CBSE Test Paper 04
Chapter 7 System of Particles and Rotational Motion

1. A ring of radius r and mass m rotates about its central axis. The kinetic energy is
   a. mr ω^2 /2
   b. $\mathrm{mr}^2\omega^2$ /2
   c. mr ω^2
   d. $\mathrm{mr}^2\omega^2$
2. A wheel is rotating about an axis through its centre at 720 r.p.m. When acted upon by a constant torque opposing its motion for 8 seconds it stops rotating. The value of this torque in Nm is (given I = $\frac{24}{\pi}$ kg m^2)
   a. 72
   b. 48
   c. 96
   d. 120
3. The angular velocity of a body changes form 1 rev / sec to 25 rev/sec. without applying any external torque. The ratio of the radii of gyration in the two cases is
   a. it is 1: 25
   b. it is 25:1
   c. it is 5:1
   d. it is 1: 5
4. The vector product of two vectors a and b is a vector c such c is perpendicular to the plane containing a and b and the direction is given by is given by
   a. left hand rule
   b. left handed screw rule
   c. index finger rule
   d. right handed screw rule
5. A particle is orbiting in a vertical plane. Its linear momentum will be directed

a. vertically
b. tangential to the orbit
c. at 45^{0} to the vertical
d. horizontally

6. In a flywheel, most of the mass is concentrated at the rim. Explain why?

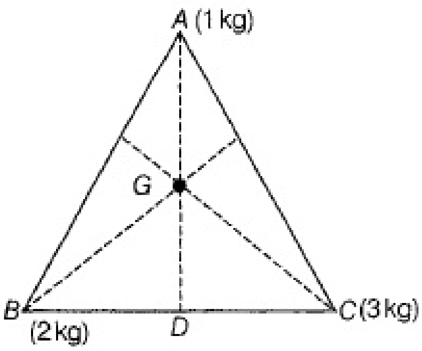
7. The bottom of a ship is made heavy. Why?

8. A metre stick is balanced on a knife-edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm, what is the mass of the metre stick?

9. State the factors on which the moment of inertia of a body depends.

10. Show that cross product of two parallel vectors is zero.

11. Three balls of masses 1kg, 2kg and 3kg are arranged at the corners of an equilateral triangle of side 1m. What will be the moment of inertia of the system about an axis through the centroid and perpendicular to the plane of the triangle.



12. What is torque? Give its unit. Show that it is equal to the product of force and the perpendicular distance of its line of action from the axis of rotation.

13. A cord of negligible mass is wound round the rim of a flywheel of mass 20 kg and radius 20 cm. A steady pull of 25 N is applied on the cord as shown in the figure. The flywheel is mounted on a horizontal axle with frictionless bearings.

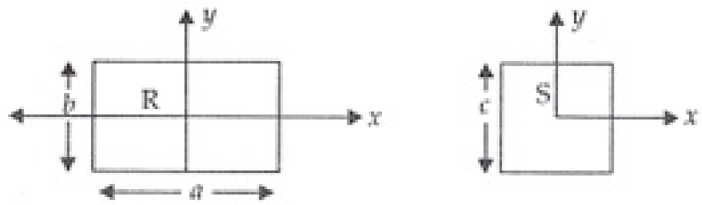
i. Compute angular acceleration of the flywheel.
ii. Find the work done by the pull, when 2 m of the cord is unwound.
iii. Find also the KE of the flywheel at this point. Assume that the flywheel starts from

rest.

iv. Compare the answers of parts (ii) and (iii).

14. Define centre of mass of a n-particle system. State the coordinates of centre of mass of a n-particle system.

15. A uniform square plate S (side c) and a uniform rectangular plate R (sides b, a) have identical areas and masses (Figure).



Show that

i. $\frac{I_{xR}}{I_{xs}} < 1$
ii. $\frac{I_{yR}}{I_{ys}} > 1$
iii. $\frac{I_{zR}}{I_{zs}} > 1$

Answer

1. b. $mr^2\omega^2/2$

   Explanation: The kinetic energy of body in rotational motion is
   $KE = \frac{1}{2}I\omega^2 = \frac{1}{2}mr^2\omega^2$ as moment of inertia of ring about its central axis is
   $I = mr^2$

2. a. 72

   Explanation: $n = \frac{720}{60} = 12$ rev/s
   angular velocity $\omega = 2\pi n = 2\pi \times 12 = 24\pi$ rad/s
   moment of inertia
   $I = \frac{24}{\pi}$ kg m^2
   torque
   $T = I\alpha$
   $T = I\frac{\Delta\omega}{\Delta t} = \frac{24}{\pi} \times \left(\frac{24-0}{8}\right) = \frac{24}{\pi} \times \frac{24\pi}{8}$ = 72.0 Nm

3. c. it is 5:1

   Explanation: $I_1\omega_1 = I_2\omega_2$
   $\frac{I_1}{I_2} = \frac{\omega_2}{\omega_1}$
   $\omega_1 = 1rev/s$
   $\omega_2 = 25rev/s$
   if radius of gyration is k_1 and k_2 then
   $\frac{M{k_1}^2}{M{k_2}^2} = \frac{\omega_2}{\omega_1}$
   $\frac{k_1}{k_2} = \sqrt{\frac{\omega_2}{\omega_1}} = \sqrt{\frac{25}{1}} = \frac{5}{1}$
   $k_1 : k_2 = 5 : 1$

4. d. right handed screw rule

   Explanation: If vectors A and B lie in the plane of this page, the vector C will be perpendicular to this plane.
   The sense (upward or downward) of the direction of the vector product is given

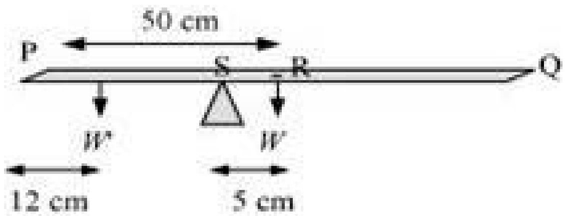
by the direction of the advance of the tip of a right-handed screw when rotated from A to B through angle θ between them, the screw being placed with its axis perpendicular to the plane containing the two vectors.

5. b. tangential to the orbit

   Explanation: As the direction of velocity at any point moving in circular orbit is tangent at that point, thus momentum would be tangential to the orbit.

6. The concentration of mass is at the rim so as to have moment of inertia same for the whole mass. If such a wheel gains or loses rotational energy, the change in the angular velocity is very small.

7. The bottom of a ship is made heavy so that its centre of gravity remains low towards its base. This ensures the stability of ship in its equilibrium position.

8. 

   Net torque about S must be zero for equilibrium.

   33cm$\times$10W' = 5cm$\times$W

   since W' = 2$\times$5 = 10g

   hence W = 66g

9. i. Mass of body
   ii. Size and shape of body
   iii. Mass distribution w.r.t. axis of rotation
   iv. position and orientation of rotational axis

10. $\overline{A} \times \overline{B} = AB\sin\theta\hat{x}$

    If $\overline{A}$ and $\overline{B}$ are parallel to each other

    $\theta = 0^\circ$

    $\Rightarrow \overline{A} \times \overline{B} = 0$

11. In the given figure, G is centroid of the equilateral triangle ABC.

    Now, median AD = $\sqrt{AB^2 - BD^2}$

= $\sqrt{(1)^2-(0.5)^2}$ = $\sqrt{0.75}$

$AG=BG=CG=\frac{2}{3}AD=\frac{2}{3}\sqrt{0.75}$

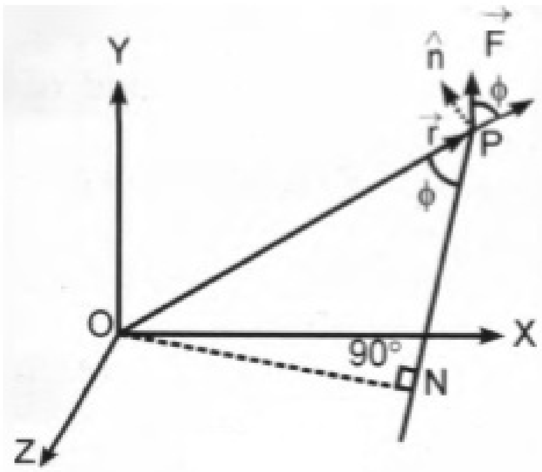
Moment or inertia of the system about an axis through centroid G and perpendicular to plane of $\triangle$ ABC is

$I=1\times(AG)^2+2\times(BG)^2+3\times(CG)^2$

$=(1+2+3)(AG)^2$ [$\because AG=CG$]

=6$\left(\frac{2}{3}\sqrt{0.75}\right)^2$

= 6 × $\frac{4}{9}$ $\times 0.75=2kg\ m^2$

12. 

Torque, also called moment of a force, is the tendency of a force to rotate the body to which it is applied. The torque specified with regard to the axis of rotation, is equal to the magnitude of the component of the force vector lying in the plane perpendicular to the axis multiplied by the shortest distance between the axis and the direction of the force component. Turning moment of a force about a fixed axis of rotation is called its torque. SI unit of torque is newton-metre (N-m).

If a force $\vec{F}$ is acting at a point P whose position vector is $\vec{r}$, then torque of given force about the origin O is given by

torque $\vec{\tau}=\vec{r}\times\vec{F}$

As shown in the Figure, if $\vec{r}$ and $\vec{F}$ are inclined at an angle, then

$|\vec{\tau}|=rF\sin\phi$= $F(r\sin\phi)$= $F(OP\sin\phi)$= F (ON)

= Force × (perpendicular distance of line of action of force from the axis of rotation)

Vectorially, $\vec{\tau}=rF\sin\phi\,\hat{n}$, where $\hat{n}$ lies perpendicular to the plane of $\vec{r}$ and $\vec{F}$ as given right-handed corkscrew rule.

13. i. Torque, $\tau=FR=25\times0.20=5Nm$

$[\because R = 20cm = 0.2m]$

Moment of inertia,

I = $\frac{MR^2}{2}$ = $\frac{20\times(0.2)^2}{2}$ = 0.4kg-m^2

Angular acceleration, $\alpha = \frac{\tau}{I}$ = $\frac{5}{0.4}$ $= 12.5 rad/s^2$

ii. Work done by the pull, $w = F \times s = 25 \times 2 = 50J$

iii. KE = $\frac{1}{2}I\omega^2$

$\omega^2 = \omega_0^2$ + 2 $\alpha\, \theta = 0 + 2 \times 12.5 \times 10$

$[\because \omega_0 = 0, \theta = \frac{2}{0.2} = 10\text{rad}]$

$\therefore KE = \frac{1}{2}\times 0.4 \times 250 = 50J$

iv. From parts (ii) and (iii),

$KE = W$

$\therefore$ No loss of energy due to friction.

14. The centre of mass (CoM) is the point relative to the system of particles in an object. This is that point of the system of particles that embarks the average position of the system in relation to the mass of the object. At the centre of mass, the weighted mass gives a sum equal to zero.

If in an n-particle system particle of masses m_1, m_2 ... m_n be situated at $\overrightarrow{r_1}, \overrightarrow{r_2}, \ldots, \overrightarrow{r_n}$ respectively, then the position vector of the centre of mass $\vec{r}_{cm}$ is given by,

$\vec{r}_{cm}$ = $\frac{m_1\vec{r}_1 + m_2\overrightarrow{r_2} + \ldots + m_n\vec{r}_n}{m_1 + m_2 + \ldots + m_n}$ = $\frac{m_1\vec{r}_1 + m_2\overrightarrow{r_2} + \ldots + m_n r_n}{M}$ ....(1)

As position vector r in terms of coordinates may be expressed as $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, hence in coordinate forms, the centre of mass of n-particle system may be specified as,

$X_{cm} = \frac{m_1x_1 + m_2x_2 + \ldots + m_nx_n}{M}, Y_{cm} = \frac{m_1y_1 + m_2y_2 + \ldots + m_ny_n}{M}$ and

$Z_{cm} = \frac{m_1z_1 + m_2z_2 + \ldots + m_nz_n}{M}$ ....(2)

where, M = m_1 + m_2 + ... + m_n = the total mass of entire n-particle system.

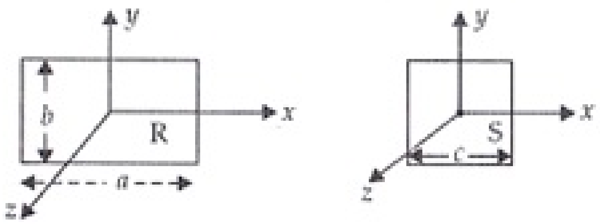
15. Moment of inertia, in physics, quantitative measure of the rotational inertia of a body-i.e., the opposition that the body exhibits to having its speed of rotation about an axis altered by the application of a torque (turning force). The axis may be internal or external and may or may not be fixed. The moment of inertia (*I*), however, is always specified with respect to that axis and is defined as the sum of the products obtained by multiplying the mass of each particle of matter in a given body by the square of its

distance from the axis. The unit of moment of inertia is a composite unit of measure. In the International System (SI), m is expressed in kilograms and r in metres, with I (moment of inertia) having the dimension kilogram-metre square.

$m_R = m_S = m$

Area of square = Area of rectangle

$c^2 = ab$ ...(i)



a. $\because I = mr^2$

$\frac{I_{xR}}{I_{xz}} = \frac{m\cdot\left(\frac{b}{2}\right)^2}{m\left(\frac{c}{2}\right)^2} = \frac{b^2}{4}\frac{4}{c^2} = \frac{b^2}{c^2}$

$\because c > b$ [from (i)]

Or $c^2 > b^2$

$1 > \frac{b^2}{c^2} : \frac{I_{xR}}{I_{xs}} < 1$

Hence proved.

b. $\frac{I_{yR}}{I_{ys}} = \frac{m\left(\frac{a}{2}\right)^2}{m\left(\frac{c}{2}\right)^2} = \frac{a^2}{4}\cdot\frac{4}{c^2} = \frac{a^2}{c^2}$

$\because a > c \Rightarrow \frac{a^2}{c^2} > 1$

$\frac{I_{yR}}{I_{ys}} > 1$

c. $I_{\text{zR}} - L_{zs} = m\left(\frac{d_R}{2}\right)^2 - m\left(\frac{ds}{2}\right)^2$

$I_{zR} - I_{zS} = \frac{m}{4}\left[d_R^2 - d_S^2\right] = \frac{m}{4}\left[a^2 + b^2 - 2c^2\right]$

$\therefore I_{zR} - I_{zS} = \frac{m}{4}\left(a^2 + b^2 - 2ab\right) = \frac{m}{4}(a-b)^2\ (c_2 = ab)$

$\therefore I_{zR} - I_{zS} > 0 \because \frac{m}{4}(a-b)^2 > 0$

$\Rightarrow \frac{I_{\text{zR}}}{I_{zS}} > 1$ Hence proved.