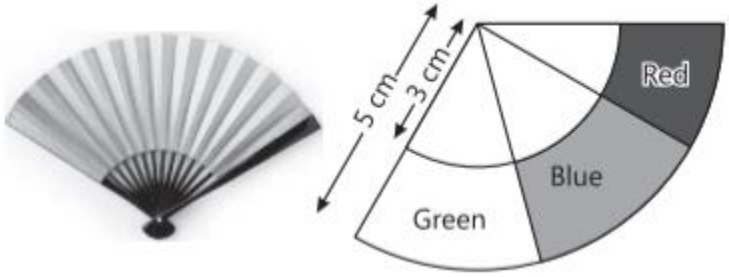
Areas Related to Circles

Case Study Based Questions

Case Study 1

Sara hold a Japanese folding fan in her hand as shown in the figure. It is shaped like a sector of a circle and made of a thin material such as paper or feather. The inner and outer radii are 3 cm and 5 cm respectively. The fan has three colours, i.e., red, blue and green.



Based on the above information, solve the following questions:

Q1. If the region containing blue colour makes an angle of 80° at the centre, the area of the region having blue colour is:

a. 9.17 cm^2

b. 10.1 cm^2

c. 11.17 cm^2

d. 13.17 cm^2

Q2. If the region containing green colour makes an angle of 60° at the centre, the area of the region having green colour is:

a. 6.2 cm^2

b. 8.38 cm^2

c. 9.9 cm^2

d. 11.12 cm^2

Q3. If the region containing red colour makes an angle of 20° at the centre, the perimeter of the region containing red colour is:

a. 2.9 cm

b. 4.2 cm

c. 5.4 cm

d. 6.79 cm

Q4. The area of the region having radius 3 cm is:

a. 12.57 cm^2

b. 14.8 cm^2

c. 20 cm^2

d. 26.57 cm^2

Q5. The region given in the figure represents:

a. minor sector

b. major sector

c. minor segment

d. major segment

Solutions

1. Area of the region containing blue colour

$$= \frac{22}{7} \times 5 \times 5 \times \frac{80^\circ}{360^\circ} - \frac{22}{7} \times 3 \times 3 \times \frac{80^\circ}{360^\circ}$$

$$= \frac{22}{7} \times \frac{2}{9} \times [25 - 9] = \frac{44}{63}(16) = 11.17 cm^2$$

So, option (c) is correct.

2. Area of the region containing green colour

$$= \frac{22}{7} \times \frac{60^\circ}{360^\circ}(5 \times 5 - 3 \times 3) = \frac{22}{7} \times \frac{1}{6} \times 16 = 8.38\ cm^2$$

So, option (b) is correct.

3. Perimeter of the region containing red colour (5-3)+(5-3)+ length of arc of sector having radius 3 cm + length of arc of sector having radius 5 cm

$$=2+2+2\times\frac{22}{7}\times3\times\frac{20^\circ}{360^\circ}+2\times\frac{22}{7}\times5\times\frac{20^\circ}{360^\circ}$$

$$=4+\frac{44}{7}\times\frac{1}{18}(3+5)$$

$$=4+\frac{44}{7}\times\frac{1}{18}\times8=4+\frac{176}{63}=4+2.79=6.79\text{ cm}$$

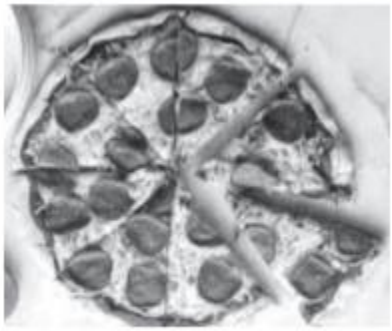
So, option (d) is correct.

5. (a) Angle of given sector =80° +60° +20° = 160°
Thus, the given region represents minor sector of a circle.
So, option (a) is correct.

Case Study 2

We all love to eat pizzas, especially kids and a variety of pizzas are available in India which have been modified according to Indian taste and menu. From the Greeks to the Egyptians, from the Persians to the Indians, there have been incarnations of pizza served throughout history, Flatbreads, naan and plakountos are all early preparations that could be considered cousins to the modern pizza, but there isn't a consensus as to which is first and whether these could even be considered precursors to pizza at all. Consider two pizzas, both of equal diameter, namely, 12 inches. The first pizza marked (I) has been cut into six equal slices, whereas the second pizza, marked (II) has been cut into eight equal slices.

(I)



(II)

Based on the above information, solve the following questions:

Q1. The area of one slice in pizza, marked (I) is:

a. 6πsq. inches

b. 8π sq. inches

c. 10π sq. inches

d. None of these

Q2. The perimeter of the pizza slice shown in (1) is:

a. (π +12) inch

b. (π+10) inch

c. (2π+10) inch

d. (2π +12) inch

Q3. The ratio of area of slice to the area of remaining pizza in (1) is:

a. 5:1

b. 1:5

c. 2:5

d. 5:3

Q4. The ratio of areas of each slice of pizza (1) and (II) is:

a. 3:4

b. 5:3

c. 4:3

d. 2:5

Q5. The relation between area of a sector (A), length of the arc (4), angle (0) subtended by the arc at the centre and radius of circle is:

a. $\frac{1}{2}lr$

b. lr

c. $\frac{1}{3}lr$

d. $\frac{1}{2}lr^2$

1. We have,

$$\text{Area of sector of a circle } = \frac{\theta}{360^\circ} \times \pi r^2$$

where, is the angle subtended by the sector at the centre. The radius of pizza (I) 6 inches

and as the pizza has been cut into six equal slices, the angle (0)

subtended at the centre is equal $= \frac{360^\circ}{6}$, *i.e.*, to 60°.

$\therefore$ Required area $= \frac{60^\circ}{360^\circ} \times \pi \times 6^2 = 6\pi$ sq. inches.

So, option (a) is correct.

2. Perimeter of a sector = 1 + 2r, where I is the length

of the arc given by $\frac{\theta}{360^\circ} \times 2\pi r$

Here, $\theta = 60^\circ$ and $r = 6$ inch

$\therefore$ $l = \frac{60^\circ}{360^\circ} \times 2 \times \pi \times 6 = 2\pi$

and $2r = 2 \times 6 = 12$

$\therefore$ Perimeter of pizza $= (2\pi + 12)$ inch

So, option (d) is correct.

3. The angle subtended by a slice of pizza (1) at the centre is 60°.
Therefore, angle subtended by the remaining pizza at the centre is 300°.
:- Ratio of areas of minor sector and major sector

$$= \frac{60^\circ}{360^\circ} \times \pi \times (6)^2 : \frac{300^\circ}{360^\circ} \times \pi \times (6)^2$$

$$= 6^\circ : 30^\circ = 1 : 5$$

So, option (b) is correct.

4. The angle subtended by a slice in pizza (1) is 60°,
and the angle subtended by a slice in pizza (II) is 45°

i.e., $\frac{360^\circ}{8}$.

Therefore, ratio of area of slice in the two cases

$$= \frac{60^\circ}{360^\circ} \times \pi \times (6)^2 : \frac{45^\circ}{360^\circ} \times \pi \times (6)^2$$

$$= 60^\circ : 45^\circ = 4 : 3$$

So, option (c) is correct.

5. The area of sector is given by $A = \frac{\theta}{360^\circ} \times \pi r^2$ and

length of the arc is given by $l = \frac{\theta}{360^\circ} \times 2\pi r$

Multiplying and dividing A by 2, we get

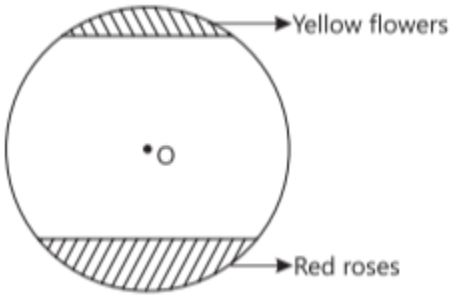
$$A = \frac{1}{2} \times \frac{\theta}{360^\circ} \times 2 \times \pi r^2$$

$$= \frac{1}{2} \times \left(\frac{\theta}{360^\circ} \times 2\pi r \right) \times r = \frac{1}{2} \times l \times r \quad = \frac{1}{2} lr$$

So, option (a) is correct.

Case Study 3

Flower beds look beautiful growing in gardens. One such circular park of radius 'r' m, has two segments with flowers. One segment which subtends an angle of 90° at the centre is full of red roses, while the other segment with central angle 60° is full of yellow coloured flowers. [See figure]



It is given that the combined area of the two segments (of flowers) is $256\frac{2}{3}$ sq m. *[CBSE 2023]*

Based on the above information, answer the following questions:

Q1. Write an equation representing the total area of the two segments in terms of 'r'.

Q2. Find the value of 'r'.

Q3. Find the area of the segment with red roses.

Or

Find the area of the segment with yellow flowers.

Solutions

1. Given, radius of circular park = rm.

Central angle for segment of red roses (01) = 90°

Central angle for segment of yellow roses (02) = 60°

Now, area of segment of yellow roses

$$= \left[\frac{\theta_2 \pi}{180^\circ} - \sin\theta_2\right] \times \frac{r^2}{2}$$

$$= \left\{\frac{60^\circ \pi}{180^\circ} - \sin 60^\circ\right\} \times \frac{r^2}{2}$$

$$= \left(\frac{\pi}{3} - \sin 60^\circ\right) \times \frac{r^2}{2} = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right) \times \frac{r^2}{2}$$

$$= \frac{r^2}{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right) m^2$$

$$\text{and area of segment of red roses} = \left[\frac{\theta_1 \pi}{180^\circ} - \sin\theta_1\right] \times \frac{r^2}{2}$$

$$= \left[\frac{90^\circ \pi}{180^\circ} - \sin 90^\circ\right] \times \frac{r^2}{2} = \left(\frac{\pi}{2} - \sin 90^\circ\right) \times \frac{r^2}{2}$$

$$= \left(\frac{\pi}{2} - 1\right) \times \frac{r^2}{2} = \frac{r^2}{2}\left(\frac{\pi}{2} - 1\right) m^2$$

Given, total area of the two segments $= 256\frac{2}{3}\,m^2$

$\Rightarrow$ Area of segment of (red roses + yellow roses)

$$= \frac{770}{3}\,m^2$$

$$\therefore \quad \frac{r^2}{2}\left(\frac{\pi}{2}-1\right)+\frac{r^2}{2}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)=\frac{770}{3} \qquad \ldots(i)$$

2. From, eq (i),

$$\frac{r^2}{2}\left(\frac{\pi}{2}-1+\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)=\frac{770}{3}$$

$$\Rightarrow \quad \frac{r^2}{2}\left(\frac{5\pi}{6}-1-\frac{1.732}{2}\right)=\frac{770}{3}$$

$$\Rightarrow \quad \frac{r^2}{2}\left\{\frac{5}{6}\times\frac{22}{7}-1-0.866\right\}=\frac{770}{3}$$

$$\Rightarrow \quad \frac{r^2}{2}(2.619-1.866)=\frac{770}{3}$$

$$\Rightarrow \quad \frac{r^2}{2}\times 0.753=\frac{770}{3} \;\Rightarrow\; r^2=\frac{770\times 2}{0.753\times 3}=\frac{1540}{2.259}$$

$$\Rightarrow \quad r^2 = 681.72 \;\Rightarrow\; r = 26.11 \text{ m.}$$

3. The area of segment with red roses

$$=\frac{r^2}{2}\left(\frac{\pi}{2}-1\right)=\frac{681.72}{2}\left(\frac{22}{2\times 7}-1\right)$$

$$= 340.86 \times 0.57 = 194.3\ m^2.$$

Or

The area of segment with yellow roses

$$=\frac{r^2}{2}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)$$

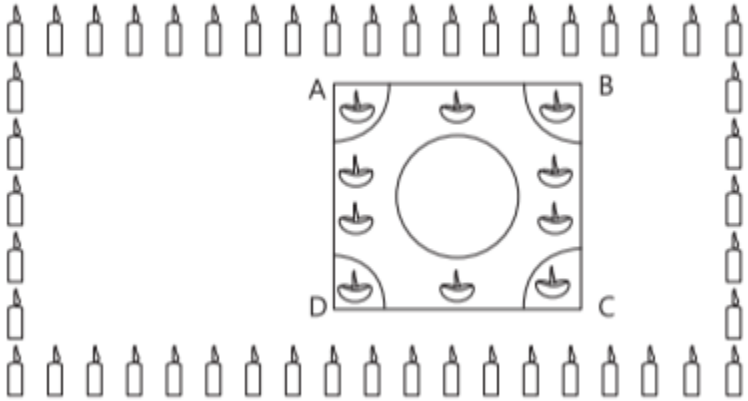
$$=\frac{681.72}{2}\left(\frac{22}{3\times 7}-\frac{1.732}{2}\right)$$

$$= 340.86\ (1.0476 - 0.866)$$

$$= 340.86 \times 0.1816 = 61.9\ m^2.$$

Case Study 4

Interschool Rangoli Competition was organized by one of the reputed schools of odissa. The theme of the Rangoli Competition was Diwali celebrations where students were supposed to make mathematical designs. Students from various schools participated and made beautiful Rangoli designs. One such design is given below.



Rangoli is in the shape of square marked as ABCD, side of square being 40 cm. At each corner of a square, a quadrant of circle of radius 10 cm is drawn (in which diyas are kept). Also a circle of diameter 20 cm is drawn inside the square. [CBSE 2023]

Based on the above information, solve the following questions:

Q1. What is the area of square ABCD?

Q 2. Find the area of the circle.

Q3. If the circle and the four quadrants are cut off from the square ABCD and removed, then find the area of remaining portion of square ABCD.

Or

Find the combined area of 4 quadrants and the circle, removed.

Solutions

1. Given, side of square ABCD, 'a' = 40 cm.

$\therefore$ Area of square ABCD = $a^2 = (40)^2$

$= 1600\ cm^2.$

2. Given, diameter of inside circle = 20 cm

$\therefore$ Radius of circle $(r) = \frac{20}{2} = 10\,cm$

So, area of the circle = r^2

= $л(10)^2 = 100л$

= 100 x 3.14 = 314 cm^2

3. Given, radius of a quadrant (R) = 10 cm

All quadrants are equal at each corner of a square ABCD.

Now, area of a quadrant $= \frac{1}{4}\pi R^2$

$= \frac{1}{4}\pi(10)^2$

$= \frac{1}{4}\pi \times 100 = 25\pi\ cm^2$

$\therefore$ area of four quadrants $= 4 \times 25\pi = 100\ \pi$

$= 100 \times 3.14 = 314\ cm^2$

So, area of remaining portion of square ABCD Area of square ABCD - Area of the circle - Area of four quadrants

1600-314-314

= 1600-628972 cm^2.

Or

Combined area of 4 quadrants and the circle, removed Area of four quadrants + Area of the circle

= 314 + 314

= 628 cm^2.

Case Study 5

Kritika bought a pendulum clock for her living room. The clock contains a small pendulum of length 15 cm. The minute hand and hour hand of the clock are 9 cm and 6 cm long respectively.

Based on the given information, answer the following questions:

Q1. Find the area swept by the minute hand in 10 minutes.

Q2. If the pendulum covers a distance of 22 cm in the complete oscillation, then find the angles described by pendulum at the centre.

Q3. Find the area swept by the hour hand in 1 hour.

Or

Find the area swept by the hour hand between 11 am and 5 pm.

Solutions

1. Angles made by minutes hand in 60 minutes = 360°
:- Angle made by minute hand in 10 minutes

$= \frac{360^\circ}{60} \times 10 = 60^\circ$

Length of minute hand (r) = 9 cm

$\therefore$ Area swept by minute hand in 10 minutes

= Area of sector having central angles 60°

$= \pi r^2 \left(\frac{60^\circ}{360^\circ} \right) = \frac{22}{7} \times 9 \times 9 \times \frac{1}{6} = \frac{297}{7}$

$= 42.42 \text{ cm}^2$

3. Angle made by hour hand in 1 hour $= \frac{360^\circ}{12} = 30^\circ$

Also, $r_1 = 6$ cm

:- Area swept by hour hand in 1 hour

= Area of sector having central angle 30°

$= \pi r_1^2 \times \left(\frac{30^\circ}{360^\circ} \right) = \frac{22}{7} \times 6 \times 6 \times \frac{1}{12} = \frac{66}{7}$

$= 9.428 \text{ cm}^2$

Or

Number of hours from 11 am to 5 pm = 6

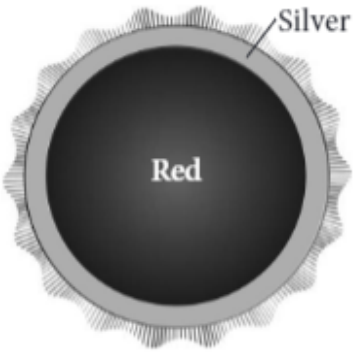
Area swept by hour hand in 1 hour = 9.428 cm^2

:- Area swept by hour hand in 6 hours = 9.428 × 6

= 56.568 cm^2

Solutions for Questions 6 to 20 are Given Below

Case Study 6



Choosing Representative of the Students

Principle of a school decided to give badges to students who are chosen for the post of Head boy, Head girl, Prefect and Vice Prefect. Badges are circular in shape with two colour area, red and silver, as shown in figure. The diameter of the region representing red colour is 22 cm and silver colour is filled in 10.5 cm wide ring.
Based on the above information, answer the following questions.

(i) The radius of circle representing the red region is
(a) 9 cm (b) 10 cm (c) 11 cm (d) 12 cm

(ii) Find the area of the red region.
(a) 380.28 cm^2 (b) 382.28 cm^2 (c) 384.28 cm^2 (d) 378.28 cm^2

(iii) Find the radius of the circle formed by combining the red and silver region.
(a) 20.5 cm (b) 21.5 cm (c) 22.5 cm (d) 23.5 cm

(iv) Find the area of the silver region.
(a) 172.50 cm^2 (b) 1062.50 cm^2 (c) 1172.50 cm^2 (d) 1072.50 cm^2

(v) Area of the circular path formed by two concentric circles of radii r_1 and r_2 $(r_1 > r_2)$ =
(a) $\pi(r_1^2 + r_2^2)$ sq. units (b) $\pi(r_1^2 - r_2^2)$ sq. units
(c) $2\pi\,(r_1 - r_2)$ sq. units (d) $2\pi\,(r_1 + r_2)$ sq. units

Case Study 7

Area of Button

While doing dusting a maid found a button whose upper face is of black colour, as shown in the figure. The diameter of each of the smaller identical circles is 1/4 of the diameter of the larger circle whose radius is 16 cm.
Based on the above information, answer the following questions.

(i) The area of each of the smaller circle is
(a) 40.28 cm^2 (b) 46.39 cm^2 (c) 50.28 cm^2 (d) 52.3 cm^2

(ii) The area of the larger circle is

(a) 804.57 cm^2 (b) 704.57 cm^2 (c) 855.57 cm^2 (d) 990.57 cm^2

(iii) The area of the black colour region is

(a) 600.45 cm^2 (b) 603.45 cm^2 (c) 610.45 cm^2 (d) 623.45 cm^2

(iv) The area of quadrant of a smaller circle is

(a) 11.57 cm^2 (b) 13.68 cm^2 (c) 12 cm^2 (d) 12.57 cm^2

(v) If two concentric circles are of radii 2 cm and 5 cm, then the area between them is

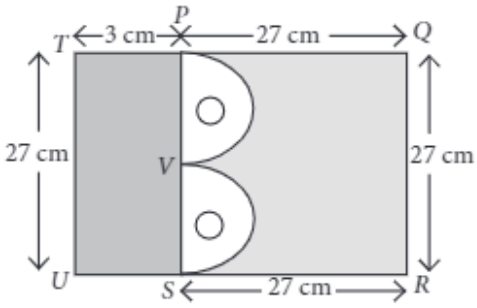
(a) 60 cm^2 (b) 63 cm^2 (c) 66 cm^2 (d) 68 cm^2

Case Study 8

Layout of a House

Mr Ramanand purchased a plot *QRUT* to build his house. He leave space of two congruent semicircles for gardening and a rectangular area of breadth 3 cm for car parking.

Based on the above information, answer the following questions.



(i) Area of square *PQRS* is

(a) 700 cm^2 (b) 729 cm^2 (c) 732 cm^2 (d) 735 cm^2

(ii) Area of rectangle left for car parking is

(a) 64 cm^2 (b) 76 cm^2 (c) 81 cm^2 (d) 100 cm^2

(iii) Radius of semi-circle is

(a) 6.75 cm (b) 7 cm (c) 7.75 cm (d) 8.75 cm

(iv) Area of a semi-circle is

(a) 61.59 cm^2 (b) 66.29 cm^2 (c) 70.36 cm^2 (d) 71.59 cm^2

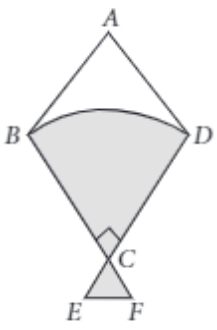
(v) Find the area of the shaded region.

(a) 660.82 cm^2 (b) 666.82 cm^2 (c) 669.89 cm^2 (d) 700 cm^2

Case Study 9

Kite Flying Competition

Makar Sankranti is a fun and delightful occasion. Like many other festivals, the kite flying competition also has a historical and cultural significance attached to it. The following figure shows a kite in which *BCD* is the shape of quadrant of a circle of radius 42 cm, *ABCD* is a square and ΔCEF is an isosceles right angled triangle whose equal sides are 7 cm long.



Based on the above information, answer the following questions.

(i) Find the area of the square.
(a) 1700 cm^2 (b) 1764 cm^2 (c) 1800 cm^2 (d) 1864 cm^2

(ii) Area of quadrant *BCD* is
(a) 1290 cm^2 (b) 1380 cm^2 (c) 1386 cm^2 (d) 1390 cm^2

(iii) Find the area of ΔCEF.
(a) 24.5 cm^2 (b) 25 cm^2 (c) 25.5 cm^2 (d) 26 cm^2

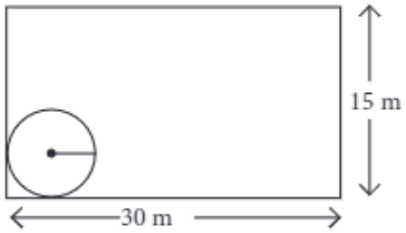
(iv) Area of the shaded portion is
(a) 1377 cm^2 (b) 1390 cm^2 (c) 1400 cm^2 (d) 1410.5 cm^2

(v) Area of the unshaded portion is
(a) 370 cm^2 (b) 378 cm^2 (c) 380 cm^2 (d) 384 cm^2

Case Study 10

Rain Water Harvesting

A farmer has a rectangular field of length 30 m and breadth 15 m. By the farmer a pit of diameter 7 m is dug 12 m deep for rain water harvesting. The earth taken out is spread in the field.



Based on the above information, answer the following questions.

(i) Find the volume of the earth taken out.
(a) 460 m^3 (b) 462 m^3 (c) 465 m^3 (d) 468 m^3

(ii) The area of the rectangular field is
(a) 420 m^2 (b) 430 m^2 (c) 440 m^2 (d) 450 m^2

(iii) Find the area of the top of the pit.
(a) 38.5 m^2 (b) 40.5 m^2 (c) 41.5 m^2 (d) None of these

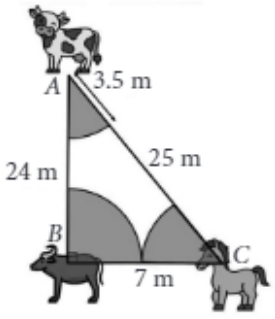
(iv) The area of the remaining field is
(a) 402.3 m^2 (b) 405 m^2 (c) 410 m^2 (d) 411.5 m^2

(v) Find the level rise in the field.
(a) 0.5 m (b) 3 m (c) 1.12 m (d) 2.12 m

Case Study 11

Grazing Management for Animals

Gayatri have a triangular shaped grass field. At the three corners of the field, a cow, a buffalo and a horse are tied separately to the pegs by means of ropes of 3.5 m each to graze in the field, as shown in the figure. Sides of the triangular field are 25 m, 24 m and 7 m. Based on the above information, answer the following questions.



(i) Area of triangular field is
(a) 82 m^2 (b) 84 m^2 (c) 86 m^2 (d) 88 m^2

(ii) Area of the region grazed by the cow is

(a) $\frac{\angle A}{360^\circ} \times \pi \times (3.5)^2$ (b) $\frac{\angle B}{360^\circ} \times \pi \times (24)^2$ (c) $\frac{\angle C}{360^\circ} \times \pi \times (3.5)^2$ (d) None of these

(iii) Area of region grazed by the buffalo and the horse is

(a) $\frac{(\angle A + \angle C)}{360^\circ} \times \pi \times (5.5)^2$ (b) $\frac{(\angle B + \angle C)}{360^\circ} \times \pi \times (5.6)^2$

(c) $\frac{(\angle A + \angle C)}{360^\circ} \times \pi \times (3.5)^2$ (d) $\frac{(\angle B + \angle C)}{360^\circ} \times \pi \times (3.5)^2$

(iv) Total area grazed by the cow, the buffalo and the horse is

(a) 16.25 m^2 (b) 17.3 m^2 (c) 18.25 m^2 (d) 19.25 m^2

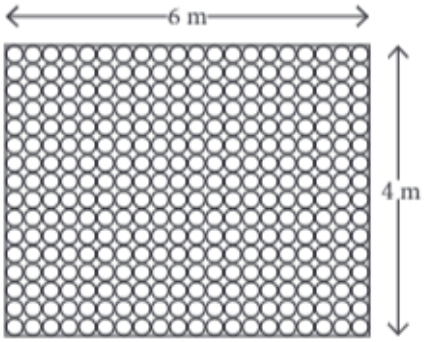
(v) Find the area of the field that cannot be grazed.

(a) 60.75 m^2 (b) 64.75 m^2 (c) 68 m^2 (d) 69.75 m^2

Case Study 12

Design of a Floor

Shweta wants to change the design of the floor of her living room which is of dimensions 6 m × 4 m and it is covered with circular tiles of diameters 50 cm each, as shown in the figure.



Based on the above information, answer the following questions.

(i) Number of circular tiles along length of room is

(a) 11 (b) 12 (c) 13 (d) 14

(ii) Total number of circular tiles equals to

(a) 90 (b) 92 (c) 94 (d) 96

(iii) Area covered by each circular tile is

(a) 1954.28 cm^2 (b) 1960.08 cm^2 (c) 1964.28 cm^2 (d) 1980 cm^2

(iv) Area of rectangular floor is

(a) 240000 cm^2 (b) 204000 cm^2 (c) 420000 cm^2 (d) None of these

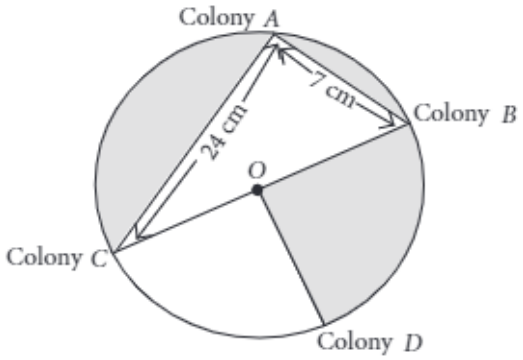
(v) Find the area of the floor that remains uncovered with tiles.

(a) 3.6 m^2 (b) 4.6 m^2 (c) 5 m^2 (d) 5.142 m^2

Case Study 13

Identifying Polluted Region

To find the polluted region in different areas of Dwarka (a part of Delhi represented by the circle given below) a survey was conducted by the students of class X. It was found that the shaded region is the polluted region, where O is the centre of the circle.



Based on the above information, answer the following questions.

(i) Find the radius of the circle.

(a) 12.5 cm (b) 13.5 cm (c) 15 cm (d) 16.5 cm

(ii) Find the area of the circle.

(a) 481.7 cm^2 (b) 490 cm^2 (c) 491.07 cm^2 (d) 495.6 cm^2

(iii) If D lies at the middle of arc BC, then area of region COD is

(a) 121 cm^2 (b) 122.76 cm^2 (c) 126 cm^2 (d) 129.8 cm^2

(iv) Area of the ΔBAC is

(a) 77 cm^2 (b) 79 cm^2 (c) 81 cm^2 (d) 84 cm^2

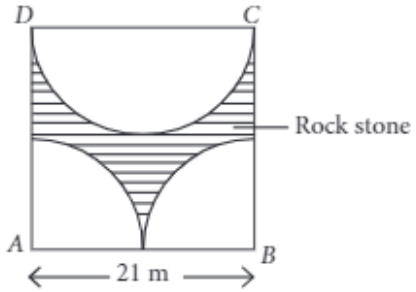
(v) Find the area of the polluted region.

(a) 280.31 cm^2 (b) 284.31 cm^2 (c) 285.31 cm^2 (d) 240.31 cm^2

Case Study 14

Construction with Virtue

A builder of residential project have a vacant square land of side 21 m. He wants to make a temple in the shape of semi-circle and a park in the shape of two quadrants of a circle as shown in the figure.



Based on the above information, answer the following questions.

(i) Find the area of square.

(a) 436 m^2 (b) 438 m^2 (c) 441 m^2 (d) 444 m^2

(ii) Area of two quadrants, shown in figure, is

(a) 170.25 m^2 (b) 173.25 m^2 (c) 175 m^2 (d) 178.25 m^2

(iii) Find the area of semi-circular temple.

(a) 163.25 m^2 (b) 168.25 m^2 (c) 173.25 m^2 (d) 178.25 m^2

(iv) Find the area of unshaded region.

(a) 340.5 m^2 (b) 346.5 m^2 (c) 350.5 m^2 (d) 355.65 m^2

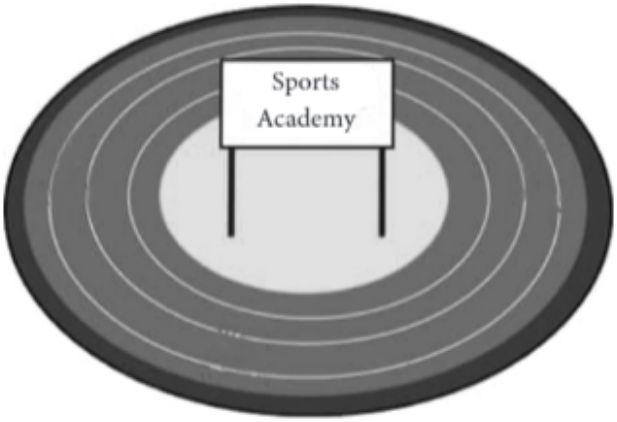
(v) Find the area of shaded region.

(a) 88.5 m^2 (b) 90.5 m^2 (c) 92.5 m^2 (d) 94.5 m^2

Case Study 15

Race Competition

There is a race competition between all students of a sports academy, so that the sports committee can choose better students for a marathon. The race track in the academy is in the form of a ring whose inner most circumference is 264 m and the outer most circumference is 308 m.



Based on the above information, answer the following questions.

(i) Find the radius of the outer most circle.

(a) 48 m (b) 49 m (c) 50 m (d) 51 m

(ii) Find the radius of the inner most circle.

(a) 38 m (b) 40 m (c) 42 m (d) 44 m

(iii) Find the width of the track.

(a) 7 m (b) 8 m (c) 9 m (d) 10 m

(iv) Find the area of the race track.

(a) 2010 m^2 (b) 2006 m^2 (c) 2000 m^2 (d) 2002 m^2

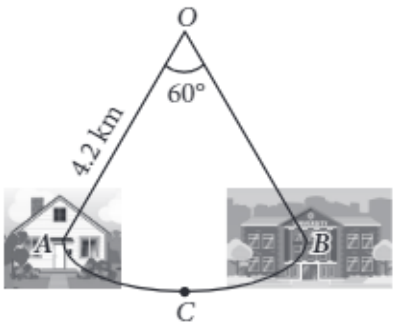
(v) If the cost of painting on the race track is ₹ 6 per m^2, then find the total cost for painting the whole race track.

(a) ₹ 12000 (b) ₹ 12012 (c) ₹ 12550 (d) ₹ 12850

Case Study 16

Analysing Distance and Expense

Kartik has his home located at A and his college located at B. Kartik drives his motorbike three days in a week and rides his bicycle in the remaining 3 days, to go to his college and back to home. AOB is a sector of a circle with centre O, central angle 60° and radius 4.2 km. Path AOB is the route for driving by motorbike and path ACB is for bicycle only.



(i) Find the total distance travelled by Kartik through the motorbike in a week to go to college.

(a) 50.4 km (b) 55 km (c) 56.4 km (d) 58 km

(ii) Find the total distance travelled by Kartik through the bicycle in a week to go to college.

(a) 24.4 km (b) 26.4 km (c) 28 km (d) 29.4 km

(iii) Find the area of sector AOB.

(a) 7.88 km^2 (b) 8.24 km^2 (c) 9.24 km^2 (d) 10.14 km^2

(iv) If the cost of fuel for the motorbike is ₹ 20 per km, then find the total cost of fuel used in a week in going college.

(a) ₹ 1008 (b) ₹ 1120 (c) ₹ 1200 (d) ₹ 1240

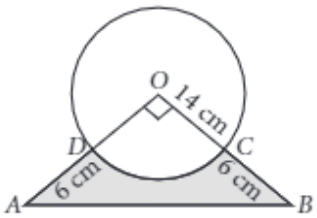
(v) If the angle of sector changed from 60° to 90°, then find the total length of the available paths.

(a) 12 km (b) 13 km (c) 14 km (d) 15 km

Case Study 17

Prize Distribution

Director of a company select a round glass trophy for awarding their employees on annual function. Design of each trophy is made as shown in the figure, where its base $ABCD$ is golden plated from the front side at the rate of ₹ 6 per cm^2.



(i) Find the area of sector $ODCO$.

(a) 154 cm^2 (b) 155 cm^2 (c) 156 cm^2 (d) 157 cm^2

(ii) Find the area of ΔAOB.

(a) 150 cm^2 (b) 200 cm^2 (c) 250 cm^2 (d) 300 cm^2

(iii) Find the total cost of golden plating.

(a) ₹ 276 (b) ₹ 280 (c) ₹ 284 (d) ₹ 200

(iv) Find the area of major sector formed in the given figure.

(a) 400 cm^2 (b) 450 cm^2 (c) 462 cm^2 (d) 472 cm^2

(v) Find the length of arc *DC*.

(a) 16 cm (b) 18 cm (c) 20 cm (d) 22 cm

Case Study 18

Pendulum Clock

Kritika bought a pendulum clock for her living room. The clock contains a small pendulum of length 15 cm. The minute hand and hour hand of the clock are 9 cm and 6 cm long respectively.

Based on the above information, answer the following questions.

(i) Find the area swept by the minute hand in 10 minutes.

(a) 24.24 cm^2 (b) 42.42 cm^2

(c) 44 cm^2 (d) 44.42 cm^2

(ii) If the pendulum covers a distance of 22 cm in one complete oscillation, then find the angle described by pendulum at the centre.

(a) 40° (b) 42° (c) 45° (d) 48°

(iii) Find the angle described by hour hand in 10 minutes.

(a) 5° (b) 10° (c) 15° (d) 20°

(iv) Find the area swept by the hour hand in 1 hour.

(a) 7.68 cm^2 (b) 8.2 cm^2 (c) 8.86 cm^2 (d) 9.428 cm^2

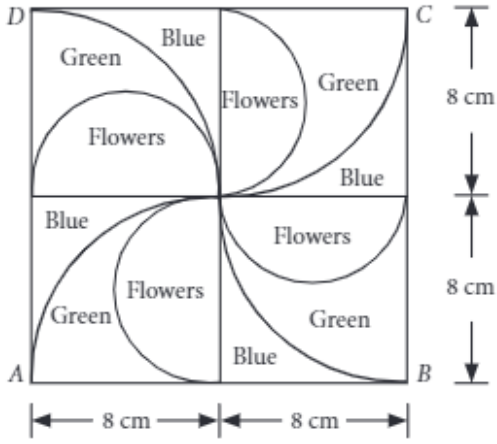
(v) Find the area swept by the hour hand between 11 a.m. and 5 p.m.

(a) 56.568 cm^2 (b) 62 cm^2 (c) 70 cm^2 (d) 72 cm^2

Case Study 19

Creating a Painting

Shiva made a painting on a square chart paper *ABCD*. The painting is made up of squares, semicircular arcs (painted with flowers) and arcs of quadrant of circles as shown below. He painted the same type of regions with same colours.



Based on the above information, answer the following questions.

(i) Find the total area of the region which is painted with flowers.

(a) 90.8 cm^2 (b) 100.57 cm^2 (c) 105.6 cm^2 (d) 111.20 cm^2

(ii) Find the area of all the quadrants given in the figure.

(a) 190 cm^2 (b) 198.14 cm^2 (c) 201.14 cm^2 (d) 222.14 cm^2

(iii) Find the area of the region which is painted green

(a) 100.57 cm^2 (b) 111.57 cm^2 (c) 120 cm^2 (d) 128.57 cm^2

(iv) Find the area of the region which is painted blue.

(a) 46.86 cm^2 (b) 48 cm^2 (c) 50.86 cm^2 (d) 54.86 cm^2

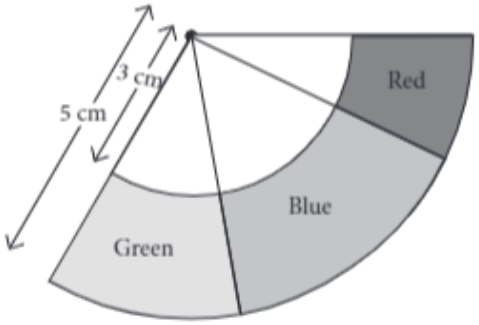
(v) Find the total length of the boundary of the region which is painted green.

(a) 128.57 cm (b) 132.56 cm (c) 145.57 cm (d) 150 cm

Case Study 20

Japanese Folding Fan

Sara hold a japanese folding fan in her hand as shown in the figure. It is shaped like a sector of a circle and made of a thin material such as paper or feather. The inner and outer radii are 3 cm and 5 cm respectively. The fan has three colours *i.e.*, red, blue and green.



Based on the above information, answer the following questions.

(i) If the region containing blue colour makes an angle of 80° at the centre, then find the area of the region having blue colour.

(a) 9.17 cm^2 (b) 10.1 cm^2 (c) 11.17 cm^2 (d) 13.17 cm^2

(ii) If the region containing green colour makes an angle of 60° at the centre, then find the area of the region having green colour.

(a) 6.2 cm^2 (b) 8.38 cm^2 (c) 9.9 cm^2 (d) 11.12 cm^2

(iii) If the region containing red colour makes an angle of 20° at the centre, then find the perimeter of the region containing red colour.

(a) 2.9 cm (b) 4.2 cm (c) 5.4 cm (d) 6.79 cm

(iv) Find the area of the region having radius 3 cm.

(a) 12.57 cm^2 (b) 14.8 cm^2 (c) 20 cm^2 (d) 26.57 cm^2

(v) The region given in the figure represents

(a) minor sector (b) major sector (c) minor segment (d) major segment

6. **(i)** **(c)** : Radius of circle representing red region

$= \frac{22}{2} = 11$ cm [$\because$ Diameter = 22 cm (Given)]

(ii) **(a)** : Area of red region = πr^2

$= \frac{22}{7} \times 11 \times 11 = 380.28$ cm^2

(iii) **(b)** : Radius of circle formed by combining red and silver region = Radius of red region + width of silver sign

= (11 + 10.5) cm = 21.5 cm

(iv) **(d)** : Area of silver region

= Area of combined region – Area of red region

$= \frac{22}{7} \times 21.5 \times 21.5 - 380.28$

= 1452.78 – 380.28 = 1072.50 cm^2

(v) **(b)** : Area of circular path formed by two concentric circles = $\pi(r_1^2 - r_2^2)$ sq. units

7. Let r and R be the radii of each smaller circle and larger circle respectively.

We have, $d = \frac{1}{4}D$

$\Rightarrow r = \frac{1}{4}R \Rightarrow r = \frac{1}{4} \times 16 \Rightarrow r = 4$ cm

(i) **(c)** : Area of smaller circle = πr^2

$= \frac{22}{7} \times 4 \times 4 = 50.28$ cm^2

(ii) **(a)** : Area of larger circle = πR^2

$= \frac{22}{7} \times 16 \times 16 = \frac{5632}{7} = 804.57$ cm^2

(iii) **(b)** : Area of the black colour region = Area of larger circle – Area of 4 smaller circles

= 804.57 – 4 × 50.28 = 603.45 cm^2

(iv) **(d)** : Area of quadrant of a smaller circle

$= \frac{1}{4} \times 50.28 = 12.57$ cm^2

(v) **(c)** : Area between two concentric circles

$= \pi(R^2 - r^2) = \frac{22}{7}(5^2 - 2^2)$

$= \frac{22}{7}(25 - 4) = \frac{22}{7} \times 21 = 66$ cm^2

8. **(i)** **(b)** : Side of square $PQRS$ = 27 cm

$\therefore$ Area of square $PQRS = 27 \times 27 = 729$ cm^2

(ii) **(c)** : Area of rectangle left for car parking is area of region $PSUT = 27 \times 3 = 81$ cm^2

(iii) **(a)** : Diameter of semi-circle = $PV = \frac{PS}{2} = \frac{27}{2}$

= 13.5 cm

$\therefore$ Radius of semi-circle = $\frac{13.5}{2} = 6.75$ cm

(iv) **(d)** : Area of a semi-circle = $\frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 6.75 \times 6.75 = 71.59$ cm^2

(v) **(b)** : Area of shaded region = area of rectangular plot $QRUT$ – area of two semi-circles

= 30 × 27 – 2 × 71.59 = 666.82 cm^2

9. **(i)** **(b)** : Area of square $ABCD = 42 \times 42$

= 1764 cm^2

(ii) **(c)** : Area of quadrant BCD

$= \frac{1}{4} \times \frac{22}{7} \times 42 \times 42 = 1386$ cm^2

(iii) **(a)** : Area of $\Delta CEF = \frac{1}{2} \times CE \times CF$

$= \frac{1}{2} \times 7 \times 7 = 24.5$ cm^2

(iv) **(d)** : Area of shaded region = Area of quadrant BCD + Area of ΔCEF

= 1386 + 24.5 = 1410.5 cm^2

(v) **(b)** : Area of the unshaded region = Area of square $ABCD$ – Area of quadrant BCD

= 1764 – 1386 = 378 cm^2

10. **(i)** **(b)** : Volume of the earth taken out

$= \pi\left(\frac{7}{2}\right)^2 \times 12 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12 = 462$ m^3

(ii) **(d)** : Area of the rectangular field

= 30 × 15 = 450 m^2

(iii) **(a)** : Area of top of the pit $= \pi\left(\frac{7}{2}\right)^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{77}{2} = 38.5$ m^2

(iv) **(d)** : Area of the remaining field = Area of rectangular field – area of top of pit

= 450 – 38.5 = 411.5 m^2

(v) (c) : The rise in the level of field $= \frac{462}{411.5} = 1.12$ m

11. (i) (b): Since ΔABC is a right angled triangle.

$\therefore$ Area of triangle $ABC = \frac{1}{2} \times 7 \times 24 = 84$ m^2

(ii) (a): Area of region grazed by the cow

$$= \frac{\angle A}{360^\circ} \times \pi \times (3.5)^2$$

(iii) (d): Area of the region grazed by the buffalo and the horse $= \frac{\angle B}{360^\circ} \times \pi \times (3.5)^2 + \frac{\angle C}{360^\circ} \times \pi \times (3.5)^2$

$$= \frac{(\angle B + \angle C)}{360^\circ} \times \pi \times (3.5)^2$$

(iv) (d): Total area grazed by the cow, the horse and the buffalo $= \frac{\angle A}{360^\circ} \times \pi \times (3.5)^2 + \frac{(\angle B + \angle C)}{360^\circ} \times \pi \times (3.5)^2$

$$= \frac{(\angle A + \angle B + \angle C)}{360^\circ} \times \pi \times (3.5)^2$$

$$= \frac{22}{7} \times \frac{180^\circ}{360^\circ} \times 3.5 \times 3.5$$

($\because$ sum of interior angles of a triangle is 180°)

$$= \frac{77}{4} = 19.25 \text{ m}^2$$

(v) (b): Area of the field that cannot be grazed = Area of ΔABC – Area of region grazed by all the three animals

$= 84 - 19.25 = 64.75$ m^2

12. (i) (b): Number of circular tiles along length of room $= \frac{600}{50} = 12$

(ii) (d): Number of circular tiles along breadth of room $= \frac{400}{50} = 8$

$\therefore$ Total number of circular tiles $= 12 \times 8 = 96$

(iii) (c) : Area covered by each circular tile $= \pi(25)^2$

$$= \frac{22}{7} \times 25 \times 25 = 1964.28 \text{ cm}^2$$

(iv) (a): Area of rectangular floor $= 600 \times 400$

$= 240000$ cm^2

(v) (d): Area of the floor that remains uncovered with tiles = Area of rectangular floor – Area of 96 circular tiles

$= 240000 - 96 \times 1964.28$

$= 51429.12$ cm^2 $= 5.142$ m^2

13. (i) (a): Since BOC is the diameter and $\angle BAC = 90^\circ$

$\therefore$ $BC^2 = AB^2 + AC^2$

$= 7^2 + 24^2 = 625$

$\Rightarrow$ $BC = 25$ cm

$\therefore$ Radius of circle $= \frac{25}{2}$ cm $= 12.5$ cm

(ii) (c) : Area of circle $= \pi(12.5)^2 = \frac{22}{7} \times 12.5 \times 12.5$

$= 491.07$ cm^2

(iii) (b): Clearly, $\angle COD = 90^\circ$

[$\because$ $\angle COB = 180^\circ$ and equal arcs subtends equal angles at the centre]

Area of region $COD = \frac{90^\circ}{360^\circ} \times \pi r^2$

$$= \frac{1}{4}(491.07) = 122.76 \text{ cm}^2$$

(iv) (d): Area of $\Delta BAC = \frac{1}{2} \times AB \times AC$

$$= \frac{1}{2} \times 7 \times 24 = 84 \text{ cm}^2$$

(v) (b): Area of the polluted region = Area of circle – Area of sector COD – Area of ΔABC

$= 491.07 - 122.76 - 84$

$= 284.31$ cm^2

14. (i) (c) : Area of square $ABCD = 21 \times 21 = 441$ m^2

(ii) (b): Area of two quadrants $= 2\left(\pi r^2 \times \frac{90^\circ}{360^\circ}\right)$

$$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{1}{2} = 173.25 \text{ m}^2$$

(iii) (c) : Area of semi-circular temple $= \frac{1}{2}(\pi r^2)$

$$= \frac{1}{2} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} = 173.25 \text{ m}^2$$

(iv) (b): Area of unshaded region = Area of semi-circle + Area of two quadrants

$= 173.25 + 173.25 = 346.5$ m^2

(v) (d): Area of shaded region = Area of square – (Area of two quadrants + Area of semi-circle)

$= 441 - 346.5 = 94.5$ m^2

15. (i) (b): Let the radius of outer most circle be R.

Outer most circumference = 308 m [Given]

$\Rightarrow$ $2\pi R = 308$ $\Rightarrow$ $2 \times \frac{22}{7} \times R = 308$

$\Rightarrow$ $R = \frac{308 \times 7}{2 \times 22} = 49$ m

(ii) **(c)** : Let the radius of inner most circle be r
Inner most circumference = 264 m [Given]
$\Rightarrow \quad 2\pi r = 264$
$\Rightarrow \quad 2\times\frac{22}{7}\times r = 264 \Rightarrow r = \frac{264\times 7}{2\times 22} = 42$ m

(iii) **(a)** : Width of the track = Radius of outer most track – Radius of inner most track = 49 – 42 = 7 m

(iv) **(d)**: Area of the race track = Area of outer circle – Area of inner circle
$= \pi(R^2 - r^2) = \pi[(49)^2 - (42)^2]$
$= \frac{22}{7}[2401 - 1764] = 2002$ m^2

(v) **(b)**: Cost of painting the whole race track
= ₹ (6 × 2002) = ₹ 12012.

16. **(i)** **(a)** : In a week, Kartik drives his motorbike 3 days to go to college
∴ Total distance travelled by Kartik through motorbike = 2 × 4.2 × 6 = 50.4 km

(ii) **(b)**: In a week Kartik rides his bicycle 3 days to go to college.
∴ Total distance travelled by Kartik through bicycle = Length of arc $\overset{\frown}{ACB} \times 6$
$= \frac{\theta}{360^\circ}\times 2\pi r \times 6 = \frac{60^\circ}{360^\circ}\times 2\times\frac{22}{7}\times 4.2\times 6 = 26.4$ km

(iii) **(c)** : Area of sector $AOB = \frac{\theta}{360^\circ}\times\pi r^2$
$= \frac{60^\circ}{360^\circ}\times\frac{22}{7}\times(4.2)^2 = 9.24$ km^2

(iv) **(a)** : Total cost of fuel used for a week
= ₹ (20 × 50.4) = ₹ 1008

(v) **(d)**: Total length of available paths
$= 4.2 + 4.2 + \frac{\theta}{360^\circ}\times 2\pi r$
$= 8.4 + \frac{90^\circ}{360^\circ}\times 2\times\frac{22}{7}\times 4.2 = 8.4 + 6.6 = 15$ km

17. **(i)** **(a)** : Area of sector $ODCO = \frac{1}{4}\pi r^2$
$= \frac{1}{4}\times\frac{22}{7}\times 14\times 14 = 154$ cm^2

(ii) **(b)**: Area of $\Delta AOB = \frac{1}{2}\times OA\times OB = \frac{1}{2}(20\times 20)$
$= 200$ cm^2

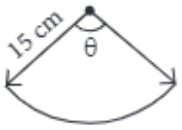
(iii) **(a)** : Area of region which is golden plated
= area of ΔOAB – area of sector $ODCO$.
= 200 – 154 = 46 cm^2
∴ Total cost of golden plating = ₹ (6 × 46) = ₹ 276

(iv) **(c)** : Area of major sector = area of circle – area of minor sector
$= \pi r^2 - \frac{1}{4}\pi r^2 = \frac{3\pi r^2}{4} = \frac{3}{4}\times\frac{22}{7}\times 14\times 14 = 462$ cm^2

(v) **(d)**: Length of arc $DC = \frac{90^\circ}{360^\circ}\times 2\times\frac{22}{7}\times 14$
$= 22$ cm

18. **(i)** **(b)**: Angle made by minute hand in 60 minutes = 360°
∴ Angle made by minute hand in 10 minutes
$= \frac{360^\circ}{60}\times 10 = 60^\circ$
Length of minute hand = 9 cm
∴ Area swept by minute hand in 10 minutes = Area of sector having central angle 60°
$= \pi r^2\left(\frac{60^\circ}{360^\circ}\right) = \frac{22}{7}\times 9\times 9\times\frac{1}{6}$
$= \frac{297}{7} = 42.42$ cm^2

(ii) **(b)**: We have, r = 15 cm
and $l = \frac{1}{2}(22) = 11$ cm



We known that $l = 2\pi r\left(\frac{\theta}{360^\circ}\right)$
$\Rightarrow \quad \theta = \frac{11\times 360^\circ}{2\times\frac{22}{7}\times 15} = \frac{90^\circ\times 7}{15} = 6^\circ\times 7 = 42^\circ$

(iii) **(a)** : Angle made by hour hand in 12 hours = 360°
∴ Angle made by hour hand in 10 minutes
$= \left(\frac{360^\circ}{12}\times\frac{1}{6}\right) = 5^\circ$

(iv) **(d)**: Angle made by hour hand in 1 hour
$= \frac{360^\circ}{12} = 30^\circ$
Also, r = 6 cm
∴ Area swept by hour hand in 1 hour = Area of sector having central angle 30°
$= \pi r^2\times\left(\frac{30^\circ}{360^\circ}\right) = \frac{22}{7}\times 6\times 6\times\frac{1}{12} = \frac{66}{7} = 9.428$ cm^2

(v) **(a)** : Number of hours from 11 a.m. to 5 p.m. = 6
Area swept by hour hand in 1 hour = 9.428 cm^2
∴ Area swept by hour hand in 6 hours = 9.428 × 6
= 56.568 cm^2

19. **(i)** **(b)**: Region which is painted with flowers are 4 semi-circles of diameter 8 cm, *i.e.*, 2 circles of radius 4 cm each.

$\therefore$ Total area of region which is painted with flowers

$= 2 \times \pi r^2 = 2 \times \frac{22}{7} \times 4 \times 4 = 100.57 \text{ cm}^2$

(ii) **(c)** : In the figure, we have 4-quadrants of radius 8 cm

$\therefore$ Area of all quadrants $= 4 \times \frac{1}{4} \pi r^2 = \pi r^2$

$= \frac{22}{7} \times 8 \times 8 = 201.14 \text{ cm}^2$

(iii) **(a)** : Area of region which is painted green
= area of all quadrants – total area of region which are painted with flowers
= 201.14 – 100.57 = 100.57 cm^2

(iv) **(d)**: Area of region which is painted blue
= Area of whole chart paper – area of all quadrants
= 16 × 16 – 201.14 = 256 – 201.14 = 54.86 cm^2

(v) **(b)**: Total length of boundary of region which is painted green = 4 [circumference of semi-circle of radius 4 cm each + length of arc of quadrant of radius 8 cm + radius of quadrant]

$= 4\left[\frac{1}{2} \times 2\pi(4) + \frac{1}{4} \times 2 \times \pi \times 8 + 8\right]$

$= 4\left[4\pi + 4\pi + 8\right] = 4(8\pi + 8) = 4\left(8 \times \frac{22}{7} + 8\right)$

= 4(33.14) = 132.56 cm

20. **(i)** **(c)** : Area of the region containing blue colour

$= \frac{22}{7} \times 5 \times 5 \times \frac{80^\circ}{360^\circ} - \frac{22}{7} \times 3 \times 3 \times \frac{80^\circ}{360^\circ}$

$= \frac{22}{7} \times \frac{2}{9} \times [25 - 9] = \frac{44}{63}(16) = 11.17 \text{ cm}^2$

(ii) **(b)** : Area of the region containing green colour

$= \frac{22}{7} \times \frac{60^\circ}{360^\circ}[5 \times 5 - 3 \times 3] = \frac{22}{7} \times \frac{1}{6} \times 16 = 8.38 \text{ cm}^2$

(iii) **(d)** : Perimeter of the region containing red colour
= 2 + 2 + length of arc of sector having radius 3 cm + length of arc of sector having radius 5 cm

$= 4 + 2 \times \frac{22}{7} \times 3 \times \frac{20^\circ}{360^\circ} + 2 \times \frac{22}{7} \times 5 \times \frac{20^\circ}{360^\circ}$

$= 4 + \frac{44}{7} \times \frac{1}{18} \times 8 = 4 + \frac{176}{63} = 4 + 2.79 = 6.79$ cm

(iv) **(a)** : Required area $= \frac{22}{7} \times 3 \times 3 \times \frac{160^\circ}{360^\circ}$

$= \frac{88}{7} = 12.57 \text{ cm}^2$

(v) **(a)** : Angle of given sector = 80° + 60° + 20° = 160°
Thus, the given region represents minor sector of a circle.