Sample Question Paper - 33
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours ***Maximum Marks : 40***

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
3. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
4. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. If the sum of two natural numbers is 8 and their product is 15, then find the numbers.

2. Find the mean of the following data :

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	2	1	6	8	3

3. Find the 25th term of the A.P. $-5, \frac{-5}{2}, 0, \frac{5}{2}, ...$

OR

How many terms of the A.P. 27, 24, 21, ... should be taken so that their sum is zero?

4. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 13 cm. Find the length PQ.

5. Find the value of k so that the quadratic equation $kx\,(3x - 10) + 25 = 0$, has two equal roots.

6. Find the volume of the largest sphere that can be cut from a cylindrical log of wood of base radius 1 cm and height 5 cm.

OR

Three cubes each of edge 3 cm are joined end to end. Find the surface area of the resulting cuboid.

SECTION - B

7. Let ABC be a right angled triangle in which $AB = 12$ cm, $BC = 5$ cm and $\angle B = 90°$. BD is the perpendicular from B on AC. The circle through B, C and D is drawn. Construct the tangents from A to this circle.

8. From the top of a 10 m high tower, the angle of depression of a point on the ground is found to be 30°. Find the distance of the point from the base of the tower.

OR

On the level ground, the angle of elevation of a tower is 30°. On moving 20 m nearer to the tower, the angle of elevation becomes 60°. What is the height of the tower?

9. Look at the frequency distribution table given below.

Class interval	35-45	45-55	55-65	65-75
Frequency	8	12	20	10

Find the median of the above distribution.

10. Calculate the mode for the following frequency distribution.

Class interval	1-4	5-8	9-12	13-16	17-20	21-24	25-28	29-32	33-36	37-40
Frequency	2	5	8	9	12	14	14	15	11	10

SECTION - C

11. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle, which touches the smaller circle.

OR

In the given figure, AB and AC are tangents to a circle with centre O and radius 8 cm. If $OA = 17$ cm, then find the length of AC.

12. If the sum of first m terms of an A.P. is the same as the sum of its first n terms, show that the sum of its first $(m + n)$ terms is zero.

Case Study - 1

13. Ritu packed a football as a gift for her brother's birthday in a cuboidal box whose diameter is same as that of length of base of the box having length, breadth and height respectively 23 cm, 23 cm and 28 cm.

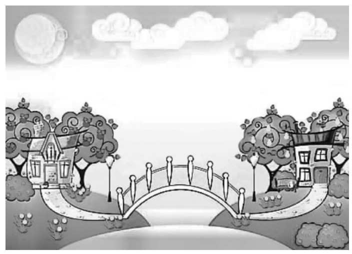
Based on the above information, answer the following questions.

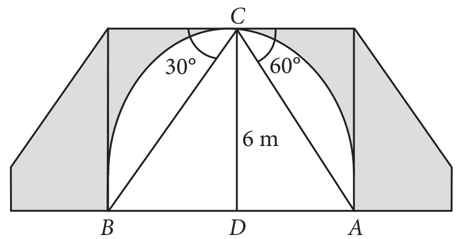
(i) What is the volume of the football?

(ii) Ritu covers the box with a wrapping sheet. Find the area of the wrapping sheet that covers the box exactly.

Case Study - 2

14. One day while sitting on the bridge across a river Arun observes the angles of depression of the banks on opposite sides of the river are 30° and 60° respectively as shown in the figure. (Take $\sqrt{3} = 1.73$)





Based on the above information, answer the following questions.

(i) If the bridge is at a height of 6 m, then find the length of AD.

(ii) Find the width of the river.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

1. Let the two natural numbers be a and b.

So, $a + b = 8$ (Given) $\Rightarrow$ $a = 8 - b$ ...(1)

and $ab = 15$ (given)

$\Rightarrow$ $(8 - b)b = 15$ [Using (1)]

$\Rightarrow$ $8b - b^2 = 15$ $\Rightarrow$ $b^2 - 8b + 15 = 0$

$\Rightarrow$ $b^2 - 5b - 3b + 15 = 0$ $\Rightarrow b(b-5) - 3(b-5) = 0$

$\Rightarrow$ $(b-3)(b-5) = 0$ $\Rightarrow$ $b = 3$ or $b = 5$

When $b = 3$, then $a = 8 - b = 8 - 3 = 5$

When $b = 5$, then $a = 8 - 5 = 3$

Thus, the required natural numbers are 3 and 5.

2. The frequency distribution table from the given data can be drawn as:

Class interval	Class mark (x_i)	Frequency (f_i)	$f_i x_i$
0–10	5	2	10
10–20	15	1	15
20–30	25	6	150
30–40	35	8	280
40–50	45	3	135
		$\sum f_i = 20$	$\sum f_i x_i = 590$

$\therefore$ Mean, $\overline{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{590}{20} = 29.5$

3. Given, $-5, -\dfrac{5}{2}, 0, \dfrac{5}{2}, \ldots$ are in A.P.

$\Rightarrow$ $a = -5,\ d = \left(\dfrac{-5}{2}\right) - (-5) = \dfrac{5}{2}$

We know that, $a_n = a + (n-1)d$

$\therefore$ $a_{25} = a + (25 - 1)d$

$= (-5) + 24 \times \left(\dfrac{5}{2}\right) = -5 + 60 = 55$

OR

Let a be the first term and d be the common difference of A.P.

Sum of n terms is given as $S_n = \dfrac{n}{2}[2a + (n-1)d]$

Here, $a = 27$, $d = 24 - 27 = -3$

According to question,

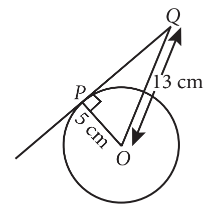
$$0 = \frac{n}{2}[2(27) + (n-1)(-3)]$$

$\Rightarrow$ $n[54 - 3n + 3] = 0$

$\Rightarrow$ $3n = 57$ $\Rightarrow$ $n = 19$ $(\because\ n \neq 0)$

4. We have, $OP = 5$ cm, $OQ = 13$ cm, $\angle OPQ = 90°$

[Since, radius is perpendicular to the tangent at the point of contact]



$\therefore$ In right ΔOPQ,

$(OP)^2 + (PQ)^2 = (OQ)^2$ [By Pythagoras Theorem]

$\Rightarrow$ $5^2 + (PQ)^2 = 13^2$

$\Rightarrow$ $(PQ)^2 = 169 - 25 = 144$

$\Rightarrow$ $PQ = 12$ cm

5. Given, $kx(3x - 10) + 25 = 0$

$\Rightarrow$ $3kx^2 - 10kx + 25 = 0$ ...(i)

For equal roots, discriminant, $D = 0$

$\Rightarrow$ $(-10k)^2 - 4 \cdot 3k \cdot 25 = 0$

$\Rightarrow$ $100k^2 - 300k = 0$ $\Rightarrow$ $100k(k - 3) = 0$

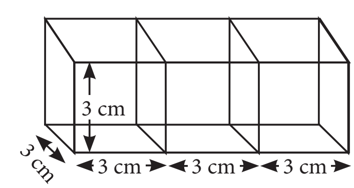
$\Rightarrow$ $k = 3$ $[\because\ k \neq 0]$

6. Radius of largest sphere, r = Radius of cylindrical log of wood = 1 cm

$\therefore$ Volume of largest sphere =

$\dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi \times 1^3 = \dfrac{4}{3}\pi \text{ cm}^3$

OR



Length of the resulting cuboid = 3 + 3 + 3 = 9 cm

Breadth of the resulting cuboid = 3 cm

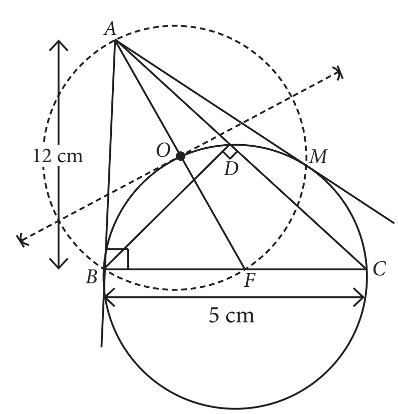
Height of the resulting cuboid = 3 cm

$\therefore$ Surface area of the resulting cuboid

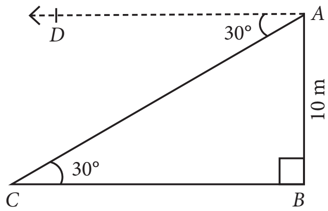
$= 2(lb + bh + hl)$

$= 2(9 \times 3 + 3 \times 3 + 3 \times 9) = 2 \times 63 = 126 \text{ cm}^2$

7.



8. Let AB be the tower and C be the point on the ground, such that $\angle ACB = 30°$.
$\angle DAC = \angle ACB = 30°$ (Alternate angles)



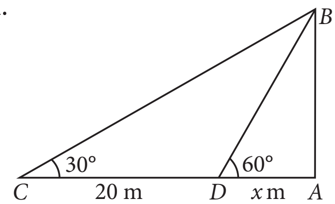
In ΔABC, $\tan 30° = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{BC} \Rightarrow BC = 10\sqrt{3}$ m

Hence, the distance of the point C from the base of tower is $10\sqrt{3}$ m.

OR

Let AB be the tower and C, D be the points of observation such that $\angle BCA = 30°$, $\angle BDA = 60°$ and $CD = 20$ m. Let $DA = x$ m.



In ΔABD, $\frac{AB}{AD} = \tan 60°$

$\Rightarrow \frac{AB}{x} = \sqrt{3}$

$\Rightarrow AB = \sqrt{3}x$ ...(i)

In ΔABC, $\frac{AB}{AC} = \tan 30°$

$\Rightarrow \frac{AB}{20+x} = \frac{1}{\sqrt{3}} \Rightarrow AB = \frac{20+x}{\sqrt{3}}$ ...(ii)

From (i) and (ii), we get $\sqrt{3}x = \frac{20+x}{\sqrt{3}}$

$\Rightarrow 3x = 20 + x \Rightarrow 2x = 20 \Rightarrow x = 10$

$\therefore$ Height of the tower, $AB = \sqrt{3}x = 10\sqrt{3}$ m

9. The cumulative frequency distribution table from the given data can be drawn as :

Class interval	Frequency	Cumulative frequency
35-45	8	8
45-55	12	20
55-65	20	40
65-75	10	50

Here, $n = 50 \Rightarrow \frac{n}{2} = 25$, which lies in class interval 55-65.

$\therefore$ Median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h = 55 + \left(\frac{25-20}{20}\right) \times 10$
$= 57.5$

10. Here, the class intervals are not in inclusive form. So, we first convert them in inclusive form by subtracting 0.5 from lower limit and adding 0.5 to the upper limit. The given frequency distribution in inclusive is as follows.

Class interval	Frequency
0.5-4.5	2
4.5-8.5	5
8.5-12.5	8
12.5-16.5	9
16.5-20.5	12
20.5-24.5	14
24.5-28.5	14
28.5-32.5	15
32.5-36.5	11
36.5-40.5	10

Here, maximum frequency is 15, which lies in the interval 28.5-32.5.

$\therefore$ Modal class is 28.5 – 32.5.

So, $l = 28.5, f_1 = 15, f_0 = 14, f_2 = 11, h = 4$

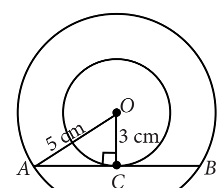
$\therefore$ Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 28.5 + \left(\frac{15-14}{2 \times 15 - 14 - 11}\right) \times 4 = 28.5 + \frac{1}{5} \times 4$

$= 28.5 + 0.8 = 29.3$

11. Let chord AB of larger circle is a tangent to the smaller circle.

$\therefore$ $OC \perp AB$



In right ΔOAC,

$(OA)^2 = (OC)^2 + (AC)^2$

$\Rightarrow (5)^2 = (3)^2 + (AC)^2$

$\Rightarrow AC^2 = 25 - 9 = 16$

$\Rightarrow AC = 4$ cm

As, perpendicular drawn from the centre to the chord bisects the chord.

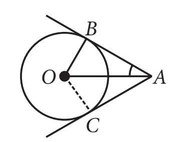
$\therefore$ Length of chord $AB = (2 \times 4)$ cm $= 8$ cm

OR

We have, AB and AC are the tangents to a circle with centre O. Join OC.

Since tangent is perpendicular to radius through point of contact

$\therefore$ $\angle OCA = 90°$



In right ΔAOC,

$(OA)^2 = (AC)^2 + (OC)^2$

and $OC = 8$ cm, $OA = 17$ cm

$\Rightarrow (AC)^2 = (17)^2 - (8)^2 = 289 - 64 = 225$

$\Rightarrow AC = 15$ cm

12. Let a be the first term and d be the common difference of the A.P.

Sum of m and n terms of A.P. are

$S_m = \frac{m}{2}[2a + (m-1)d]$

and $S_n = \frac{n}{2}[2a + (n-1)d]$ respectively.

Given that $S_m = S_n$

$\therefore \quad \frac{m}{2}[2a + (m-1)d] = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad 2a(m - n) + \{m(m - 1) - n(n - 1)\}d = 0$

$\Rightarrow \quad 2a(m - n) + \{(m^2 - n^2) - (m - n)\}d = 0$

$\Rightarrow \quad (m - n)\{2a + (m + n - 1)d\} = 0$

$\Rightarrow \quad 2a + (m + n - 1)d = 0 \qquad [\because\ m - n \neq 0] \quad ...(i)$

Now, $S_{m+n} = \frac{m+n}{2}[2a + (m+n-1)d]$

$= \frac{m+n}{2}[0] = 0 \qquad$ [Using (i)]

13. Diameter of football = Length of base of the box = 23 cm

$\therefore$ Radius of football = $\left(\frac{23}{2}\right)$ cm

(i) Volume of the football = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{23}{2} \times \frac{23}{2} \times \frac{23}{2} = 6373.19 \text{ cm}^3$

(ii) Area of wrapping sheet = Total surface area of the cuboidal box

$= 2(lb + bh + hl) = 2(23 \times 23 + 23 \times 28 + 28 \times 23)$

$= 2(529 + 644 + 644) = 3634 \text{ cm}^2$

14. (i) Clearly, $\angle DAC = 60°$

So, in ΔADC, we have

$\tan 60° = \frac{CD}{AD} \Rightarrow \sqrt{3} = \frac{6}{AD}$

$\Rightarrow \quad AD = \frac{6}{\sqrt{3}}$ m $= 2\sqrt{3}$ m

(ii) Clearly, $\angle DBC = 30°$

So, in ΔBDC, we have

$\tan 30° = \frac{CD}{BD}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{6}{BD}$

$\Rightarrow \quad BD = 6\sqrt{3}$ m

$\therefore$ Width of the river = $AB = AD + BD$

$= \frac{6}{\sqrt{3}} + 6\sqrt{3}$

$= 6\left(\frac{1}{\sqrt{3}} + \sqrt{3}\right) = 6\left(\frac{4}{\sqrt{3}}\right) = \frac{24}{\sqrt{3}}$ m $= 13.87$ m