SIGNALS AND SYSTEMS TEST I

Number of Questions: 35 **Section Marks: 90**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. The step response of the system
$y[n] = \beta y[n-1] + x[n]$, $-1 < \beta < 1$
If the initial condition is $y[-1] = 1$

(A) $\frac{\left(1+\beta^{n+2}\right)}{(1-\beta)}u[n]$

(B) $\frac{\left(1-\beta^{n+2}\right)}{(1-\beta)}u[n]$

(C) $\left(\frac{1-\beta^{n+1}}{1-\beta}\right)u[n]$

(D) $\beta^{n+1}u[n]-\frac{1+\beta^{n+1}}{1-\beta}u[n]$

2. If a cosine signal is multiplied with a sinc signal then the resultant frequency characteristic look like _________.
(A) Band pass filter (B) Low pass filter
(C) High pass filter (D) All pass filter

3. A random variable is known to have a cumulative distribution function $F_x(x) = u[x]\left[1-\frac{x^2}{a}\right]$ its density function is ________.

(A) $u(x)-\frac{x}{a}\delta(x)$ (B) $u(x)\frac{2x}{a}e^{\frac{-x^2}{a}}$

(C) $u(x)\left[1-\frac{x^2}{a}\right]\delta(x)$ (D) $\delta(x)-\left(\frac{2x}{a}\right)u(x)$

4. The impulse response of a discrete time system is given by $h[n] = \frac{1}{4}\left[\delta[n]+\delta[n-4]\right]$
The magnitude of the response can be expressed as _________.

(A) $\frac{1}{4}|\cos 4\Omega|$ (B) $|\cos 4\Omega|$

(C) $\frac{1}{2}|\cos 2\Omega|$ (D) $|\cos 2\Omega|$

5. Match list – I & list – II

	List – I		List – II
1.	Odd signal	(P)	$x(n) = \left(\frac{1}{2}\right)^n u(n)$
2.	Energy signal	(Q)	x(–n) = –x(n)
3.	Periodic signal	(R)	x(t) u(t)
4.	Causal signal	(S)	x(n) = x(n + N)

(A) $1-Q$, $2-P$, $3-S$, $4-R$
(B) $1-S$, $2-R$, $3-Q$, $4-P$
(C) $1-S$, $2-R$, $3-P$, $4-Q$
(D) $1-R$, $2-P$, $3-Q$, $4-S$

6. The correct relation is

(A) $x(bt) \xleftrightarrow{F.T} bX(\omega/b)$

(B) $x(bt) \xleftrightarrow{F.T} bX(b\omega)$

(C) $x(t/b) \xleftrightarrow{F.T} bX(\omega/b)$

(D) $x(bt) \xleftrightarrow{F.T} 1/b\, X(\omega/b)$

7. Match list I & List II

	List – I		List – II
(P)	y(n + 4) + y(n + 2) + y(n) = 4x(n + 4) + x(n)	1	Linear, time – invariant, dynamic
(Q)	(n2 + 1) y2(n) + y(n) = 4x2 (n)	2	Non – linear, time – variable, dynamic
(R)	y(n + 3) + y(n + 1) = 4x(n + 1)	3	Non – linear, time – variable, memoryless
(S)	y(n + 3) + ny(n) = 3nx(n)	4	Linear, time – variable, dynamic
		5	Non – linear, time – invariant, dynamic

Codes
(A) $P-2$ $Q-3$ $R-5$ $S-1$
(B) $P-2$ $Q-5$ $R-1$ $S-4$
(C) $P-3$ $Q-5$ $R-2$ $S-1$
(D) $P-3$ $Q-2$ $R-5$ $S-4$

8. A linear system has the transfer function $H(j\omega) = \frac{1}{(j\omega+2)}$.

When it is subjected to an input white noise process with a constant spectral density 'A' the spectral density of the output will be ________.

(A) $\frac{A}{(j\omega+2)}$ (B) $\frac{A}{\sqrt{\omega^2+4}}$

(C) $\frac{A}{(\omega^2+4)}$ (D) $\frac{A}{(\omega^2+2)}$

9. Consider two signals $x_1(t) = e^{j20t}$ and $x_2(t) = e^{(-2+j)t}$
Which one of the following statement is correct.
(A) Both $x_1(t)$ and $x_2(t)$ are periodic.
(B) $x_1(t)$ is periodic but $x_2(t)$ is not periodic
(C) $x_2(t)$ is periodic but $x_1(t)$ is not periodic
(D) Neither $x_1(t)$ nor $x_2(t)$ is periodic

10. Period of the sinusoidal signal
$x[n] = 10\cos[0.5\pi n]$ is _______.
(A) 3 (B) 4
(C) 2 (D) 1

11. Which one of the following is a Dirichlet condition?

(A) $\int_{t_1}^{\infty} |x(t)| < \infty$

(B) signal $x(t)$ must have a finite number of Maxima and minima in the expansion interval.

(C) $x(t)$ can have an infinite number of finite discontinuities in the expansion interval.

(D) $x^2(t)$ must be absolutely summable.

12. A random sinusoidal signal $x(t) = \sin(\omega_o t + \phi)$ where a random variable 'ϕ' is uniformly distributed in the range $\pm\frac{\pi}{2}$. The mean value of $x(t)$ is

(A) $\frac{2\cos\omega_o t}{\pi}$ (B) $\frac{2\sin\omega_0 t}{\pi}$

(C) zero (D) $\frac{2}{\pi}$

13. Consider the following statements regarding a linear discrete time system.

$$H(z) = \frac{z^2+2}{(z+1)(z-0.2)}$$

(1) The initial value $h(0)$ of the impulse response is –10.

(2) The system is stable

(3) The steady state output is zero for a sinusoidal discrete time input of frequency equal to one – fourth the sampling frequency.

The correct statements are

(A) 1, 2 and 3 (B) 1 and 2

(C) 2 and 3 (D) 1 and 3

14. The discrete time system described by $y[n] = x(n^2)$ is

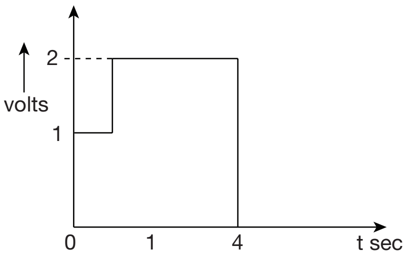
(A) causal, linear and time varying.

(B) non – Causal, linear and time – variant.

(C) causal, non – linear, and time varying

(D) non – causal, non – linear and time – variant.

15.



The Laplace transform of the waveform shown in the figure is ______.

(A) $\frac{1}{s}\left[1+e^{-s}-2e^{-4s}\right]$ (B) $\frac{1}{s}\left[1+e^{s}-2e^{4s}\right]$

(C) $\frac{1}{s}\left[1+e^{-s}-2e^{+4s}\right]$ (D) $\frac{1}{s}\left[1+e^{+s}-2e^{-4s}\right]$

16. The sum of two or more arbitrary sinusoids is

(A) Periodic under certain conditions

(B) Never periodic

(C) Always periodic

(D) Periodic only if all the sinusoids are identical in frequency and phase.

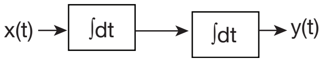
17. The Nyquist rate for the signal

$x(t) = \cos 1000\pi t + 4\sin 3000\pi t$

(A) 2 KHz (B) 3 KHz

(C) 6 KHz (D) 1 KHz

18. Magnitude & phase of the given network is



(A) $\frac{1}{\omega^2}, 0$ (B) $\frac{1}{\omega^2}, \frac{\pi}{2}$

(C) $\frac{1}{\omega^2}, \pi$ (D) $-\frac{1}{\omega^2}, 0$

19. If a LTI system s is given with impulse response $h[n]$ and Z – transform $H(z)$ and

(1) $h[n]$ is real

(2) $h[n]$ is left sided

(3) $H(Z)$ has one of its poles at a non – real location on the circle defined by $|Z| = 2$

(4) $H(Z)$ has two zeros

Then the system S is

(A) non – causal & unstable

(B) stable & causal

(C) non – causal & stable

(D) unstable & causal

20. If $x(t)$ be a signal with Nyquist rate ω_0. Then the Nyquist rate for the signal $y(t) = x^2(t)\cos\omega_0 t + x^2(t)\sin\omega_0 t$ is

(A) $8\omega_0$ (B) $2\omega_0$

(C) $3\omega_0$ (D) $4\omega_0$

21. Two systems S_1 & S_2 are cascaded and their input – output relationship is given as

$S_1 : y_1[n] = 3x_1[n-2] + 4x_1[n-4]$

$S_2 : y_2[n] = 5x_2[n-3] + 3x_2[n-5] + 7x_2[n-7]$

If the overall system is S, then the overall impulse response $h[n]$ for the system S is ____.

(A) $[\underset{\uparrow}{0}, 0, 0, 0, 0, 15, 20, 9, 21, 12, 0, 28]$

(B) $[\underset{\uparrow}{0}, 0, 0, 0\ 0, 15, 0, 29, 0, 33, 0, 28]$

(C) $[\underset{\uparrow}{15}, 0, 29, 0, 33, 0, 28]$

(D) $[\underset{\uparrow}{0}, 0, 0, 0, 15, 0, 33, 0, 29, 0, 28]$

22. A signal $x(t)$ is given as

$$x(t) = \sin\left(\frac{3}{8}t + 30^\circ\right) + \cos\left(\frac{3}{4}t + 45^\circ\right)$$

is periodic. The harmonics present in $x(t)$ are

(A) Only 1st Harmonic
(B) 1st and 2nd Harmonic
(C) 2nd and 3rd Harmonics
(D) 1and and 3rd Harmonics

23 Time delay of a sequence $x[n]$ is given by

$$D = \frac{\sum_{n=-\infty}^{+\infty} nx[n]}{\sum_{n=-\infty}^{+\infty} x[n]}. \text{ For } x[n] = \left(\frac{1}{3}\right)^n u[n],$$

value of D at ω = 0 is

(A) 2/3 (B) 1
(C) 1/2 (D) 2

24. N–point DFT of $x[n] = u[n–N]$ is

(A) $X[k] = \begin{cases} 0 & for\ k \neq 0 \\ N & for\ k = 0 \end{cases}$ (B) $X[k] = 0$ for all k

(C) $X[k] = N$ for all k (D) $X[k] = \begin{cases} N & for\ k \neq 0 \\ 0 & for\ k = 0 \end{cases}$

25. A signal $x(t)$ has a duration of 5ms and an essential bandwidth of 25KHz and it is desirable to have frequency resolution of 125Hz in the DFT. The period N_o for discrete signal $x[n]$ is ________.

(A) 100 (B) 200
(C) 400 (D) 800

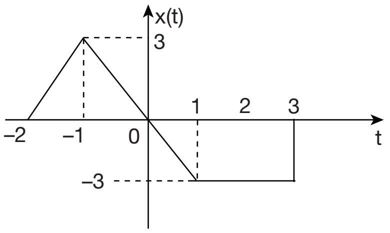
26. The phase spectrum of a function is

(A) discrete function
(B) odd function
(C) symmetric function
(D) anti – symmetric function

27. $x[n]$ is a real periodic sequence, with period N and fourier co efficient of $x[n]$ is given as $C_K = a_k + jb_k$, if a_k, b_k are real then

(A) $a_{-k} = -a_k$, $b_{-k} = b_k$ (B) $a_{-k} = a_k$, $b_{-k} = -b_k$
(C) $a_{-k} = -a_k$, $b_{-k} = -b_k$ (D) $a_{-k} = a_k$, $b_{-k} = b_k$

28. The equation for the waveform shown in the figure is _________.



(A) $3(t + 2)\, u(t + 2) – 6(t + 1)\, u(t + 1) + 3(t – 1)\, u(t – 1) + 3\, u(t – 3)$
(B) $3(t + 2)\, u(t + 2) – 3(t + 1)\, u(t + 1) + 2(t – 1)\, u(t – 1) + 3U(t – 3)$
(C) $3(t + 2)\, u(t + 2) + 3(t + 1)\, u(t + 1) + 3(t – 1)\, u(t – 1) – 2u(t – 3)$
(D) None of the above

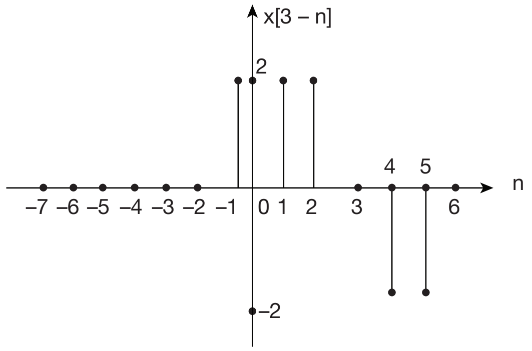
29. The impulse response of a continuous time LTI system is $h(t) = e^{-4t}u(4 – t)$

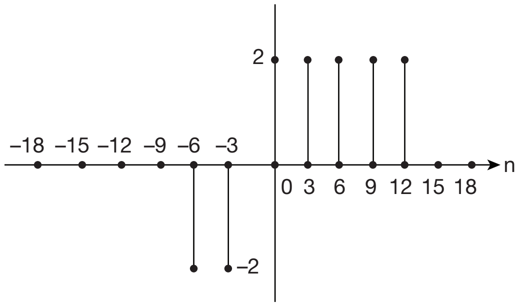
The system is

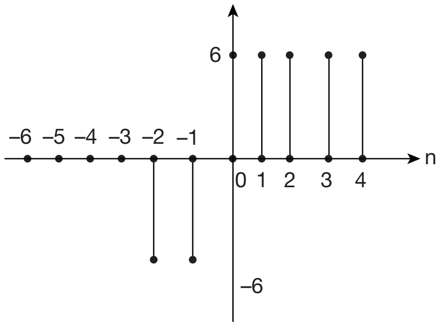
(A) stable but not causal
(B) causal but not stable
(C) causal and stable
(D) neither causal nor stable

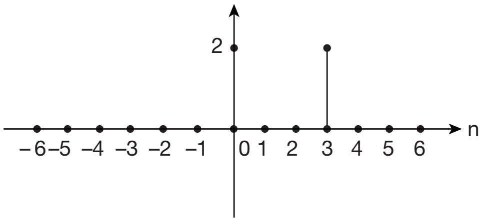
30. The discrete time signal $x[3 – n]$ shown below

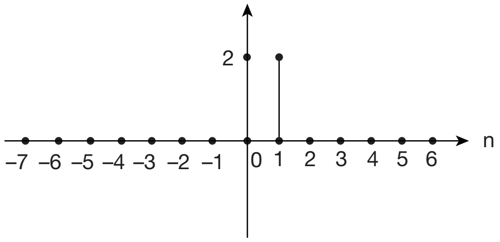
The signal $x\,[3n]$ is



(A)


(B)


(C)


(D)


31. Consider the three LTI systems with impulse response

$h_1(t) = u(t)$

$h_2(t) = -2\delta(t) + 5e^{-2t}u(t)$

$h_3(t) = 2te^{-}_{t}\, u(t)$

The response to $x(t) = cost$ of above systems are

$y_1(t) = x(t) * h_1(t)$

$y_2(t) = x(t) * h_2(t)$

$y_3(t) = x(t) * h_3(t)$

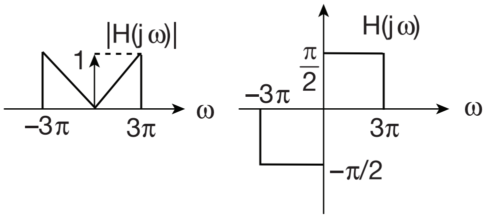
which of the following gives same response

(A) $y_1(t)$ & $y_2(t)$ (B) $y_2(t)$ & $y_3(t)$

(C) $y_3(t)$ & $y_1(t)$ (D) All $y_1(t)$, $y_2(t)$ & $y_3(t)$

Common data for questions 32 and 33:

The frequency response $H(j\omega)$ of a continuous time filter is shown below.



32. If the input to this system is $x(t) = \cos(2\pi t + \theta)$, then output will be

(A) $\frac{-2}{3}\sin(2\pi t+\theta)$ (B) $\frac{2\pi}{3}\sin(2\pi t+\theta)$

(C) $\frac{2}{3\pi}\sin(2\pi t+\theta)$ (D) None of these

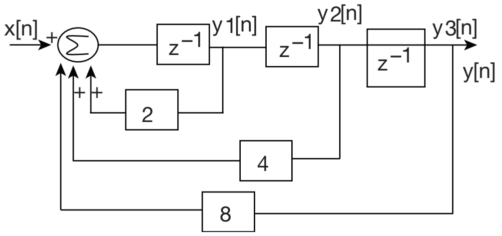
33. If the input to the system is $x(t) = \cos(4\pi t + \theta)$. Then the output will be

(A) $-\frac{4\pi}{3}\cos(4\pi t+\theta)$ (B) $\frac{-4}{3}\cos(4\pi t+\theta)$

(C) $\frac{4}{3\pi}\cos(4\pi t+\theta)$ (D) 0

Statement for Linked Answer questions 34 and 35:

An LTI system is given below for $n \geq 0$



34. Transfer function $H(Z)$ for the system is

(A) $\dfrac{z^{-3}}{2z^{-2}+4z^{-3}+8}$

(B) $\dfrac{1}{z^3-2z^2-4z+8}$

(C) $\dfrac{1}{z^3+2z^2+4z-8}$

(D) $\dfrac{Z^{-3}}{1-2z^{-1}-4z^{-2}-8z^{-3}}$

35. If $x[n] = \delta[n]$ then $y_3[n]$ is

(A) [1, 2, 8, 16] (B) $\left[\underset{\uparrow}{\frac{1}{32}}\ \frac{1}{16}\ \frac{1}{16}\ \frac{1}{8}.......\right]$

(C) $\left[....\frac{1}{32}\ \frac{1}{16}\ \frac{1}{16}\ \underset{\uparrow}{\frac{1}{8}}\right]$ (D) [0, 0, 0, 1, 2, 8, 16...]

ANSWER KEYS

1. B	**2.** A	**3.** D	**4.** C	**5.** A	**6.** D	**7.** B	**8.** C	**9.** B	**10.** B
11. B	**12.** B	**13.** C	**14.** B	**15.** A	**16.** A	**17.** B	**18.** C	**19.** C	**20.** D
21. B	**22.** B	**23.** C	**24.** A	**25.** C	**26.** C	**27.** B	**28.** A	**29.** D	**30.** D
31. D	**32.** A	**33.** D	**34.** B	**35.** D					

HINTS AND EXPLANATIONS

1. By taking z – transform on both sides,

$Y(z) = \beta[(z^{-1}\,Y(z) + y(-1)] + X(z)$

$(1 - \beta z^{-1})\,Y(z) = \beta + \dfrac{1}{1-z^{-1}}$ [∵ $x[n]$ is unit step]

$$Y(z) = \frac{\beta}{1-\beta z^{-1}} + \frac{1}{(1-\beta z^{-1})(1-z^{-1})}$$

By taking Inverse z – transform

$$y[n] = \beta\,.\,\beta^n\,u[n] + \frac{(1-\beta^{n+1})}{(1-\beta)}u[n]$$

$$= \beta^{n+1}u[n] + \left(\frac{1-\beta^{n+1}}{1-\beta}\right)u[n]$$

$$= \frac{(1-\beta^{n+2})}{(1-\beta)}u[n]$$

Choice (B)

2. Fourier transform of cosine signal is delta function.

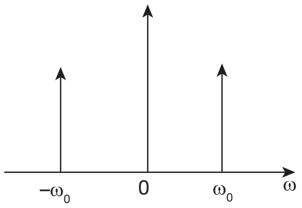
⇒ Fourier transform of sinc signal is gate function.

⇒ We know that multiplication in time domain is equal to convolution in frequency domain.

⇒ When delta function is convoluted with gate signal then the gate signal will itself present at the frequency where delta function was present.

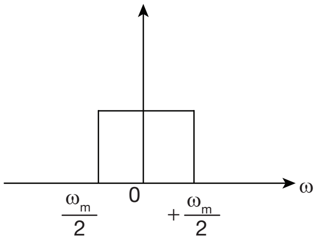
Let cosine signal is cos $\omega_o t$

F. T. of $\cos\omega_0 t$ is



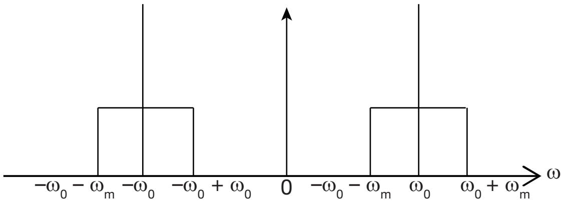
Let sin *c* signal is $\frac{\omega_m}{2\pi}\sin C\left(\frac{\omega_m t}{2}\right)$

Then *F. T* of $\frac{\omega_m}{2\pi}\left(\sin c\left(\frac{\omega_m t}{2}\right)\right)$ is



So after convolution

Resultant will look like



So like band pass filter. Choice (A)

3. CDF = $F_X(x) = u[x]\left[1-\frac{x^2}{a}\right]$

We know that

$$\frac{d}{dx}(CDF) = pdf$$

$$\frac{d}{dx}F_x(x) = \frac{d}{dx}\left[u[x]\left(1-\frac{x^2}{a}\right)\right]$$

$$= \left[\frac{d}{dx}u(x)\right]\left[1-\frac{x^2}{a}\right] + u(x)\frac{d}{dx}\left(1-\frac{x^2}{a}\right)$$

$$= \delta(x)\left[1-\frac{x^2}{a}\right] + u(x)\left(\frac{-2x}{a}\right) \because \left[\begin{matrix}\delta(x).\frac{x^2}{a} = 0\\ at x = 0\end{matrix}\right]$$

$$= \delta(x) - \frac{2x}{a}u(x)$$

Choice (D)

4. $h[n] = \frac{1}{4}[\delta(n)+\delta(n-4)]$

Taking *z* – transform $H(z) = \frac{1}{4}[1+z^{-4}]$

Put $z = e^{j\Omega}$

$$H(e^{j\Omega}) = \frac{1}{4}\left[1+e^{-j4\Omega}\right]$$

$$= \frac{1}{4}e^{-j2\Omega}\left[e^{+j2\Omega}+e^{-j2\Omega}\right]$$

$$H(e^{j\Omega}) = \frac{1}{2}e^{-j2\Omega}[\cos 2\Omega]$$

$$H(e^{j\Omega}) = \frac{1}{2}\left|e^{-j2\Omega}\right|\left|\cos 2\Omega\right|$$

$$= \frac{1}{2}\left|\cos 2\Omega\right|$$

Choice (C)

5. odd signal = $x(n) = -x(-n)$

Energy signal is absolutely summable

$$\sum_{n=0}^{\infty}\left|\left(\frac{1}{2}\right)^n u(n)\right| < \infty$$

periodic signal satisfies $x(n) = x(n+N)$

Where N = Fundamental period

Causal system is one which output at any time depends only on present and/ or past values of input.

Choice (A)

6. By scaling property

$$x(bt) \xleftrightarrow{FT} \frac{1}{b}X\left(\frac{\omega}{b}\right)$$

Choice (D)

7. Choice (B)

8. $S_0(\omega) = |H(\omega)|^2\, Si(\omega)$

$$= \left[\frac{1}{(\omega^2+4)}\right]A = \frac{A}{(\omega^2+4)}$$

Choice (C)

9. $x_1(t)$ is periodic with period $\frac{\pi}{10}$ but $x_2(t)$ is not periodic.

Choice (B)

10. $x[n] = 10\cos[0.5\pi n]$

$$\frac{2\pi}{N} = \omega$$

Where N = time period

So, $N = \frac{2\pi}{0.5\pi} = 4.$

Choice (B)

11. Choice (B)

12. $\overline{x(t)} = \overline{\sin(\omega_0 t+\varphi)}$

$$= \int_{-\infty}^{+\infty}\sin(\omega_0 t+\varphi)\int\varphi(\varphi)\,d\varphi$$

$$= \frac{1}{\pi}\int_{-\pi/2}^{+\pi/2}\sin(\omega_0 t+\varphi)\,d\varphi$$

$$= \frac{1}{\pi}\left[-\cos\left(\omega_0 t+\varphi\right)\right]_{\frac{-\pi}{2}}^{\frac{+\pi}{2}}$$

$$= \frac{1}{\pi}\left[-\cos\left(\omega_0 t+\frac{\pi}{2}\right)+\cos\left(\omega_0 t-\frac{\pi}{2}\right)\right]$$

$$= \frac{1}{\pi}\left[\sin\omega_0 t+\sin\omega_0 t\right] = \frac{2\sin\omega_0 t}{\pi}$$ Choice (B)

13. (1) Roots of $H(z)$ are at $z = -1$ and $z = 0.2$ so the one root is inside the unit circle and second root is on the unit circle. So system is marginally stable.

(2) Initial value of $h(t)$ by Initial value theorem

$$\underset{t\to 0}{Lim}^{h(t)} = \underset{Z\to\infty}{Lim}^{H(z)}$$

$$= \underset{Z\to\infty}{Lim}\frac{z^2+2}{(z+1)(z-0.2)}$$

$$= \underset{Z\to\infty}{Lim}\frac{1+\frac{2}{z^2}}{\left(1+\frac{1}{z}\right)\left(1-\frac{0.2}{Z}\right)} = 1.$$ Choice (C)

14. $y[n] = x[n^2]$

(i) For $n = -2$, $y(-2) = x(4)$

Since, $y[n]$ depends on future states also, it is a non – causal system

(ii) let $x_3[n] = ax_1[n] + bx_2[n]$

$y_3[n] = x_3[n^2]$

$= ax_1[n^2] + bx_2[n^2]$

$= ay_1[n] + by_2[n]$

∴ system is linear

(iii) $x_1[n] \Rightarrow y_1[n] = x_1[n^2]$

Let $x_2[n] = x_1[n - n_0]$

$\Rightarrow$ $y_2[n] = x_2[n^2]$

$= x_1[n^2 - n_0]$

$y_1[n - n_0] = x_1[n - n_0]^2$

$y_2[n] \neq y_1[n - n_0]$

∴ The system is time variant. Choice (B)

15. Let $v(t) = u(t) + u(t - 1) - 2u(t - 4)$

By taking laplace transform

$$V(s) = \frac{1}{s}+\frac{e^{-s}}{s}\frac{-2e^{-4s}}{s}$$

$$= \frac{1}{s}\left[1+e^{-s}-2e^{-4s}\right].$$ Choice (A)

16. Periodic under certain conditions. Choice (A)

17. $x(t) = \cos 1000\pi t + 4\sin 3000\pi t$

Maximum frequency component

$$f_m = \frac{3000\pi}{2\pi} = 1500 \text{ Hz}$$

$$f_{Nyquist} = 2f_m$$

$= 2 \times 1500 = 3000$Hz or 3 KHz.

Choice (B)

18. $H(s) = H_1(s) \times H_2(s) = \frac{1}{s}\times\frac{1}{s}$

$$H(j\omega) = \frac{1}{j\omega}\times\frac{1}{j\omega} = \frac{1}{-\omega^2} = \left|\frac{1}{\omega^2}\right|e^{+j\pi}$$

So $\left|H(j\omega)\right| = \frac{1}{\omega^2}$

And $H(j\omega) = \pi$. Choice (C)

19. $\Rightarrow$ Since $h[n]$ is left sided sequence so system S is non-causal because ROC is inside the circle.

$\Rightarrow$ For stability, ROC should include unit circle & if it is left sided sequence & one of its poles at $|Z| = 2$ so ROC includes the unit circle so stable.

Choice (C)

20. $y(t) = x^2(t)\cos\omega_o t + x^2(t)\sin\omega_o t$

Let $x^2(t) = Z(t)$

Then $Z(t)$ will have double frequency in comparison to $x(t)$

Let maximum component present in $Z(t)$ is ω_z then $\omega_z = \omega_0 = 2\,\omega_x$

Where ω_x = maximum frequency component present in signal $x(t)$

$Y(\omega) = \frac{1}{2}[Z(\omega_Z + \omega_O) + Z(\omega_Z - \omega_O)] + \frac{j}{2}[Z(\omega_z + \omega_o) - (\omega_z - \omega_o)]$

Maximum frequency component present is $Y(\omega)$ is $(\omega + \omega_o)$

& $\omega_Z = \omega_o$

So $\omega_{\max}$ in $Y(\omega) = 2\omega_0$

Nyquist rate $= 4\omega_0$. Choice (D)

21. $y_1[n] = 3x_1[n - 2] + 4x_1[n - 4]$

$y_2[n] = 5x_2[n - 3] + 3x_2[n - 5] + 7x_2[n - 7]$

$Y_1[z] = 3z^{-2} X_1(z) + 4z^{-4} X_1(z)$

$Y_2(z) = 5z^{-3} X_2(Z) + 3z^{-5} X_2(z) + 7z^{-7} X_2(z)$

$H_1(z) = 3z^{-2} + 4z^{-4}$

$H_2(z) = 5z^{-3} + 3z^{-5} + 7z^{-7}$

$H_1(z)$ & $H_2(z)$ are cascaded

$H(z) = H_1(z) H_2(z)$

$= (3z^{-2} + 4z^{-4})(5z^{-3} + 3z^{-5} + 7z^{-7})$

$= 15z^{-5} + 9z^{-7} + 21z^{-9} + 20z^{-7} + 12z^{-9} + 28z^{-11}$

$H(z) = 15z^{-5} + 29z^{-7} + 33z^{-9} + 28z^{-11}$

$h[n] = [\ldots.0, 0, 0, 0, 0, 15, 0, 29, 0, 33, 0, 28, 0, 0\ldots.]$

$h[n] = [0, 0, 0, 0, 0, 15, 0, 29, 0, 33, 0, 28]$

Choice (B)

22. ∵ $x(t)$ is periodic, so $x(t)$ is periodic with fundamental

period ω_0 = LCM of $\left(\frac{3}{8}, \frac{3}{4}\right)$

Fundamental period $\omega_0 = \frac{3}{8}$

& $\omega_1 = \frac{3}{8}$ & $\omega_2 = \frac{3}{4}$

So Harmonic = $\frac{\omega_1}{\omega_0}$ and $\frac{\omega_2}{\omega_0}$

= 1st and 2nd Harmonics. Choice (B)

23. Let $X\left(e^{j\omega}\right)\xleftrightarrow{f} x[n]$

From the differentiation in frequency property of DTFT

$$\sum_{n=-\infty}^{+\infty} nx[n]e^{-j\omega n} = \frac{jdX\left(e^{j\omega}\right)}{d\omega}$$

So, $\sum_{-\infty}^{+\infty} nx[n] = \frac{J.d\times\left(e^{j\omega}\right)}{dw}\Big|_{\omega=0}$ ———— (1)

DFTF of $x[n]$ is $x\left[e^{j\omega}\right] = \sum_{-\infty}^{+\infty} x[n]e^{-j\omega n}$

$\sum_{n=-\infty}^{+\infty} x[n] = X\left(e^{j0}\right)$ ———— (2)

$$D = \frac{j\frac{dX\left(e^{j\omega}\right)}{dw}\Big|_{w=0}}{X\left(e^{j0}\right)}$$

for $x[n] = \left(\frac{1}{3}\right)^n u[n]$

$$X(e^{j\omega}) = F\left\{\left(\frac{1}{3}\right)^n u[n]\right\} = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$\text{So } \frac{jdX\left(e^{j\omega}\right)}{d\omega}\Big|_{\omega=0} = \frac{\frac{1}{3}e^{-j\omega}}{\left(1-\frac{1}{3}e^{-j\omega}\right)^2}\Bigg|_{\omega=0}$$

$$= \frac{\frac{1}{3}}{\left(1-\frac{1}{3}\right)^2} = \frac{3}{4}$$

and $X(e^{j0}) = \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$

So $D = \frac{\frac{3}{4}}{\frac{3}{2}} = \frac{1}{2}$ Choice (C)

24. As we know that N-point DFT of $x[n]$ is

$$X[k] = \sum_{n=0}^{N-1} x[n]\omega_N^{kn} = \sum_{n=0}^{N-1} \omega_N^{kn}$$

$$\frac{1-\omega_N^{kn}}{1-\omega_N^{k}} = 0, k \neq 0$$

$$\Rightarrow \omega_N^{kn} = e^{-j\left(\frac{2\pi}{N}\right)kN} = e^{-jk2\pi} = 1$$

And $X[0] = \sum_{n=0}^{N-1}\omega_N^0 = \sum_{n=0}^{N-1} 1 = N$ for $k = 0$.

Choice (A)

25. $f_0 = 125\text{Hz}$

So $T_0 = \frac{1}{125} = 8\text{ms}$

Since signal duration is of 8ms. So for 3ms, we use padding.

Bandwidth $B = 25\text{KHz}$

So sampling frequency $f_s = 2B = 50\text{KHz}$

So $N_o = \frac{f_s}{f_0} = \frac{50\times10^3}{125} = 400$. Choice (C)

26. Phase – spectrum of a function is symmetric function. Choice (C)

27. Choice(B)

28. at $t = -2$, the slope of the signal changes from 0 to 3, for a change in slope 3.

At $t = -1$, the slope of the signal changes from 3 to –3, for a changes is slope of –6

At $t = 1$ the slope becomes 0 for a change of 3

At $t = 3$, the function steps from –3 to 0 for a change in amplitude of 3,

Hence equation for $x(t)$ is

$3(t + 2)\,u(t + 2) - 6(t + 1)\,u(t + 1) + 3(t - 1)\,u(t - 1) + 3u(t - 3)$ Choice (A)

29. Not causal because $h(t) \neq 0$ for $t < 0$,

unstable because $\int_{-\infty}^{+\infty}|h(t)|dt = \infty$. Choice (D)

30. Let $x[3 - n] = v[n]$

$n = 0, x[3 - 0] = v[0] \Rightarrow x[3] = v[0] = 2$

$n = 1, x[3 - 1] = v[1] \Rightarrow x[2] = v[1] = 2$

$n = 2, x[3 - 2] = v[2] \Rightarrow x[1] = v[2] = 2$

$n = 3, x[3 - 3] = v[3] \Rightarrow x[0] = v[3] = 0$

Let $g[n] = x[3n] = g[0] = x[0] = 0$

$g[1] = x[3] = 2$

$g[-1] = x[-3] = 0$. Choice (D)

31. $X(j\omega) = \pi\,[\delta(\omega + 1) + \delta(\omega - 1)]$

$$H_1(j\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$$

$$Y_1(j\omega) = \left[\frac{1}{j\omega} - \pi\delta(\omega)\right]\pi\left[\delta(\omega+1)+\delta(\omega-1)\right]$$

$= j\omega\,[\delta(\omega + 1) - \delta(\omega - 1)]$ (at $\omega = 1$ & $\omega = 1$ only)

$y_1(t) = \sin(t)$

$$H_2(j\omega) = -2 + \frac{5}{2+j\omega} = \frac{1-2j\omega}{2+j\omega}$$

$$Y_2(j\omega) = \left[\frac{1-2j\omega}{2+j\omega}\right]\pi\left[\delta(\omega+1)+\delta(\omega-1)\right]$$

$$= \left(\frac{1+2j}{2-j}\right)\pi\delta(\omega+1) + \left(\frac{1-2j}{2+j}\right)\pi\delta(\omega-1)$$

$= j\pi\ \{\delta(\omega + 1) - \delta(\omega - 1)$ (at $\omega = -1$ & $\omega = 1$ only)

$y_2(t) = \sin t$

$$H_3(j\omega) = \frac{2}{(1+j\omega)^2}$$

$$Y_3\ (j\omega) = \frac{2}{(1+j\omega)^2}\pi\{\delta(\omega+1)+\delta(\omega-1)\}$$

$$= \frac{2\pi}{(1-j)^2}\delta(\omega+1)+\frac{2\pi}{(1+j)^2}\delta(\omega-1)$$

$= j\pi\ [\delta(\omega + 1) - \delta(\omega - 1\}\ ,\ y_3(t) = \sin t$

(at $\omega = -1$ & $\omega = 1$ only). Choice (D)

32. $$H(J\omega) = \begin{cases} \dfrac{\omega}{3\pi}e^{\frac{j\pi}{2}}, & 0 \le \omega \le 3\pi \\ \dfrac{-\omega}{3}e^{\frac{-j\pi}{2}} & -3\pi \le \omega \le 0 \\ 0 & otherwise \end{cases}$$

$$\Rightarrow \quad H(j\omega) = \begin{cases} \dfrac{j\omega}{3\pi}; & -3\pi \le \omega \le 3\pi \\ 0 & otherwise \end{cases}$$

$x(t) = \cos(2\pi t + \theta)$

$X(j\omega) = e^{j\theta}\ \pi\ \delta(\omega - 2\pi) + e^{-j\theta}\ \pi\ \delta(\omega + 2\pi)$

This is zero outside the region $-3\pi \le \omega \ge 3\pi$

Thus $Y(\omega) = H(j\omega) \times (j\omega)$

$$= \frac{j\omega}{3\pi}X(j\omega)$$

$$y(t) = \frac{1}{3\pi}\frac{dx(t)}{dt} = \frac{-2}{3}\sin(2\pi t+\theta)$$ Choice (A)

33. $x(t) = \cos(4\pi t + \theta)$

$X(j\omega) = e^{j\theta}\ \pi\ \delta(\omega - 4\pi) + e^{-j\theta}\ \pi\ \delta(\omega + 4\pi)$

The non – zero position of $X(jw)$ lie outside the range $-3\pi \le \omega \le 3\pi$. This implies that $Y(j\omega) = X(j\omega)\ H(j\omega) = 0$, therefore $y(t) = 0$.

Choice (D)

34. $y_3[n] = y_2[n - 1]$ ——— (1)

$y_2[n] = y_1[n - 1]$ ——— (2)

$y_1[n] = x[n - 1] + 2y_1[n - 1] + 4y_2[n - 1] - 8y_3[n - 1]$ ——— (3)

$y_3[n] = y_1[n - 2]$ ——— (4)

$= x[n - 3] + 2y_2[n - 3] + 4y_2[n - 3] - 8y_3[n - 3]$

By eq (4) & (2)

$y_3[n] = x[n - 3] + 2y_3[n - 1] + 4y_3[n - 2] -8y_3[n - 3]$

By taking z – Transform

$Y_2(z) = [1 - 2z^{-1} - 4z^{-2} + 8z^{-3}] = z^{-2}\ X(z)$

$$Y_3(z) = \frac{z^{-3}}{1-2z^{-1}-4z^{-2}+8z^{-3}}$$

$$= \frac{1}{z^3-2z^2-4z+8}$$ Choice (B)

35. $$H(z) = \frac{1}{z^3-2z^2-4z+8}$$

$$Y_3(z) = \frac{X(z)}{z^3-2z^2-4z+8}$$

$$= \frac{1}{z^3-2z^2-4z+8}$$

Since given LTI system is for $n \ge 0$ so it is causal system (Right sided sequence)

$Z^3 - 2Z^2 - 4Z + 8$

$$\begin{array}{r} z^{-3}+2z^{-4}+8z^{-5}+16z^{-6}.... \\ \hline \big) \quad 1 \qquad\qquad\qquad\qquad \\ 1-2z^{-1}-4z^{-2}+8z^{-3} \\ \hline +2z^{-1}-4z^{-2}+8z^{-3} \\ +2z^{-1}-4z^{-2}+8z^{-3}+16z^{-4} \\ \hline 8-2z^{-2}-16z^{-4} \\ 8-2z^{-2}-32z^{-4}-16z^{-3}+64z^{-5} \\ \hline +16z^{-3}+16z^{-4}-64z^{-5} \end{array}$$

$Y_3(z) = Z^{-3} + 2Z^{-4} + 8Z^{-5} + 16Z^{-6}$ …….

$y_3[n] = [0, 0, 0, 1, 2, 8, 16, ….]$ Choice (D)