $\frac{1}{3}$, $G_1$, $G_2$, $G_3$, 432 is a G.P.

Thus, we have

First term $= a = \frac{1}{3}$

$5^{th}$ term of the G.P. $= ar^4 = 432$

$\Rightarrow \frac{1}{3} \times r^4 = 432$

$\Rightarrow r^4 = 1296$

$\Rightarrow r^4 = 6^4$

$\Rightarrow r = 6$

$\therefore G_1 = ar = \frac{1}{3} \times 6 = 2$

$G_2 = ar^2 = \frac{1}{3} \times 6 \times 6 = 12$

$G_3 = ar^3 = \frac{1}{3} \times 6 \times 6 \times 6 = 72$

**Solution 3(i).**

Let $G_1$, $G_2$ be two geometric means between $a = 2$ and $b = 16$.

Then, 2, $G_1$, $G_2$, 16 is a G.P.

Thus, we have

First term $= a = 2$

$4^{th}$ term of the G.P. $= ar^3 = 16$

$\Rightarrow 2 \times r^3 = 16$

$\Rightarrow r^3 = 8$

$\Rightarrow r^3 = 2^3$

$\Rightarrow r = 2$

$\therefore G_1 = ar = 2 \times 2 = 4$

$G_2 = ar^2 = 2 \times 2 \times 2 = 8$

**Solution 3(ii).**

Let $G_1$, $G_2$, $G_3$, $G_4$ be four geometric means between $a = 3$ and $b = 96$.

Then, 3, $G_1$, $G_2$, $G_3$, $G_4$, 96 is a G.P.

Thus, we have

First term $= a = 3$

$6^{th}$ term of the G.P. $= ar^5 = 96$

$\Rightarrow 3 \times r^5 = 96$

$\Rightarrow r^5 = 32$

$\Rightarrow r^5 = 2^5$

$\Rightarrow r = 2$

$\therefore G_1 = ar = 3 \times 2 = 6$

$G_2 = ar^2 = 3 \times 4 = 12$

$G_3 = ar^3 = 3 \times 8 = 24$

$G_4 = ar^4 = 3 \times 16 = 48$

## Solution 3(iii).

Let $G_1, G_2, G_3, G_4, G_5$ be five geometric means between

$a = 3\frac{5}{9} = \frac{32}{9}$ and $b = 40\frac{1}{2} = \frac{81}{2}$.

Then, $\frac{32}{9}, G_1, G_2, G_3, G_4, G_5, \frac{81}{2}$ is a G.P.

Thus, we have

First term $= a = \frac{32}{9}$

$7^{th}$ term of the G.P. $= ar^6 = \frac{81}{2}$

$\Rightarrow \frac{32}{9} \times r^6 = \frac{81}{2}$

$\Rightarrow r^6 = \frac{81}{2} \times \frac{9}{32}$

$\Rightarrow r^6 = \frac{729}{64}$

$\Rightarrow r^6 = \left(\frac{3}{2}\right)^6$

$\Rightarrow r = \frac{3}{2}$

$\therefore G_1 = ar = \frac{32}{9} \times \frac{3}{2} = \frac{16}{3}$

$G_2 = ar^2 = \frac{32}{9} \times \frac{9}{4} = 8$

$G_3 = ar^3 = \frac{32}{9} \times \frac{27}{8} = 12$

$G_4 = ar^4 = \frac{32}{9} \times \frac{81}{16} = 18$

$G_5 = ar^5 = \frac{32}{9} \times \frac{243}{32} = 27$

**Solution 4.**

Let the numbers be $\frac{a}{r}$, a and ar.

$\Rightarrow \frac{a}{r} \times a \times ar = 1$

$\Rightarrow a^3 = 1$

$\Rightarrow a = 1$

Now, $\frac{a}{r} + a + ar = \frac{39}{10}$

$\Rightarrow \frac{1}{r} + 1 + r = \frac{39}{10}$

$\Rightarrow \frac{1 + r + r^2}{r} = \frac{39}{10}$

$\Rightarrow 10 + 10r + 10r^2 = 39r$

$\Rightarrow 10r^2 - 29r + 10 = 0$

$\Rightarrow 10r^2 - 25r - 4r + 10 = 0$

$\Rightarrow 5r(2r - 5) - 2(2r - 5) = 0$

$\Rightarrow (2r - 5)(5r - 2) = 0$

$\Rightarrow r = \frac{5}{2} \text{ or } r = \frac{2}{5}$

Thus, required terms are:

$\frac{a}{r}, a, ar = \frac{1}{5/2}, 1, 1 \times \frac{5}{2} \quad \text{OR} \quad \frac{1}{2/5}, 1, 1 \times \frac{2}{5}$

$= \frac{2}{5}, 1, \frac{5}{2} \quad \text{OR} \quad \frac{5}{2}, 1, \frac{2}{5}$

## Solution 5.

Let the numbers be a, ar and $ar^2$.

$\Rightarrow a + ar + ar^2 = 52 \qquad \ldots(i)$

And, $(a \times ar) + (ar \times ar^2) + (ar^2 \times a) = 624$

$\Rightarrow a^2r + a^2r^3 + a^2r^2 = 624$

$\Rightarrow ar(a + ar^2 + ar) = 624$

$\Rightarrow ar \times 52 = 624 \qquad \ldots[\text{From (i)}]$

$\Rightarrow ar = 12$

$\Rightarrow a = \frac{12}{r}$

Substituting in (i), we get

$\frac{12}{r} + \frac{12}{r} \times r + \frac{12}{r} \times r^2 = 52$

$\Rightarrow \frac{12}{r} + 12 + 12r = 52$

$\Rightarrow \frac{12 + 12r + 12r^2}{r} = 52$

$\Rightarrow 12 + 12r + 12r^2 = 52r$

$\Rightarrow 12r^2 - 40r + 12 = 0$

$\Rightarrow 3r^2 - 10r + 3 = 0$

$\Rightarrow 3r^2 - 9r - r + 3 = 0$

$\Rightarrow 3r(r - 3) - 1(r - 3) = 0$

$\Rightarrow (3r - 1)(r - 3) = 0$

$\Rightarrow r = \frac{1}{3} \text{ or } r = 3$

$\Rightarrow a = \frac{12}{1/3} = 36 \text{ or } 4$

Thus, required terms are:

$a, ar, ar^2 = 36, 36 \times \frac{1}{3}, 36 \times \frac{1}{9} \quad \text{OR} \quad 4, 4 \times 3, 4 \times 9$

$= 36, 12, 4 \qquad \text{OR} \quad 4, 12, 36$

## Solution 6.

Let the numbers be a, ar and $ar^2$.

$\Rightarrow (a)^2 + (ar)^2 + (ar^2)^2 = 189$

$\Rightarrow a^2 + a^2r^2 + a^2r^4 = 189$

And, $a + ar + ar^2 = 21$

$\Rightarrow (a + ar + ar^2)^2 = 21^2$

$\Rightarrow a^2 + a^2r^2 + a^2r^4 + 2a^2r + 2a^2r^3 + 2a^2r^2 = 441$

$\Rightarrow 189 + 2ar(a + ar^2 + ar) = 441$

$\Rightarrow 2ar \times 21 = 441 - 189$

$\Rightarrow 42ar = 252$

$\Rightarrow ar = 6$

$\Rightarrow r = \frac{6}{a}$

Now, $a + ar + ar^2 = 21$

$\Rightarrow a + a \times \frac{6}{a} + a \times \frac{36}{a^2} = 21$

$\Rightarrow a + 6 + \frac{36}{a} = 21$

$\Rightarrow a^2 + 6a + 36 = 21a$

$\Rightarrow a^2 - 15a + 36 = 0$

$\Rightarrow a^2 - 12a - 3a + 36 = 0$

$\Rightarrow a(a - 12) - 3(a - 12) = 0$

$\Rightarrow (a - 12)(a - 3) = 0$

$\Rightarrow a = 12 \quad \text{or} \quad a = 3$

$\Rightarrow r = \frac{6}{12} = \frac{1}{2} \quad \text{or} \quad r\frac{6}{3} = 2$

Thus, required terms are:

$a,\ ar,\ ar^2 = 12,\ 12 \times \frac{1}{2},\ 12 \times \frac{1}{4} \quad \text{OR} \quad 3,\ 3 \times 2,\ 3 \times 4$

$= 12,\ 6,\ 3 \quad \text{OR} \quad 3,\ 6,\ 12$

## Solution 7.

$y = x + x^2 + x^3 + \dots\dots\dots\infty$, where $|x| < 1$

$\Rightarrow y = x(1 + x + x^2 + \dots\dots\dots\infty)$

$\Rightarrow y = x\left[\frac{1}{1-x}\right]$

$\Rightarrow y(1 - x) = x$

$\Rightarrow y - xy = x$

$\Rightarrow y = x + xy$

$\Rightarrow y = x(1 + y)$

$\Rightarrow \frac{y}{1+y} = x$

$\Rightarrow x = \frac{y}{1+y}$

## Solution 8.

Let $G_1, G_2, G_3, G_4, G_5$ be five geometric means between $a = 1$ and $b = 27$.

Then, $1, G_1, G_2, G_3, G_4, G_5, 27$ is a G.P.

Thus, we have

First term $= a = 1$

$7^{th}$ term of the G.P. $= ar^6 = 27$

$\Rightarrow 1 \times r^6 = 27$

$\Rightarrow r^6 = 27$

$\Rightarrow (r^2)^3 = (3)^3$

$\Rightarrow r^2 = 3$

$\Rightarrow r = \sqrt{3}$

$\therefore G_1 = ar = 1 \times \sqrt{3} = \sqrt{3}$

$G_2 = ar^2 = 1 \times (\sqrt{3})^2 = 3$

$G_3 = ar^3 = 1 \times (\sqrt{3})^3 = 3\sqrt{3}$

$G_4 = ar^4 = 1 \times (\sqrt{3})^4 = 9$

$G_5 = ar^5 = 1 \times (\sqrt{3})^5 = 9\sqrt{3}$

## Solution 9.

For a G.P.,

First term, $a = -3$

It is given that,

$(2^{nd}\ \text{term})^2 = 4^{th}\ \text{term}$

$\Rightarrow (ar)^2 = ar^3$

$\Rightarrow a^2r^2 = ar^3$

$\Rightarrow a = r$

$\Rightarrow r = -3$

Now, $7^{th}$ term $= ar^6 = -3 \times (-3)^6 = -3 \times 729 = -2187$

## Solution 10.

Sum of an infinite G.P. $= S = \frac{80}{9}$

Common ratio $= r = -\frac{4}{5}$ $\quad \ldots.. \left[\left|-\frac{4}{5}\right| = \frac{4}{5} < 1\right]$

$\therefore \frac{a}{1-r} = \frac{80}{9}$

$\Rightarrow \frac{a}{1-\left(-\frac{4}{5}\right)} = \frac{80}{9}$

$\Rightarrow \frac{5a}{9} = \frac{80}{9}$

$\Rightarrow 5a = 80$

$\Rightarrow a = 16 =$ First term