Mathematics

Class – XII

Time allowed: **3** hours | Maximum Marks: **100**

General Instructions:

a) All questions are compulsory.
b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d) Use of calculators is not permitted.

Section A

(1 marks)

1. find the unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$.

2. find x if $\tan^{-1}4 + \cot^{-1} x = \frac{\pi}{2}$

3. If A is a square matrix of order 3 and |A| = -2, find the value of |-3a|.

4. Solve $\int \frac{1}{x-\sqrt{x}} dx$

Section B

(2 marks)

5. Show that the relation R in the set (1, 2,3} given by R {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.

6. Show that local maximum value of $x+\frac{1}{x}$ is less the local minimum.

7. Find the equation of a plane which is at a distance $3\sqrt{3}$ units from origin and the normal to which is equally inclined to co-ordinate axis.

8. If $P(A)=\frac{3}{10}$, $P(B)=\frac{2}{5}$ and $P(AUB)=\frac{3}{5}$, then P(B/A) + P(A/B) =?

9. Differentiate sec-1 $\left(\frac{1}{4x^3}-3x\right)$, $0<x<\frac{1}{\sqrt{2}}$

10. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$, then show that $A^2=\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$

11. Solve the equation cos(tan-1 x) $=\sin\left(\cot^{-1}\frac{3}{4}\right)$

12. find $\int\left(\frac{1-x}{1+x^2}\right)^2 / e^x / dx \int e^n\left[\frac{x^2+1}{(n+1)^2}\right]dx$

Section C

(4 marks)

13. $\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}=(a+b+c)^3$

14. The probability distribution of a random variable X is Given below:

X	0	1	2	3
P(x)	K	k/2	k/4	k/8

(i) Determine the value of k.

(ii) Determine P(x ≤ 2) and P(X ≥ 2)

(iii) find P(X ≤ 2) + P(X > 2).

15. $\int\frac{\text{Sin}\,x+\cos x}{\sqrt{1+\sin 2x}}dx$

OR

$\int\frac{1}{5\cos x-12\,\text{Sin}\,x}dx$

16. Find the general solution of (1 + tany)(dx – dy) + 2x dy = 0.

17. Find the equations of two lines through the origin-

Which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at angles of $\frac{\pi}{3}$ each.

18. If sin y = x Sin (a + y) then show that $\frac{dy}{dx}=\frac{\text{Sin}^2(a+y)}{\text{Sin}\, a}$

OR

If X = a $\cos^3\theta$, y = a $\sin^3\theta$ then find $\frac{d^2y}{dx^2}$

19. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the No. of diamond card drawn. Also, find the mean and the variance of the distribution.

20. Prove that $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\text{Sin}^{-1}\frac{1}{5}$

OR

Prove that $\cot^{-1}7 + \text{Cot}^{-1}8 + \text{Cot}^{-1}18 = \text{Cot}^{-1}18 = \text{Cot}^{-1}3$

21. find volume of parallelepiped whose conterminous edges are-

Given by vectors $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}, \vec{b}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{C}=3\hat{i}-\hat{j}+2\hat{k}$.

22. For what value of P, $f(x)=\{x^p \sin(1/x)$ x ≠0 x = 0 is derivable at x = 0.

23. Using the properties of definite integrals, evaluate

$$\int_0^{\pi}\frac{xdx}{4-\cos^2 x}$$

Sector D

(6 marks)

24. Using matrix method, solve the system of linear equations:

2x – y = 4, 2y + z = 5, z + 2x = 7

25. Show that the semi vertical angle of right circular cone of Given surface area and max. Volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

OR

Find a point on the parabola $y^2 = 4x$ which is meant to the point (2, -8).

26. There are 5 cards numbered 1 to 5 one number on one card. Two cards are drawn at random w/out replacement. Let X denote the sum of the number on two cards drawn. Find the mean and variance of X.

27. Find the area of the region bounded by the curve $y^2 = 2x$ and $x^2 + y^2 = 4x$

Or

Find the area bounded by the curve $y = 2 \cos x$ and the x – axis from $x = 0$ to $x = 2\pi$

28. A manufacturing company makes two models A and B of a product. Each piece of model A requires 9 labour hours for fabricating and 1 labour hour for finishing. Each piece of model B requires 12 labour hours for fabricating and 3 labour hour for finishing. For fabricating and finishing the maximum labour hours available are 180 and 30 respectively. The company makes a profit of rs. 8000 on each piece of model A and 12000 on each piece of model B.

29. Find the angle between the planes $4x + 8y + z - 8 = 0$ and $y + z - 4 = 0$

(Solution)
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Section A
(1 marks)

Ans.1 Let $\vec{c}$ denote the sum of $\vec{a}$ and $\vec{b}$. We have $\vec{c}=(2\hat{i}-\hat{j}+2\hat{k})+(-\hat{i}+\hat{j}+3\hat{k})$

$=\hat{i}+5\hat{k}$

Now $|\vec{c}|=\sqrt{1^2+5^2}=\sqrt{26}$

Thus, the required unit vector is

$$\hat{c}=\frac{\vec{c}}{|\vec{c}|}=\frac{1}{\sqrt{26}}(\hat{i}+5\hat{k})$$

$$=\frac{1}{\sqrt{26}}\hat{i}+\frac{5}{\sqrt{26}}\hat{k}$$

Ans.2 we know

$$\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}$$

So, x = 4

Ans.3 We know

$|KA| = K^n |A|$

Here n = 3

$\Rightarrow |-3A| = (-3)^3 |A|$

= - 27 |A|

= - 27 (-2)

= + 54

Ans.4

$$\int\frac{1}{x-\sqrt{x}}dx$$

$$\int\frac{1}{\left(\sqrt{x}\right)^2-\sqrt{x}}dx$$

$$\int\frac{1}{\sqrt{x}-\left(\sqrt{x}-1\right)}dx$$

Put $\sqrt{x}-1=t$

$$\frac{1}{2\sqrt{x}}dx=dt$$

$$\frac{1}{\sqrt{x}}dx = 2dt$$

$$= 2\int \frac{1}{t}dt$$

$$= 2\log t + c$$

$$= 2\log(\sqrt{x} - 1) + c$$

OR

$$= \log(\sqrt{x} - 1)^2 + c$$

Ans.5 R = {(1, 2), (2, 1)}

Here, since (1, 1) ∉R

Thus, R is not reflexive

Here, (1, 2) ∈R and (2, 1) ∈R

Thus, R is symmetric

Again, (1, 2) Er and (2, 1) ∈R

But (1, 1) is ∉R

Thus, R is not transitive

Ans.6 Let $y = x + \frac{1}{x}$

$$\Rightarrow \frac{dy}{dx} = 1 - \frac{1}{x^2}$$

$$\frac{dy}{dx} = 0 \Rightarrow \quad x^2 = 1 \quad \Rightarrow x = \pm 1$$

$$\frac{d^2y}{dx^2} = \pm \frac{2}{x^3}$$

And $\frac{d^2y}{dx^2}(a + x = -1) < 0$

Local max x = - 2

Local minimum = 2

Ans.7 Let the equation of the required plane be ax + by+ c_z + d = 0

It is given that the plane is equally inclined coordinate axes.

Hence its direction cosines are (1, 1, 1)

⇒ The equation of plane is (x + y + z + d) = 0

It is also given that ⊥ :-

Distance from the origin is $3\sqrt{3}$

i.e. $\left|\frac{d}{\sqrt{a^2 + b^2 + c^2}}\right| = 3\sqrt{3}$

Substituting for (a, b, c) as (1, 1, 1)

$$\left|\frac{d}{\sqrt{1^2+1^2+1^2}}\right| = 3\sqrt{3}$$

$$\frac{d}{\sqrt{3}} = \pm 3\sqrt{3}$$

$$\Rightarrow \quad d = \pm 3\sqrt{3} \times \sqrt{3} = \pm 9$$

Therefore the equation of plane is

X + y + z = ±9

Ans.8 $P(A) = \frac{3}{10}, \;\; P(B) = \frac{2}{5}$

$$P(A \cup B) = \frac{3}{5}$$

We know that

$$P(A \cup B) = P(A) - P(A \cap B)$$

$$\frac{3}{5} = \frac{3}{10} + \frac{2}{5} - P(A \cap B)$$

$$P(A \cap B(= \frac{3}{10} + \frac{2}{5} - \frac{3}{5}$$

$$= \frac{3+4-6}{10}$$

$$P(A \cap B) = \frac{1}{10}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/10}{\frac{2}{5}}$$

$$P(A/B) = \frac{1 \times 5}{10 \times 2} = \frac{1}{4}$$

$$P(B/A) = \frac{1}{3}$$

Now P(B/A) + P(A/B)

$$= \frac{1}{3} + \frac{1}{4} = \frac{4+3}{12}$$

$$= \frac{7}{12}$$

Ans.9 $y = \sec^{-1}\left(\frac{1}{4x^3 - 3x}\right)$

Let x = cos θ

Then

$\Rightarrow \quad y=\sec^{-1}\left(\frac{1}{4\cos^3\theta-3\cos\theta}\right)$

$\Rightarrow \quad y=\sec^{-1}\left(\frac{1}{\cos 3\theta}\right)$

$\therefore \quad \left[4\cos^3\theta 3\cos\theta=\cos 3\theta\right]$

$\Rightarrow \quad y=\sec^{-1}(\sec 3\theta)$

⇒ y = 3θ

Where θ = $\cos^{-1}$ x

⇒ y = 3$\cos^{-1}$ x

Now differentiating on both side

$\Rightarrow \frac{dy}{dx}=3\left(\frac{-1}{\sqrt{1-x^2}}\right)$

Ans.10 $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$A^2=A\cdot A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$

$=\begin{bmatrix}\cos^2-\sin^2\theta & \cos\theta\sin\theta+\sin\cos\theta\\ -\sin\theta\cos\theta-\sin\theta\cos\theta & -\sin^2\theta+\cos^2\theta\end{bmatrix}$

$=\begin{bmatrix}\cos^2-\sin^2\theta\\ -\sin\theta\cos\theta-\sin\theta\cos\theta\end{bmatrix}$

$\therefore \quad \begin{bmatrix}we\ know\ that\\ \cos^2\theta-\sin^2\theta=\cos 2\theta\\ and\ 2\sin\theta\cos\theta=\sin 2\theta\end{bmatrix}$

$=\begin{bmatrix}\cos^2\theta-\sin^2\theta & 2\sin\theta\cos\theta\\ -2\sin\theta\cos\theta & \cos^2\theta-\sin^2\theta\end{bmatrix}$

$=\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$

Hence Proved

Ans.11 $\cos(\tan^{-1}x)=\sin\left(\tan^{-1}\frac{3}{4}\right)$ .....(1)

As we know that $\sin x=\cos\left(\frac{\pi}{2}-x\right)$

So, $\sin\left[\cot^{-1}\frac{3}{4}\right]$

$$= \cos\left[\frac{\pi}{2} - \cot^{-1}\left(\frac{3}{4}\right)\right]$$

Now we know that

$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$, So

$$\frac{\pi}{2} - \cot^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\frac{3}{4}$$

Putting this, we get

$$\sin\left[\cot^{-1}\left(\frac{3}{4}\right) = \cos\left[\frac{\pi}{2} - \cot^{1}\left(\frac{3}{4}\right)\right]\right.$$

$$= \cos\left[\tan^{-1}\left(\frac{3}{4}\right)\right]$$

Now putting in equation Ist.
We get

$$\cos(\tan^{-1} x) \neq \cos\left(\tan^{-1}\frac{3}{4}\right)$$

$$\Rightarrow \quad \tan^{-1} x = \tan^{-1}\frac{3}{4}$$

$$\Rightarrow \quad x = \frac{3}{4}$$

Ans.12 $\int\left(\frac{1+x^2}{(1+x)^2}\right)e^n dx$

$$\int e^n\left[\frac{n^2-1+2}{(1+n)^2}\right]e^n dx$$

$$\int e^n\left(\frac{n-1}{n+1}\right)dx + 2$$

$$\int \frac{e^n}{(n+1)^2}dx$$

$$\Rightarrow \quad e^n\left(\frac{n-1}{n+1}\right) + 2\int\frac{e^n}{(n+1)}dx + 2 - \int\frac{e^n}{(n+1)^2}$$

$$\Rightarrow \quad e^n\left(\frac{n-1}{n+1}\right) + c$$

Ans.13 $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$= (a + b + c)^3$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$

$$= \begin{vmatrix} a+bc & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

(a + b + c) common from R_1, then

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_2 \rightarrow C_2 \rightarrow C_1$, $C_3 \rightarrow C_3 - C_2$

$$= (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c & a+b+c \\ 2c & 0 & -(a+b+c) \end{vmatrix}$$

Expanding

$$= (a+b+c)\left[(a+b+c)^2 - 0\right]$$

$= (a + b + c) (a + b + c)^2$

$= (a + b + c)^3$

Hence proved

Ans.14

X	0	1	2	3
P(x)	K	K/2	K/4	K/8

i) It is know that the sum of probabilities of a probability distribution of random variables is one.

$\therefore \quad k + \frac{k}{2} + \frac{k}{4} + \frac{k}{8} = 1$

$\Rightarrow \quad 15k = 8$

$\Rightarrow \quad k = \frac{8}{15}$

ii) $P(x \leq 2)$ and $P(x \geq 2)$

$P(x \leq 2) = P(0) + P(1) + P(2)$

$= \quad k + \frac{k}{2} + \frac{k}{4}$

$= \quad \frac{8}{15} + \frac{8}{30} + \frac{8}{60}$

$= \quad \frac{32+16+8}{60} = \frac{56}{60}$

$= \quad \frac{14}{15}$

P(x ≥ 2) = P(2) + P(3)

$= \quad \frac{k}{4} + \frac{k}{8}$

$= \quad \frac{8}{60} + \frac{8}{20} = \frac{16+8}{120}$

$= \quad \frac{24}{120} = \frac{1}{5}$

(iii) P(x ≤ 2) + P(x > 2)

$= \quad \frac{14}{15} + \frac{8}{120}$

$= \quad \frac{112+8}{120} = \frac{120}{120} = 1$

Ans.15 $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}} dx$

⇒ We know that $\cos^2 x + \sin^2 x = 1$

⇒ $\int \frac{\sin x + \cos x}{\sqrt{\sin^2 x + \cos^2 x + 2\sin \cos x}} dx$

⇒ $\int \frac{\sin x + \cos x}{\sqrt{(\sin x + \cos x)^2}} dx$

∵ $a^2 + b^2 + 2ab = (a + b)^2$

$= \int \frac{\sin x + \cos x}{\sin x + \cos x} dx$

$= \int 1 dx$

= x + c

OR

$= \int \frac{1}{5\cos x - 12\sin x} dx$

Let $I = \int \frac{dx}{5\cos x - 12\sin x}$

$$= \int \frac{dx}{5\left[\frac{1-\tan^2 \frac{x}{2}}{1+\tan^2 \frac{x}{2}}\right] - 12\left[\frac{\frac{2\tan x}{2}}{\frac{1+\tan^2 x}{2}}\right]}$$

$$= \int \frac{\sec^2(x/2)dx}{5 - 5\tan^2\frac{x}{2} - 24\tan\frac{x}{2}}$$

Put $\tan\left(\frac{x}{2}\right) = t$

$\Rightarrow \quad \sec^2\left(\frac{x}{2}\right)dx = 2dt$

So, $I = 2\int \frac{dt}{5 - 5t^2 - 24t}$

$$= \frac{-2}{5}\int \frac{dt}{t^2 + \frac{24}{5}t - 1}$$

$$= \frac{-2}{5}\int \frac{dt}{t^2 \frac{24}{5}t + \frac{144}{25} - \frac{144}{25} - 1}$$

$$= \frac{-2}{5}\int \frac{dt}{\left[t + \frac{12}{5}\right]^2 - \left(\frac{13}{5}\right)^2}$$

$$= \frac{2}{5}\int \frac{dt}{\left(\frac{13}{5}\right)^2 - \left(t + \frac{12}{5}\right)^2}$$

$$= \frac{2}{5} \times \frac{5}{26}\log\left|\frac{t+5}{\frac{1}{5} - t}\right|$$

$$= \frac{1}{13}\log\left|\frac{\tan(x/2) + 5}{\frac{1}{5} - \tan\left(\frac{x}{2}\right)}\right| + C$$

Ans.16 We have

$\Rightarrow$ (1 + tany) (dx – dy) + 2x dy = 0

(1 + tany)dx – (1 + tany)dy + 2x dy = 0

$\Rightarrow$ (1 + tany)dx – (1 + tany – 2x)dy = 0

$\Rightarrow$ (1 + tany)dx = (1 + tany – 2x)dy

$\Rightarrow \quad \frac{dx}{dy} = \frac{1 + \tan y - 2x}{1 + \tan y}$

$\Rightarrow \quad \frac{dx}{dy} - \left(1 - \frac{2x}{1 + \tan y}\right) = 0$

$$\Rightarrow \quad \Rightarrow \qquad \frac{dx}{dy}+\frac{2x}{1+\tan y}=1$$

This is a linear differential equation of the form

$$\frac{dx}{dy}+Px=Q$$

Where $P=\dfrac{2}{1+\tan y}$ and Q = 1

$$I.F=e^{\int\frac{2}{1+\tan y}dy}$$

$$=e^{\int\frac{2}{1+\frac{\sin y}{\cos y}}dy}$$

$$=e^{\int\frac{2\cos y}{\cos y+\sin y}dy}$$

$$=e^{\int\frac{(\cos y+\sin y)+(\cos y-\sin y)}{\cos y+\sin y}dy}$$

$$=e^{\int 1+\frac{\cos y-\sin y}{\cos y+\sin y}dy}$$

$$=e^{\int y+\log e(\cos y+\sin y)}$$

$$=e^{y}\log(\cos y+\sin y)$$

= e^y (cosy + siny)

Mutiply both side by I.F

We botain

$$e^{y}(\cos y+\sin)\left[\frac{dx}{dy}+\frac{2}{1+\tan y}x\right]$$

$$e^{y}(\cos y+\sin y)$$

$$\Rightarrow \qquad x\cdot e^{y}(\cos y+\sin y)=\int e^{y}(\cos y+\sin y)dy$$

$$\Rightarrow \qquad x\cdot e^{y}(\cos y+\sin y)=\int(n)+f^{1}(y)e^{y}dy$$

⇒ Where f(y) = siny

And f(y) = cosy

$$\Rightarrow \qquad x\cdot e^{y}(\cos y+\sin y)=e^{y}f(y)+c$$

$$\Rightarrow \qquad x\cdot e^{y}(\cos y+\sin y)=e^{y}\sin y+c$$

Ans.17 Given that

$$\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}=k$$

X = 2k + 3, y = k + 3, z = k

Let print of intersection b/w

This line and line :-

Through origin be: (2k + 3, k + 3, k)

Direction vector of original line : a = <2, 1, 1>

$|a| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$

Direction vector of line b is

$b = \langle 2k+3, k+3, k \rangle$

$|b| = \sqrt{(2k+3)^2 + (k+3)^2 + k^2}$

$= \sqrt{4k^2 + 9 + 12k + k^2 + 9 + 6k + k^2}$

$= \sqrt{6k^2 + 18k + 18}$

It two intersecting line form an acute angle of π/3.

So, we need to find k s·t.

$$\frac{(a \cdot b)}{|a||b|} = \cos\frac{\pi}{3}$$

$$\frac{\langle 2,1,1 \rangle \cdot \langle 2k+3, k+3, k \rangle}{\sqrt{6}\sqrt{6k^2 + 18k + 18}} = \frac{1}{2}$$

$$\frac{4k + 6 + k + 3 + k}{\sqrt{6}\sqrt{6k^2 + 18k + 18}} = \frac{1}{2}$$

$$2(6k+9) = 6\left(\sqrt{k^2 + 3k + 3}\right)$$

$$2 \times 3(2k+3) = 6\sqrt{k^2 + 3k + 3}$$

Sq. on both side

$4(2k + 3)^2 = 4(k^2 + 3k + 3)$

$4(4k^2 + 9 + 12k) = 4(k^2 + 3k + 3)$

$16k^2 - 4k^2 + 48k - 12k + 36 - 12 = 0$

$12k^2 + 36k + 24 = 0$

$12(k^2 + 3k + 2) = 0$

$K^2 + 2k + k + 2 = 0$

K(k + 2) + 1(k + 2) = 0

(k + 2) (k + 1) = 0

K = - 1, -2

If k = -1

Then x = 2(-1) + 3 = 1

Y = - 1 + 3 = 2

Z = -1

Or k = - 2

Then

X = - 4 + 3 = - 1

Y = - 2 + 3 = 1

Z = -2

Ans.18 If siny = x sin(a + y)

Show that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

⇒ sin = x sin(a + y)

$$\frac{\sin y}{\sin(a+y)} = x$$

Diff. on both side

$$\frac{\sin(a+y)\cos\frac{dy}{dx} - \sin y\cos(a+y\frac{dy}{dx}}{\sin^2(a+y)} = 1$$

$$\frac{dy}{dx}\left[\sin(a+y)\cos y - \sin y\cos(a+y)\right] = \sin^2(a+y)$$

$\because \quad \sin a\cos b - \cos a\sin b = \sin(a-b)$

Hence

$$\frac{dy}{dx}\left[\sin(a+y-y)\right] = \sin^2(a+y)$$

$$\frac{dy}{dx}\sin a = \sin^2(a+y)$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

OR

X = a$\cos^3\theta$, y = a $\sin^3\theta$

Differentiate w·r·t· θ

$$\frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta \qquad ......(1)$$

$$\frac{dx}{d\theta} = 3a\sin^2\theta\cos\theta \qquad .....(2)$$

Now

$$\frac{dx}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$= \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta}$$

$$\frac{dy}{dx} = \frac{\sin\theta}{\cos\theta} \qquad ....(3)$$

Now again differentiating

$$\frac{d^2y}{dx^2} = \sin^2\theta\frac{d\theta}{dx}$$

$$= \sec^2\theta \times \frac{-1}{3a\cos^2\theta\sin\theta}$$

$$= \frac{-1}{3a}\sin^4\theta\cos ec\theta$$

Ans.19 Let x denote the No. diamond card x can take values of 0, 1, 2.

Let S be sample space

⇒ n(s) = 52

Let E be the event of getting a diamond probability of getting a diamond card

$(P) = \frac{13}{52} = \frac{1}{4}$

Not getting a diamond card $(q) = 1 - \frac{1}{4} = \frac{3}{4}$

We know that n = 2

$mean = np = 2 \times \frac{1}{4} = \frac{1}{2}$

Variance = npq

$= 2 \times \frac{1}{4} \times \frac{3}{4}$

$= \frac{3}{8}$

Ans.20 $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \sin^{-1}\frac{1}{\sqrt{5}}$

$\because \quad \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

L.H.S

$= \tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\times\frac{2}{9}}\right)$

$= \tan^{-1}\left(\frac{\frac{9+8}{36}}{\frac{36-2}{36}}\right)$

$= \tan^{-1}\left(\frac{\frac{17}{36}}{\frac{34}{36}}\right)$

$= \tan^{-1}\left(\frac{1}{2}\right)$

Now

Using the formula

$= \tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

$$= \sin^{-1}\left(\frac{\frac{1}{2}}{\sqrt{1+\left(\frac{1}{2}\right)^2}}\right)$$

$$= \sin^{-1}\left(\frac{\frac{1}{2}}{\sqrt{\frac{4+1}{2}}}\right)$$

$$= \sin^{-1}\frac{1}{\sqrt{5}},$$ Hence proved.

OR

$$\cot^{-1}(7)+\cot^{-1}(8)+\cot^{-1}(18)$$

$$= \tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{8}\right)+\tan^{-1}\left(\frac{1}{18}\right)$$

$$= \left[\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{8}\right]+\tan^{-1}\left(\frac{1}{18}\right)$$

$$= \tan^{-1}\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\times\frac{1}{8}}+\tan^{-1}\left(\frac{1}{18}\right)$$

$$= \tan^{-1}\frac{8+7}{56-1}+\tan^{-1}\frac{1}{18}$$

$$= \tan^{-1}\frac{3}{11}+\tan^{-1}\frac{1}{18}$$

$$\Rightarrow \quad \tan^{-1}\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{3}{11}\times\frac{1}{18}}$$

$$\Rightarrow \quad \tan^{-1}\frac{54+11}{198-3}$$

$$= \tan^{-1}\frac{65}{195}$$

$$= \tan^{-1}\frac{1}{3}$$

$$= \cot^{-1}3$$

= R.H.S Hence proved

Ans.21 Let $\vec{a} = 2\hat{i}+3\hat{j}+4\hat{k}$

$\vec{b} = \hat{i}+2\hat{j}-\hat{k}$

$\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$

$$Volume = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

= 2[4 – 1] – 3[2 + 3] + 4[- 1 – 6]
= 2(3) – 3(5) + 4(-7)
= 6 – 15 – 28
= 6 – 43
= - 37 cubic unit.

Ans.22 $f(x) = \begin{cases} x^p \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0 & n = 0 \end{cases}$

Now L $f^1(0) = \lim_{h\to 0^-} \frac{f(0+h) - f(0)}{h}$

$= \lim_{h+0^-} \frac{h^p \sin\left(\frac{1}{h}\right) - 0}{h}$

$\Rightarrow$ 0

Now R $f^1(0) = \lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h}$

$= \lim_{h\to 0^+} \frac{h^p \sin\left(\frac{1}{h}\right)}{h}$

$\Rightarrow$ 0

Since the left hand derivative and right hand derivative both are equal, hence f is derivable at x = 0

Ans.23 $I = \int_0^{\pi} \frac{x dx}{4 - \cos^2 x}$ ......(1)

$\int_0^a f(x) = \int_0^a f(a-x)$

$= \int_0^{\pi} \frac{\pi - x}{4 - (\cos(\pi - x))^2} dx$

$= \int_0^{\pi} \frac{\pi - x}{4 - (-\cos x)^2} dx$

$= \int_0^{\pi} \frac{\pi - x}{4 - \cos^2 x} dx$ ....(2)

By adding Ist + 2nd

$$2I = \int_0^{\pi} \frac{x+\pi-x}{4-\cos^2 x} dx$$

$$= \pi \int_0^{\pi} \frac{x+\pi-x}{4-\cos^2 x}$$

$$= \pi \int_0^{\pi} \frac{1}{(2)^2-\cos^2 x} dx$$

$$= \frac{\pi}{2}\left[\frac{1}{2a} \lim \left|\frac{a+x}{a-x}\right|\right.$$

$$= \frac{\pi}{2}\left[\frac{1}{2\times 2} - \log\left|\frac{2+\cos x}{2-\cos x}\right| + c\right.$$

Ans.24 Given equation are :-

2x – y = 4 ....(1)

2y – y = 5 ....(2)

Z + 2x = 7 .....(3)

AX = B

Where

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

$$x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$X = A^{-1} B$ ......(4)

|A| = 2(2 – 0) + 1(0 – 2) + 0 = 4 – 2 = 2

Now

$A_{11} = 2, A_{12} = +2, A_{13} = -4$

$A_{21} = +1, A_{22} = 2, A_{23} = -2$

$A_{31} = -1, A_{32} = -2, A_{33} = 4$

$$\therefore \quad adj\ A = \begin{bmatrix} 2 & 1 & -1 \\ 2 & 2 & -2 \\ -4 & -2 & 4 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} adj\ A$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & 1 & -1 \\ 2 & 2 & -2 \\ -4 & -2 & 4 \end{bmatrix}$$

Use in equation 4th

X = A-1 B

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 1 & -1 \\ 2 & 2 & -2 \\ -4 & -2 & 4 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 8+5-7 \\ 8+10-14 \\ -16-10+28 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 13-7 \\ 18-14 \\ -26+28 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$$

After dividing by 2 we get

X = 3, y = 2, z = 1

Ans.25 Let θ be the semi vertical angle of cone.

$$= \theta \in \left[0, \frac{\pi}{2}\right]$$

It is clear that Let r, h and l be the radius, height and slant height of cone respectively.

Now r = l sinθ and h = l cos θ

Now the volume of cone is given by,

$$V = \frac{1}{3}\pi r^2 h \qquad ......(1)$$

$$S = \pi r^2 + \pi r L$$

$\Rightarrow \quad l = \frac{S - \pi r^2}{\pi r}$

We know that

$h = \sqrt{l^2 r^2}$

$v^2 = \frac{1}{9}\pi^2 r^4 (l^2 - r^2)$

$= \frac{1}{9}\pi^2 r^4 \left[\left(\frac{s}{\pi r} - r\right)^2 - r^2\right]$

$= \frac{1}{9} s^2 r^2 - \frac{2\pi}{9} s r^4$

Diff. U^2 we get

$\frac{2vdu}{dr} = \frac{2rs}{9}(s - 4\pi r^2) \quad$ .....(2)

For max. or min.

$\frac{dv}{dr} = 0$

$\Rightarrow$ either r = 0

Or s – 4πr² = 0

$\Rightarrow$ S = 4πr²

$\frac{s}{4\pi} = r^2$

Again diff. 2nd

$2\left(\frac{du}{dr}\right)^2 + 2v\frac{d^2 0}{dr^2} = \frac{1}{9}s(25 - 24\pi r^2)$

$\Rightarrow \quad \frac{2vd^2u}{dr^2} = \frac{1}{9}s(25 - 65)$

$\Rightarrow \quad \frac{2vd^2U}{dr^2} = \frac{1}{9}s(-45)$

= - ve

Vis max. when

$r^2 = \frac{S}{4\pi}$

Also $r^2 = \frac{5}{4\pi} = \frac{\pi r^2 + \pi r l}{4\pi}$

$\Rightarrow \quad 4\pi r^2 = \pi r^2 + \pi r l$

$\Rightarrow \quad l = 3r$

Semi vertical angle of cone, then

$\sin\theta = \frac{r}{r} = \frac{r}{3r}$

$\sin\theta = \frac{1}{3}$

$\theta = \sin^{-1}\left(\frac{1}{3}\right)$

OR

Let (x, y_1) be the point on the parabola

$\therefore \quad y_1^2 = 4x_1$

And $x_1 = \frac{y_1^2}{4}$ ……(1)

Let l be the distance b/w

(x, y_1) and (2, -8)

$l = \sqrt{(x,-2)^2 + \left(y_1 - (-8)\right)^2}$

$= \sqrt{\left(\frac{y_1^2}{4} - 2\right)^2 + \left(y_1 + 8\right)^2}$

$l = \frac{1}{2}\sqrt{(y_1^2 - 8)^2 + 4(y_1 + 8)^2}$ …..(2)

Since (x_1, y_1) is nearest to the point (2, -8) the distance should be minimum.

$\therefore \quad \frac{dl}{dy_1}$ should be equal too diff. 2nd

$\frac{dl}{dy_1} = \frac{1}{2} \times \frac{1}{2} \times \frac{2(y_1^2 - 8) \times 2y_1 + 8(y_1 + 8)}{\sqrt{(y_1^2 - 8)^2 + 4(y_1 + 8)^2}}$

$\frac{dl}{dy_1} = 0$

Then after solving

$y_1^3 - 6y_1 + 16 = 0$ ….(3)

Consider a general cubic equation

$ax^3 + bx^2 + cx + d = 0$

Define $f = \frac{\frac{3c}{a} - \frac{b_2}{a^2}}{3}$

$g = \frac{\left(\frac{2b^3}{a^3}\right) - \frac{9bc}{a^2} + 27\left(\frac{d}{2}\right)}{27}$

$h = \left(\frac{9^2}{4}\right) + \left(\frac{f3}{27}\right),$

$k = \sqrt{\frac{9^2}{4} - h},$

$m = (k)^{1/3}, n = \cos^{-1}\left(\frac{-9}{2k}\right)$

Q = n 3 gives that

A = 1, b = 0, c = -6, d = 16

Substituting the value we get

g = 16, f = -6, h = 56

Only one root is real other 2 are imaginary.

We get $y_1 \approx -3\cdot 3$

$x_1 \approx 2\cdot 7$

Ans.26 Let X defined as sum of numbers on two cards.

X = (3, 4, 5, 6, 7, 8, 9)

P(x = 3) = P[(1, 2) or (2, 1)]

$= \frac{2}{20} = \frac{1}{10}$

P(x = 4) = P[(1, 3) or (3, 1)]

$= \frac{2}{20} = \frac{1}{10}$

P(x = 5) = P[(1, 4) or (4, 1) or (3, 2) or (2, 3)]

$= \frac{4}{20} = \frac{1}{5}$

$P(x = 6) = \frac{4}{20} = \frac{1}{5}$

$P(x = 7) = \frac{4}{20} = \frac{1}{5}$

$P(x = 8) = \frac{2}{20} = \frac{1}{10}$

$P(x = 9) = \frac{2}{20} = \frac{1}{10}$

$\sum pixi = \frac{60}{10} = 6$

$\sum pixi^2 = 39$

Variance = 39 – 6^2 = 39 – 36 = 3

Ans.27. The given equation of circle

$X^2 + y^2 = 4x$ .....(1)

And $y^2 = 2x$ .....(2)

Use (2) in Ist

$X^2 + 2x = 4x$

$X^2 + 2x - 4x = 0$

$X^2 - 2x = 0$

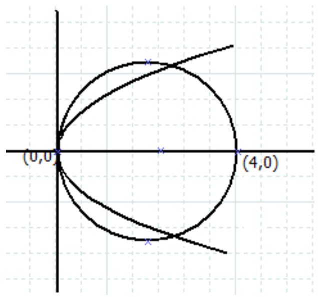
$X(x - 2) = 0$

$X = 0, 2$

Now It can be expressed in $(x - 2)^2 + y^2 = 8$

Thus, the centre of circle is (2, 0) and radius is $2\sqrt{2}$.

Thus, the points of



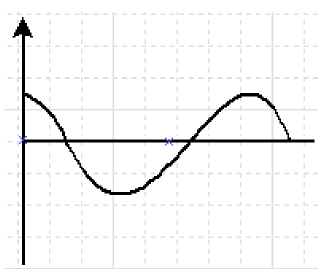
$$= \int_0^2 y dx + \int_2^4 y dx$$

$$= \sqrt{2}\int_0^2 \sqrt{x} dx + \int_2^4 \sqrt{8-(x-2)^2}$$

$$= \sqrt{2}\left[\frac{\frac{x^3}{2}}{\frac{3}{2}}\right]_0^2 + \int_2^4 (2\sqrt{2})^2 - t^2$$ where x – 2 = t

OR

The area from x = 0 to x = 2pi can be divided into 4 equal segments. It we find the area from x = 0 to $x = \frac{\pi}{2}$ and multiple it by 4.



$$2\int_0^{2\pi} \cos x \, dx$$

$$= 8\int_0^{\frac{\pi}{2}} \cos x \, dx$$

$$= 8[\sin x]_0^{\frac{\pi}{2}}$$

$= 8\left[\sin\frac{\pi}{2} - \sin 0\right]$

= 8 ×1 = 8 units

Ans.28 Suppose x is the number of pieces of model A and y is the number of pieces of model B.

Then, Profit Z= 8000x + 12000y

The mathematical formulation of the problem is as follows:

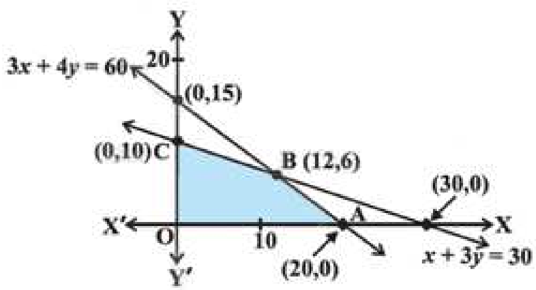
Max Z= 8000x+12000y

$9x + 12y \leq 180(\textit{fabricating constra}\text{int})$

$3x + 4y \leq 60$

$x + 3y \leq 30(\textit{finishing constra}\text{int})$

$x \geq 0, y \geq 0$



We graph the above inequalities. The feasible region is as shown in the figure. The corner points are O, A, B and C. The co-ordinates of the corner points are (0,0), (20,0), (12,6), (0,10).

Corner Point	Z=8000x +12000y
(0,0)	0
(20,0)	16000
(12,6)	**16800**
(0,10)	12000

Thus profit is maximized by producing 12 units of A and 6 units of B and maximum profit is 16800.

Ans.29 Given planes are

$4x + 8y + z - 8 = 0$ ....(1)

And y + z – 4 = 0

$\overrightarrow{x_1} = (A_1, B_1, C_1)$

$= (4, 8, 1)$

$\overrightarrow{x_2} = (A_2, B_2, C_2)$

= (0,1,1)

$$\cos\theta = \frac{|A_1 \cdot A_2 + B_1 \cdot B_2 + C_1 \cdot C_2|}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}}$$

$$= \frac{|4 \cdot 0 + 8 + 1|}{\sqrt{16 + 64 + 1}\sqrt{0 + 1 + 1}}$$

$$= \frac{13}{\sqrt{81}\sqrt{2}}$$

$$= \frac{13 \times \sqrt{2}}{9\sqrt{2} \times \sqrt{2}}$$

$$\text{Cos}\,\theta = \frac{13\sqrt{2}}{18}$$

$$\theta = \text{Cos}^{-1}\frac{13\sqrt{2}}{18}$$