CBSE Test Paper 02
Chapter 1 Relations and Functions

1. If $f(x) = \frac{1}{4x-3}$, then D_f =.
   a. $\left(\frac{3}{4}, \infty\right)$
   b. R
   c. R - $\left\{\frac{3}{4}\right\}$
   d. $\left(-\infty, \frac{3}{4}\right)$
2. A relation R in a set A is called transitive, if
   a. $(a_1, a_2) \in R \Rightarrow (a_2, a_3) \in R\ \forall (a_1, a_2, a_3) \in A$
   b. $(a_1, a_3) \in R, (a_2, a_3) \in R \Rightarrow (a_1, a_2) \in R\ \forall\ a_1, a_2, a_3 \in A$
   c. $(a_1, a_1) \in R, (a_2, a_2) \in R \Rightarrow (a_1, a_2) \in R\ \forall a_1, a_2 \in A$
   d. $(a_1, a_2) \in R, (a_2, a_3) \in R \Rightarrow (a_1, a_3) \in R \forall a_1, a_2, a_3 \in A$
3. If the mappings f: A $\rightarrow$B and g: B $\rightarrow$C are both bijective, then the mapping A$\rightarrow$C is
   a. one – one but not onto
   b. one – one and onto
   c. onto, but not one – one
   d. neither one – one nor onto
4. A function $f : X \rightarrow Y$ is defined to be one – one (or injective), if
   a. the images of distinct elements of X under f are not distinct
   b. the images of distinct elements of X under f are distinct
   c. the images of distinct elements of X under f are identical
   d. the images of distinct elements of X under f are not defined
5. A binary operation * : $R \times R \rightarrow R$ defined by a $*$ b = a + 2b is
   a. Not well defined
   b. Not associative

c. A unary operation
d. Commutative

6. A relation R in a set X is called an ________ relation, if no element of X is related to any element of X.
7. A relation R from a set X to a set Y is defined as a ________ of the cartesian product X $\times$ Y.
8. A relation R in a set X is called ________ relation, if each element of X is related to every element of X.
9. If f: R $\rightarrow$ R be given by f(x) $= \left(3 - x^3\right)^{\frac{1}{3}}$, find fof (x) **(1)**
10. Give examples of two functions f : N $\rightarrow$ N and g : N $\rightarrow$ N such that gof is onto but f is not onto.
11. Let * be a binary operation defined by a * b = 2a + b - 3. find 3 * 4
12. If f : R $\rightarrow$ R is defined by f(x) = x^2 - 3x + 2 write f(f(x))
13. Let f : X $\rightarrow$ Y be an invertible function. Show that f has unique inverse.
14. Check whether the relation R in R defined by R = {(a,b) : a < b^3} is reflexive, symmetric or transitive.
15. Let L be the set of all lines in plane and R be the relation in L define if R = {(l_1, L_2) : L_1 is $\perp$ to L_2}. Show that R is symmetric but neither reflexive nor transitive.
16. Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = {(a, b): b = a+1} is reflexive, symmetric or transitive.
17. Prove that the relation R in set A = {1, 2, 3, 4, 5} given by R = {(a, b) : Ia - bI is even} is an equivalence relation.
18. Discuss the commutativity and associativity of binary operation '*' defined on A = Q - {1} by the rule a * b = a - b + ab for all a, b $\in$ A Also, find the identity element of * in A and hence find the invertible elements of A.

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Solution

1. c. R - $\left\{\frac{3}{4}\right\}$

   Explanation: Domain of given function f(x) is given by all real values of x except those values of x for which 4x – 3 = 0. i.e. all reals except x = ¾.

2. d. $(a_1, a_2) \in R, (a_2, a_3) \in R \Rightarrow (a_1, a_3) \in R\ \forall a_1, a_2, a_3 \in A$

   Explanation: A relation R on a non empty set A is said to be transitive if xRy and yRz $\Rightarrow$ xRz, for all x,y,z $\in$ A.

3. b. one - one and onto

   Explanation: If the mappings f: A $\rightarrow$B and g: B $\rightarrow$C are both bijective, then the mapping A$\rightarrow$C is defined as composition from A to C. And every composite mapping is bijective , i.e. one-one and onto.

4. b. the images of distinct elements of X under f are distinct

   Explanation: A function f: X $\rightarrow$ Y is defined to be one – one (or injective), if f $\$x_1 \neq x_2\$in\$$A $\Rightarrow f(x_1) \neq f(x_2)\$$ in A in B.

5. b. Not associative

   Explanation: Here, a * (b * c) = a * (b+ 2c) = a+ 2(b+2c) = a + 2b + 4c and (a *b)*c = (a + 2b) *c = a + 2b + 2c. Since a * (b * c) $\neq$ (a * b)* c. Therefore, *: $R \times R \rightarrow R$ is not associative.

6. fof(x) =f(f(x))

   = (3-(3-x^3))$^{1/3}$

   $= \left(3 - 3 + x^3\right)^{\frac{1}{3}}$

   = x

   $\therefore$ fof(x) = x

7. empty

8. subset

9. universal

10. Let f(x) = x + 1

$$\therefore g(x) = \begin{cases} x-1 & if\ x>1 \\ 1 & if\ x=1 \end{cases}$$

These are two examples in which gof is onto but f is not onto.

11. 3 * 4 = 2 (3) + 4 - 3 = 7

12. Given that f(x) = x^2 - 3x + 2,

f(f(x)) = f(x^2 - 3x + 2),

= $(x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$

= $x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2 - 3x^2 + 9x - 6 + 2$

= $x^4 + 10x^2 - 6x^3 - 3x$

f(f(x)) = $x^4 - 6x^3 + 10x^2 - 3x$

13. Given: f : X $\rightarrow$ Y be an invertible function.

Thus f is 1 - 1 and onto and therefore f^{-1} exists.

Let g_1 and g_2 be two inverses of f. Then for all y $\in$ Y,

$fog_1(y) = I_y(y) = fog_2(y) \therefore fog_1(y) = fog_2(y)$

$\Rightarrow f[g_1(y)] = f[(g_2(y)]$

$\Rightarrow g_1(y) = g_2(y)$

$\therefore$ The inverse is unique and hence f has a unique inverse.

14. i. For (a, a), a < a^3 which is false. $\therefore$ R is not reflexive.

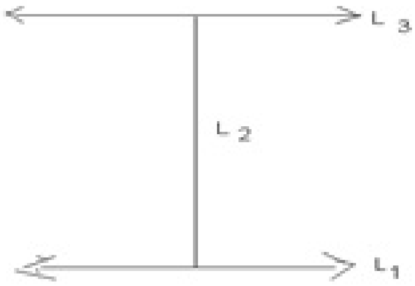
ii. For (a, b), a < b^3 and (b, a), b > a^3 which is false. $\therefore$R is not symmetric.

iii. For a < b^2 b < c^3. Now b < c^3 implies $b^3 < c^9$

Thus, we get a < c^9, therefore (a,c) does not belong to R and hence R is not transitive.

Therefore, R is neither reflexive, nor symmetric and nor transitive.

15. R is not reflexive, as a line L_1 cannot be $\perp$ to itself i.e $(L_1,L_1) \notin R$



Let $(L_1, L_2) \in R$

$\Rightarrow$ $L_1 \perp L_2$

$\Rightarrow$ $L_2 \perp L_1$

$\Rightarrow$ $(L_2, L_1) \in R$

$\Rightarrow$ R is symmetric

Let $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$,then

$L_1 \perp L_2$ and $L_2 \perp L_3$

Then L_1 can never be $\perp$ to L_3 in fact $L_1 \,||\, L_3$

i.e $(L_1,L_2) \in R$, $(L_2,L_3) \in R$.

But $(L_1, L_3) \notin R$

R is not transitive.

16. R = {(a,b): b= a+1}

Symmetric or transitive

R = {(1,2) (2,3) (3,4) (4,5) (5,6) }

R is not reflective , because (1,1) $\notin R$

R is not symmetric because (1,2)$\in R$ but (2,1) $\notin R$

(1, 2) $\in R$ and (2,3) $\in R$

But (1,3) $\notin R$ Hence it is not transitive

17. The given relation is R = {(a, b) : $|a-b|$ is even} defined on set A = {1, 2, 3, 4, 5}.

Reflexive

As $|x-x|$ = 0 is even, $\forall$ x$\in$A.

$\Rightarrow$ (x, x)$\in$R,$\forall$ x $\in$ A

$\therefore$ R is reflexive.

Symmetric Let (x, y) $\in$ R = Ix - yI is even [by the definition of relation]

$\Rightarrow$ $|y-x|$is also even. [$\because$ $|a| = |-a|$,$\forall$a$\in$R]

$\Rightarrow$ (y, x)$\in$R

Thus, (x, y)$\in$R

$\Rightarrow$ (y, x) $\in$ R,$\forall$ x, y$\in$A

$\therefore$ R is symmetric.

Transitive Let (x, y)$\in$ R and (y, z)$\in$R

$\Rightarrow |x-y|$ is even and $|y-z|$ is even. [by using.definition of relation]

Now, $|x-y|$ is even

$\Rightarrow$ x and y both are even or odd.

and $|y-z|$ is even.

$\Rightarrow$ y and z both are even or odd.

Clearly, two cases arises

Case I When y is even. Then, both x and z are even.

$\Rightarrow |x-z|$ is even. [$\because$ difference of two even numbers is even]

$\Rightarrow$ (x, z) $\in$ R

Case II When y is odd.

Then, both x and z are odd = $|x-z|$is even. [$\because$difference of two odd numbers is even]

$\Rightarrow$ (x, z) $\in$ R

Thus, (x, y) $\in$ R and (y, z) $\in$ R

$\Rightarrow$ (x, z) $\in$ R , $\forall$ x, y, z $\in$ A

$\therefore$ R is transitive.

Since, R is reflexive, symmetric and transitive, so it is an equivalence relation.

18. According to the question, We have a binary operation » defined on A = Q- {1} by the rule a * b = a - b + ab.

    Commutative

    Let a, b $\in$ A are arbitrary elements.

    $\therefore a*b=a-b+ab$ and $b*a=b-a+ba$

    $\because a-b+ab\neq b-a+ba$ for some a, b $\in$ A

    $\therefore$ a * b $\neq$ b * a, for some a, b $\in$ A

    $\Rightarrow$ * is not commutative. ...(i)

    Associativity

    Let a, b, c $\in$ A are arbitrary elements.

    Then $a*(b*c)=a*(b-c+bc)$

    = $a-(b-c+bc)+a(b-c+bc)$

= a - b + c - bc + ab - ac + abc

$(a * b) * c = (a - b + ab) * c$

= $(a - b + ab) - c + (a - b + ab)c$

= a - b + ab - c + ac - bc + abc

= a - b - c - bc + ab + ac + abc

a * (b * c)$\neq$ (a * b) * c for some a, b, c $\in$ A

$\Rightarrow$ * is not associative. .....(ii)

From equation (i) and (ii)

* is neither commutative nor associative.

Identity element

Let e be the identity element of * in A.

$\Rightarrow$ we have a * e = a = e * a, $\forall$a $\in$ A

Consider, a * e = a

$\Rightarrow$ a - e + ae = a

$\Rightarrow$ e(a - 1) = 0

$\Rightarrow$ e = 0 [$\because$a $\in$A, $\therefore$ a $\neq$ 1 and so divide both sides by a -1]

Substitute, e = 0 in e * a = a

$\Rightarrow$ 0 * a = a

$\Rightarrow$ 0 - a + 0 = a$\Rightarrow$ -a = a,

which is not true for some a $\in$ A.

$\therefore$ Identity element does not exist

$\therefore$ A does not have any invertible element.