CBSE Board

Class XII Mathematics

Sample Paper 5

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. **All** the questions are **compulsory**.
2. The question paper consists of **37** questions divided into **three parts** A, B, and C.
3. **Part A** comprises of **20** questions of **1 mark** each. **Part B** comprises of **11** questions of **4 marks** each. **Part C** comprises of **6** questions of **6 marks** each.
4. There is no overall choice. However, an internal choice has been provided in **three questions of 4 marks** each, **four questions of 6 marks** each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is **not** permitted.

Part A

Q 1 – Q 20 are multiple choice type questions. Select the correct option.

1. If $A = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$, then for what value of α is A an identity matrix?

A. $0°$

B. $30°$

C. $45°$

D. $60°$

2. Integral of $\frac{1}{\sqrt{9-25x^2}}$ is

A. $3\cos^{-1}\left(\frac{3x}{5}\right)+c$

B. $3\sin^{-1}\left(\frac{3x}{5}\right)+c$

C. $\frac{1}{3}\cos^{-1}\left(\frac{5x}{3}\right)+c$

D. $\frac{1}{3}\sin^{-1}\left(\frac{5x}{3}\right)+c$

3. Find the equation of a line through (−2, 1, 3) and parallel to $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$.

A. $\frac{x-2}{3}=\frac{y+1}{5}=\frac{z+3}{6}$

B. $\frac{x+2}{3}=\frac{y-1}{5}=\frac{z-3}{6}$

C. $\frac{x-2}{-3}=\frac{y+1}{-5}=\frac{z+3}{-6}$

D. $\frac{x+2}{-3}=\frac{y-1}{-5}=\frac{z-3}{-6}$

4. A vector normal to the plane x + 2y +3z – 6 = 0 will be

A. $2\hat{i}+3\hat{j}+\hat{k}$

B. $-\hat{i}-2\hat{j}-3\hat{k}$

C. $\hat{i}+2\hat{j}+3\hat{k}$

D. $-2\hat{i}-3\hat{j}-\hat{k}$

5. If the vectors $\vec{a}=3\hat{i}+\hat{j}-2k$ and $\vec{b}=\hat{i}+\lambda\hat{j}-3k$ are perpendicular to each other, then the value of λ is

A. –1

B. 3

C. –9

D. 18

6. The equation of the plane with intercept 3 on the y-axis and parallel to ZOX plane is

A. x = 3

B. y = 3

C. z = 3

D. x = –3

7. Two men A and B appear for an interview in a company. The probabilities of A's and B's selection are $\frac{1}{4}$ and $\frac{1}{6}$ respectively. Then the probability that both of them are selected is

A. $\frac{3}{2}$

B. $\frac{1}{6}$

C. $\frac{1}{4}$

D. $\frac{1}{24}$

8. The value of $\begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha\cos\delta - \sin\alpha\sin\delta \\ \sin\beta & \cos\beta & \cos\beta\cos\delta - \sin\beta\sin\delta \\ \sin\gamma & \cos\gamma & \cos\gamma\cos\delta - \sin\gamma\sin\delta \end{vmatrix}$ is

A. −1
B. 0
C. 1
D. 2

9. Principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is

A. $\frac{\pi}{6}$

B. $-\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. $-\frac{\pi}{3}$

10. Find the value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{3\pi}{2}$

11. Let $R = \{(a, b): a = b^2\}$ for all $a, b \in N$. Then R satisfies which of the following?

A. Reflexivity
B. Symmetric
C. Transitivity
D. None of these

12. The value of k, so that the function $f(x)=\begin{cases}\dfrac{x^2-25}{x-5}, & x \neq 5 \\ k, & x=5\end{cases}$ is continuous at x = 5 is

A. 0
B. 10
C. 5
D. 15

13. Range of the real function f defined by $f(x)=\dfrac{x^2}{(1+x^2)}$, is

A. R
B. $\{y \in R : 0 \leq y < \infty\}$
C. $\{y \in R : 0 \leq y < 1\}$
D. $R - \{1\}$

14. The total revenue received from the sale of x units of a product is given by $R(x) = 3x^2 + 40x + 10$. When x = 5, the marginal revenue will be

A. 46
B. –10
C. 60
D. 70

15. Find the area of the curve $y = x^2$ bounded by the line $x = y$.

A. $\frac{1}{6}$ units
B. $\frac{1}{3}$ units
C. $\frac{1}{2}$ units
D. $\frac{5}{6}$ units

16. Solve the differential equation: $\dfrac{dy}{dx}+y=1$

A. $\log|1 + y| - x = C$
B. $\log|1 - y| + x = C$
C. $|1 - y| + x + C = 0$
D. $\log|1 + x| - y = C$

17. Find solution of the differential equation: $(x^2+1)\dfrac{dy}{dx}=xy$

A. $y = C_1\sqrt{x^2+1}$

B. $x = \sqrt{y^2+1}$

C. $y = C_1\sqrt{x^2-1}$

D. $x = C_1\sqrt{y^2-1}$

18. Find $\int \frac{dx}{1+\sin x}$.

A. sec x – tan x + c

B. tan x – sec x + c

C. tan x – cos x + c

D. 2sec x – sin x + c

19. The value of $\frac{d}{dx}\left(e^{x^3}\right)$ is

A. $e^{3x^2} e^{x^3}$

B. e^{x^3}

C. $3x^2 e^{x^3}$

D. $3x^2$

20. Find the interval in which the function f given by $f(x) = 2x^2 - 3x$ is increasing.

A. $\left(-\infty, \frac{3}{4}\right)$

B. $\left(\frac{3}{4}, \infty\right)$

C. $(3, \infty)$

D. $(-\infty, 3)$

Part B

21. Evaluate: $\int_0^{\frac{\pi}{2}} (2\log\sin x - \log\sin 2x)\,dx$

OR

Evaluate: $\int_0^{\frac{\pi}{2}} \frac{x\sin x\cos x}{\sin^4 x + \cos^4 x}\,dx$

22. Solve the given differential equation $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$ if y (0) = 0.

23. Let A = Q × Q, Q being the set of rational numbers. Let '*' be a binary operation on A, defined by (a, b) * (c, d) = (ac, ad + b). Show that

(i) '*' is not commutative (ii) '*' is associative

(iii) The identity element with respect to '*' is (1, 0)

24. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$.

25. Find the interval in which the value of the determinant of the matrix A lies, given $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$.

26. If $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, prove that $\frac{d^2y}{dx^2} = -\frac{a}{(2a-y)^2}$

27. Find the distance of the point (2, 4, −1) from the line having the following equation: $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$

28. Evaluate $\int \frac{x}{\sqrt{8+x-x^2}}dx$

29. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

30. Check the continuity and differentiability of the function f(x) = |x – 2| at x = 2.

OR

Show that the function f defined by that $f(x) = |1 - x + |x||$, $x \in R$ is continuous.

31. Find the value of λ which makes the vectors $\vec{a}, \vec{b}$, and $\vec{c}$ coplanar, where $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 3\hat{i} - \lambda\hat{j} + 5\hat{k}$.

OR

Find a, if the points A (10, 3), B (12, –5) and C (a, 11) are collinear.

Part C

32. A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash award of Rs. 6,000. Three times the award money for hard work added to that given for honesty amounts to Rs. 11,000. The award money given for Honesty and Hard work together is double the one given for Regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. Apart from these values, namely, Honesty, Regularity and Hard work, suggest one more value which the school must include for awards.

33. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$. Also find the distance of the plane obtained above, from the origin.

OR

Find the image of the point $2\hat{i} + 3\hat{j} - 4\hat{k}$ in the plane $\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) = 3$.

34. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants per day while B can stitch 10 shirts and 4 pants per day. How many days shall each work, if it is desired to produce atleast 60 shirts and 32 pants at a minimum labour cost? Solve the problem graphically.

OR

A catering agency has two kitchen at A and B. From these places, supply is made to three schools situated at P, Q and R for mid-day meals. The monthly requirements of the three schools are respectively 40, 40 and 50 food packets while the production capacity of the kitchens at A and B are 60 and 70 packets respectively. The transportation cost per packet from the kitchens to the schools is given below.

Transportation Cost per packet (in Rupees)		
To	From	
	A	B
P	5	4
Q	4	2
R	3	5

How many packets from each kitchen should be transported to each school so that the cost of transportation is minimum? Also find the minimum cost.

35. Find the area bounded by the curve $y = 2x - x^2$ and the line $y = -x$.

OR

Find the area of the region $\{(x, y): 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, 0 \leq x \leq 2\}$.

36. Find the intervals on which the function $f(x) = \frac{4x^2 + 1}{x}, (x \neq 0)$ is

(a) increasing (b) decreasing.

37. A doctor is to visit a patient. Form past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus and scooter respectively. But if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that the doctor came by train?

OR

In a factory which manufactures bulbs, machines X, Y and Z manufacture 1000, 2000, 3000 bulbs respectively. Of their outputs, 1%, 1.5% and 2 % are defective bulbs. A bulb is drawn at random and is found to be defective. What is the probability that the machine X manufactures it?