Chapter 6

Filters and Field Theory

LEARNING OBJECTIVES

After reading this chapter, you will be able to understand:

- Ideal filters
- Filter fundamentals
- Low pass filter
- High pass filer
- Circuit parameters
- Band pass filter
- Band elimination filter
- Poly phase circuits
- Inter connection of three phases
- Delta or mesh connection
- Power in delta connected system
- Star to delta and delta to star transformation
- Electrical fields
- Gradient, divergence and curl in all the co-ordinate system
- Electric field of continuous space distribution of charges
- Gauss's law
- Dielectrics inductance and capacitance
- Magneto statics
- Magnetic field on the axis of a circular coil
- Induced EMF
- Maxwells equations

Basic Filter Concepts

A network which freely passes a desired band of frequency while suppressing other band of frequency is called a filter.

It has the ability to discriminate between signals which differ in frequency.

Classification of Filters

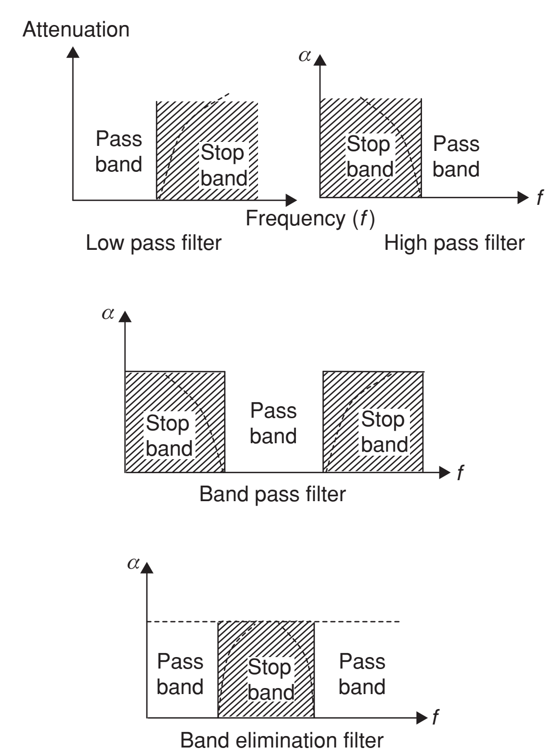
1. **Active filters:** These filters consist of active elements such as transistor, op-amp along with resistors, capaitors and inductors as active elements. In these filters voltage, current and power gain is possible. But the main disadvantage of this filter is that they require additional power supplies for the operation of active elements.
2. **Passive filters:** These filters use only passive elements like resistors, capacitors and inductors. In passive filters, none of the gain is possible. As inductors are very heavy and bulky, they are very costly. They don't require any additional power supplies.

Basic Filter Section

1. **Low pass filter:** It passes the frequency upto the cut off frequency and attenuates all frequencies above it.
2. **High pass filter:** It attenuates all frequencies upto cut off frequency and passes all frequencies above it.
3. **Band pass filter:** This type of filter passes all frequencies between two cut off frequencies and attenuates all other frequencies.
4. **Band elimination filter:** This type of filter attenuates all frequencies between two cut off frequencies and passes all other frequencies.

Ideal Filters

An ideal filter should have zero attenuation in the pass band and infinite attenuation in the top band. But practically in stop band attenuation gradually changes. Dark lines represent ideal characteristics and dotted lines show practical characteristics.



Filter Fundamentals

For T Network

1. Characteristic impedence

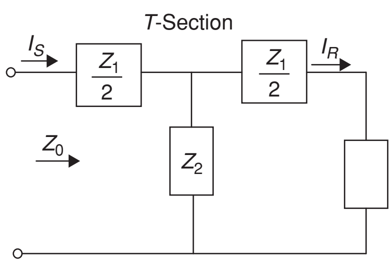
$$Z_{\sigma T} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

If we consider all elements of $Z_{OT} = j\sqrt{\frac{X_1^2}{4} + X_1 X_2}$

T network are reactive.

2. Propagation constant (γ)

$$\cos h\gamma = 1 + \frac{Z_1}{2Z_2}$$



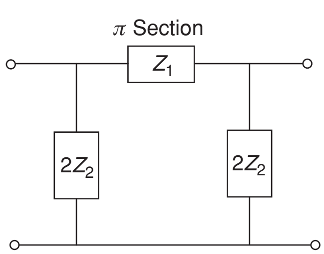
For π Network

1. Characteristic impedance

$$Z_{0\pi} = \frac{Z_1 Z_2}{\sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}}$$

2. Propagation constant (γ)

$$\cosh\gamma = 1 + \frac{Z_1}{2Z_2}$$



Cut-off Frequency

$$Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

If $Z_1 = jX_1$ and $Z_2 = jX_2$

$$Z_0 = \sqrt{\frac{-X_1^2}{4} - X_1 X_2} = \sqrt{-X_1\left(\frac{X_1}{4} + X_2\right)}$$

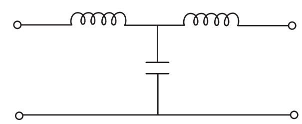
If X_1 and $\left(\frac{X_1}{4} + X_2\right)$ have same sign Z_0 is imaginary and gives STOP BAND.

If X_1 and $\left(\frac{X_1}{4} + X_2\right)$ have oppositie sign Z_0 is real (purely resistive) and gives a PASS BAND.

CUT-off Frequency: It is the frequency at which Z_0 changes from real to imaginary.

$$\text{At } f_c : X_1\left(\frac{X_1}{4} + X_2\right) = 0 \quad \Rightarrow \quad X_1 = -4X_2$$

Low-pass Filter



T-type prototype low-pass filter

Circuit Parameters

1. Total series impedance $Z_1 = j\omega L$
2. Total shunt impedance $Z_2 = \frac{j}{\omega C}$

$$Z_1 Z_2 = (j\omega L)\left(\frac{-j}{\omega C}\right) = \frac{L}{C}$$

$$R_0^2 = \frac{L}{C} \quad \Rightarrow \quad R_0 = \sqrt{\frac{L}{C}}.$$

3. Cut-off frequency

$$Z_1 = j\omega L \quad X_1 = \omega L$$

$$Z_2 = \frac{-j}{\omega C} \quad X_2 = \frac{-1}{\omega C}$$

$$\therefore \quad \left(\frac{X_1}{4} + X_2\right) = j\left(\frac{\omega L}{4} - \frac{1}{\omega C}\right)$$

Point A marks cut off frequency at which $\omega = \omega_C$

Hence $\frac{\omega_c L}{4} - \frac{1}{\omega_c C} = 0$

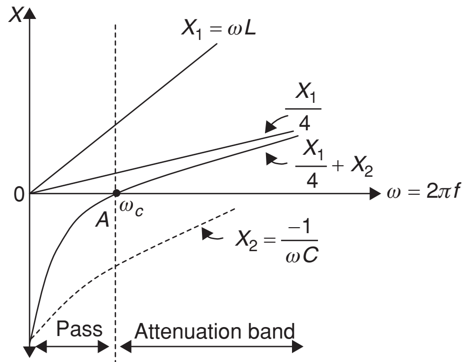
$$\omega_c^2 = \frac{4}{LC}$$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

4. Characteristic impedance

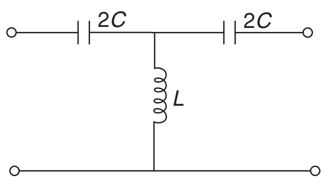
$$Z_{OT} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{\frac{-\omega^2 L^2}{4} + \frac{L}{C}}$$

$$Z_{OT} = R_0\sqrt{1 - \frac{\omega^2 LC}{4}}$$



Reactance curve

High-pass Filter



T-section prototype high-pass filter

Circuit Parameters

1. Total series arm impedance $Z_1 = \dfrac{-j}{\omega C}$
2. Total shunt arm impedance $Z_2 = j\omega L$

$$Z_1 Z_2 = \left(-\frac{j}{\omega C}\right)(j\omega L) = \frac{L}{C}$$

$$R_0^2 = \frac{L}{C} \quad \Rightarrow \quad R_0 = \sqrt{\frac{L}{C}}$$

3. Reactance curve and cut-off frequency

$$Z_1 = \frac{-j}{\omega C} X_1 = \frac{-1}{\omega C}$$

$$Z_2 = j\omega L, X_2 = \omega L$$

$$\frac{X_1}{4} + X_2 = \frac{-1}{4\omega C} + \omega L = \omega L - \frac{1}{4\omega C}$$

At point B the curve $\left(\dfrac{X_1}{4} + X_2\right)$ crosses the frequency axis.

$$\omega_c L - \frac{1}{4\omega_c C} = 0$$

$$\omega_c^2 = \frac{1}{4LC}$$

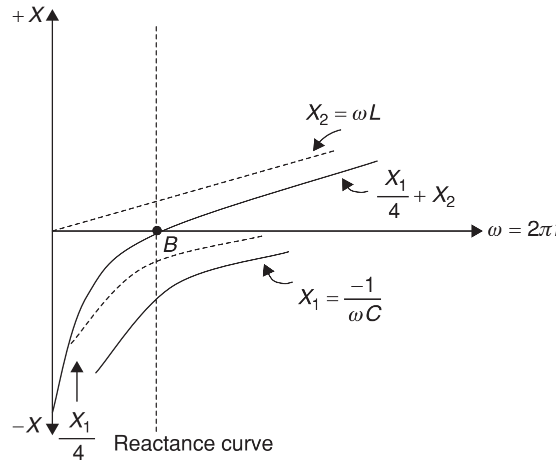
$$f_c = \frac{1}{2\sqrt{LC}}$$

4. Characteristic impedance

$$Z_{OT} = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2} = \sqrt{\frac{-1}{4\omega^2 C^2} + \frac{L}{C}}$$

$$= \sqrt{\frac{L}{C}}\sqrt{1 - \frac{1}{4\omega^2 LC}}$$

$$Z_{OT} = R_o\sqrt{1 - \frac{1}{4\omega^2 LC}}$$

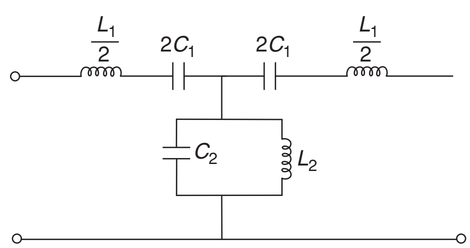


Reactance curve

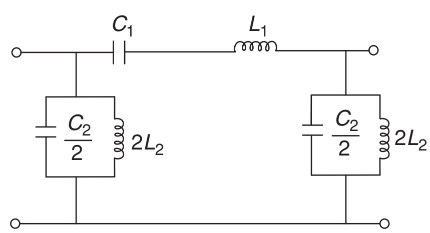
Reactance Curve

All frequencies below $B \longrightarrow$ stop Band
Above $B \longrightarrow$ Pass band
At point $B \longrightarrow$ cut off frequency

Band Pass Filter



(a)



(b)

Circuit Parameters

1. Series arm impedance $Z_1 = j\omega L_1 - \dfrac{j}{\omega C_1}$

$$= j\left(\frac{\omega^2 L_1 C_1 - 1}{\omega C_1}\right)$$

2. Shunt arm impedence Z_2

$$= \frac{j\omega L_2 \cdot \dfrac{1}{j\omega C_2}}{j\omega L_2 + \dfrac{1}{j\omega C_2}} = \frac{j\omega L_2}{1-\omega^2 L_2 C_2}$$

$$Z_1 Z_2 = \frac{L_2}{C_1}$$

$$= \frac{L_1}{C_2} = K^2$$

3. Condition for resonance and cut-off frequency for series arm $\dfrac{\omega_0 L_1}{2} = \dfrac{1}{2\omega_0 C_1} \omega_0^2 L_1 C_1 = 1$ for shunt arm

$$\frac{1}{\omega_0 C_2} = \omega_0 L_2 \omega_0^2 L_2 C_2 = 1$$

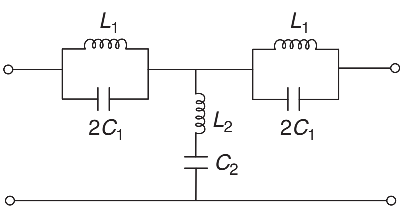
$L_1 C_1 = L_2 C_2$

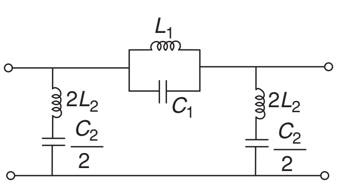
Cut-off frequency $f_0 = \sqrt{f_1 f_2}$.

4. Circuit components

$C_1 = \dfrac{f_2 - f_1}{4\pi k f_1 f_2}$	$C_2 = \dfrac{L_1}{k^2} = \dfrac{1}{\pi(f_2 - f_1)k}$
$L_1 = \dfrac{K}{\pi(f_2 - f_1)}$	$L_2 = C_1 K^2 = \dfrac{(f_2 - f_1)k}{4\pi f_1 f_2}$

Band Elimination Filter





Condition for Equal Resonance Frequencies

For series arm $\dfrac{\omega_0 L_1}{2} = \dfrac{1}{2\omega_0 C_1} \Rightarrow \omega_0^2 = \dfrac{1}{L_1 C_1}$

For shunt arm $\omega_0 L_2 = \dfrac{1}{\omega_0 C_2} \Rightarrow \omega_0^2 = \dfrac{1}{L_2 C_2}$

$$\frac{1}{L_1 C_1} = \frac{1}{L_2 C_2} = K$$

1. Cut-off frequency $f_0 = \sqrt{f_1 f_2}$
2. Circuit components

$$C_1 = \frac{L_2}{k^2} = \frac{1}{4\pi k(f_2 - f_1)}$$

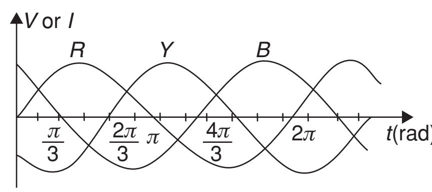
$$L_1 = k^2 C_2 = \frac{K}{\pi}\left(\frac{f_2 - f_1}{f_1 f_2}\right)$$

$$C_2 = \frac{1}{k\pi}\left|\frac{f_2 - f_1}{f_1 f_2}\right|$$

$$L_2 = \frac{k}{4\pi(f_2 - f_1)}$$

Poly Phase Circuits

A three phase system of voltages (currents) is a combination of three single phase system of voltages (currents) of which the three voltages differ in phase by 120 electrical degrees from each other in a particular sequence.



Advantages of Three Phase System

1. Power in 1ϕ system is pulsating. If *pf* is unity power pulsates at a frequency of 100 Hz. The total three phase power supplied to a balanced three phase circuit is constant at every instant of time.
2. Three phase transmission circuit requires less conductor material than a single phase circuit.
3. In a given frame size, a three phase motor or generator produces more output than the single phase counter part.

Phase Sequence

If the three phases are named *R*, *Y*, *B* then the order in which there phase attain their maximum values is the phase sequence of the system. In the system given, the phase sequence is *RYB*. The sequence depends on the rotation of the field in the generator.

Note that the sequence *RYB*, *BRY* and *YBR* are the same.

If the direction of rotation of field is changed, the phase sequence changes. The other phase sequence is *RBY*, *BYR* or *YRB*.

In phase sequence *RYB*

$V_R = V_m \sin \omega t$

$V_Y = V_m \sin(\omega t - 120°)$

$V_B = V_m \sin(\omega t - 240°)$

Note: The sum of three vectors equal is magnitude but displaced from each other by 120° is zero.

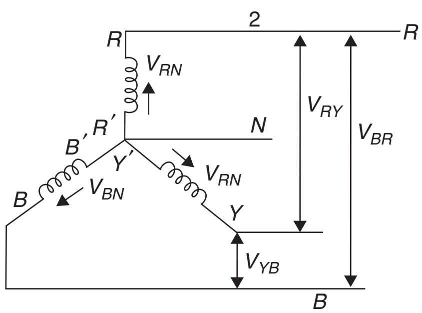
So in a balanced three phase system, sum of phase as well as line quantities is zero.

Inter Connection of Three Phases

The coils in a three phase system may be connected to form a wye (Y) or delta (Δ) system to achieve economy and to reduce the number of conductors.

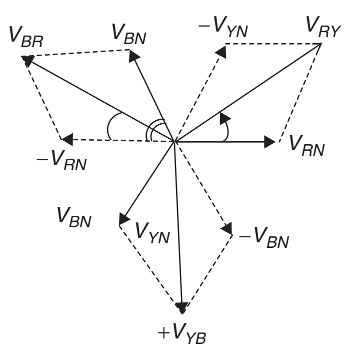
Wye or Star Connection

In this connection similar ends (start or finish) of the three phases are joined together. The common terminal is called is the neutral. The terminals R, Y, B are called the line terminals. The voltage between any line and the neutral point is called phase voltage (V_{RN}, V_{YN}, V_{BN}), while voltage between any two lines is called line voltage (V_{RY}, V_{YB}, V_{BR}).



The double subscript is purposefully used to represent currents and voltage is a 3ϕ system.

Voltage relation in Y connected circuit



$$\begin{aligned} V_{RY} &= V_{RN} - V_{YN} = V_{RN} + (-V_{YN}) \\ V_{YB} &= V_{YN} - V_{BN} = V_{YN} + (-V_{BN}) \\ V_{BR} &= V_{BN} - V_{RN} = V_{BN} + (-V_{RN}) \end{aligned}$$

Since $|V_{RN}| = |-V_{YN}| = V_{PL}$

$$V_{RY} = 2V_{Ph}\cos\left(\frac{60}{2}\right) = \sqrt{3}\,V_{Ph}$$

$$\therefore\ |V_{RY}| = |V_{YB}| = |V_{BR}| = \sqrt{3}\,V_{Ph}$$

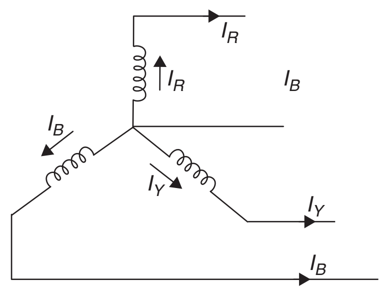
In a star connected circuit

1. Line voltage $= \sqrt{3} \times$ phase voltage
2. All line voltages are equal in magnitude but displaced in phase by 120°.
3. All line voltages are 30° ahead of their respective phase voltages.

Current Relation

All the phase currents are displaced by 120° from each other. For a balanced load, all the phase currents are equal in magnitude. Each live conductor is connected in series with its individual phase winding. So current in a line conductor is the same as that is phase to which the line conductor is connected.

$$I_L = I_{ph} = I_R = I_Y = I_B$$



If load is lagging, the angle between line (phase) current and line voltage is $(30 + \phi)°$ and it is $(30 - \phi)$ if the load is leading.

Power in a star connected network

Total power in the three phase = 3 × power each phase

$$V_L = \sqrt{3}V_{ph} I_L = I_{ph}$$

$$P = \sqrt{3}V_L I_L \cos\phi \text{ W}$$

Total reactive power in the circuit is

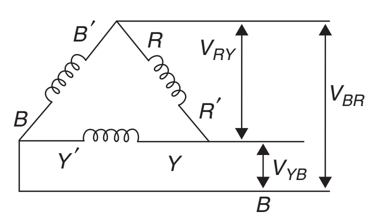
$$Q = \sqrt{3}V_L I_L \sin\phi \text{ VAR}$$

Apparent power (or) Active Power

$$S = \sqrt{3}V_L I_L$$

Delta or Mesh Connection

Here, the dissimilar ends are joined together as in figure. Here there is no common terminal only there line voltages V_{RY}, V_{YB}, and V_{BR} are available. These voltages are displaced 120° from each other.



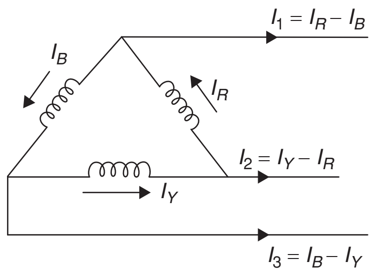
Voltage relation in Delta circuit

Voltage between two lines $V_L = V_{ph}$ the phase voltage.

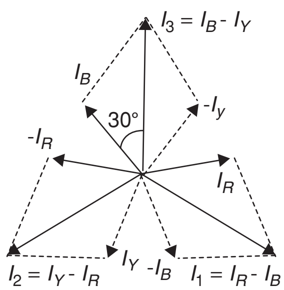
$$\therefore \quad V_{RY} = V_L = V_{Ph} = V_{YB} = V_{BR}$$

Since the system is balanced, all phase voltages are equal but displaced by 120° from one another.

Current Relation



$$I_1 = I_R - I_B = \sqrt{3} I_{ph}$$



The line currents are 30° behind the respective phase currents.

Power in delta connected system

The power per phase = $V_{Ph}\, I_{ph} \cos \phi$ is the phase angle between phase voltage and phase current.

Total power $P = 3 \times V_{ph} I_{ph} \cos \phi$

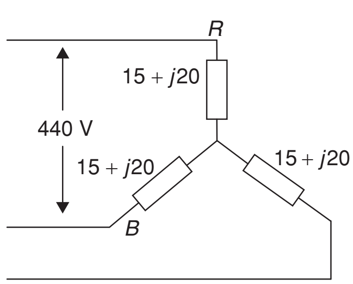
In terms of line quantities

$$P = \sqrt{3} V_L I_L \cos\phi$$

$$V_{ph} = V_L \text{ and } I_{ph} \frac{I_L}{\sqrt{3}}$$

Solved Examples

Example 1: A balanced star connected load having impedance (15 + 20 *j*) Ω per phase is connected to a three phases 440 V, 50 Hz supply. Find the line currents and power aborted by the load.



Solution: $V_{RN} = \frac{440\angle 0°}{\sqrt{3}} = 254\angle o°$

$$V_{YN} = 254 \angle -120°$$
$$V_{BN} = 254 \angle -240°$$

Impedance per phase

$$Z_{ph} = 15 + j\,20 = 25\angle 53.\,13°$$

The phase currents are

$$I_R = \frac{V_{RN}}{Z_{ph}} = \frac{254\angle o}{25\angle 53.13°}$$

$$= 10.16 \angle -53.13° \text{ A}$$

$$I_Y = 10.16 \angle -\text{A}3.13° \text{ A}$$

$$I_B = 10.16 \angle -293.13° \text{ A}$$

Since the load is star connected, these current also represent the line quantities.

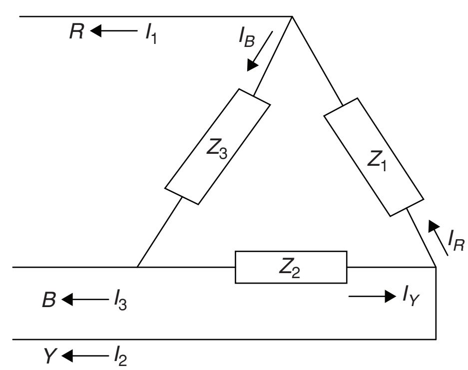
$$S = 3\text{ VI} = 3 \times 254 \times 10.16 \angle 53.13°$$
$$= 7741.92\angle 53.13°$$
$$= 4645 + j\,61\,93$$
$$\therefore \quad P = Re\,(S) = 4645 \text{ W}$$

Example 2: Three impedances $Z_1 = 20 \angle 30°\ \Omega$, $Z_2 = 40 \angle 60°\ \Omega$ and $Z_3 = 10 \angle 90°\ \Omega$ are delta connected to a 400 V. 3ϕ system as shown in figure. Determine the

(i) Phase currents

(ii) Line currents and

Total power consumed by the load.



Solution*:* Let the phase currents be I_R, I_Y, I_B and the line currents are I_1, I_2, and I_3.

Let

$$V_{RY} = 400 \angle 0°$$
$$V_{YB} = 400 \angle -120°$$
$$V_{BR} = 400 \angle -240°$$

$$I_R = \frac{V_{RY}}{Z_1}\frac{400\angle 0}{40\angle 30°} = 20\angle -30°\text{A}$$

$$I_Y = \frac{V_{YB}}{Z_2} = \frac{400\angle -120}{40\angle 60°} = 10\angle -180°\text{A}$$

$$I_B = \frac{V_{BR}}{Z_3} = \frac{400\angle -240}{10\angle -90} = 40\angle -150°\text{A}$$

The line currents are

$$I_1 = I_R - I_B = 20\angle{-30°} - 40\angle{-150°}$$
$$= 52.91\angle 10.89°\text{ A}$$
$$I_2 = I_Y - I_R = 20.09\angle 159.89\text{ A}$$
$$I_3 = I_B - I_Y = 31.73\angle{-140.94}\text{ A}$$

The total power is calculated by adding the powers in the individual phases [$R = Z\cos\phi$]

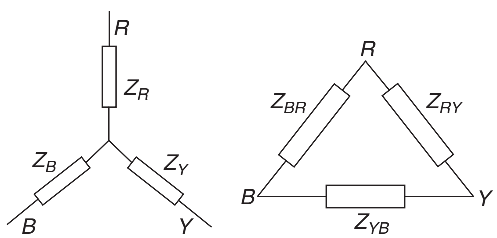
Power in R phase $= I_R^2 \times R_R = 20^2 \times 17.32 = 6928$ W

Power in Y phase $= I_Y^2 \times R_Y = 10^2 \times 20 = 2000$ W

Power in B phase $= I_B^2 \times R_B = 40^2 \times 0 = 0$ W

Total power in the load = 6928 + 2000 = 8928 W

Star to Delta and Delta to Star Transformation



Delta impedances in terms of star impedances are

$$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B} = Z_R + Z_Y + \frac{Z_R Z_Y}{Z_B}$$

$$Z_{YB} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R} = Z_Y + Z_B + \frac{Z_y Z_B}{Z_R}$$

$$Z_{BR} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y} = Z_B + Z_R + \frac{Z_B Z_R}{Z_Y}$$

Similarly we can replace the delta load of figure by equivalent star load

$$Z_R = \frac{Z_{RY} Z_{BR}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$Z_Y = \frac{Z_{RY} Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

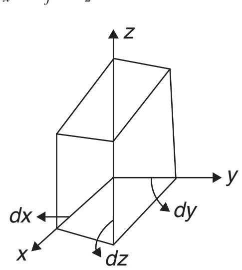
$$Z_B = \frac{Z_{BR} Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}.$$

Electrical Fields

Co-ordinate System and Vector Calculus

Cartesian coordinate system (x, y, z)

Unit vectors are $\hat{a}_x, \hat{a}_j, \hat{a}_z$



Differential length

$$\vec{d\ell} = (dx)\hat{a}_x + (dy)\hat{a}_y + (dz)\hat{a}_z$$

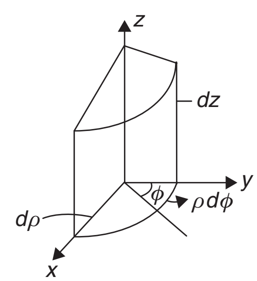
Differential surface

$$\vec{ds_1} = (dxdy)\hat{a}_z, \vec{ds_2} = (dydz)\hat{a}_x, \vec{ds_3} = (dxdz)\hat{a}_y$$

Differential volume: $dv = dx\, dy\, dz$

Cylindrical co-ordinate system (ρ, ϕ, z)

$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$z = Z$$
$$\rho = \sqrt{x^2 + y^2}$$
$$\phi = \tan^{-1}\frac{y}{x}$$



Differential length

$$\vec{d\ell} = (d\rho)\hat{a}_\rho + (\rho\, d\phi)\hat{a}_\phi + (dz)\hat{a}_z$$

Differential surface

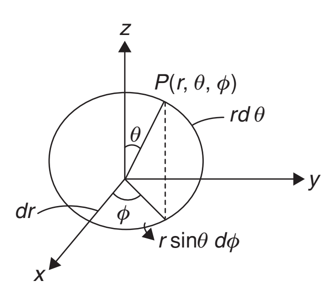
$$ds = (\rho\, d\phi dZ)\hat{a}_\rho + (d\rho\, dz)\hat{a}_\phi + (\rho\, d\rho\, d\phi)\hat{a}_z$$

Differential volume

$$dv = \rho d\rho d\phi dz$$

Spherical coordinate system (r, θ, ϕ)

$$\vec{d\ell} = (dr)\hat{a}_r + (rd\theta)\hat{a}_\theta + (r\sin\theta\, d\phi)\hat{a}_\phi$$
$$\vec{ds} = (r^2\sin^2\theta\, d\theta\, d\phi)\hat{a}_r + (r\sin\theta\, drd\phi)\hat{a}_\theta + (rdrd\theta)\hat{a}_\phi$$
$$\vec{dv} = r^2\sin\theta d\theta drd\phi$$



$x = r \sin\theta \cos\phi$
$y = r \sin\theta \sin\phi$
$z = r \cos\theta$

$$r = \sqrt{x^2 + y^2 + z^2}\,\theta = \cos^{-1}\left(\frac{z}{r}\right) = \cos^{-1}\frac{Z}{\sqrt{x^2+y^2+z^2}}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

The del operator (∇)

$$\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z}\hat{k}$$

$$\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial\phi}{\partial z}\hat{k} \text{ where } \phi \text{ is a scalar quantity.}$$

Gradient of a scalar field

Gradient is defined as the vector quantity whose magnitude is given by maximum rate of change of scalar quantity with respect to space variable and its direction is where maximum change occurs.

If ϕ is any scalar field, then its gradient is given by
Grad $\phi = \nabla\,\phi$

Divergence of a vector

Divergence of a vector quantity is a scalar quantity whose magnitude is equal to net outflow of flux from a closed surface when volume shrinks to zero.

If $\vec{f}$ is a vector function whose first partial derivatives exist, then

$$\text{div } f = \vec{\nabla}\cdot\vec{f} = \frac{\partial f}{\partial y} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}$$

Curl of a vector

Curl of a vector is a vector quantity whose magnitude is given by maximum circulation per unit area when area tends to zero and its direction is normal to the surface.

If V is vector function who's first partial derivative exist, then

$$\text{Curl } V = \nabla\times V = \begin{vmatrix} i & j & K \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial y} \\ V_1 & V_2 & V_3 \end{vmatrix}$$

GRADIENT, DIVERGENCE AND CURL IN ALL THE CO-ORDINATE SYSTEM

Gradient

$$\nabla V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_j + \frac{\partial V}{\partial z}\hat{a}_z$$

$$\nabla V = \frac{\partial V}{\partial\rho}\hat{a}_p + \frac{1}{\rho}\frac{\partial V}{\partial\phi}\hat{a}_\phi + \frac{\partial V}{\partial z}\hat{a}_z$$

$$\nabla V = \frac{\partial V}{\partial r}\hat{a}_r + \frac{1}{r}\frac{\partial V}{\partial\theta}\hat{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial v}{\partial\phi}\hat{a}_\phi$$

Divergence

$$\nabla\cdot\vec{V} = \frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z}$$

$$\nabla\cdot\vec{V} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho V_\rho) + \frac{1}{\rho}\frac{\partial V\phi}{\partial\phi} + \frac{\partial V_z}{\partial_z}$$

$$\nabla\cdot\vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(V_\theta\sin\theta) + \frac{1}{r^2\sin\theta}\frac{\partial V\phi}{\partial\phi}$$

Curl

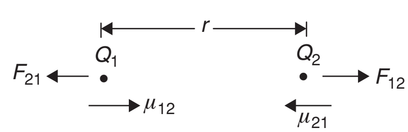
$$\nabla\times\vec{V} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

$$\nabla\times\vec{V} = \frac{1}{\rho}\begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ V_\rho & \rho v_\phi & V_z \end{vmatrix}$$

$$\nabla\times\vec{V} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_p & r\hat{a}_\phi & r\sin\theta\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial\phi} \\ V_r & rV_\theta & r\sin\theta V\phi \end{vmatrix}$$

Coulomb's Law

The force between two charged particles at rest is proportional to product of charges and inversely proportional to square of distance between them. The force acts along the line joining the mid point of the particles.



$$F_{12} = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{Q_1Q_2}{r^2}\cdot U_{12}$$ where U_{12} and U_{21} are unit vectors ε_r is relative permittivity of medium. ε_o is absolute permittivity = 8.854×10^{-12} F/n

Example 3: Calculate the distance of separation between two electrons in vacuum for which electric force between them is equal to gravitational force on one of them at earth surface.

Solution: Gravitational force on one electron = mg Newton

Electrostatic force in vaccum $= \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r^2}$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = \text{mg} \quad \therefore \quad r = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{q^2}{\text{mg}}}$$

$$= 5.08 \text{ m}$$

Example 4: Find the force are Q_1 (20μ c) due to Q_2 (–300 μc) Q_1 is at (0, 1, 2) and Q_2 is at (2, 0, 0)

Solution: $F_2 = \dfrac{1}{4\pi \in_o}\dfrac{q_1 q2}{r^2}\hat{U}_2$

$$\hat{U}_{21} = \frac{-2\hat{i} + \hat{j} + 2\hat{k}}{3} = \frac{-2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$

$$F_{21} = \frac{1}{4\pi \in_o}\frac{20 \times -300 \times 10^{-12}}{3^2} \times \left(\frac{-2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}\right)$$

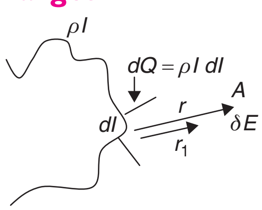
$$= -6\left(\frac{-2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}\right) = 4\hat{i} - 2\hat{j} - 4\hat{k}N.$$

Electric Field of Continuous Space Distribution of Charges

Electric field intensity $E = \left[\dfrac{Q}{4\pi\varepsilon_0 r^2}\right]\hat{r}_1$ Newtons/Coloumb

Line Distribution of Charges

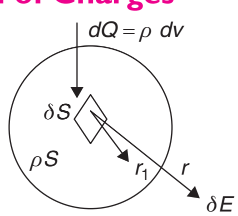
$$E = \frac{1}{4\pi\varepsilon_0}\left[\int_l \left\{\frac{\rho l dl}{r^2}\right\}\right]\hat{r}_1$$



$\rho_l \rightarrow$ charge per unit length

Surface Distribution of Charges

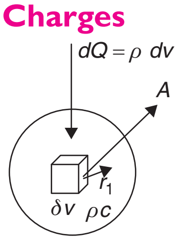
$$E = \frac{1}{4\pi\varepsilon_0}\left[\iint_s \left\{\frac{\rho s ds}{r^2}\right\}\right]\hat{r}_1$$



ρ_s = charge/unit area

Volume Distribution of Charges

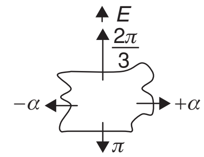
$$E = \frac{1}{4\pi\varepsilon_0}\left[\iiint_v \left\{\frac{\rho_c dv}{r^2}\right\}\right]\hat{r}_1$$



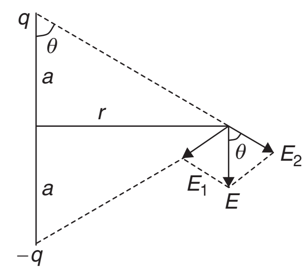
$$\rho_c = \frac{\text{charge}}{\text{unit volume}}.$$

Infinite Plane Charge

$$\vec{E} = \frac{\rho s}{2E_0}\hat{a}_x$$



Electric Field Due to Dipole



$$\bar{E} = \bar{E}_1 + \bar{E}_2$$

$$E_1 = E_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(a^2 + r^2)}$$

$$\cos\theta = \frac{a}{\sqrt{a^2 + r^2}}$$

$$\therefore \quad \bar{E} = 2 \cdot \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2 + r^2} \cdot \frac{a}{\sqrt{a^2 + r^2}}$$

$$\bar{E} = \frac{2aq}{4\pi\varepsilon_0 (a^2 + r^2)^{\frac{3}{2}}} = \frac{\bar{P}}{4\pi\varepsilon_0 (a^2 + r^2)^{\frac{3}{2}}}$$

For $r >> a$

$$E = \frac{2aq}{4\pi\varepsilon_0 r^3} = \frac{\bar{P}}{4\pi\varepsilon_0 r^3}$$

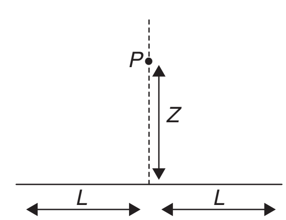
$$|E| \,\alpha\, \frac{1}{r^3}.$$

Electric Field at a point P on the axis of a circular loop of radius a, carrying charge density λ (c/m)

$$\vec{E} = \frac{qz}{4\pi \in_0} \cdot \frac{1}{(r^2 + z^2)^{\frac{3}{2}}} \cdot \hat{a}_z \text{ at } z = 0 \,|\, \vec{E}\,| = 0$$

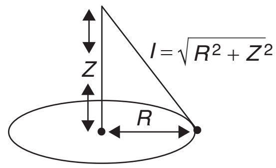
Electric field at point P from a line of charge of length $2L$ carrying a uniform charge λ

$$E = \frac{1}{2\pi \in_0}\frac{\lambda L}{z\sqrt{z^2 + L^2}}$$



Electric Field at a distance z above the center of a flat disc of radius R carrying uniform surface charge σ.

$$\vec{E} = \frac{\sigma}{2\in_0}\left(1 - \frac{z}{\sqrt{R^2+Z^2}}\right)$$



Gauss's Law

The total electric flux enclosed by a surface surrounding charge is equal to the amount of charge enclosed.

$\int_s \bar{D}\cdot d\bar{s} = Q$ where Q is the total charge and D is the electric flux density.

$$\int_s \in_0 \bar{E}\cdot d\bar{s} = Q$$

$$\therefore \int_s \bar{E}\cdot d\bar{s} = \frac{Q}{\in_0} = \frac{1}{\in_0}\int_v \rho dv$$

Where ρ is the charge/unit volume?

The total flux emanating from the surface is independent of

1. Shape of the closed surface
2. Position of charge within the surface

Electric Potential

Electric potential at any point in an electric field near a charged body is defined as the amount of work done in bringing a unit positive charge from α to that point against the electric field.

If a charge q is moved through a distance dl is a uniform electric field E.

Electric force on the charge $= q\bar{E}$

Work done $= \bar{F}\cdot\overline{dl} = -q\bar{E}\cdot\overline{dl}$

Work done in moving the test charge from ∞ to a point P

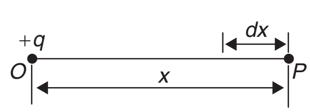
$$W = \int_\infty^P -q\bar{E}\cdot dl = -q\int_\infty^P E\cdot dl$$

Potential of point $P, V_p = \int_{-\alpha}^{p} E\cdot dl$

i.e., work done by unit positive charge.

Electric field $E = \text{grad}V$

Potential due to a point charge



Consider a point charge $+q$ situated at O and let a unit positive charge is placed at P, x meters from O.

Force of repulsion experienced by unit positive charge at P is $|F| = \frac{q}{4\pi\in_0 x^2}$.

If this unit charge is moved by a short distance dx, work done in doing so against the force of repulsion is $dW = \frac{-q}{4\pi\in_0 x^2}dx$ Joule.

If the point is at ∞, $W = \int_\infty^r \frac{-q}{4\pi\in_0 x^2}dx$ Joules

$$= \frac{q}{4\pi\in_0}\left[\frac{1}{x}\right]_\infty^r = \frac{q}{4\pi\in_0 r}$$

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

Potential due to a number of charges

If charges $q_1, q_2, \ldots$ etc. are at distances $r_1, r_2, r_3, \ldots$ etc from the point at which potential is to be found, then the resultant potential at this point,

$$V = \frac{q_1}{4\pi\in_0 r_1} + \frac{q_2}{4\pi\in_0 r_2} + \frac{q_3}{4\pi\in_0 r_3} + \cdots$$

$$= \frac{1}{4\pi\in_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right)$$

$$V = \frac{1}{4\rho\varepsilon_0}\sum\frac{q}{p}$$

Potential due to a line charge distribution

When the charge is uniformly distributed along a line then potential at any point P due to this linear charge distribution of λ coulombs/meter is given by

$$V_p = \frac{1}{4\pi\in_0}\int_l \frac{\lambda}{r}dl$$

Potential due to surface charge distribution

Potential at any point P due to surface charge distribution ρ C/sq m is

$$V_p = \frac{1}{4\pi\in_0}\int_s \frac{\sigma}{r}ds$$

Potential due to volume charge distribution

Potential due to uniform volume charge distribution ρ coulombs per cubic meter is

$$V_p = \frac{1}{4\pi\in_0}\int_v \frac{\rho}{r}dv$$

Dielectrics Inductance and Capacitance

Behavior of Conductors and Dielectrics in Electrostatic Fields

In conductors, there is abundance of conduction or free electrons which give rise to conduction current under the influence of applied electric field. However in dielectric substances, electrons are tightly bound to nuclei of atoms and so dielectrics do not conduct current through them. In dielectrics, charges are not able to move freely and they are bound by finite forces.

Polarisation

When a dielectric is placed in an electric field, the nucleus gets displaced in the direction of electric field and the electrons opposite to the field. In this way, a dipole is formed by a positive and negative charge with distance d separating them. This gives a dipole moment

$$P = qd$$

In each atom of the dielectric material, a tiny dipole moment is induced tending to align the atoms in the direction of electric field. The dielectric is said to be polarized and polarization P, is defined as the dipole moment per unit volume.

$$P \alpha \bar{E}$$

$\bar{E}$ = total field, ie field due to polarization and external field.

Flux density in dielectrics

$$D = \in_0 \bar{E} + P \frac{C}{m^2}$$

Inductance

It is defined as the property of the electric circuit by the virtue of which a varying current induces an emf in the neighbouring circuit or the property of a conductor or circuit is that establishes magnetic flux linkages.

Inductance of simple geometries

1. Coaxial conductor $L = \frac{\mu l}{2\pi} \ln\left(\frac{b}{a}\right)$

   Where l – length of conductor
   b – outer radius
   a – inner radius

2. Toroid $L = \frac{\mu_0 N^2 a}{2\pi} \ell_n \left(\frac{r_2}{r_1}\right)$

   N – number of turns
   a – axial thickness
   r_2 – outer radius
   r_1 – inner radius
   If average radius is given

   $$L = \frac{\mu_0 N^2 s}{2\pi r}, \ r \text{ – average radius}$$

3. Parallel conductors of radius a,

   $$L = \frac{\mu_0 l}{\pi} \cosh^{-1}\left(\frac{d}{2a}\right)\left(\frac{H}{m}\right)$$

   $$L = \frac{\mu_0 l}{\pi} \ln\left(\frac{d}{a}\right) \ (H/m) \text{ when } d \text{ is larger}$$

4. Long solenoid of small cross-sections

   $$L = \frac{\mu_0 N^2 S}{l} Hl >> a$$

   N – number of turns
   S – area (πr^2)
   l – length

Capacitance

Capacitance is the ability of the device to store energy in an electric field.

For a two conductor capacitor it is the ratio of charge on one of the conductors to the magnitude.

i.e., $C = \frac{Q}{V} \frac{C}{V}$ or Farad

For a parallel plate of surface A and separated by a distance d.

$$C = \frac{\in A}{d} \in \text{ is the permittivity of the medium} \in = \in_0 \in_r.$$

For concentric spheres with inner radius a and outer radius b

$$C = 4\pi \in \left(\frac{ab}{b-a}\right)$$

For concentric cylinders of length L and inner and outer radii a and b.

$$C = \frac{2\pi \in dL}{\ln\left(\frac{b}{a}\right)}$$

Energy stored in the capacitor

$$W = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV$$

Magnetostatics

Electric current flowing through a conductor produces a magnetic field which is concentric circles around it.

Magnetic Flux (ϕ)

It is the number of magnetic lines of force passing through a given area perpendicular to the area. If the lines of force are not normal to the area, we have to take the component which is normal to the area. It is denoted by ϕ and unit is Webers (Wb)

Magnetic Flux Density (B)

It is the number of times of force passing per unit area i.e., magnetic flux per unit area is flux density

$B = \frac{\phi}{A}$. It is measured in Webers/m^2 or Tesla (T).

$$\phi = \int_s \bar{B} \cdot d\bar{s} Wb$$

Magnetic Field Intensity or Field Strength (H)

It is the force experienced by a unit North pole placed at that point in the magnetic field.

$$H = \frac{NI}{l}\frac{A}{m}$$

Also $\vec{B} = \mu\vec{H}$

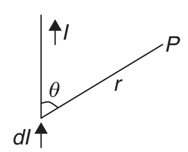
Where μ is the permeability of the medium $\mu = \mu_0\mu_r$

Biot Swat's Law

The Biot savart law gives the magnetic flux density at a point P at a certain distance from a small current carrying element.

$$\Delta B = \left(\frac{\mu_0 I}{4\pi}\right)\left(\frac{\bar{dl} \times \bar{u}}{r^2}\right)$$

$$B = \frac{\mu_0 I}{4\pi}\int_c \left(\frac{d\bar{l} \times \bar{u}}{r^2}\right)$$

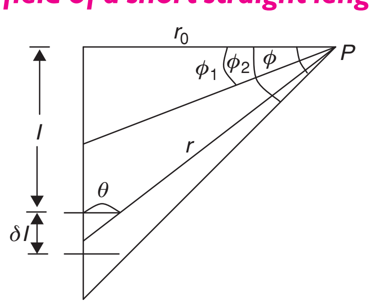


Where unit vector $\bar{u}$ is directed towards the fixed point at which the flux density is determined.

The magnitude of flux density is

$$|\Delta B| = \frac{\mu_0 I dl \sin\theta}{4\pi r^2}$$

Magnetic field of a short straight length or wire



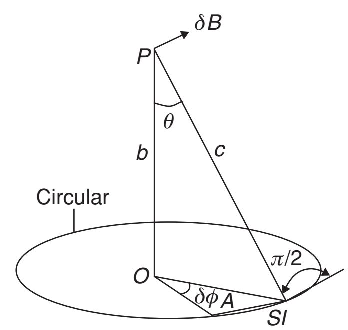
$$B_P = \frac{\mu_0 I}{4\pi r_0}(\sin\phi_2 - \sin\phi_1)$$

The direction of B_P would be normal to plane containing dl and r. The magnetic field due to an infinitely long wire carrying a current I, we have to substitute $\phi_1 = \frac{-\pi}{2}, \phi_2 = \frac{\pi}{2}$, so,

$$B_P = \frac{\mu_0 I}{2\pi r_0}$$

Magnetic field on the axis of a circular coil

The magnetic field ∂B at P due to circumferential element ∂l is



$$\partial B = \frac{\mu_0 I \partial l \sin\left(\frac{\pi}{2}\right)}{4\pi c^2} = \frac{\mu_0 I \partial l}{4\pi c^2}$$

The components of B normal to the axis OP cancel out and components along the axis add up

$$B_{ax} = \frac{\mu_0 I}{2a}\sin^3\theta$$

When P is on centre of the coil, i.e., at o

$$B = \frac{\mu_0 I}{2r}$$

Ampere's law

The magnetic field B at a point P at a distance r from an infinitely long straight conductor

$$B = \frac{\mu_0 I}{2\pi r} \text{ or } B \times (2\pi r) = \mu_0 I$$

i.e., B multiplied by the length of the contour is proportional to the current in the wire.

Vector H is such that $H = \frac{B}{\mu}\left(\frac{B}{\mu_0} \text{ in free space}\right)$

$$\Rightarrow H \times 2\pi r = I$$

A contour of any shape can be built up of infinite number of circumferential elements and for a contour of any arbitrary shape.

$$\oint_c H \cdot dl = I$$

I is the current enclosed by contour C.

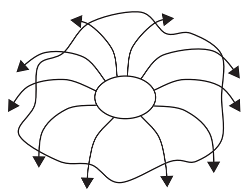
By using the principle of superposition, we now extend the law to conductors of finite cross section and any conducting medium having current distribution in it.

$$\oint_c H \cdot dl = \Sigma I$$

This is Ampere's law or Magnetic circuit law.

Example 5: What net flux crosses the surface S shown in figure which contain a charge distribution in the form of a plane disk of radius 4 m and $\rho_s = \frac{\sin^2\phi}{2r}\frac{C}{m^2}$.

Solution:



$$\psi = Q_{enc} = \int_s \rho_s d_s$$

$$= \int_0^{2\pi}\int_0^4 \frac{\sin^2\phi}{2r} r dr d\phi$$

$$= \int_0^{2\pi} \frac{\sin^2\phi}{2}[r]_0^4 d\phi = 2\int_0^{2\pi}\sin^2\phi d\phi$$

$$= 2\left[\phi + \frac{\sin 2\phi}{2}\right]_0^{2\pi} = 4\pi C.$$

Example 6: A point Q is at origin of spherical coordinate system. Find the flux which crosses the portion of a spherical shell described by $\alpha \le \theta \le \beta$

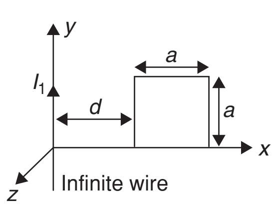
Solution: Area A of the shell $A = \int_0^{2\pi}\int_\alpha^\beta r^2 \sin\theta d\theta d\phi$

$$= 2\pi r^2 (\cos\alpha - \cos\beta)$$

$$\psi_{\text{net}} = \frac{A}{4\pi r^2}\cdot Q = \frac{Q}{2}(\cos\alpha - \cos\beta).$$

Mutual inductance between infinite current carrying conductor and square current carrying loop separated by a distance

$$M_{ab} = \frac{\mu a}{2\pi}\ln\left(\frac{d+a}{d}\right)H$$



Force and torques in magnetic fields

$$\vec{F} = q\vec{V}\times\vec{B}$$

$\vec{F} = q(\vec{E} + \vec{V}\times\vec{B})$ is called as the Lorentz force.

Induced EMF

Faraday's law (varying field and fixed conductor)

(i) $\phi\vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_s \vec{B}\cdot d\vec{s}$

$$\Rightarrow \oint \vec{E}\cdot d\vec{l} = \int_s\left(\frac{-\partial B}{\partial t}\right)\cdot d\vec{s}$$

Field is fixed but conductor is moving

$$V_{ab} = \int_0^a \overline{E_m}\cdot\overline{dl} = \int_0^a (\vec{V}\times\vec{B})\cdot d\vec{l}$$

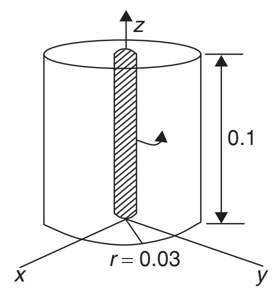
$$\Rightarrow V = \oint(\vec{V}\times\vec{B})\cdot d\vec{l}$$

Conductor in motion through time dependent fields

$$V = -\int\frac{\partial\vec{B}}{\partial t}\cdot d\vec{s} + \oint(\vec{V}\times\vec{B})\cdot d\vec{l}$$

Example 7: Work and power required to move the conductor shown in figure, one full revolution in 0.02 sec. If the magnetic flux density $\vec{B} = 2.5\times10^{-3} a\hat{r}\ T$ and current 45 A.

Solution:



$$\vec{F} = I(\vec{1}\times\vec{B}) = -1.13\times10^{-2}\hat{a}_\phi N$$

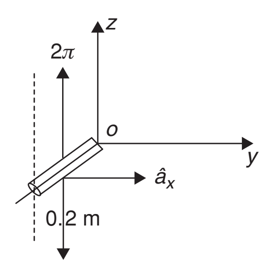
$$W = \int_{2\pi}\overline{Fa}\cdot d\vec{l} = \int_0^{2\pi}(-1.13\times10^{-2})\hat{a}_\phi\cdot r d\phi\hat{a}_\phi$$

$$= -2.13\times10^{-3}J$$

$$P = \frac{W}{t} = -0.107\text{ W}.$$

Example 8: Find the induced voltage in the conductor of the figure where $\overline{B} = 0.04\hat{a}_y\ T$ and $V = 2.5\sin(10^3 t)\hat{a}_z$.

Solution:

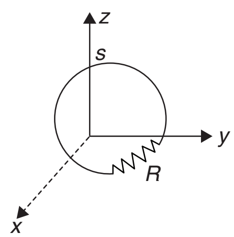


$$E_m = \vec{V} \times \vec{B} = 0.1\sin(10^3 t)(-\hat{a}_x)$$

So, induced voltage $V = \int \overrightarrow{E_m} \cdot d\vec{l}$

$$= -0.02\sin(10^3 t)\text{ V.}$$

Example 9: The circular loop conductor shown in the figure lies in the $z = 0$ plane, has a radius of 0.1 m and resistance of 5 Ω. Given $\vec{B} = 0.02\sin(10^3 t)a_z$ determine the current in the resistance R.



Solution:

$$\phi = \vec{B} \cdot \vec{S}$$

$$\Rightarrow \quad \phi = 0.00628\sin(10^3 t)$$

$$V = -\frac{d\phi}{dt} = -6.28\cos(10^3 t)$$

$$i = \frac{V}{R} = -1.256\cos(10^3 t)\text{ A}$$

Maxwells Equations

1. Total displacement through the surface enclosing a volume is equal to the total charge within the volume. It is given in differential and integral form as

$$\nabla \cdot \bar{E} = \frac{\rho}{\varepsilon_0} \text{ and } \oint_s \bar{E} \cdot \overline{dS} = \frac{1}{\varepsilon_0}\int_v \rho dv$$

2. Net magnetic field emerging through any closed surface is zero.

$$\nabla \cdot \bar{B} = 0 \text{ and } \oint B d\bar{s} = 0$$

3. Electromotive force around a closed path is equal to the time derivative of the magnetic displacement through any surface bounded by the path.

$$\nabla \cdot \bar{E} = \frac{\rho}{\varepsilon_0} \text{ and } \oint_s \bar{E} \cdot \overline{dS} = \frac{1}{\varepsilon_0}\int_v \rho dv$$

4. M.M.F around a closed path is equal to the conduction current and the time derivative of electric displacement though any surface bounded by the path.

$$\nabla \cdot \bar{B} = 0 \text{ and } \oint B d\bar{s} = 0$$

$$\oint \bar{B} \cdot \overline{dl} = \int_s \left(\mu_0 \bar{j} + \mu_0 \varepsilon_0 \frac{\partial \bar{E}}{\partial t} \right) \cdot d\bar{s}$$

Exercises

Practice Problems I

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. An air filled parallel plate capacitor has a stored energy W_0 and a charge of Q_0 when connected to a voltage source V_0. If air space is now filled completely by a dielectric having a dielectric constant of 2 and connected to a voltage source of 2 V_0, then stored energy will be

(A) W_0 (B) $2W_0$
(C) $4W_0$ (D) $8W_0$

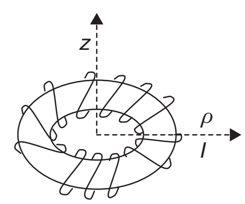
2. An insulated metal sphere of 10cm radius is charged by rubbing with a charge of 2×10^{-8}C. Potential developed will be

(A) 1800 V (B) 900 V
(C) 0 V (D) 200 V

3. The energy stored in the magnetic field of a solenoid 30 cm long and 3 cm diameter wound with 100 turns of wire carrying a current of 100 A is

(A) 0.015 J (B) 0.15 J
(C) 0.5 J (D) 1.15 J

4. For the N term toroid with I amperes, H within the toroid and outside toroid is



(A) $\frac{NI}{\pi l}\hat{a}_\phi$ and 0 (B) 0 and $\frac{NI}{\pi l}\hat{a}_\phi$

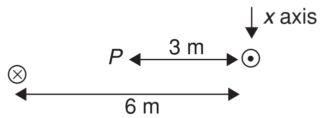
(C) $\frac{NI}{2\pi l}\hat{a}_\phi$ and 0 (D) 0 and $\frac{NI}{2\pi l}\hat{a}_\phi$

5. If a conductor of cross-section A and carrying a current $I\ \hat{a}_y$ is oriented along the y-axis in the magnetic field $B = B_0\hat{a}_x + B_0\hat{a}_y$, the force density exerted on the conductor is

(A) $\frac{B_0 I}{a}\hat{a}_z \frac{N}{m^2}$ (B) $\frac{B_0 I}{a}\hat{a}_x \frac{N}{m^2}$

(C) $\frac{-B_0 I}{a}\hat{a}_z \frac{N}{m^2}$ (D) $\frac{B_0 I}{a}\hat{a}_y \frac{N}{m^2}$

6. Two infinitely long wires separated by distance 6 m carry currents I in opposite directions along z axis as shown in figure. Magnetic field intensity at point P is given by

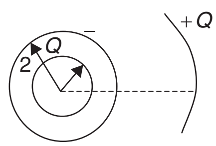


(A) $\frac{I}{3\pi}\hat{a}_\phi \frac{A}{m}$ (B) $\frac{I}{6\pi}\hat{a}_\phi \frac{A}{m}$

(C) $\frac{I}{12\pi}\hat{a}_\phi$ (D) 0

7. If in a 1 μF capacitor, an instantaneous displacement current of 1 A is to be established between its plates, then it is possible by
(A) 10^6 A (B) 10^6 A/s
(C) 10^6 V/s (D) 10^6 V

8. An infinite number of concentric rings carry a charge Q each alternately positive and negative. Their radii are 1, 2, 4, 8 meter etc in geometric progression as shown. Potential at the centre of the rings will be



(A) $\frac{Q}{2\pi \in_0}$ (B) $\frac{Q}{4\pi \in_0}$

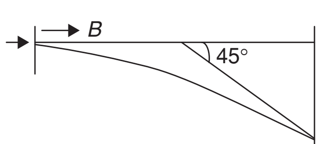
(C) 0 (D) $\frac{Q}{6\pi \in_0}$

9. In the infinite plane, y = 6 m, there exists a uniform surface charge density of $\left(\frac{1}{600\pi}\right)\frac{\mu C}{m^2}$. The associated electric field strength is

(A) $30\hat{i}\frac{V}{m}$ (B) $30\hat{j}\frac{V}{m}$

(C) $30\hat{k}\frac{V}{m}$ (D) $60\hat{j}\frac{V}{m}$

10. A slab of uniform magnetic field deflects a moving charged particle by 45° as shown in the figure. The kinetic energy of the charged particle at the entry and exit points in the magnetic fields will change in the ratio of



(A) $1:\sqrt{2}$
(B) $\sqrt{2}:1$
(C) 1 : 1
(D) 1 : 2

11. A radial field $\overrightarrow{H} = \frac{2.39\times 105}{r}\cos\phi\,\hat{a}_r \frac{A}{m}$ exists in free space. Magnetic flux ϕ crossing the surface defined by $-\frac{\pi}{4} \le \phi \le \frac{\pi}{4}$, $0 \le z \le 1$ m will be
(A) 6.3 Wb
(B) 8.48 Wb
(C) 4.24 Wb
(D) 3.16 Wb

12. A plane electromagnetic wave in free space is specified by the electric field $\hat{a}_x[(20\cos(\omega t - \beta z) + 5\cos(\omega t + \beta z)]$ V/m
The associated magnetic field is

(A) $\frac{\hat{a}_x}{120\pi}[20\cos(\omega t - \beta z) + 5\cos(\omega t + \beta z)]$ A/m

(B) $\frac{\hat{a}_x}{120\pi}[20\cos(\omega t - \beta z) - \cos(\omega t + \beta z)]$ A/m

(C) $\frac{\hat{a}_y}{120\pi}[20\cos(\omega t - \beta z) + 5\cos(\omega t + \beta z)]$ A/m

(D) $\frac{\hat{a}_y}{120\pi}[20\cos(\omega t - \beta z) - \cos(\omega t + \beta z)]$ A/m

13. A rectangular loop in the x-z plane bounded by the lines $x = 0$, $x = a$, $z = 0$ and $z = b$, is in a time varying magnetic field given by $\vec{B} = B_0 \cos\omega t\, \hat{a}_y$
B_0 is a constant, ω is the angular frequency and $\hat{a}_y$ is the unit vector in the y direction. The emf induced in the loop is given by
(A) $ab\,B_0 \cos\omega t$
(B) $ab\,B_0\, \omega \sin\omega t$
(C) $B_0\, \omega \sin\omega t$
(D) Zero

14. A parallel plate air capacitor has plates of 1500 cm² separated by 5 mm. If a layer of dielectric 2 mm thick and relative permittivity 3 is now introduced between the plates then new separation in mm between the plates so that capacitive value is unchanged will be
(A) 7 (B) 6.33
(C) 4.33 (D) 8

15. Let $\overline{D} = 1\cdot\hat{a}_r \frac{c}{m^2}$. Find ρ_v, hence calculate amount of flux leaving the spherical surface $r = 1$.
(A) π C
(B) 2π C
(C) 6π C
(D) 4π C

Practice Problems 2

Directions for questions 1 to 15: Select the correct alternative from the given choices.

1. The equation that is desired from Gauss law is

(A) $\nabla \cdot \vec{D} = \rho_v$ (B) $\vec{D} = \epsilon_0 \vec{E}$
(C) $\nabla \times \vec{E} = 0$ (D) None of these

2. There is a charged metal sphere and a thin circular plate. Distribution of charge around the surface is

(A) Uniform in both
(B) Uniform in sphere and Non uniform in plate
(C) Non uniform in sphere and uniform in plate
(D) Non-uniform in both

3. A charged hollow sphere is suspended in a uniform electric field of strength E, then the field strength

(A) Inside the sphere is zero
(B) Gradually decreases to zero inside the sphere
(C) E and V inside the sphere are zero
(D) All of these

4. As a result of reflections from a plane conducting wall, electro magnetic waves acquire an apparent velocity greater than the velocity of light in space. This is called the

(A) Velocity of propagation
(B) Normal velocity
(C) Group velocity
(D) Phase velocity

5. Two infinite parallel metal plates are charged with equal surface charge density of the same polarity. The electric field in the gap between the plates is

(A) The same as that produced by one plate
(B) Double of the field produced by one plate
(C) Dependent on coordinates of the field point
(D) Zero

6. Copper behaves as a

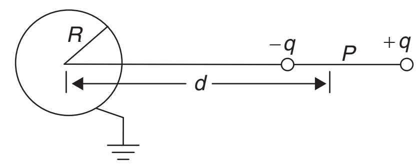
(A) Conductor always
(B) Conductor or dielectric depending on applied electric field strength
(C) Conductor or dielectric depending on the frequency
(D) Conductor or dielectric depending on the electric current density

7. Match the following

List I	List II
P. $\nabla \times \vec{H} = J + \dfrac{\partial s}{\partial t}$	X Gauss Law.
Q. $\nabla \cdot \vec{D} = \rho$	Y Biot Savart Law
R. $d\vec{H} = \dfrac{\mu I d\vec{l} \times \vec{a}_{12}}{4\pi d^2}$	Z Ampere's Law

	X	Y	Z
(A)	P	Q	R
(B)	R	P	Q
(C)	Q	P	R
(D)	R	Q	P

8. An electric dipole of moment P is placed in front of a grounded sphere as shown in the figure. The charge induced on the surface of the sphere is



(A) Zero (B) $\dfrac{PR}{d^2}$
(C) $\dfrac{PR}{d^3}$ (D) $\dfrac{PR^3}{d^4}$

9. Maxwell's divergence equation for the magnetic field is given by

(A) $\nabla \times B = 0$ (B) $\nabla \cdot B = 0$
(C) $\nabla \times \beta = \rho$ (D) $\nabla \cdot B = \rho$

10. When a lossy capacitor with a dielectric of permittivity $\in$ and conductivity σ operates at a frequency ω, the loss tangent for the capacitor is given by

(A) $\dfrac{\omega\sigma}{\in}$ (B) $\dfrac{\omega \in}{\sigma}$
(C) $\dfrac{\sigma}{\omega \in}$ (D) $\sigma\omega \in$

11. Which of the following is Laplace's equation?

(A) $\nabla^2 v = 0$ (B) $\nabla^2 v = \dfrac{-\rho_v}{\in}$
(C) $\nabla \cdot D = \rho_v$ (D) $\nabla \cdot B = 0$

12. The potential difference V_{LM}, the potential at M with reference to L, is

(A) $V_{LM} = -\int_L^M E \cdot dl$ (B) $V_{LM} = \int_L^M E \cdot dl$
(C) $V_{LM} = -\int_L^M E \cdot dl$ (D) $V_{LM} = \int_L^M E \cdot dl$

13. Work done in moving a charge from one point to another along an equipotential line or surface is

(A) Infinite (B) Unity
(C) Zero (D) None

14. Which of the following is Ohm's law

(A) $D = \in E$ (B) $B = \mu H$
(C) $J = \dfrac{E}{\sigma}$ (D) $J = \sigma E$

15. Identify, which of the following expressions are not Maxwell's equation for time – varying fields

(A) $\nabla \cdot D = \rho_v$ (B) $\nabla \cdot J + \dfrac{\partial \rho_v}{\partial t} = 0$
(C) $\nabla \times E = \dfrac{-\partial B}{\partial t}$ (D) $\nabla \times H = J + \dfrac{\partial D}{\partial t}$

Previous Years' Questions

1. A capacitor consists of two metal plates each 500 × 500 mm^2 and spaced 6 mm apart. The space between the metal plates is filled with a glass plate of 4 mm thickness and a layer of paper of 2 mm thickness. The relative permittivity of the glass and paper and 8 and 2 respectively. Neglecting the fringing effect, the capacitance will be (Given that $E_0 = 8.85 \times 10^{-12}$ F/m) **[2008]**

(A) 983.33 pF (B) 1475 pF
(C) 6637.5 pF (D) 9956.25 pF

2. A coil of 300 turns is wound on a non-magnetic core having a mean circumference of 300 mm and a cross-sectional area of 300 mm^2. The inductance of the coil corresponding to a magnetizing current of 3A will be **[2008]**

(Given that $\mu_0 = 4\pi \times 10^{-7}$ H/m)
(A) 37.68 μH (B) 113.04 μH
(C) 37.68 mH (D) 113.04 mH

3. $F(x, y) = (x^2 + xy)\hat{a}_x + (y^2 + xy)\hat{a}_y$. It's line integral over the straight line from $(x, y) = (0, 2)$ to $(x, y) = (2, 0)$ evaluates to **[2009]**

(A) –8 (B) 4
(C) 8 (D) 0

4. Divergence of the three-dimensional radial vector field $\bar{r}$ is **[2010]**

(A) 3 (B) $1/r$
(C) $\bar{t} + \bar{j} + \bar{k}$ (D) $3(\bar{i} + \bar{j} + \bar{k})$

5. A capacitor is made with a polymeric dielectric having an ε_r of 2.26 and dielectric breakdown strength of 50 kV/cm. The permittivity of free space is 8.85 pF/m. If the rectangular plates of capacitor have a width of 20 cm and a length of 40cm, then the maximum electric charge in capacitor is **[2011]**

(A) 2 μC (B) 4 μC
(C) 8 μC (D) 10 μC

6. The direction of vector $\bar{A}$ is radially outward from the origin, with $|A| = kr^n$ where $r^2 = x^2 + y^2 + z^2$ and k is a constant. The value of n for which $\nabla \cdot A = 0$ is **[2012]**

(A) −2 (B) 2
(C) 1 (D) 0

7. The flux density at a point in space is given by $B = 4a_x, + 2kya_y + 8a_z$, Wb/m^2. The value of constant k must be equal to **[2013]**

(A) −2 (B) −0.5
(C) +0.5 (D) +2

8. A dielectric slab with 500 mm × 500 mm cross-section is 0.4 m long. The slab is subjected to a uniform electric field of $E = 6a_x + 8a_y$ kV/mm. The relative permittivity of the dielectric material is equal to 2. The value of constant ε_0 is 8.85×10^{-12} F/m. The energy stored in the dielectric in Joules is **[2013]**

(A) 8.85×10^{-11} (B) 8.85×10^{-5}
(C) 88.5 (D) 885

9. C_0 is the capacitance of a parallel plate capacitor with air as dielectric (as in Figure (a)). If, half of the entire gap as shown in Figure (b) is filled with a dielectric of permittivity $\in_r$, the expression for the modified capacitance is **[2014]**

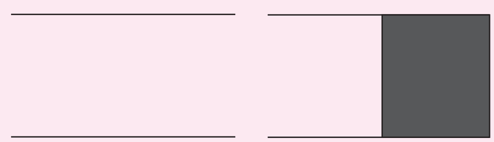


Figure (a) Figure (b)

(A) $\frac{C_0}{2}(1+\in_r)$ (B) $(C_0 + \in_r)$

(C) $\frac{C_0}{2}\in_r$ (D) $C_o(1 + \in_r)$

10. The undesirable property of an electrical insulating material is **[2014]**

(A) High dielectric strength
(B) High relative permittivity
(C) High thermal conductivity
(D) High insulation resistively

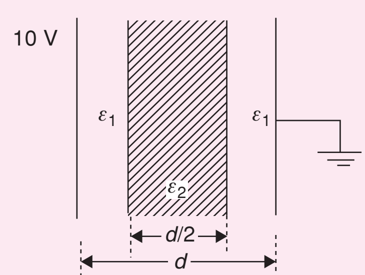
11. The line integral of function $F = yzi$, in the counter-clockwise direction, along the circle $x^2 + y^2 = 1$ at $z = 1$ is **[2014]**

(A) $-\pi$ (B) $-\pi$
(C) π (D) 2π

12. The following four vector fields are given in Cartesian co-ordinate system. The vector filed which does not satisfy the property of magnetic flux density is **[2014]**

(A) $y^2a_x + z^2a_y + x^2a_z$
(B) $z^2a_x + x^2a_y + y^2a_z$
(C) $x^2a_x + y^2a_y + z^2a_z$
(D) $y^2z^2a_x + x^2z^2a_y + x^2y^2a_z$

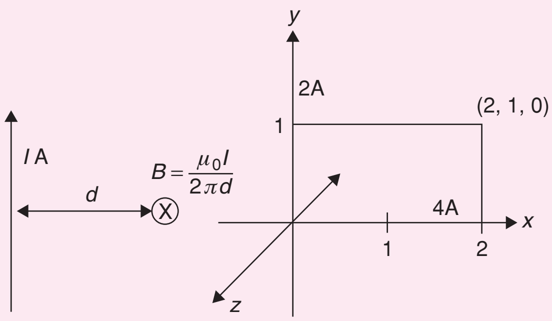
13. A parallel plate capacitor consisting two dielectric materials is shown in the figure. The middle dielectric slab is placed symmetrical with respect to the plates. **[2014]**



If the potential difference between one of the plates and the nearest surface of dielectric interface is 2 Volts, then the ratio $\varepsilon_1 : \varepsilon_2$ is **[2014]**

(A) 1 : 4 (B) 2 : 3
(C) 3 : 2 (D) 4 : 1

14. The magnitude of magnetic flux density $(\vec{B})$ at a point having normal distance d meters from an infinitely extended wire carrying current of I A is $\frac{\mu_0 I}{2\pi d}$ (in SI units). An infinitely extended wire is laid along the x-axis and is carrying current of 4 A in the $+ve$ x direction. Another infinitely extended wire is laid along the y-axis and is carrying 2 A current in the +ve y direction. μ_o is permeability of free space. Assume $\hat{i}$, $\hat{j}$, $\hat{k}$ to be unit vectors along x, y and z axes respectively. **[2014]**



Assuming right handed coordinate system, magnetic field intensity, $\vec{H}$ at coordinate (2, 1, 0) will be **[2014]**

(A) $\frac{3}{2\pi}\hat{k}$ Wb/m (B) $\frac{4}{3\pi}\hat{i}$ A/m

(C) $\frac{3}{3\pi}\hat{k}$ A/m (D) 0 A/m

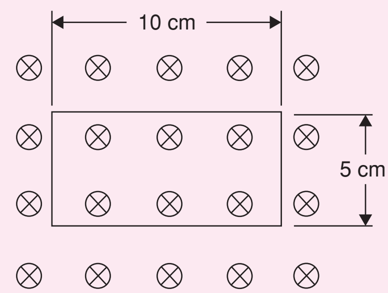
15. The horizontally placed conductors of a single phase line operating at a 50 Hz are having outside diameter of 1.6 cm, and spacing between centers of the conductors is 6 m. The permittivity of free space is 8.854×10^{-12} F/m. The capacitance to ground per kilometer of each line is **[2014]**

(A) 4.2×10^{-9} F (B) 8.4×10^{-9} F
(C) 4.2×10^{-12} F (D) 8.4×10^{-12} F

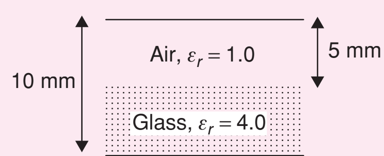
16. Consider a function $\vec{f} = \frac{1}{r^2}\hat{r}$, where r is the distance from the origin and $\hat{r}$ is the unit vector in the radial direction. The divergence of this function over a sphere of radius R, which includes the origin, is **[2015]**

(A) 0 (B) 2π
(C) 4π (D) $R\pi$

17. Consider a one-turn rectangular loop of wire placed in a uniform magnetic field as shown in the figure. The plane of the loop is perpendicular to the field lines. The resistance of the loop is 0.4 |, and its inductance is negligible. The magnetic flux density (in Tesla) is a function of time, and is given by $B(t) = 0.25 \sin \omega t$, where $\omega = 2\pi \times 50$ radian/second. The power absorbed (in Watt) by the loop from the magnetic field is _____. **[2015]**



18. A parallel plate capacitor is partially filled with glass of dielectric constant 4.0 as shown below. The dielectric strengths of air and glass are 30 kV/cm and 300 kV/cm, respectively. The maximum voltage (in kilovolts), which can be applied across the capacitor without any breakdown, is _____. **[2015]**



19. Match the following. **[2015]**

P. Stoke's Theorem 1. $\oiint D.ds = Q$

Q. Gauss's Theorem 2. $\oint f(z)dz = 0$

R. Divergence Theorem 3. $\iiint (\nabla.A)dv = \oiint A.ds$

S. Cauchy's Integral Theorem 4. $\iint (\nabla \times A).ds = \oint A.dl$

(A) P–2 Q–1 R–1 S–3
(B) P–4 Q–1 R–3 S–2
(C) P–4 Q–3 R–1 S–2
(D) P–3 Q–4 R–2 S–1

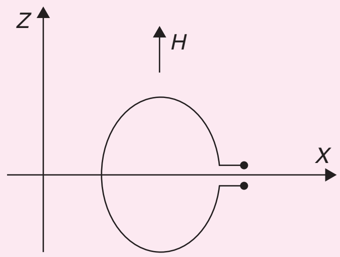
20. Two semi-infinite dielectric regions are separated by a plane boundary at $y = 0$. The dielectric constants of region 1($y < 0$) and region 2($y > 0$) are 2 and 5, respectively. Region 1 has uniform electric field $\vec{E} = 3\hat{a}_y + 2\hat{a}_z$, where $\hat{a}_x, \hat{a}_y$ and $\hat{a}_z$ are unit vectors along the x, y and z axes, respectively. The electric field in region 2 is **[2015]**

(A) $3\hat{a}_x + 1.6\hat{a}_y + 2\hat{a}_z$ (B) $1.2\hat{a}_x + 4\hat{a}_y + 2\hat{a}_z$

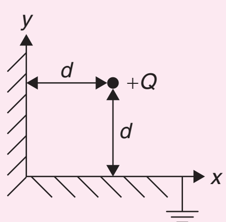
(C) $1.2\hat{a}_x + 4\hat{a}_y + 0.8\hat{a}_z$ (D) $3\hat{a}_x + 10\hat{a}_y + 0.8\hat{a}_z$

21. A circular turn of radius 1 m revolves at 60 rpm about its diameter aligned with the x-axis as shown in the figure. The value of μ_0 is $4\pi \times 10^{-7}$ in SI unit. If a uniform magnetic field intensity

$\vec{H} = 10^7\ \hat{z}$ A/m is applied, then the peak value of the induced voltage, V_{turn}(in Volts), is______. **[2015]**



22. Two semi-infinite conducting sheets are placed at right angles to each other as shown in the figure. A point charge of +Q is placed at a distance of d from both sheets. The net force on the charge is $\frac{Q^2}{4\pi \in_0} \frac{K}{d^2}$. where K is given by **[2015]**



(A) 0

(B) $-\frac{1}{4}i - \frac{1}{4}j$

(C) $-\frac{1}{8}i - \frac{1}{8}j$

(D) $\frac{1-2\sqrt{2}}{8\sqrt{2}}i + \frac{1-2\sqrt{2}}{8\sqrt{2}}j$

23. In cylindrical coordinate system, the potential produced by a uniform ring charge is given by $\phi = f(r, z)$, where f is continuous function of r and z. Let $\vec{E}$ be the resulting electric field. Then the magnitude of $\nabla \times \vec{E}$ **[2016]**

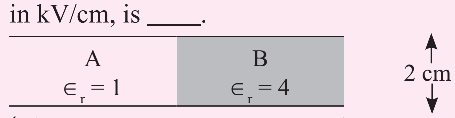
(A) increase with r (B) is 0
(C) is 3 (D) decrease with z

24. A soft-iron toroid is concentric with a long straight conductor carrying a direct current/. If the relative permeability μ_r of soft-iron is 100, the ratio of the magnetic flux densities at two adjacent points located just inside and just outside the toroid, is _____. **[2016]**

25. Two electric charges z and −2z are placed at (0,0) and (6,0) on the x-y plane. The equation of the zero equipotential curve in the x-y plane is ________. **[2016]**

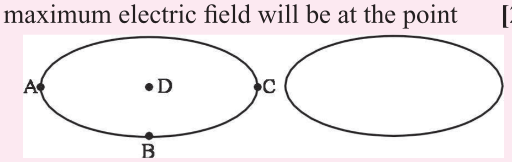
(A) $x = -2$ (B) $y = 2$
(C) $x^2 + y^2 = 2$ (D) $(x + 2)^2 + y^2 = 16$

26. A parallel plate capacitor filled with two dielectrics is shown in the figure below. If the electric field in the region A is 4kV/cm, the electric field in the region B, in kV/cm, is ____. **[2016]**



(A) 1 (B) 2
(C) 4 (D) 16

27. Two electrodes, whose cross-sectional view is shown in the figure below, are at the same potential. The maximum electric field will be at the point **[2016]**



(A) A (B) B
(C) C (D) D

Answer Keys

Exercises

Practice Problems 1

1. D	**2.** A	**3.** B	**4.** C	**5.** C	**6.** A	**7.** C	**8.** D	**9.** B	**10.** C
11. C	**12.** D	**13.** B	**14.** B	**15.** D					

Practice Problems 2

1. A	**2.** B	**3.** A	**4.** D	**5.** D	**6.** A	**7.** B	**8.** B	**9.** B	**10.** C
11. A	**12.** A	**13.** C	**14.** D	**15.** B					

Previous Years' Questions

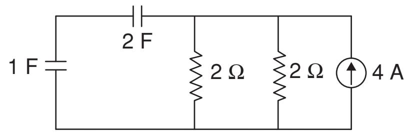
1. B	**2.** B	**3.** D	**4.** A	**5.** C	**6.** D	**7.** A	**8.** C	**9.** A	**10.** B
11. B	**12.** C	**13.** C	**14.** C	**15.** B	**16.** A	**17.** 0.193	**18.** 17 to 20		**19.** B
20. A	**21.** 246 to 250		**22.** D	**23.** B	**24.** 100	**25.** D	**26.** C	**27.** A	

Test
Electric Circuits and Fields

Time: 60 min.

Directions for questions 1 to 25: Select the correct alternative from the given choices.

1. The time constant for the given circuit will be



(A) $\frac{2}{3}s$ (B) $3s$
(C) $\frac{4}{3}s$ (D) $\frac{1}{4}s$

2. The V-I characteristic as seen from the terminal pair (A, B) of the network of Figure (a) is shown in the figure (b). If an inductance of 5 mH is connected across the terminal pair (A, B), the time constant of the system will be

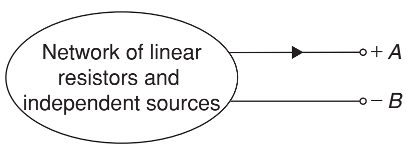


Figure: (a)

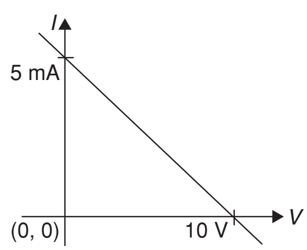
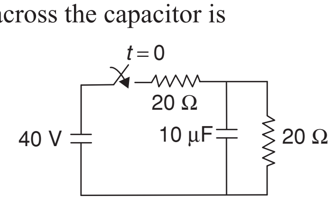


Figure: (b)

(A) 2.5 μsec (B) 10 μsec
(C) 5 μsec (D) 15 μsec

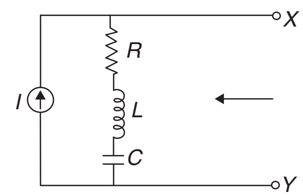
3. In a R-L-C series resonant circuit at the half power points,
(A) the impedance is half the impedance at resonance
(B) the current is half of the current at resonance
(C) power factor is 0.5
(D) the resistance is equal to resultant reactance

4. In the figure given below, the initial capacitor voltage is zero. The switch is closed at $t = 0$. The final steady state voltage across the capacitor is



(A) 5 V (B) 20 V
(C) 10 V (D) 0 V

5. In the given figure, the current source is $1 \angle 0$. $R = 1\ \Omega$, $Z_C = -2j\ \Omega$, $Z_L = j2\ \Omega$. The Thevenin equivalent looking into the circuit across $X - Y$.

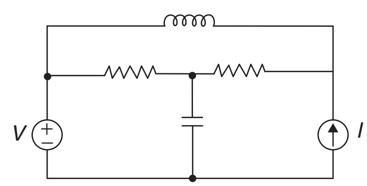


(A) $1 \angle 0$ V, $1 + j2\ \Omega$ (B) $1 \angle 45$ V, $1 - j2\ \Omega$
(C) $1 \angle 0$ V, 1 Ω (D) $1.414 \angle 45$ V, 1 Ω

6. Form factor of the symmetrical square wave is ______.
(A) zero (B) 1
(C) 2 (D) ∞

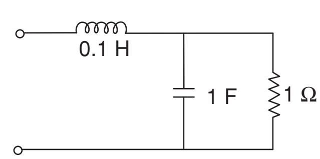
7. Which of the following statements holds for the divergence of electric and magnetic flux densities?
(A) Both are zero
(B) These are zero for static densities but non zero for time varying densities
(C) It is zero for the electric flux density.
(D) It is zero is for the magnetic flux density.

8. The number of chords in the graph of the given circuit will be



(A) 3 (B) 4
(C) 5 (D) 2

9. The resonant frequency for the given circuit will be



(A) 1 rad/sec (B) 2 rad/sec
(C) 3 rad/sec (D) 4 rad/sec

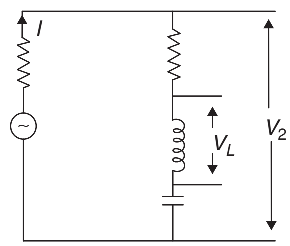
10. The resistance of a strip of copper of rectangular cross-section is 4 Ω. A metal of resistivity twice that of copper is coated on its upper surface to a thickness, equal to that of copper strip. The resistance of the composite strip will be______.

(A) 12 Ω (B) $\frac{8}{3}$ Ω

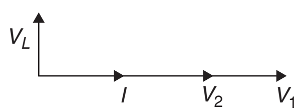
(C) $\frac{14}{5}$ Ω (D) $\frac{3}{8}$ Ω

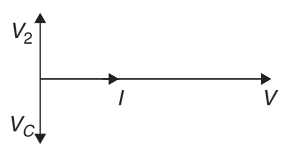
11. A lag network achieves the desired result through its
(A) Attenuation property at high frequencies
(B) Attenuation property at low frequencies
(C) Amplification property
(D) None of these

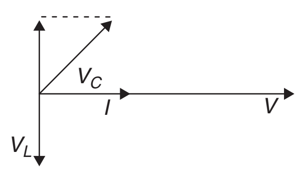
12. The circuit shown in the figure is energized by the sinusoidal voltage source at a frequency which causes resonance with current of I ampere.

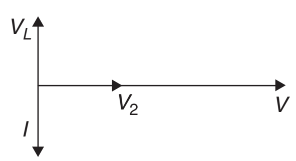


The phasor diagram which is applicable is

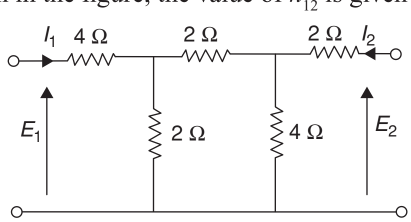
(A)


(B)


(C)


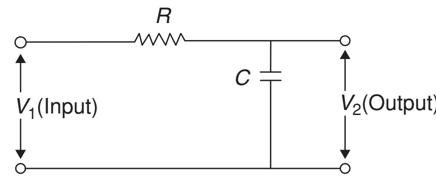
(D)


13. The h parameters of a two-port network are defined by: $\begin{bmatrix} E_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ E_2 \end{bmatrix}$. For the two port network shown in the figure, the value of h_{12} is given by



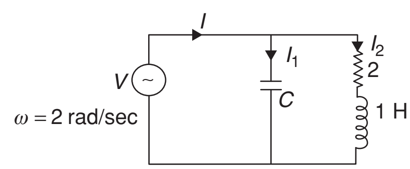
(A) 0.125 (B) 0.167
(C) 0.625 (D) 0.25

14.



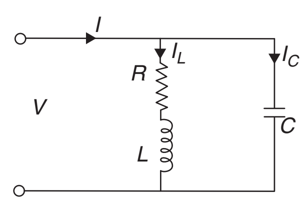
The two-port network shown in the above figure can be described as
(A) phase lead network, differentiating circuit or low-pass filter
(B) phase lag network, integrating circuit or low-pass filter
(C) phase lag network, differentiating circuit or high-pass filter
(D) phase lead network, integrating circuit or high-pass filter

15. For the given ac circuit, if the value of C is chosen such that V and I are in phase, then I_1 leads I_2 by an angle given by



(A) 0° (B) 45°
(C) 90° (D) 135°

16. For the given parallel resonant circuit, match list I with II and select the correct answer using the codes given below in the lists:

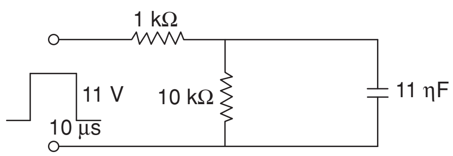


	List - I		List - II
a	I at resonance	1	$\frac{\omega}{R}$
b	I_L	2	In phase with voltage
c	Dynamic impedance	3	$\frac{L}{CR}$
		4	Lags the applied voltage

Codes:

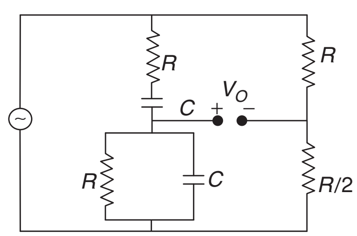
	a	b	c
(A)	4	2	3
(B)	2	4	3
(C)	4	2	1
(D)	2	4	1

17. An 11 V pulse of 10 μs duration is applied to the circuit shown in the figure. Assuming that the capacitor is completely discharged prior to applying the pulse, the peak value of the capacitor voltage is



(A) 11 V (B) 5.5 V
(C) 6.32 V (D) 0.86 V

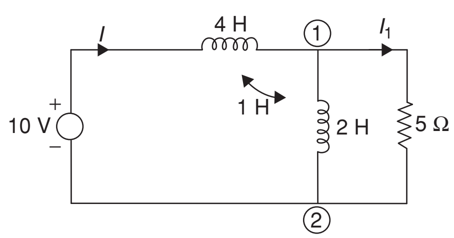
18. In the circuit shown in figure, V_o will be zero at a frequency of



(A) $\dfrac{1}{2\pi RC}$ Hz (B) $\dfrac{1}{4\pi RC}$ Hz

(C) $\dfrac{1}{\pi RC}$ Hz (D) $\dfrac{4}{3\pi RC}$ Hz

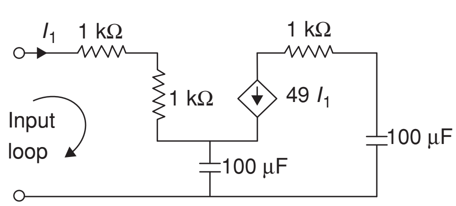
19. The tie set matrix corresponding to the coupled circuit shown below is



(A) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$

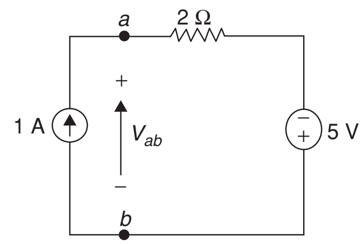
(C) $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$

20. The equivalent capacitance of the input loop of the circuit shown is ________.



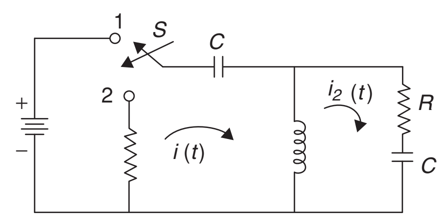
(A) 2 µF (B) 100 µF
(C) 200 µF (D) 4 µF

21. Assuming ideal elements in the circuit show below, V_{ab} will be



(A) 3 V (B) −3 V
(C) 5 V (D) 0 V

Common Data for Questions 22 and 23: For the circuit shown in the below figure:



22. At $t = 0^+$, the current is

(A) $\dfrac{-V}{2R}$ (B) $\dfrac{-V}{R}$

(C) $\dfrac{-V}{4R}$ (D) Zero

23. The equations for the loop currents after the switch is brought from position 1 to position 2 at $t = 0$; are

(A) $\begin{bmatrix} R+Ls+\frac{1}{s} & -Ls \\ -Ls & R+\frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{V}{s} \\ 0 \end{bmatrix}$

(B) $\begin{bmatrix} R+Ls+\frac{1}{s} & -Ls \\ -Ls & R+\frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{-V}{s} \\ 0 \end{bmatrix}$

(C) $\begin{bmatrix} R+Ls+\frac{1}{s} & -Ls \\ -Ls & R+Ls+\frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{-V}{s} \\ 0 \end{bmatrix}$

(D) $\begin{bmatrix} R+Ls+\frac{1}{s} & -Ls \\ -Ls & R+Ls+\frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{V}{s} \\ 0 \end{bmatrix}$

Common Data for Questions 24 and 25: A two-port network shown below is excited by external dc source. The voltages and the currents are measured with voltmeters V_1, V_2 and ammeters A_1, A_2, (all assumed to be ideal), as indicated.

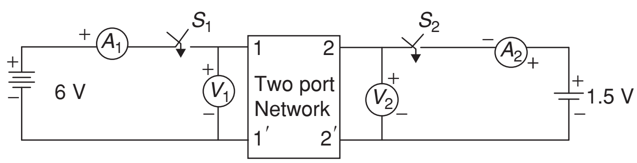
Under following switch conditions, the readings obtained are:

(i) S_1 – open, S_2 – closed

$A_1 = 0$ A, $V_1 = 4.5$ V, $V_2 = 1.5$ V, $A_2 = 1$ A

(ii) S_1 – closed, S_2 – open

$A_1 = 4$ A, $V_1 = 6$ V, $V_2 = 6$ V, $A_2 = 0$ A



24. The z-parameter matrix for this network is

(A) $\begin{bmatrix} 1.5 & 1.5 \\ 4.5 & 1.5 \end{bmatrix}$ (B) $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 4.5 \end{bmatrix}$

(C) $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 1.5 \end{bmatrix}$ (D) $\begin{bmatrix} 4.5 & 1.5 \\ 1.5 & 4.5 \end{bmatrix}$

25. The h-parameter matrix for this network is

(A) $\begin{bmatrix} -3 & 3 \\ -1 & 0.67 \end{bmatrix}$ (B) $\begin{bmatrix} -3 & -1 \\ 3 & 0.67 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 3 \\ 1 & 0.67 \end{bmatrix}$ (D) $\begin{bmatrix} 3 & 1 \\ -3 & -0.67 \end{bmatrix}$

Answers Keys

1. A	**2.** A	**3.** D	**4.** B	**5.** C	**6.** B	**7.** D	**8.** D	**9.** C	**10.** B
11. A	**12.** A	**13.** D	**14.** B	**15.** D	**16.** B	**17.** D	**18.** A	**19.** B	**20.** A
21. B	**22.** A	**23.** C	**24.** C	**25.** A					