Class: XII

SESSION : 2022-2023

SUBJECT: Mathematics

SAMPLE QUESTION PAPER - 4 with SOLUTION

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions :

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment (**4 marks each) with sub parts.

Section A

1. $\int \frac{x^2-1}{x^4+3x^2+1} dx$ is equal to **[1]**

a) $\tan\left(x+\frac{1}{x}\right)+C$

b) $\tan^{-1}\left(x+\frac{1}{x}\right)+C$

c) $\tan^{-1}\left(3x^2+2x\right)+C$

d) $\tan^{-1}\left(x^2+1\right)+C$

2. The angle between a line with direction ratios 2 : 2 : 1 and a line joining (3, 1, 4) to (7, 2, 12) **[1]**

a) $cos^{-1}(\frac{2}{3})$

b) $tan^{-1}(-\frac{2}{3})$

c) none of these

d) $cos^{-1}(\frac{3}{2})$

3. If a unit vector $\vec{a}$ makes angles $\frac{\pi}{3}$ $with$ $\hat{i}$ $\frac{\pi}{4}$ $with$ $\hat{j}$ $and\ an\ acute\ angle\ \theta\ with$ $\hat{k}$, then find θ **[1]**

a) $2\frac{\pi}{3}$

b) $\frac{\pi}{10}$

c) $\frac{\pi}{3}$

d) $\frac{\pi}{5}$

4. If one ball is drawn at random from each of three boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, then the probability that 2 white and 1 black balls will be drawn is **[1]**

a) $\frac{1}{4}$

b) $\frac{13}{32}$

c) $\frac{1}{32}$

d) $\frac{3}{16}$

5. $\int 2x^3 e^{x^2}$ dx = ? **[1]**

a) e^{x^2} $(x^2 - 1) + C$

b) e^{x^2} $(x^2 + 1) + C$

c) e^{x^2} (x + 1) + C

d) None of these

6. The probability that an event E occurs in one trial is 0.4. Three independent trials of the experiment are performed. What is the probability that E occurs at least once? **[1]**

a) 0.936

b) 0.784

c) None of these

d) 0.964

7. The area of the region (in square units) bounded by the curve $x^2 = 4y$, line x = 2 and x-axis is **[1]**

a) $\frac{8}{3}$

b) 1

c) $\frac{2}{3}$

d) $\frac{4}{3}$

8. The lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{5}$ are **[1]**

a) none of these

b) parallel

c) intersecting

d) skew

9. If θ is the angle between two vectors $\vec{a}\ and\ \vec{b}$, then $\vec{a}.\vec{b} \geqslant 0$ only when **[1]**

a) $0 < \theta < \frac{\pi}{2}$

b) $0 \leqslant \theta \leqslant \pi$

c) $0 < \theta < \pi$

d) $0 \leqslant \theta \leqslant \frac{\pi}{2}$

10. Which of the following is a homogeneous differential equation? **[1]**

a) $y^2 dx + \left(x^2 - xy - y^2\right) dy = 0$

b) $(xy)\ dx - \left(x^3 + y^3\right) dy = 0$

c) $\left(x^3 + 2y^2\right) dx + 2xy\, dy = 0$

d) $(4x + 6y + 5)\ dy - (3y + 2x + 4)\ dx = 0$

11. The area bounded by the lines y = 2 + x , y = 2 – x and x = 2 is **[1]**

a) 16

b) 8

c) 3

d) 4

12. $\int \frac{x^2}{(a^6 - x^6)} dx = ?$ **[1]**

a) $\frac{1}{3a^3} \log\left|\frac{a^3 + x^3}{a^3 - x^3}\right| + C$

b) $\frac{1}{6a^3} \log\left|\frac{a^3 - x^3}{a^3 + x^3}\right| + C$

c) $\frac{1}{6a^3} \log\left|\frac{a^3 + x^3}{a^3 - x^3}\right| + C$

d) None of these

13. If the function f (x) = 2 tan x + (2a + 1) $\log_e$ |sec x| + (a - 2) x is increasing on R, then **[1]**

$a \in R$

a) a = $\frac{1}{2}$

b)

c) $a \in (\frac{1}{2}, \infty)$

d) $a \in (-\frac{1}{2}, \frac{1}{2})$

14. If A = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then A^4 = **[1]**

a) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

15. If A is a matrix of order 3 and |A| = 8, then |adj A| = **[1]**

a) 2

b) 1

c) 2^6

d) 2^3

16. The system of equations **[1]**
x + y + z = 2,
3 x - y + 2z = 6
3x + y + z = -18
has:

a) zero solution as the only solution

b) an infinite number of solutions

c) a unique solution

d) no solution

17. Range of $\text{coses}^{-1}x$ is **[1]**

a) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

b) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - {0}

c) None of these

d) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

18. Find a solution of $\cos\left(\frac{dy}{dx}\right) = a\ (a \in R)$ which satisfy the condition y = 1 when x = 0. **[1]**

a) $cos\frac{y-10}{x} = a$

b) $cos\frac{y-1}{x} = a$

c) $cos\frac{y-4}{x} = a$

d) $cos\frac{y-3}{x} = a$

19. **Assertion (A):** The function f(x) = x^2 - 4x + 6 is strictly increasing in the interval $(2, \infty)$. **[1]**

Reason (R): The function f(x) = x^2 - 4x + 6 is strictly decreasing in the interval $(-\infty, 2)$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

20. **Assertion (A):** If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then x = ±6. **[1]**

Reason (R): If A is a skew-symmetric matrix of odd order, then |A| = 0.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ **[2]**

22. Solve the differential equation: $x\frac{dy}{dx}$ = x + y **[2]**

23. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular? **[2]**

OR

Solve the system of equations by Cramer's rule:
2x - y = 17
3x + 5y = 6

24. If $\vec{a} = (2\hat{i} - 4\hat{j} + 5\hat{k})$ then find the value of λ so that $\lambda\vec{a}$ may be a unit vector. **[2]**

25. A bag contains 5 white, 7 red and 4 black balls. If four balls are drawn one by one with replacement, what is the probability that none is white? **[2]**

Section C

26. If $f\left(\frac{3x-4}{3x+4}\right) = x + 2$, then find value of $\int f(x)dx$ **[3]**

27. Solve the differential equation: cos x $\frac{dy}{dx}$ - cos 2x = cos 3x **[3]**

OR

Solve the differential equation: $(2x^2 y + y^3)$ dx + $(xy^2 - 3x^3)$ dy = 0

28. Find λ when the projection of $\overrightarrow{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$ on $\overrightarrow{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units. **[3]**

OR

Show that the points A,B,C with position vectors $(3\hat{i} - 2\hat{j} + 4\hat{k})$, $(\hat{i} + \hat{j} + \hat{k})$ and $(-\hat{i} + 4\hat{j} - 2\hat{k})$ respectively are collinear.

29. Evaluate the integral: $\int(x+1)\sqrt{x^2 - x + 1}\,dx$ **[3]**

OR

Evaluate $\int_0^{\pi} \frac{1}{3+2\sin x+\cos x}dx$

30. Find $\frac{dy}{dx}$, when x = ae^{θ}(sin θ - cos θ), y = ae^{θ}(sin θ + cos θ) **[3]**

31. Find the area of the segment of the parabola $y = x^2 - 5x + 15$ cut off by the straight line y = 3x + 3. **[3]**

Section D

32. Solve the following LPP graphically: **[5]**
Maximize Z = 5x + 7y
Subject to
$x + y \leq 4$
$3x + 8y \leq 24$
$10x + 7y \leq 35$
$x, y \geq 0$

33. Let R be a relation on $N \times N$, defined by (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in N \times N$. Show that R is an equivalence relation. **[5]**

OR

Let n be a fixed positive integer. Define a relation R in Z as follows $\forall$a, b $\in$ Z aRb if and only if a-b is divisible by n. Show that R is an equivalence relation.

34. Find the shortest between the l_1 and l_2 whose vector equations are **[5]**
$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k})$
and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

OR

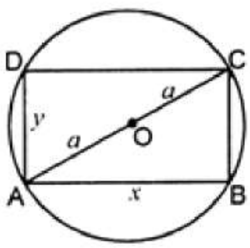
Find the distance of the point (-1,-5,-10) from the point of intersection of the line $\overrightarrow{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\overrightarrow{r} \cdot \left(\hat{i} - \hat{j} + \hat{k}\right) = 5$.

35. Differentiate $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, if 0 < x < 1. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)



(i) Find the perimeter of rectangle in terms of any one side and radius of circle.

(ii) Find critical points to maximize the perimeter of rectangle?

(iii) Check for maximum or minimum value of perimeter at critical point.

OR

If a rectangle of the maximum perimeter which can be inscribed in a circle of radius 10 cm is square, then the perimeter of region.

37. **Read the text carefully and answer the questions:** **[4]**

Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats, and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold by school A, B, C are given below.

Article	School	A	B	C
Fans		40	25	35
Mats		50	40	50
Plates		20	30	40

(i) Represent the sale of handmade fans, mats and plates by three schools A, B and C and the sale prices (in ₹) of given products per unit, in matrix form.

(ii) Find the funds collected by school A, B and C by selling the given articles.

(iii) If they increase the cost price of each unit by 20%, then write the matrix representing new price.

OR

Find the total funds collected for the required purpose after 20% hike in price.

38. **Read the text carefully and answer the questions:** **[4]**

In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03.

(i) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, find the probability that the form is NOT processed by Govind.

(ii) Find the probability that Priyanka processed the form and committed an error.

SOLUTION

Section A

1. **(b)** $\tan^{-1}\left(x+\frac{1}{x}\right)+C$

Explanation: Divide num. and deno. by x^2

Substitute $x+\frac{1}{x}=t$ then $(1-\frac{1}{x^2})dx=dt$

$\Rightarrow \int \frac{dt}{t^2+1}$

$\Rightarrow \tan^{-1}(x+\frac{1}{x})+C$

2. **(a)** $cos^{-1}(\frac{2}{3})$

Explanation: The angle between a line with direction ratios 2 : 2 : 1 and a line joining (3, 1, 4) to (7, 2, 12)

Direction ratios of the line joining the points A(3, 1, 4), B(7, 2, 12) is <x2-x1 , y2-y1, z2-z1> = < 7-3 , 2-1 , 12-4> = <4 ,1 ,8>

Now as the angle between two lines having direction ratios <a1,b1,c1> and <a2,b2,c2> is given by

$\text{Cos}^{-1}\frac{a1a2+b1b2+c1c2}{\sqrt{a1^2+b1^2+c1^2}\sqrt{a2^2+b2^2+c2^2}}$

Using the vuales we have

$\cos^{-1}\frac{2\times 4+2\times 1+1\times 8}{\sqrt{2^2+2^2+1^2}\sqrt{4^2+1^2+8^2}}=\cos^{-1}\frac{18}{27}=\cos^{-1}\frac{2}{3}$

3. **(c)** $\frac{\pi}{3}$

Explanation: Let

$\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$,

It is given that $\left|\vec{a}\right|=1$

, then,

$\Rightarrow a_1^2+a_2^2+a_3^2=1....(1)$

$\therefore \vec{a}.\hat{i}=(a_1\hat{i}+a_2\hat{j}+a_3\hat{k}).\hat{i}$

$\Rightarrow |\vec{a}||\hat{i}|cos\frac{\pi}{3}=a_1 \Rightarrow a_1=\frac{1}{2}$

$\therefore \vec{a}.\hat{j}=(a_1\hat{i}+a_2\hat{j}+a_3\hat{k}).\hat{j}$

$\Rightarrow |\vec{a}||\hat{j}|cos\frac{\pi}{4} = a_2 \Rightarrow a_2 = \frac{1}{\sqrt{2}}$

$\therefore \vec{a}.\hat{k} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}).\hat{k}$

$\Rightarrow |\vec{a}||\hat{k}|cos\theta = a_3 \Rightarrow a_3 = \cos\theta$

Putting these values in (1), we get:

$\frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$

$\Rightarrow \frac{3}{4} = 1 - \cos^2\theta \Rightarrow \sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 60^\circ$

4. **(b)** $\frac{13}{32}$

Explanation: Here, the three boxes contain 3 white and 1 black (3 W, 1 B), 2 white and 2 black (2 W, 2 B) and 1 white and 3 black balls (1 W, 3 B), respectively

P(2 W, 1 B) = $\frac{3}{4} \times \frac{2}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{2}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{2}{4} \times \frac{1}{4}$

$= \frac{18}{64} + \frac{6}{64} + \frac{2}{64}$

$= \frac{26}{64}$

$= \frac{13}{32}$

5. **(a)** $e^{x^2}(x^2 - 1) + C$

Explanation: $I = \int(2x)x^2e^{x^2}dx = \int tI\ e^tIIdt$, where $x^2 = t$

after solving we get

$I = e^{x^2}(x^2 - 1) + C$

6. **(b)** 0.784

Explanation: p = 0.4, q = (1 - 0.4) = 0.6 and n = 3.

Required probability = P(E occuring at least once)

$= {}^3C1.(0.4)^1 \times (0.6)^2 + {}^3C_2.(0.4)^2 \times (0.6)^1 + {}^3C_3.(0.4)^3$

$= \left(\frac{54}{125} + \frac{36}{125} + \frac{8}{125}\right)$

$= \frac{98}{125}$

= 0.784.

7. **(c)** $\frac{2}{3}$

Explanation: The area of the region bounded by

the curve $x^2 = 4y$ and line x = 2 and x-axis

$$\Rightarrow \int_0^2 y dx = \int_0^2 \frac{x^2}{4} dx$$

$$\Rightarrow \int_0^2 y dx = \left[\frac{x^3}{12}\right]_0^2$$

$$\Rightarrow \int_0^2 y dx = \frac{8}{12} = \frac{2}{3}$$

8. **(c)** intersecting

Explanation: Here $(a_1, b_1, c_1) = (2, 3, 4)$

and, $(a_2, b_2, c_2) = (3, 4, 5)$

Consider $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

We get

$$\begin{vmatrix} 0 & 0 & 0 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$$

Since the shortest distance is zero hence the lines are intersecting.

9. **(d)** $0 \leqslant \theta \leqslant \frac{\pi}{2}$

Explanation: $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|cos\theta,$

Also, $\vec{a}.\vec{b} \geqslant 0$

$$\Rightarrow |\vec{a}||\vec{b}|cos\theta \Rightarrow cos\theta \leqslant 0 \Rightarrow 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

10. **(a)** $y^2dx + \left(x^2 - xy - y^2\right) dy = 0$

Explanation: it is a homogeneous differential equation ,because the degree of each individual term is same i.e. 2.

11. **(d)** 4

Explanation: Reqd. area sq.units

$= \int_0^2 (y - 2)dy + \int_2^0 (2 - y)dy + \int_0^4 2dy$

$$= \left[\frac{y^2}{2} - 2y\right]_0^2 + \left[2y - \frac{y^2}{2}\right]_2^0 + [2y]_0^4$$

= (2 - 4) - (4 - 2) + 8 = 4

12. **(c)** $\frac{1}{6a^3}\log\left|\frac{a^3+x^3}{a^3-x^3}\right|+C$

Explanation: $I=\int\frac{x^2}{\left(a^3\right)^2-\left(x^3\right)^2}dx$

Let $x^3 = t$

$\Rightarrow\ 3x^2\,dx = dt$

$\Rightarrow\ x^2dx=\frac{dt}{3}$

$\Rightarrow I=\frac{1}{3}\int\frac{dt}{\left(a^3\right)^2-t^2}$

We know, $\int\frac{1}{a^2-x^2}=\frac{1}{2a}\log\frac{a+x}{a-x}+c$

$=\frac{1}{6a^3}\log\frac{a^3+t}{a^3-t}+c$

put $t = x^3$

$=\frac{1}{6a^3}\log\frac{a^3+x^3}{a^3-x^3}+c$

13. **(a)** $a=\frac{1}{2}$

Explanation: $a=\frac{1}{2}$

14. **(b)** $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Explanation: In the given question the given matrix is an Identity matrix, and I^n = I.I.I .... .I(n times) = I.

15. **(c)** 2^6

Explanation: |A| = d

$|adj\ A| = |A|^{n-1}$

Here, n = 3, |A| = 8

$|adj\ A| = 8^2$

$|adj\ A| = (2^3)^2 = 2^6$

16. **(c)** a unique solution

Explanation: a unique solution

The given system of equations can be written in matrix form as follows:

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ -18 \end{bmatrix}$$

AX = B

Here,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 3 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 \\ 6 \\ -18 \end{bmatrix}$$

|A| = 1(- 1 - 2) - 1 (-3 - 6) + 1(3 + 3)

= -3 + 3 + 6

= 6 not equal to 0.

So, the given system of equations has a unique solution.

17. **(b)** $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - {0}

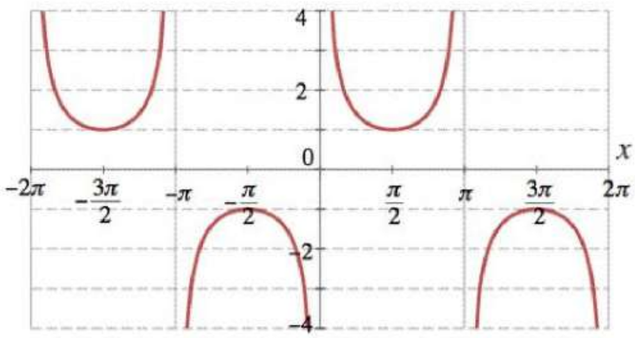
Explanation: To Find: The range of coses^{-1}(x)

Here, the inverse function is given by y = f^{-1}(x)

The graph of the function coses^{-1}(x) can be obtained from the graph of

Y = coses^{-1}(x) by interchanging x and y axes.i.e, if a, b is a point on Y = cosec x then b, a is the point on the function y = coses^{-1}(x)

Below is the Graph of the range of coses^{-1}(x)



From the graph, it is clear that the range of coses^{-1}(x) is restricted to interval

$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ - {0}

18. **(b)** $cos\frac{y-1}{x} = a$

Explanation: $\frac{dy}{dx} = cos^{-1}a$

$\int dy = cos^{-1}a\int dx$

$y = xcos^{-1}a + c$

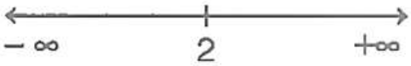
When y = 1 , x = 0 , then $1=0\ cos^{-1}a + c \quad c = 1$

$\therefore y = x\cos^{-1}a + 1$

$\therefore \frac{y-1}{x} = \cos^{-1}a$

19. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation: We have, $f(x) = x^2 - 4x + 6$
or $f'(x) = 2x - 4 = 2(x - 2)$



Therefore, $f'(x) = 0$ gives $x = 2$.
Now, the point $x = 2$ divides the real line into two disjoint intervals namely, $(-\infty, 2)$ and $(2, \infty)$.
In the interval $(-\infty, 2)$, $f'(x) = 2x - 4 < 0$.
Therefore, f is strictly decreasing in this interval.
Also, in the interval $(2, \infty)$, $f'(x) > 0$ and so the function f is strictly increasing in this interval.
Hence, both the statements are true but Reason is not the correct explanation of Assertion.

20. **(b)** Both A and R are true but R is not the correct explanation of A.
Explanation: Both A and R are true but R is not the correct explanation of A.

Section B

21. Let $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = y$

$\Rightarrow \cos y = -\frac{1}{\sqrt{2}}$

$\Rightarrow \cos y = -\cos\frac{\pi}{4}$

$\Rightarrow \cos y = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$

Since, the principal value branch of $\cos^{-1}$ is $[0, \pi]$.

Therefore, Principal value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

22. Given that, $x\frac{dy}{dx} = x + y$

$\Rightarrow \frac{dy}{dx} = \frac{x+y}{x}$

This is a homogeneous differential equation

Substituting, $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get

$v + x\frac{dv}{dx} = \frac{x + vx}{x}$

$\Rightarrow v + x\frac{dv}{dx} = 1 + v$

$\Rightarrow x\frac{dv}{dx} = 1 + v - v$

$\Rightarrow x\frac{dv}{dx} = 1$

$\Rightarrow dv = \frac{1}{x} dx$

Integrating both sides, we get

$\int dv = \int \frac{1}{x} dx$

$\Rightarrow v = \log |x| + C$

Putting $v = \frac{y}{x}$, we get

$\Rightarrow \frac{y}{x} = \log |x| + C$

$\Rightarrow y = x \log |x| + Cx$

Hence, $y = x \log |x| + Cx$ is the required solution..

23. Here

$A = \begin{bmatrix} 5 - x & x + 1 \\ 2 & 4 \end{bmatrix}$

Hence, $|A| = \begin{vmatrix} 5 - x & x + 1 \\ 2 & 4 \end{vmatrix}$

$= (5 - x) \times 4 - (x + 1) \times 2$ ...(Expanding along R_1)

$\Rightarrow |A| = 18 - 6x$

For A to be a singular matrix, |A| has to be 0.

Therefore, $18 - 6x = 0 \Rightarrow x = 3$.

OR

For the given system, we have

$D = \begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix} = 2 \times 5 - (-1) \times 3 = 13 \neq 0$

$D_1 = \begin{vmatrix} 17 & -1 \\ 6 & 5 \end{vmatrix} = 85 + 6 = 91$ and $D_2 = \begin{vmatrix} 2 & 17 \\ 3 & 6 \end{vmatrix} = 12 - 51 = -39$.

So, by Cramer's rule, we obtain

$x = \frac{D_1}{D} = \frac{91}{13} = 7$ and $y = \frac{D_2}{D} = \frac{-39}{13} = -3$

Hence, x = 7 and y = - 3 is the required solution

24. Given, $\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$

$\therefore \lambda\vec{a} = 2\lambda\hat{i} - 4\lambda\hat{j} + 5\lambda\hat{k}$

For a unit vector, its magnitude equals to 1.

We know that for any vector $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the magnitude $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

$\therefore\ |\lambda\vec{a}| = \sqrt{(2\lambda)^2 + (4\lambda)^2 + (5\lambda)^2} = 1$

$\Rightarrow\ 45\lambda^2 = 1$

$\Rightarrow\ \lambda^2 = \frac{1}{45} = \frac{1}{(3\sqrt{5})^2}$

$\Rightarrow\ \lambda = \pm\frac{1}{3\sqrt{5}}$

25. Let A, B, C and D denote the events of not getting a white ball in first, second, third and fourth draw respectively. Since the balls are drawn with replacement.
Therefore, A, B, C and D are independent events such that
P (A) = P (B) = P(C) = P (D)
There are 16 balls, out of which 11 are not white. Therefore, P (A) = P (B) = P (C) = P (D) $=\frac{11}{16}$
Therefore, the required probability is given by, $P(A \cap B \cap C \cap D) = P(A)P(B)P(C)P(D) = \left(\frac{11}{16}\right)^4$.

Section C

26. According to the question, $f\left(\frac{3x-4}{3x+4}\right) = x + 2$

Let $\frac{3x-4}{3x+4} = t \Rightarrow 3x - 4 = 3xt + 4t$

$3x - 3xt = 4t + 4$

$\Rightarrow x = \frac{4t+4}{3(1-t)}$

$\therefore f(t) = \frac{4t+4}{3(1-t)} + 2$

$\therefore\ f(x) = \frac{4x+4}{3(1-x)} + 2$

$= \frac{4x+4+4-4}{3(1-x)} + 2$

$= \frac{4x-4+8}{3(1-x)} + 2$

$= \frac{4(x-1)+8}{3(1-x)} + 2$

$= \frac{4(x-1)}{3(1-x)} + \frac{8}{3(1-x)} + 2$

$= -\frac{4}{3} - \frac{8}{3(x-1)} + 2$

$f(x) = \frac{2}{3} - \frac{8}{3(x-1)}$

$\therefore \int f(x)dx = \frac{2}{3}x - \frac{8}{3} ln\ |x-1| + c$

27. The given differential equation is,

$\cos x \frac{dy}{dx}$ - cos 2x = cos 3x

$\Rightarrow dy = \frac{\cos 3x + \cos 2x}{\cos x} dx$

$\Rightarrow dy = \frac{4\cos^3 x - 3\cos x + 2\cos^2 x - 1}{\cos x} dx$

$\Rightarrow$ dy = (4 $\cos^2$ x - 3 + 2 cos x - sec x) dx

$\Rightarrow$ dy = [2 (2 $\cos^2$ x - 1) - 1 + 2 cos x - sec x] dx

$\Rightarrow$ dy = (2 cos 2x - 1 + 2 cos x - sec x) dx

integrating both sides, we get

$\int$ dy = $\int$ (2 cos 2x - 1 + 2 cos x - sec x) dx

$\Rightarrow$ y = sin 2x - x + 2 sin x - log |sec x + tan x| + C

Hence, y = sin 2x - x + 2 sin x - log|sec x + tan x| + C is the solution to the given differential equation.

OR

The given differential equation is,

$(2x^2 y + y^3)$ dx + $(xy^2 - 3x^3)$ dy = 0

$$\frac{dy}{dx} = -\frac{\left(2x^2y + y^3\right)}{\left(xy^2 - 3x^3\right)}$$

It is a homogeneous equation

Put y = vx and $x\frac{dv}{dx} + v = \frac{dy}{dx}$

$$\text{So, } x\frac{dv}{dx} + v = \frac{\left(2x^2vx + v^3x^3\right)}{\left(-xv^2x^2 + 3x^3\right)}$$

$$x\frac{dv}{dx} = \frac{\left(2v + v^3\right)}{\left(-v^2 + 3\right)} - v$$

$$x\frac{dv}{dx} = \frac{\left(2v^3 - v\right)}{\left(-v^2 + 3\right)}$$

Integrating Both Sides We get,

$$\int \frac{\left(-v^2+3\right)}{\left(2v^3-v\right)} dv = \int \frac{1}{x} dx$$

$$\frac{\left(-v^2+3\right)}{v\left(2v^2-1\right)} = \frac{A}{v} + \frac{Bv+C}{\left(2v^2-1\right)}$$

$3 - v^2 = A(2v^2 - 1) + (Bv + C)\ v$

$3 - v^2 = (2A + B)\ v^2 + Cv - A$

Comparing the coefficient of like power of v

A = -3

C = 0

and 2A + B = -1

$\Rightarrow$ B = 5

$$\text{So, } \int \frac{-3}{v} dv + \int \frac{5v}{2v^2-1} dv = \int \frac{1}{x} dx$$

$$-3\int \frac{1}{v} dv + \frac{5}{4}\int \frac{4v}{2v^2-1} dv = \int \frac{1}{x} dx$$

$$-3 \log v + \frac{5}{4} \log (2v^2 - 1) = \log x + \log c$$

$$-12 \log v + 5 \log (2v^2 - 1) = 4 \log x + 4 \log c$$

$$\frac{\left(2v^2-1\right)^5}{v^{12}} = x^4 c^4$$

$$x^2 c^4 y^{12} = (2y^2 - x^2)^5$$

28. Given vectors are, $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}, \vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

The projection of $\vec{a}$ along $\vec{b}$

$$= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$\vec{a}.\vec{b} = (\lambda\hat{i} + \hat{j} + 4\hat{k}) \cdot (2\hat{i} + 6\hat{j} + 3\hat{k})$

$= 2\lambda + 6 + 12$

$= 2\lambda + 18$

$|\vec{b}| = \sqrt{2^2 + 6^2 + 3^2}$

$= \sqrt{49} = 7$

Given, the projection of vector a along vector b is 4.

$$\therefore \quad \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = 4$$

$\Rightarrow \quad \frac{2\lambda + 18}{7} = 4$

$\Rightarrow \quad 2\lambda + 18 = 28$

$\Rightarrow \quad 2\lambda = 10$

$\Rightarrow \quad \lambda = 5$

OR

Given position vectors of points A,B & C are:-

$\vec{A} = (3\hat{i} - 2\hat{j} + 4\hat{k}), \vec{B} = (\hat{i} + \hat{j} + \hat{k}), \vec{C} = (- \hat{i} + 4\hat{j} - 2\hat{k})$

$\Rightarrow \overrightarrow{AB}$ = Position vector of B - position vector of A

$= (\hat{i} + \hat{j} + \hat{k}) - (3\hat{i} - 2\hat{j} + 4\hat{k}) = (- 2\hat{i} + 3\hat{j} - 3\hat{k})$

$\Rightarrow \overrightarrow{AC}$ = Position vector of C - position vector of A

$= (- i + 4\hat{j} - 2\hat{k}) - (3\hat{i} - 2\hat{j} + 4\hat{k}) = (- 4\hat{i} + 6\hat{j} - 6\hat{k})$

$$\Rightarrow (\overrightarrow{AB} \times \overrightarrow{AC}) = \begin{vmatrix} \hat{i} & j & k \\ -2 & 3 & -3 \\ -4 & 6 & -6 \end{vmatrix} = \hat{i}\begin{vmatrix} 3 & -3 \\ 6 & -6 \end{vmatrix} - \hat{j}\begin{vmatrix} -2 & 3 \\ -4 & 6 \end{vmatrix} + \hat{k}\begin{vmatrix} -2 & 3 \\ -4 & 6 \end{vmatrix}$$

$= \hat{i}(-18 + 18) - \hat{j}(-12 + 12) + \hat{k}(-12 + 12) = 0$

Hence, proved that A,B and C are collinear.

29. Let the given integral be,

$I = \int (x + 1)\sqrt{x^2 - x + 1}dx$

Also, $x + 1 = \lambda \frac{d}{dx}\left(x^2 - x + 1\right) + \mu$

$\Rightarrow \quad x + 1 = \lambda(2x - 1) + \mu$

$\Rightarrow \quad x + 1 = (2\lambda)x + \mu - \lambda$

Equating the coefficient of like terms

$2\lambda = 1$

$\Rightarrow \quad \lambda = \frac{1}{2}$ And

$\Rightarrow \mu - \lambda = 1$

$\Rightarrow \quad \mu - \frac{1}{2} = 1$

$\Rightarrow \quad \mu = \frac{3}{2}$

$\therefore \quad I = \int \left[\frac{1}{2}(2x - 1) + \frac{3}{2}\right] \sqrt{x^2 - x + 1}dx$

$= \frac{1}{2}\int (2x - 1)\sqrt{x^2 - x + 1}dx + \frac{3}{2}\int \sqrt{x^2 - x + 1}dx$

$= \frac{1}{2}\int (2x - 1)\sqrt{x^2 - x + 1}dx + \frac{3}{2}\int \sqrt{x^2 - x + \frac{1}{4} - \frac{1}{4} + 1dx}$

$$= \frac{1}{2}\int (2x-1)\sqrt{x^2 - x + 1}\,dx + \frac{3}{2}\int \sqrt{\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}}\,dx$$

$$= \frac{1}{2}\int (2x-1)\sqrt{x^2 - x + 1}\,dx + \frac{3}{2}\int \sqrt{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\,\mathrm{dx}$$

Let $x^2 - x + 1 = t$

$\Rightarrow$ $(2x - 1)dx = dt$

$$\therefore\ I = \frac{1}{2}\int \sqrt{t}\,dt + \frac{3}{2}\left[\frac{x - \frac{1}{2}}{2}\sqrt{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \right.$$

$$\left. \frac{3}{8}\log\left|x - \frac{1}{2} + \sqrt{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\right| \right] + C$$

$$= \frac{1}{3}\left(x^2 - x + 1\right)^{\frac{3}{2}} + \frac{3}{8}(2x-1)\sqrt{x^2 - x + 1} + \frac{9}{16}\log\left|x - \frac{1}{2} + \sqrt{x^2 - x + 1}\right| + C$$

OR

We know that

$$\sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}},\quad \cos x = \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$$

$$\therefore\ \frac{1}{3 + 2\sin x + \cos x} = \frac{1}{3 + 2\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + \left(\frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right)}$$

$$= \frac{\left(1 + \tan^2\frac{x}{2}\right)}{3\left(1 + \tan^2\frac{x}{2}\right) + 4\tan\frac{x}{2} + \left(1 - \tan^2\frac{x}{2}\right)}$$

$$= \frac{\sec^2 \frac{x}{2} dx}{3 + 3\tan^2 \frac{x}{2} + 4\tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}}$$

$$\therefore \int_0^{x} \frac{1}{3 + 2\sin x + \cos x} dx = \int_0^{x} \frac{\sec^2 \frac{x}{2} dx}{2\tan^2 \frac{x}{2} + 4\tan \frac{x}{2} + 4}$$

Let $\tan \frac{x}{2} = t$

Differentiating w.r.t. x, we get

$\frac{1}{2}\sec^2 \frac{x}{2} dx = dt$

Now,

$x = 0 \Rightarrow t = 0$

$x = \pi \Rightarrow t = \infty$

$$\therefore \int_0^{x} \frac{\sec^2 \frac{x}{2} dx}{2\tan^2 \frac{x}{2} + 4\tan \frac{x}{2} + 4} = \int_0^{\infty} \frac{dt}{t^2 + 2t + 2}$$

$$= \int_0^{\infty} \frac{dt}{(t+1)^2 + 1}$$

$$= \left[\tan^{-1}(t+1)\right]_0^{\infty}$$

$$= \tan^{-1}(\infty) - \tan^{-1}(0+1)$$

$$= \tan^{-1}(\infty) - \tan^{-1}(1)$$

$$= \tan^{-1}\left(\tan \frac{\pi}{2}\right) - \tan^{-1}\left(\tan \frac{\pi}{4}\right)$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

$$= \frac{2\pi - \pi}{4}$$

$$= \frac{\pi}{4}$$

$$\therefore \int_0^{\pi} \frac{1}{3 + 2\sin x + \cos x} dx = \frac{\pi}{4}$$

30. ATQ,

x = $ae^{\theta}(\sin\theta - \cos\theta)$, y = $ae^{\theta}(\sin\theta + \cos\theta)$

$$\Rightarrow \frac{dx}{d\theta} = a\left[e^{\theta}\frac{d}{d\theta}(\sin\theta - \cos\theta) + (\sin\theta - \cos\theta)\frac{d}{d\theta}\left(e^{\theta}\right)\right] \text{ and}$$

$$\frac{dy}{d\theta} = a\left[e^{\theta}\frac{d}{d\theta}(\sin\theta + \cos\theta) + (\sin\theta + \cos\theta)\frac{d}{d\theta}\left(e^{\theta}\right)\right]$$

$$\Rightarrow \frac{dx}{d\theta} = a\left[e^{\theta}(\cos\theta + \sin\theta) + (\sin\theta - \cos\theta)e^{\theta}\right] \text{ and}$$

$$\frac{dy}{d\theta} = a\left[e^{\theta}(\cos\theta - \sin\theta) + (\sin\theta + \cos\theta)e^{\theta}\right]$$

$$\Rightarrow \frac{dx}{d\theta} = a\left[2e^{\theta}\sin\theta\right] \text{and } \frac{dy}{d\theta} = a\left[2e^{\theta}\cos\theta\right]$$

$$\therefore \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\left(2e^{\theta}\cos\theta\right)}{a\left(2e^{\theta}\sin\theta\right)} = \cot\theta$$

$$\frac{dy}{dx} = Cot\theta$$

The differentiation of the given function y wrt x is as above.

31. The equation of parabola is

$y = x^2 - 5x + 15$ ....(1)

The equation of line is

$y = 3x + 3$...........(2)

From (1) and (2), we get

$3x + 3 = x^2 - 5x + 15$

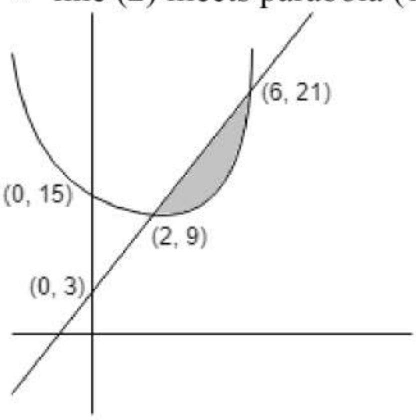
$x^2 - 8x + 12 = 0$

$(x - 2)(x - 6) = 0$

$x = 2, 6$

Therefore,from (1), $y = 9, 21$

$\therefore$ line (2) meets parabola (1) in (2, 9) and (6, 21).



Required area = the area of the segment of the parabola $y = x^2 - 5x + 15$ cut off by the straight line $y = 3x + 3$

$=\int_2^6[(3x + 3) - (x^2 - 5x + 15)]dx$ [$\because$ $Area = \int_a^b(y_2 - y_1)dx$]

$=\int_2^6(- x^2 + 8x - 12)\, dx$

$$= \left[-\frac{x^3}{3} + 4x^2 - 12x \right]_2^6$$

$$= (-72 + 144 - 72) - \left(-\frac{8}{3} + 16 - 24 \right)$$

$$= 0 + \frac{32}{3}$$

$$= \frac{32}{3} \text{ sq.units}$$

Section D

32. Converting the inequations into equations, we obtain the following equations:

x + y = 4, 3x + 8y = 24, 10x + 7y = 35, x = 0 and y = 0.

These equations represent straight lines in XOY-plane.

The line x + y = 4 meets the coordinate axes at A_1 (4, 0) and B_1 (0, 4). Join these points to obtain the line x + y = 4.

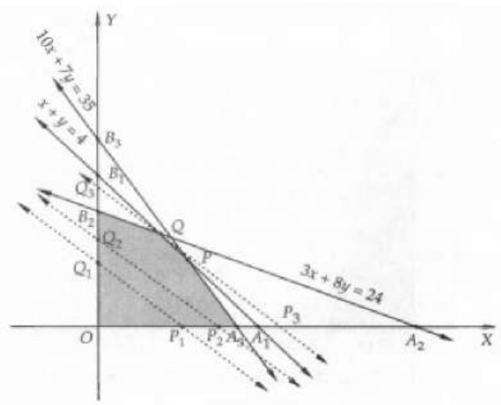
The line 3x + 8y = 24 meets the coordinate axes at A_2 (8, 0) and B_2 (0, 3). Join these points to obtain the line 3x + 8y = 24.

The line 10x + 7y = 35 cuts the coordinates axes at A_3 (3.5, 0) and B_3 (0,5). These points are joined to obtain the line 10x + 7y = 35.

Also, x = 0 is the y-axis and y = 0 is the x-axis.

The feasible region of the LPP is shaded in Figure.

The coordinates of the corner points of the feasible region OA_3PQB_2 are O (0, 0), A_3(3.5, 0), P($\frac{7}{5}$, $\frac{5}{3}$), Q($\frac{8}{5}$, $\frac{12}{5}$) and B_2(0, 3).



Now, we take a constant value, say 10 for Z. Putting Z = 10 in Z = 5x + 7y, we obtain the line 5x + 7y = 10. This line meets the coordinate axes at P_1 (2, 0) and Q(0, $\frac{10}{7}$). Join these points by a dotted line. Now, move this line parallel to itself in the increasing direction away from the origin. P_2Q_2 and P_3Q_3 are such lines. Out of these lines locate a line farthest from the origin and has at least one common point to the feasible region OA_3 PQB_2. Clearly, P_3 Q_3 is such line and it passes through the vertex Q ($\frac{8}{5}$, $\frac{12}{5}$) of the feasible region.

Hence $x = \frac{8}{5}$ and $y = \frac{12}{5}$ gives the maximum value of Z.

The maximum value of Z is given by

$Z = 5 \times \frac{8}{5} + 7 \times \frac{12}{5} = 24.8$

33. Here R is a relation on $N \times N$, defined by (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in N \times N$

We shall show that R satisfies the following properties

i. Reflexivity:

We know that a + b = b + a for all a, b $\in$ N.

$\therefore$ (a, b) R (a, b) for all (a, b) $\in (N \times N)$

So, R is reflexive.

ii. Symmetry:

Let (a, b) R (c, d). Then,

(a, b) R (c, d) $\Rightarrow$ a + d = b + c

$\Rightarrow$ c + b = d + a

$\Rightarrow$ (c, d) R (a, b).

$\therefore$ (a, b) R (c, d) $\Rightarrow$ (c, d) R (a, b) for all (a, b), (c, d) $\in N \times N$

This shows that R is symmetric.

iii. Transitivity:

Let (a, b) R (c, d) and (c, d) R (e, f). Then,

(a, b) R (c, d) and (c, d) R (e, f)

$\Rightarrow$ a + d = b + c and c + f = d + e

$\Rightarrow$ a + d + c + f = b + c + d + e

$\Rightarrow$ a + f = b + e

$\Rightarrow$ (a, b) R (e, f).

Thus, (a, b) R (c, d) and (c, d) R (e, f) $\Rightarrow$ (a, b) R (e, f)

This shows that R is transitive.

$\therefore$ R is reflexive, symmetric and transitive

Hence, R is an equivalence relation on $N \times N$

OR

Given that, $\forall$a, b $\in$ Z aRb if and only a-b is divisible by n

Now

I. Reflexive

aRa $\Rightarrow$ (a - a) is divisible by n, which is true for any integer 'a' as ‘0’ is divisible by n.

Hence, R is reflexive.

II. Symmetric

aRb

$\Rightarrow$ a-b is divisible by n.

$\Rightarrow$ -b + a is divisible by n.

$\Rightarrow$ -(b - a) is divisible by n.

$\Rightarrow$ (b - a) is divisible by n.

$\Rightarrow$ bRa

Hence, R is symmetric.

III. Transitive

Let aRb and bRc

$\Rightarrow$ (a-b) is divisible by n and (b-c) is divisible by n

$\Rightarrow$ (a-b) +(b-c) is divisible by n

$\Rightarrow$ (a-c) is divisible by n

$\Rightarrow$ aRc

Hence, R is transitive.

So, R is an equivalence relation.

34. $\vec{a}_1 = \hat{i} + \hat{j}, \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}, \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$

$\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$= \hat{i}(-2+5) - \hat{j}(4-3) + \hat{k}(-10+3)$

$= 3\hat{i} - \hat{j} - 7\hat{k}$

$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{9+1+49} = \sqrt{59}$

Also, $(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1) = (3\hat{i} - \hat{j} - 7\hat{k})(\hat{i} - \hat{k}) = 3 + 7 + 0 = 10$

$$d = \left|\frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right| = \frac{10}{\sqrt{59}}$$

OR

$\vec{r} = (2i - j + 2k) + \lambda(3i + 4j + 2k)$

Then, in Cartesian form,we have

$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = \lambda$ ..... (i)

coordinates of any point on (i) is ,

$3\lambda + 2, 4\lambda - 1, 2\lambda + 2$

The equation of plane is

$\vec{r}.\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right) \cdot \left(\hat{i} - \hat{j} + \hat{k}\right) = 5$

x - y + z = 5 .... (ii)

If the point $(3\lambda + 2, 4\lambda - 1, 2\lambda + 2)$lies on (ii),then

$(3\lambda + 2) - (4\lambda - 1) + (2\lambda + 2) = 5$

$\lambda + 5 = 5$

$\lambda = 0$

we get (2, -1, 2) as the coordinate of the point of intersection of the given line and the plane.

Now distance between the points (-1, -5, -10) and (2, -1, 2)

$= \sqrt{(2+1)^2 + (-1-5)^2 + (2+10)^2}$

$= 13$

35. Let $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put x = tan θ

$\Rightarrow u = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$\Rightarrow$ u = $\sin^{-1}(\sin 2\theta)$ ...(i)

Let $v = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$\Rightarrow v = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\Rightarrow$ v = $\cos^{-1}(\cos 2\theta)$ ...(ii)

Here, $0 < x < 1$

$\Rightarrow 0 < \tan\theta < 1$

$\Rightarrow 0 < \theta < \frac{\pi}{4}$

So, from equation (i),

$u = 2\theta \quad \left[\text{since}, \sin^{-1}(\sin\theta) = \theta, \text{ if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right]$

$\Rightarrow$ u = $2\tan^{-1}x$ ...[Since, x = tan θ]

Differentiating it with respect to x,

$\frac{du}{dx} = \frac{2}{1+x^2}$ ...(iii)

from equation (ii),

$v = 2\theta \quad \left[\text{since}, \cos^{-1}(\cos\theta) = \theta, if\theta \in [0, \pi]\right]$

$\Rightarrow$ v = $2\tan^{-1}x$ ...[Since, x = tan θ]

Differentiating it with respect to x,

$\frac{dv}{dx} = \frac{2}{1+x^2}$ ...(iv)

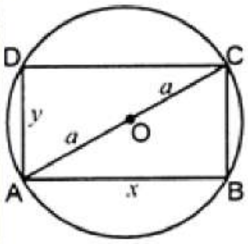
Dividing equation (iii) by (iv),

$$\frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2}{1+x^2} \times \frac{1+x^2}{2} = 1$$

Section E

36. **Read the text carefully and answer the questions:**

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)



(i) Let ‘y’ be the breadth and ‘x’ be the length of rectangle and ‘a’ is radius of given circle.

From fig $4a^2 = x^2 + y^2$

$\Rightarrow y^2 = 4a^2 - x^2$

$\Rightarrow y = \sqrt{4a^2 - x^2}$

Perimeter (P) $= 2x + 2y = 2\left(x + \sqrt{4a^2 - x^2}\right)$

(ii)

We know that $P = 2\left(x + \sqrt{4a^2 - x^2}\right)$

Critical points to maximize perimeter $\frac{dP}{dx} = 0$

$$\Rightarrow \frac{dp}{dx} = 2\left(1 + \frac{1}{2\sqrt{4a^2 - x^2}}(-2x)\right) = 0$$

$$2\left(\frac{\sqrt{4a^2 - x^2} - x}{\sqrt{4a^2 - x^2}}\right) = 0$$

$\Rightarrow \sqrt{4a^2 - x^2} = x$

$\Rightarrow 4a^2 - x^2 = x^2$

$\Rightarrow 2a^2 = x^2$

$\Rightarrow x = \pm\sqrt{2}a$

when $x = \sqrt{2}a$, $y = \sqrt{2}a$

when $x = -\sqrt{2}a$ not possible as 'x' is length critical point is $(\sqrt{2}a, \sqrt{2}a)$

(iii)

$$\frac{dp}{dx} = 2\left(1 + \frac{1}{2\sqrt{4a^2 - x^2}}(-2x)\right)$$

$$\frac{d^2P}{dx^2} = -2\left(\frac{\sqrt{4a^2-x^2} - (x)\left(\frac{-2x}{2\sqrt{4^2-x^2}}\right)}{\left(4a^2-x^2\right)}\right)$$

$$= -2\left(\frac{\left(4a^2-x^2\right)+x^2}{\left(4a^2-x^2\right)^{3/2}}\right)$$

$$\Rightarrow \frac{d^2P}{dx^2}\Big]_{x=a\sqrt{2}} = -2\left(\frac{4a^2}{\left(4a^2-2a^2\right)^{3/2}}\right) = \frac{-2}{(2\sqrt{2})a} < 0$$

Perimeter is maximum at a critical point.

OR

From the above results know that x = y = $\sqrt{2}a$

a = radius

Here, x = y = $10\sqrt{2}$

Perimeter = P = 4 × side = $40\sqrt{2}$ cm

37. **Read the text carefully and answer the questions:**

Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold handmade fans, mats, and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold by school A, B, C are given below.

Article	School	A	B	C
Fans		40	25	35
Mats		50	40	50
Plates		20	30	40

(i)

$$P = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} \textit{Fans} & \textit{Mats} & \textit{Plates} \end{array} \\ \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \end{array}$$

$$Q = \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{array}{l} \textit{Fans} \\ \textit{Mats} \\ \textit{Plates} \end{array}$$

(ii) Clearly, total funds collected by each school is given by the matrix

$$PQ = \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}$$

$$= \begin{bmatrix} 1000 + 5000 + 1000 \\ 625 + 4000 + 1500 \\ 875 + 5000 + 2000 \end{bmatrix} = \begin{bmatrix} 7000 \\ 6125 \\ 7875 \end{bmatrix}$$

∴ Funds collected by school A is ₹7000.

Funds collected by school B is ₹6125.

Funds collected by school C is ₹7875.

(iii)

$$\textit{New price matrix } Q = 20\% \times \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{array}{l} \textit{Fans} \\ \textit{Mats} \\ \textit{Plates} \end{array}$$

$$\Rightarrow Q = \begin{bmatrix} 25 + 25 \times 0.20 \\ 100 + 100 \times 0.20 \\ 50 + 50 \times 0.20 \end{bmatrix} \begin{array}{l} \textit{Fans} \\ \textit{Mats} \\ \textit{Plates} \end{array}$$

$$Q = \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix} \begin{array}{l} \textit{Fans} \\ \textit{Mats} \\ \textit{Plates} \end{array}$$

$$\text{New price matrix } Q = 20\% \times \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix} \begin{array}{l} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{array}$$

$$\Rightarrow Q = \begin{bmatrix} 25 + 25 \times 0.20 \\ 100 + 100 \times 0.20 \\ 50 + 50 \times 0.20 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$$

$$Q = \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix} \begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$$

OR

$$PQ = \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix} \begin{bmatrix} 30 \\ 120 \\ 60 \end{bmatrix}$$

$$PQ = \begin{bmatrix} 1200 + 6000 + 1200 \\ 750 + 4800 + 1800 \\ 1050 + 6000 + 2400 \end{bmatrix} = \begin{bmatrix} 8400 \\ 7350 \\ 9450 \end{bmatrix}$$

Total fund collected = 8400 + 7350 + 9450 = ₹25,200

38. **Read the text carefully and answer the questions:**
In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03.

(i) Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.
$P(E_1) = 0.5$, $P(E_2) = 0.2$, $P(E_3) = 0.3$

$P(\frac{A}{E_1}) = 0.06$, $P(\frac{A}{E_2}) = 0.04$, $P(\frac{A}{E_3}) = 0.03$

Using Bayes' theorem, we have

$$P\left(\frac{E_1}{A}\right) = \frac{P\left(E_1\right)\cdot P\left(\frac{A}{E_1}\right)}{P\left(E_1\right)\cdot P\left(\frac{A}{E_1}\right)+P\left(E_2\right)\cdot P\left(\frac{A}{E_2}\right)+P\left(E_3\right)\cdot P\left(\frac{A}{E_3}\right)}$$

$$= \frac{0.5\times 0.06}{0.5\times 0.06+0.2\times 0.04+0.3\times 0.03} = \frac{30}{47}$$

$\therefore$ Required probability $= P\left(\frac{\bar{E}_1}{A}\right)$

$$= 1 - P\left(\frac{E_1}{A}\right) = 1 - \frac{30}{47} = \frac{17}{47}$$

(ii) Let A be the event of committing an error and E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

$P(E_1) = 0.5$, $P(E_2) = 0.2$, $P(E_3) = 0.3$

$P\left(\frac{A}{E_1}\right) = 0.06$, $P\left(\frac{A}{E_2}\right) = 0.04$, $P\left(\frac{A}{E_3}\right) = 0.03$

$$P\left(A\cap E_2\right) = P\left(\frac{A}{E_2}\right).P\left(E_2\right)$$

$\Rightarrow 0.04 \times 0.2 = 0.008$