CBSE Test Paper 03

CH-10 Straight Lines

1. The point on the axis of y which is equidistant from (- 1 , 2) and (3 , 4) is
   a. (0 , 4)
   b. none of these
   c. (5 , 0)
   d. (0 , 5)
2. The point which divides the joint of (1, 2) and (3,4) externally in the ratio 1 : 1 .
   a. lies in the 2^{nd} quadrant
   b. lies in 3^{rd} quadrant
   c. cannot be found
   d. lies in the 1^{st} quadrant
3. The foot of the perpendicular from (2, 3) on the line 3x + 4y – 6 = 0 is
   a. $\left(-\frac{14}{25}, -\frac{27}{25}\right)$
   b. $\left(\frac{14}{25}, -\frac{27}{25}\right)$
   c. $\left(\frac{14}{25}, \frac{27}{25}\right)$
   d. $\left(-\frac{14}{25}, \frac{29}{25}\right)$
4. A line is equally inclined to the axis and the length of perpendicular from the origin upon the line is $\sqrt{2}$. A possible equation of the line is
   a. y = $\sqrt{2}$ x + 2 .
   b. x + y = 2
   c. y = x + 1
   d. y = x + $\sqrt{2}$.
5. The locus of the inequation xy $\geq$ 0 is
   a. none of these
   b. a straight line
   c. the set of all points either in the 1st quadrant or in the 3rd quadrant including the points on coordinate axis
   d. a pair of straight lines
6. Fill in the blanks:

   The inclination of the line x - y + 3 = 0 with the positive direction of x-axis is ________.

7. Fill in the blanks:

   Locus of the mid-points of the portion of the line x sinθ + y cosθ = p intercepted between the axes is ________.

8. Find the slope of the lines passing through the points (3, - 2) and (7, - 2).

9. If the vertices of a triangle are P(1, 3), Q(2, 5) and R(3, - 5), then find the centroid of a ΔPQR.

10. Determine the $\angle B$ of the triangle with vertices A(-2, 1), B(2, 3) and C(-2, -4).

11. The slope of a line is double the slope of another line. If tangent of the angle between them is $\frac{1}{3}$, then find the slope of the lines.

12. Show that the straight lines given by x(a + 2b) + y(a + 3b) = a + b for different values of a and b pass through a fixed point.

13. A line passes through the point (3, - 2). Find the locus of the middle point of the portion of the line intercepted between the axes.

14. Find the transformed equation of the circle $x^2 + y^2 = 9$ when the origin is shifted to (-1, -3).

15. A line 4x + y = 1 through the point A (2, -7) meets the line BC whose equation is 3x - 4y + 1 = 0 at the point B. Find the equation to the line AC so that AB = AC.

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Solution

1. (d) (0 , 5)

   Explanation: Let (0,y) be the point on Y axis which is equidistant from the points (-1,2) and (3,4)

   By applying the distance formula,

   $(0+1)^2 + (y-2)^2 = (3-0)^2 + (4-y)^2$

   on simplifying we get 4y = 20

   Therefore y = 5

   Hence the point on the y axis is (0,5)

2. (c) cannot be found

   Explanation: The point which divides the line in the ratio m:n externally is given by x = $\frac{m(x_2)-n(x_1)}{m-n}$

   Substituting the values we get,

   x = $\frac{1(3)-1(1)}{1-1}$ which is undefined.

3. (c)$\left(\frac{14}{25}, \frac{27}{25}\right)$

   Explanation: The equation of the line perpendicular to the given line 3x+4y=6 is 4x - 3y +k = 0

   Since this line passes through (2,3)

   4(2) -3(3) + k = 0

   Therefore k = 1

   Therefore the line which perpendicular to the given line is 4x-3y+1 =0

   on solving both the equations we get, x = 14/25 and y = 27/25

   Hence the foot of the perpendicular is (14/25, 27/25)

4. (b) x + y = 2

   Explanation: Since the line is equally inclined the slope of the line should be -1, because it makes 135^0 in the positive direction of the X axis

This implies the equation of the line is y= -x +c

i.e; x+y - c =0

distance of the line from the origin is given as

Therefore $\sqrt{2}$ = $\frac{|c|}{\sqrt{1^2+1^2}}$

This implies c = 2

Hence the equation of the line is x+y=2

5. (c) the set of all points either in the 1st quadrant or in the 3rd quadrant including the points on coordinate axis

   Explanation: It is the set of all points either in the 1st quadrant or in the 3rd quadrant including the points on coordiate axis. This is because the inequality $\geq$ indicates that the points belong either to 1st or 3rd quadrant.

6. 45^{o}

7. $4x^2y^2 = p^2(x^2 + y^2)$

8. Slope of the line through the points (3, - 2) and (7, - 2) is

   $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-(-2)}{7-3} = \frac{0}{4} = 0$

9. We know that, if the vertices of a triangle are (x_1, y_1), (x_2, y_2) and (x_3, y_3), then centroid of a triangle is

   $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

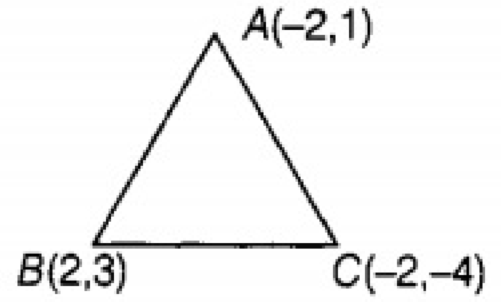
   Here, P(1, 3) = (x_1, y_1)

   Q(2, 5) = (x_2, y_2)

   and R(3, -5) = (x_3, y_3)

   $\therefore$ Centroid of a triangle

   $= \left(\frac{1+2+3}{3}, \frac{3+5-5}{3}\right) = \left(\frac{6}{3}, \frac{3}{3}\right) = (2, 1)$

10. 

Given, vertices of a triangle are A(-2, 1), B(2, 3) and (-2, -4).

Slope of line

$AB = \frac{3-1}{2+2} = \frac{2}{4} = \frac{1}{2} = m_1[\text{ say }]$

$\left[\because \text{ slope } = \frac{y_2-y_1}{x_2-x_1}\right]$

Slope of line $BC = \frac{-4-3}{-2-2} = \frac{7}{4} = m_2$ [say]

$\therefore \quad \tan B = \left|\frac{m_1-m_2}{1+m_1 \cdot m_2}\right| = \left|\frac{\frac{1}{2}-\frac{7}{4}}{\left|1+\frac{1}{2}\cdot\frac{7}{4}\right.}\right| = \left|\frac{-\frac{5}{4}}{15/8}\right| = \frac{2}{3}$

$\therefore \quad \angle B = \tan^{-1}\left(\frac{2}{3}\right)$

11. If m_1 and m_2 are the slopes of a line, tangent of angle between the line is,

$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

Let slope of one line be m, then slope of another line be 2m.

Given, the tangent of the angle between them is $\tan\theta = \frac{1}{3}$

$\therefore \quad \left|\frac{m_1-m_2}{1+m_1m_2}\right| = \frac{1}{3} \Rightarrow \frac{1}{3} = \left|\frac{m-2m}{1+m\times 2m}\right|$

$\Rightarrow \quad \frac{1}{3} = \left|\frac{-m}{1+2m^2}\right|$

$\Rightarrow (1 + 2m^2) = 3m$

$\Rightarrow 2m^2 - 3m + 1 = 0$

Factorise it by splitting the middle term.

$\Rightarrow 2m^2 - 2m - m + 1 = 0$

$\Rightarrow 2m(m - 1) - 1 (m - 1) = 0$

$\Rightarrow (2m - 1) (m -1) = 0$

$\Rightarrow 2m - 1 = 0$ or $m - 1 =0 \Rightarrow m = \frac{1}{2}, m = 1$

12. Given equation can be written as

a(x + y - 1) + b(2x + 3y - 1) =0

$\Rightarrow (x + y - 1) + \lambda(2x + 3y - 1) = 0$, where λ = b/a

This is the form of $L_1 + \lambda L_2 = 0$. So it represents a line passing through the intersection of x + y - 1 = 0 and 2x + 3y - 1 = 0.

Solving these two equations, we get the point (2, -1) which is the fixed point.

13. Let the equation of the line be $\frac{x}{a}+\frac{y}{b}=1$ ...(i)

It passes through (3, -2).

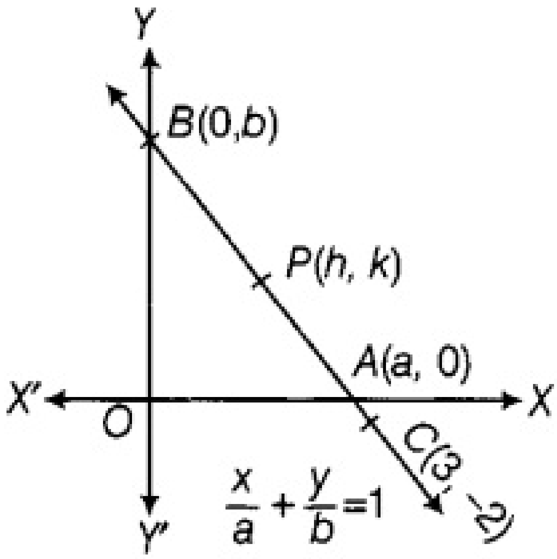
$\therefore \frac{3}{a}-\frac{2}{b}=1$ ...(ii)

The line (i) cuts the coordinate axes at A(a, 0) and B(0, b). Let P(h, k) be the mid-point of the portion AB.

Then,

$h=\frac{a+0}{2}, k=\frac{0+b}{2}$

$\Rightarrow$ a = 2h and b = 2k



On substituting the values of a and b in Eq. (ii), we get

$\frac{3}{2h}-\frac{2}{2k}=1$

Hence, locus of P(h, k) is $\frac{3}{2x}-\frac{1}{y}=1$

or 3y - 2x = 2xy

14. Let (x', y') be the new coordinates of the point (x, y) to (-1, -3)

Origin is shifted to (-1, -3)

$\therefore$ h = -1 and k = -3

Now x = x' + h = x' - 1

and y = y' + k = y' - 3

Substituting these values of x and y in equation of $x^2 + y^2 = 9$ we get

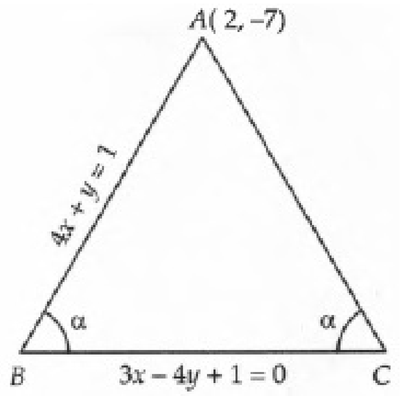
$(x' - 1)^2 + (y' - 3)^2 = 9$

$\Rightarrow x'^2 + 1 - 2x' + y'^2 + 9 - 6y' = 9$

$\Rightarrow x'^2 + y'^2 - 2x' - 6y' + 1 = 0$

Hence the equation of the given circle in new system is $x^2 + y^2 - 2x - 6y + 1 = 0$

15. The lines AB and BC meet at a point B. Let a be the angle between them. Then,
$\tan \alpha = \frac{m_1 - m_2}{1 + m_1 m_2}$, where, m_1 = Slope of AB = -4 and m_2 = Slope of BC = $\frac{3}{4}$
$\Rightarrow \tan \alpha = \frac{-4 - 3/4}{1 + (-4) \times 3/4} = \frac{19}{8}$
It is given that AB = AC. Therefore, triangle ABC is an isosceles triangle.



Clearly, AB and AC both pass through A(2, -7) and are equally inclined to 3x - 4y + 1 = 0. So, their equations are given by
$(y + 7) = \frac{m \pm \tan \alpha}{1 \mp \tan \alpha}(x - 2)$, where, m = Slope of BC = $\frac{3}{4}$ and $\tan \alpha = \frac{19}{8}$
or, $(y + 7) = \frac{\frac{3}{4} + \frac{19}{8}}{1 - \frac{3}{4} \times \frac{19}{8}}(x - 2)$ and $y + 7 = \frac{\frac{3}{4} - \frac{19}{8}}{1 + \frac{3}{4} \times \frac{19}{8}}(x - 2)$
or, $y + 7 = -4(x - 2)$ and $y + 7 = -\frac{52}{89}(x - 2)$
$\Rightarrow 4x + y = 1$ and $52x + 89y + 519 = 0$