Probability of event B $\quad P(B) = \frac{m}{n}$

$= \frac{306}{1140}$

$= \frac{51}{190}$

Required probability $= \frac{51}{190}$

(3) Event of getting all three non-defective items $= C$

Selecting 3 items from 18 non-defective items and not selecting any item from the defective items will be the favourable outcomes of event C.

The number of such outcomes $m = {}^{18}C_3 \times {}^{2}C_0 = 816 \times 1 = 816$.

Probability of event C $\quad P(C) = \frac{m}{n}$

$= \frac{816}{1140}$

$= \frac{68}{95}$

Required probability $= \frac{68}{95}$

Illustration 18 : A box contains 10 chits of which 3 chits are eligible for a prize. A boy named Kathan randomly selects two chits from this box. Find the probability that Kathan gets the prize.

There are 10 chits of which 3 chits are eligible for a prize and 7 chits are not eligible for prize. If two chits are randomly selected from these 10 chits then the number of mutually exclusive, exhaustive and equiprobable outcomes in the sample space will be $n = {}^{10}C_2 = \frac{10 \times 9}{2} = 45$.

Event of Kathan getting prize $= A$

$\therefore$ Event that Kathan does not get prize $= A'$

The outcomes in which Kathan will draw 2 chits at random from the 7 chits which are not eligible for prize will be the favourable outcomes of the event A'.

The number of such outcomes $m = {}^{7}C_2 = 21$.

Probability of A' $\quad P(A') = \frac{m}{n}$

$= \frac{21}{45}$

$= \frac{7}{15}$

Now $P(A) = 1 - P(A')$

$= 1 - \frac{7}{15}$

$= \frac{8}{15}$

Thus, probability that Kathan gets prize $= \frac{8}{15}$

Limitations : The limitations of the mathematical definition of probability are as follows :

(1) The probability of an event cannot be found by this definition if there are infinite outcomes in the sample space of a random experiment.

(2) The probability of an event cannot be found by this definition if the total number of outcomes in the sample space of a random experiment are not known.

(3) The probability of an event cannot be found by this definition if the elementary outcomes in the sample space of a random experiment are not equi-probable.

(4) The word 'equi-probable' is mentioned in the mathematical definition of probability. Equi-probable events are the events with same probability. Thus, the word probability is used in the definition of probability.

Exercise 1.2

1. A balanced coin is tossed three times. Find the probability of the following events :

(1) Getting all three heads

(2) Not getting a single head

(3) Getting at least one head

(4) Getting more than one head

(5) Getting at the most one head

(6) Getting less than two heads

(7) Getting head and tail alternately

(8) Getting more number of heads than tails

2. Two balanced dice are thrown simultaneously. Find the probability of the following events :

(1) The sum of numbers on the dice in 6

(2) The sum of numbers on the dice is not more than 10

(3) The sum of numbers on the dice is a multiple of 3

(4) The product of numbers on the dice is 12

3. One family is randomly selected from the families having two children. Find the probability that

(1) One child is a girl and one child is a boy.

(2) At least one child is a girl among the two children of the selected family.

(**Note :** Assume that the chance of the child being a boy or girl is same.)

4. One number is selected at random from the first 100 natural numbers. Find the probability that this number is divisible by 7.

5. The sample space for a random experiment of selecting numbers is $U = \{1, 2, 3, ..., 120\}$ and all the outcomes in the sample space are equiprobable. Find the probability that the number selected is

(1) a multiple of 3

(2) not a multiple of 3

(3) a multiple of 4

(4) not a multiple of 4

(5) a multiple of both 3 and 4.

6. Find the probability of getting R in the first place and M in the last place when all the letters of the word *RANDOM* are arranged in all possible ways.

7. Find the probability of getting vowels in the first, third and sixth place when all the letters of the word *ORANGE* are arranged in all possible ways.

8. Five members of a family, husband, wife and three children, are randomly arranged in a row for a family photograph. Find the probability that the husband and wife are seated next to each other.

9. Seven speakers A, B, C, D, E, F, G are invited in a programme to deliver speech in random order. Find the probability that speaker B delivers speech immediately after speeker A.

10. Find the probability of having 5 Mondays in the month of February of a leap year.

11. Find the probability of having 53 Fridays in a year which is not a leap year.

12. Find the probability of having 5 Tuesdays in the month of August of any year.

13. 4 couples (husband-wife) attend a party. Two persons are randomly selected from these 8 persons. Find the probability that the selected persons are,

(1) husband and wife (2) one man and one woman

(3) one man and one woman who are not husband and wife.

14. 8 workers are employed in a factory and 3 of them are excellent in efficiency where as the rest of them are moderate in efficiency. 2 workers are randomly selected from these 8 workers. Find the probability that,

(1) both the workers have excellent efficiency

(2) both the workers have moderate efficiency

(3) one worker is excellent and one worker is moderate in efficiency.

15. Two cards are drawn from a well shuffled pack of 52 cards. Find the probability that,

(1) both the cards are of different colour (2) both the cards are face cards

(3) one of the two cards is a king.

16. 3 bulbs are defective in a box of 10 bulbs. 2 bulbs are randomly selected from this box. These bulbs are fixed in two bulb-holders installed in a room. Find the probability that the room will be lighted after starting the electric supply.

17. For two events A and B in the sample space of a random experiment, $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.15$. Find

(1) $P(A')$ (2) $P(B - A)$ (3) $P(A \cap B')$ (4) $P(A' \cap B')$ (5) $P(A' \cup B')$

18. For two events A and B in the sample space of a random experiment, $P(A') = 2P(B') = 3P(A \cap B) = 0.6$. Find the probability of difference events $A - B$ and $B - A$.

*

1.5 Law of Addition of Probability

The rule of obtaining the probability of the occurrence of at least one of the event A and B in the sample space of a random experiment is called the law of addition of probability. We have seen earlier that the occurrence of at least one of the events A and B is denoted by $A \cup B$, the union of events A and B. Hence we can say that the law of addition of probability is the rule of obtaining the probability of $A \cup B$, the union of events A and B. This rule is stated as follows and we will accept it without proof :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

The law of addition of probability can also be used for obtaining the probability of union of more than two events. The law of addition of probability for $A \cup B \cup C$, the union of three events A, B and C is as follows :

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Some of the important results obtained from this rule are as follows :

(1) If the events A and B in the sample space of a random experiment are mutually exclusive then $A \cap B = \phi$ and $P(A \cap B) = 0$. Hence,

$$P(A \cup B) = P(A) + P(B)$$

(2) If three events A, B and C in the sample space of a random experiment are mutually exclusive then,

$A \cap B = \phi$, $A \cap C = \phi$, $B \cap C = \phi$, $A \cap B \cap C = \phi$ and

$P(A \cap B) = P(A \cap C) = P(B \cap C) = P(A \cap B \cap C) = 0$. Hence,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

(3) If two events A and B in the sample space of a random experiment are mutually exclusive and exhaustive then $A \cap B = \phi$ and $A \cup B = U$. As $P(\phi) = 0$ and $P(U) = 1$, $P(A \cap B) = 0$ and $P(A \cup B) = 1$.

$$P(A \cup B) = P(A) + P(B) = 1$$

(4) If three events A, B and C in the sample space of a random experiment are mutually exclusive and exhaustive then,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) = 1$$

Illustration 19 : A number is randomly selected from the first 50 natural numbers. Find the probability that it is a multiple of 2 or 3.

If one number is randomly selected from the first 50 natural numbers then the number of mutually exclusive, exhaustive and equi-probable outcomes in the sample space of this random experiment will be $n = {}^{50}C_1 = 50$.

If event A denotes that the number selected is a multiple of 2 and event B denotes that the number selected is a multiple of 3 then the event that the selected number is a multiple of 2 or 3 will be denoted by $A \cup B$. (This event can also be denoted as $B \cup A$. According to set theory, $A \cup B = B \cup A$). To find the probability of $A \cup B$, the union of events A and B by the law of addition of probability, we will first find $P(A), P(B)$ and $P(A \cap B)$.

A = Event that the selected number is a multiple of 2
= {2, 4, 6,..., 50}

Hence, the number of favourable outcomes of event A will be m = 25.

Probability of event A $P(A) = \frac{m}{n}$

$= \frac{25}{50}$

B = Event that the selected number is a multiple of 3
= {3, 6, 9,..., 48}

Hence, the number of favourable outcomes of event B will be m = 16.

Probability of event B $P(B) = \frac{m}{n}$

$= \frac{16}{50}$

$A \cap B =$ Event that the selected number is a multiple of 2 and 3 that is multiple the LCM of 2 and 3 which is 6.
= {6, 12, 18,..., 48}

Hence, the number of favourable outcomes of event $A \cap B$ will be m = 8.

Probability of event $A \cap B$ $P(A \cap B) = \frac{m}{n}$

$= \frac{8}{50}$

From the law of addition of probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{25}{50} + \frac{16}{50} - \frac{8}{50}$$

$$= \frac{25 + 16 - 8}{50}$$

$$= \frac{33}{50}$$

Required probability $= \frac{33}{50}$

Illustration 20 : One card is randomly selected from a pack of 52 cards. Find the probability that the selected card is

(1) club or queen card

(2) neither a club nor a queen card.

If one card is randomly selected from a pack of 52 cards then the number of mutually exclusive, exhaustive and equi-probable outcomes in the sample space of this random experiment $n = {}^{52}C_1 = 52$.

Event that the selected card is a club card = A

Event that the selected card is a queen = B

(1) Event that the selected card is club or queen card = $A \cup B$

To find the probability of event $A \cup B$ by the law of addition of probability, we will first find $P(A), P(B)$ and $P(A \cap B)$.

A = Event that the selected card is club card.

There are 13 club cards in a pack of 52 cards. Thus, the number of favourable outcomes of event A is $m = 13$.

Probability of event A $\quad P(A) = \frac{m}{n}$

$$= \frac{13}{52}$$

$B =$ Event that the selected card is a queen card.

There are 4 queen cards in a pack of 52 cards. Thus, the number of favourable outcomes of event B is $m = 4$.

Probability of event B $\quad P(B) = \frac{m}{n}$

$$= \frac{4}{52}$$

$A \cap B =$ Event that the selected card is club and queen card that is a club queen.

There is only 1 card in the pack of 52 cards which is club queen. Hence, the number of favourable outcomes of $A \cap B$ is $m = 1$.

Probability of $A \cap B$ $\quad P(A \cap B) = \frac{m}{n}$

$$= \frac{1}{52}$$

From the law of addition of probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{13 + 4 - 1}{52}$$

$$= \frac{16}{52}$$

$$= \frac{4}{13}$$

Required probability $= \frac{4}{13}$

Event $A \cup B$ can be easily explained by the following diagram :

Suit	**Type of Card**												
	A	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**J**	**Q**	**K**
♠	♠A	♠2	♠3	♠4	♠5	♠6	♠7	♠8	♠9	♠10	♠J	♠Q	♠K
♦	♦A	♦2	♦3	♦4	♦5	♦6	♦7	♦8	♦9	♦10	♦J	♦Q	♦K
♣	♣A	♣2	♣3	♣4	♣5	♣6	♣7	♣8	♣9	♣10	♣J	♣Q	♣K
♥	♥A	♥2	♥3	♥4	♥5	♥6	♥7	♥8	♥9	♥10	♥J	♥Q	♥K

(2) Event that the selected card is not of club $= A'$

Event that the selected card is not queen $= B'$

Hence, the event that the selected card is neither club nor queen is $A' \cap B'$

Thus, the probability of $A' \cap B'$

$$P(A' \cap B') = P(A \cup B)'$$

$$= 1 - P(A \cup B)$$

$$= 1 - \frac{4}{13}$$

$$= \frac{9}{13}$$

Required probability $= \frac{9}{13}$

Illustration 21 : 3 persons from medical profession and 5 persons from engineering profession offer services at a social organization. 2 persons are randomly selected from these persons with the purpose of forming a committee. Find the probability that both the persons selected belong to the same profession.

There are in all $3 + 5 = 8$ persons. Hence, 2 persons can be selected in ${}^{8}C_2 = \frac{8 \times 7}{2 \times 1} = 28$ ways.

Thus, the total number of mutually exclusive, exhaustive and equi-probable outcomes in the sample space is $n = 28$.

Event that both the persons selected belong to medical profession $= A$

Even that both the persons selected belong to the engineering profession $= B$

Event that both the persons selected belong to the same profession $= A \cup B$

The two events A and B can not occur together that is $A \cap B = \phi$

Thus, the events A and B are mutually exclusive. Hence, from the law of addition of probability,

$$P(A \cup B) = P(A) + P(B)$$

For which we first find $P(A)$ and $P(B)$.

$A =$ Event that both the persons selected belong to medical profession.

The number of favourable outcomes of A is $m = {}^{3}C_2 = 3$.

Probability of event A $P(A) = \frac{m}{n}$

$$= \frac{3}{28}$$

$B =$ Event that both the persons selected belong to engineering profession.

The number of favourable outcomes of B is $m = {}^{5}C_2 = 10$.

Probability of event B $P(B) = \frac{m}{n}$

$$= \frac{10}{28}$$

Now,

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{3}{28} + \frac{10}{28}$$

$$= \frac{3+10}{28}$$

$$= \frac{13}{28}$$

Required probability $= \frac{13}{28}$

Illustration 22 : The probability that a person from a group reads newspaper X is 0.55, the probability that he read newspaper Y is 0.69 and the probability that he reads both the newspaper X and Y is 0.27. Find the probability that a person selected at random from this group.

(1) reads at least one of the newspapers X and Y.

(2) does not read any of the newspapers X and Y.

(3) reads only one of the newspapers X and Y.

If the event that a person from the group reads newspaper X is denoted by event A and reads newspaper Y by event B then the given information can be shown as follows :

$P(A) = 0.55,\ P(B) = 0.69,\ P(A \cap B) = 0.27$

(1) Event that the selected person reads at least one of the newspapers $= A \cup B$

From the law of addition of probability,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.55 + 0.69 - 0.27$$

$$= 0.97$$

Required probability = 0.97

(2) Event that the selected person does not read newspaper A = A'

Event that the selected person does not read newspaper B = B'

Hence, event that the selected person does not read any of the newspaper X and $Y = A' \cap B'$.

Probability of $A' \cap B'$

$$P(A' \cap B') = P(A \cup B)'$$

$$= 1 - P(A \cup B)$$

$$= 1 - 0.97$$

$$= 0.03$$

Required probability = 0.03

(3) If the event that the selected person reads only one of the newspapers X and Y is denoted by C then the event C can occur as follows :

The person reads newspaper X (event A) and does not read newspaper Y (event B')

OR

The person does not read newspaper X (event A') and reads newspaper Y (event B)

Thus $C = (A \cap B') \cup (A' \cap B)$

Since the events $A \cap B'$ and $A' \cap B$ are mutually exclusive,

$$P(C) = P(A \cap B') + P(A' \cap B)$$

$$= [P(A) - P(A \cap B)] + [P(B) - P(A \cap B)]$$

$$= [0.55 - 0.27] + [0.69 - 0.27]$$

$$= 0.28 + 0.42$$

$$= 0.7$$

Required probability = 0.7

Illustration 23 : For two events A and B in the sample space of a random experiment $P(A) = 2P(B) = 4P(A \cap B) = 0.6$. Find the probability of the following events :

(1) $A' \cap B'$ (2) $A' \cup B'$ (3) $A - B$ (4) $B - A$

It is given that $P(A) = 2P(B) = 4P(A \cap B) = 0.6$. Hence,

$P(A) = 0.6$	$2P(B) = 0.6$	$4P(A \cap B) = 0.6$
	$\therefore\ P(B) = 0.3$	$\therefore\ P(A \cap B) = 0.15$

(1) Probability of event $A' \cap B'$ $= P(A' \cap B')$

$$= P(A \cup B)'$$

$$= 1 - P(A \cup B)$$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - [0.6 + 0.3 - 0.15]$$

$$= 1 - 0.75$$

$$= 0.25$$

Required probability = 0.25

(2) Probability of event $A' \cup B'$ $= P(A' \cup B')$

$$= P(A \cap B)'$$

$$= 1 - P(A \cap B)$$

$$= 1 - 0.15$$

$$= 0.85$$

Required probability = 0.85

(3) Probability of event $A-B$ $= P(A-B)$

$= P(A)-P(A\cap B)$

$= 0.6-0.15$

$= 0.45$

Required probability = 0.45

(4) Probability of event $B-A$ $= P(B-A)$

$= P(B)-P(A\cap B)$

$= 0.3-0.15$

$= 0.15$

Required probability = 0.15

Illustration 24 : For two events A and B in the sample space of a random experiment $P(A')=0.3$, $P(B)=0.6$ and $P(A\cup B)=0.83$. Find $P(A\cap B')$ and $P(A'\cap B)$.

Here, $P(A')=0.3 \therefore P(A)=1-P(A')=1-0.3=0.7$

$P(B)=0.6$ and $P(A\cup B)=0.83$

First we will find $P(A\cap B)$.

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\therefore\ 0.83=0.7+0.6-P(A\cap B)$

$\therefore\ P(A\cap B)=0.7+0.6-0.83$

$\therefore\ P(A\cap B)=0.47$

Now,

$P(A\cap B')=P(A)-P(A\cap B)$

$= 0.7 - 0.47$

$= 0.23$

Required probability = 0.23

$P(A'\cap B)=P(B)-P(A\cap B)$

$= 0.6 - 0.47$

$= 0.13$

Required probability = 0.13

Illustration 25 : Two events A and B in the sample space of a random experiment are mutually exclusive. If $3P(A)=4P(B)=1$ then find $P(A\cup B)$.

Since $3P(A)=4P(B)=1$

$3P(A)=1$	$4P(B)=1$
$\therefore\ P(A)=\frac{1}{3}$	$\therefore\ P(B)=\frac{1}{4}$

As the events A and B are mutually exclusive $(A\cap B=\phi)$,

$$P(A\cup B)=P(A)+P(B)$$

$$=\frac{1}{3}+\frac{1}{4}$$

$$=\frac{7}{12}$$

Required probability $=\frac{7}{12}$

Illustration 26 : For three mutually exclusive and exhaustive events A, B and C in the sample space of a random experiment $2P(A)=3P(B)=4P(C)$. Find $P(A\cup B)$ and $P(B\cup C)$.

Taking $2P(A)=3P(B)=4P(C)=x$,

$2P(A)=x$	$3P(B)=x$	$4P(C)=x$
$\therefore\ P(A)=\frac{x}{2}$	$\therefore\ P(B)=\frac{x}{3}$	$\therefore\ P(C)=\frac{x}{4}$

Since A, B and C are mutually exclusive and exhaustive events,

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)=1$$

$$\therefore\ \frac{x}{2}+\frac{x}{3}+\frac{x}{4}=1$$

$$\therefore\ \frac{6x+4x+3x}{12}=1$$

$$\therefore\ 13x=12$$

$$\therefore\ x=\frac{12}{13}$$

Thus,

$$P(A)=\frac{x}{2}=\frac{\frac{12}{13}}{2}=\frac{6}{13}$$

$$P(B)=\frac{x}{3}=\frac{\frac{12}{13}}{3}=\frac{4}{13}$$

$$P(C)=\frac{x}{4}=\frac{\frac{12}{13}}{4}=\frac{3}{13}$$

Now, the probability of required events,

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{6}{13} + \frac{4}{13}$$

$$= \frac{10}{13}$$

Required probability $= \frac{10}{13}$

$$P(B \cup C) = P(B) + P(C)$$

$$= \frac{4}{13} + \frac{3}{13}$$

$$= \frac{7}{13}$$

Required probability $= \frac{7}{13}$

Exercise 1.3

1. 2 cards are drawn from a pack of 52 cards. Find the probability that both the cards drawn are

(1) of the same suit

(2) of the same colour.

2. 3 books of Statistics and 4 of Mathematics are arranged on a shelf. Two books are randomly selected from these books. Find the probability that both the books selected are of the same subject.

3. One card is randomly drawn from a pack of 52 cards. Find the probability that it is

(1) Spade card or ace (2) Neighter spade nor ace.

4. A number is selected from the natural number 1 to 100. Find the probability of the event that the selected number is a multiple of 3 or 5.

5. Two balanced dice are thrown simultaneously. Find the probability that the sum of numbers on two dice is a multiple of 2 or 3.

6. The probability that the price of potato rises in the vegetable market during festive days in 0.8. The probability that the price of onion rises is 0.7. The probability of rise in price of both potato and onion is 0.6. Find the probability of rise in price of at least one of the two, potato and onion.

7. Two aircrafts drop bomb to destroy a bridge. The probability that a bomb dropped from the first aircraft hits the target is 0.9 and the probability that a bomb from the second aircraft hits the target is 0.7. The probability of one bomb dropped from both the aircrafts hitting the target is 0.63. The bridge is destroyed even if one bomb drops on it. Find the probability that the bridge is destroyed.

8. The probability that a teenager coming to a restaurant for dinner orders pizza is 0.63. The probability of ordering cold-drink is 0.54. The probability that the teenager orders at least one out of pizza and cold-drink is 0.88. Find the probability that the teenager coming for dinner on a certain day orders only one of the two items from pizza and cold-drink.

9. If A and B are mutually exclusive and exhaustive events in a sample space U and $P(A) = 2P(B)$ then find $P(A)$.

10. Three events A, B and C in a sample space are mutually exclusive and exhaustive. If $4P(A) = 5P(B) = 3P(C)$ then find $P(A \cup C)$ and $P(B \cup C)$.

11. Find $P(A \cup B \cup C)$ using the following information about three events A, B and C in a sample space.

$P(A) = 0.65,\ P(B) = 0.45,\ P(C) = 0.25,\ P(A \cap B) = 0.25,\ P(A \cap C) = 0.15,\ P(B \cap C) = 0.2,$

$P(A \cap B \cap C) = 0.05$

12. Three events A, B and C in a sample space are mutually exclusive and exhaustive. If $P(C') = 0.8$ and $3P(B) = 2P(A')$ then find $P(A)$ and $P(B)$.

*

1.6 Conditional Probability and Law of Multiplication of Probability

1.6.1 Conditional Probability

Suppose U is a finite sample space and A and B are any two events in it. The probability of occurrence of event B under the condition that A occurs is called the conditional probability. If the occurrence of event B under the condition that event A occurs is denoted by B/A then the probability $P(B/A)$ of the conditional event B/A is called the conditional probability. This probability is obtained using the following formula :

$$P(B/A) = \frac{P(A \cap B)}{P(A)}; \quad P(A) \neq 0$$

Similarly, if the occurrence of event A under the condition that event B occurs is denoted by A/B then the probability $P(A/B)$ of the conditional event A/B is obtained using the following formula :

$$P(A/B) = \frac{P(A \cap B)}{P(B)}; \quad P(B) \neq 0$$

Suppose a company produces a certain type of item in its two different factories A_1 and A_2. One item is randomly selected from a store selling the items produced by this company. Let us denote the event that the selected item is defective as D.

- If the selected item is produced at factory A_1 then the event that it is defective is denoted by D/A_1.
- If the selected item is produced at factory A_2 then the event that it is defective is denoted by D/A_2.

Thus,

$P(D/A_1)$ = Probability of occurrence of D under the condition that event A_1 has occurred and

$P(D/A_2)$ = Probability of occurrence of D under the condition that event A_2 has occurred

1.6.2 Independent Events

Suppose A and B are any two events in a finite sample space U. If the probability of occurrence of event A does not change due to occurrence (or non-occurrence) of event B then the events A and B are called the independent events.

Thus, if $P(A) = P(A/B) = P(A/B')$ and $P(B) = P(B/A) = P(B/A')$ the events A and B are called independent events.

For example, Event A = First throw of a balanced die shows number 1.

Event B = Second throw of a balanced die shows an even number.

It can be said here that the probability of getting an even number in the second throw of the die does not change because the first throw had shown the number 1. This fact can be easily understood by the following calculation :

The total number of outcomes by throwing the dice two times is $n = 6 \times 6 = 36$.

A = Event that the first throw of a balanced die shows the number 1.

The number of favourable outcomes of A is $m = 6$.

Probability of event A $P(A) = \frac{m}{n}$

$= \frac{6}{36}$

B = Event that the second throw of balanced die shows an even number.

The number of favourable outcomes of B is $m = 18$

Probability of event B $P(B) = \frac{m}{n}$

$= \frac{18}{36}$

$= \frac{1}{2}$

$A \cap B$ = Event that the first throw of a balanced die shows the number 1 and the second throw shows even number.

The number of favourable outcomes of $A \cap B$ is $m = 3$.

Probability of event $A \cap B$ $P(A \cap B) = \frac{m}{n}$

$= \frac{3}{36}$

Now, if the first throw of the die shows number 1 then the probability $P(B/A)$ for the event B/A of getting an even number in the second throw can be obtained as follows :

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{3}{36}}{\frac{6}{36}}$$

$$= \frac{1}{2}$$

Since we get $P(B) = P(B/A)$, we say that the events A and B are independent.

1.6.3 Law of Multiplication of Probability

If A and B are the two events in a sample space U then the rule of obtaining the probability of simultaneous occurrence of events A and B is called the law of multiplication of probability.

For example, Event A = Getting head when a coin is tossed for the first time.

Event B = Getting head when a coin is tossed for the second time.

If the coin is tossed two times then the probability of getting head both the times that is event $A \cap B$ can be obtained by the law of multiplication of probability. The law of multiplication of probability is as follows :

$P(A \cap B) = P(A) \times P(B/A); \; P(A) \neq 0$

$P(A \cap B) = P(B) \times P(A/B); \; P(B) \neq 0$

Some of the important results deduced from this rule are as follows which will be accepted without proof.

(1) If A and B are independent events then $P(A \cap B) = P(A) \times P(B)$

(2) If A and B are independent events then

(i) The events A' and B' are also independent. Hence, $P(A' \cap B') = P(A') \times P(B')$

(ii) The events A and B' are also independent. Hence, $P(A \cap B') = P(A) \times P(B')$

(iii) The events A' and B are also independent. Hence, $P(A' \cap B) = P(A') \times P(B)$

1.6.4 Selection with Replacement and without Replacement

When the units are to be randomly selected one by one from the population, the selection can be done in two ways :

(1) **Selection with replacement :** If the selection of a unit from the population in any trial is done by replacing the unit selected in the previous trial back to the population then that selection is called the selection with replacement.

(2) **Selection without Replacement :** If the selection of a unit from the population in any trial is done by not replacing the unit selected in the previous trial back to the population then that selection is called the selection with replacement.

Illustration 27 : A balanced coin is tossed twice. If the first toss of the coin shows head then find the probability of getting head in both the tosses.

The sample space of the random experiment of tossing a balanced coin twice is $U = \{HH, HT, TH, TT\}$, where the first symbol shows the outcome of the first toss of the coin and the second symbol shows the outcome of the second toss of the coin. The total number of outcomes in this sample space is $n = 4$.

If A denote the event of getting head in the first toss of the coin and B denotes the event that both the tosses result in head then we have to find $P(B/A)$, probability of B/A.

Event A = First toss shows head

= $\{HH, HT\}$

Hence, the number of favourable outcomes of A is $m = 2$.

Probability of event A $P(A) = \frac{m}{n}$

$= \frac{2}{4}$

Event B = Head is shown in both the tosses

= $\{HH\}$

Hence, the number of favourable outcomes of B is $m = 1$.

Probability of event B $P(B) = \frac{m}{n}$

$= \frac{1}{4}$

Event $A \cap B$ = Getting head in the first toss and getting head in both the tosses of the coin (we have $B \subset A$.)

= $\{HH\}$

Hence, the number of favourable outcomes of $A \cap B$ is $m = 1$.

Probability of $A \cap B$ $P(A \cap B) = \frac{m}{n}$

$= \frac{1}{4}$

Now,

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{1}{4}}{\frac{2}{4}}$$

$$= \frac{1}{2}$$

Required probability $= \frac{1}{2}$

Illustration 28 : A factory has received an order to prepare 50,000 units of an item in a certain time period. The probability of completing this work in the given time is 0.75 and the probability that the workers will not declare strike during that time period is 0.8. The probability that this work will be completed during the given period and the workers will not declare strike is 0.7. Find the probability that

(1) The work will be completed as per schedule under the condition that the workers have not declared strike.

(2) Find the probability that the workers do not declare strike in the given period knowing that the work is completed as per schedule.

If we denote event A that the work will be completed as per schedule and event B that the workers will not declare strike then the given information can be written as follows :

$P(A) = 0.75$, $P(B) = 0.8$, $P(A \cap B) = 0.7$

(1) Event that the work will be completed in the given period under the condition that the workers do not declare strike = A/B

Probability of A/B from the definition of condition probability,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.7}{0.8}$$

$$= \frac{7}{8}$$

Required probability = $\frac{7}{8}$

(2) If it is given that the work is completed as per schedule then the event that the workers do not declare strike = B/A

Probability of B/A from the definition of conditional probability,

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{0.7}{0.75}$$

$$= \frac{14}{15}$$

Required probability = $\frac{14}{15}$

Illustration 29 : If $P(A') = \frac{7}{25}$, $P(B/A) = \frac{5}{12}$ and $P(A/B) = \frac{1}{2}$ for two events A and B in the sample space of a random experiment then find $P(A \cap B)$ and $P(B)$.

It is given that $P(A') = \frac{7}{25}$, $P(B/A) = \frac{5}{12}$ and $P(A/B) = \frac{1}{2}$.

$$P(A) = 1 - P(A')$$

$$= 1 - \frac{7}{25}$$

$$= \frac{18}{25}$$

We will find $P(A \cap B)$ from the formula of $P(B/A)$.

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$\therefore \quad \frac{5}{12} = \frac{P(A \cap B)}{\frac{18}{25}}$$

$$\therefore \quad \frac{5}{12} \times \frac{18}{25} = P(A \cap B)$$

$$\therefore \quad P(A \cap B) = \frac{3}{10}$$

Required probability $= \frac{3}{10}$

Now, we will find $P(B)$ by substituting $P(A/B)$ and $P(A \cap B)$ in the formula of $P(A/B)$.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\therefore \quad \frac{1}{2} = \frac{\frac{3}{10}}{P(B)}$$

$$\therefore \quad P(B) = \frac{\frac{3}{10}}{\frac{1}{2}}$$

$$= \frac{3 \times 2}{10 \times 1}$$

$$= \frac{3}{5}$$

Required probability $= \frac{3}{5}$

Illustration 30 : A medicine is tested on a group of rabbits and mice to know its effect. It was observed that 7 rabbits show the effect of medicine in a group of 10 rabbits who were given the medicine and 5 mice show the effect of medicine in a group of 7 mice who were given the medicine. One animal is selected at random from each group. Find the probability that (1) both the selected animals show the effect of medicine and (2) one of the two selected animals shows the effect of medicine and the other animal does not show the effect of medicine.

The given information will be shown as follows :

Animals affected by Medicine	**Animals not affected by Medicine**
Rabbits 7	Rabbits 3
Mice 5	Mice 4
Total 12	**Total 7**

(1) Event that a rabbit shows effect of medicine $= A$

Event that a mouse shows effect of medicine $= B$

$\therefore$ Event that both the animals show the effect of medicine $= A \cap B$

The events A and B are independent. Whether the mice show the effect of medicine is not affected by the effect of medicine on rabbits. Hence,

$$P(A \cap B) = P(A) \times P(B)$$

From the total number of outcomes n = 10, m = 7 outcomes are favourable for event A.

$\therefore$ Probability of event A $P(A) = \frac{m}{n}$

$= \frac{7}{10}$

From the total number of outcomes n = 9, m = 5 outcomes are favourable for event B.

$\therefore$ Probability of event B $P(B) = \frac{m}{n}$

$= \frac{5}{9}$

$\therefore P(A \cap B) = \frac{7}{10} \times \frac{5}{9}$

$= \frac{7}{18}$

Required probability $= \frac{7}{18}$

(2) Let C denote the event that one animal is affected by the medicine and the other animal is not affected by the medicine. The event C can occur as follows :

Rabbit is affected by the medicine (event A) and mouse is not affected by the medicine (event B')

OR

Rabbit is not affected by the medicine (event A') and mouse is affected by the medicine (event B)

Thus, event $C = (A \cap B') \cup (A' \cap B)$

Since the events $A \cap B'$ and $A' \cap B$ are mutually exclusive,

$P(C) = P(A \cap B') + P(A' \cap B)$

$= [P(A) \times P(B')] + [P(A') \times P(B)]$ ($\because$ A and B are independent events)

Here, $P(A') = 1 - P(A)$

$= 1 - \frac{7}{10}$

$= \frac{3}{10}$

$P(B') = 1 - P(B)$

$= 1 - \frac{5}{9}$

$= \frac{4}{9}$

$\therefore$ $P(C) = [\frac{7}{10} \times \frac{4}{9}] + [\frac{3}{10} \times \frac{5}{9}]$

$= \frac{28}{90} + \frac{15}{90}$

$= \frac{43}{90}$

Required probability $= \frac{43}{90}$

Illustration 31 : A company produces a certain type of item in its two different factories A_1 and A_2 in the proportion 60% and 40% respectively. The proportions of defectives in the production of these factories are 2% and 3% respectively. One item is randomly selected after mixing the items produced in the two factories. Find the probability that this item is defective.

Event that the selected item is produced in factory A_1 = A_1

$$\therefore \quad P(A_1) = \frac{60}{100}$$

$$= \frac{3}{5}$$

Event that the selected item is produced in factory A_2 = A_2

$$\therefore \quad P(A_2) = \frac{40}{100}$$

$$= \frac{2}{5}$$

Let D denote the event that the item selected from the total production is defective.

Event that the selected item is defective when it is produced in factory A_1 = D/A_1

$$\therefore \quad P(D/A_1) = \frac{2}{100}$$

$$= \frac{1}{50}$$

Event that the selected item is defective when it is produced in factory A_2 = D/A_2

$$\therefore \quad P(D/A_2) = \frac{3}{100}$$

Event D can occur as follows.

The selected item is produced in factory A_1 and it is defective.

OR

The selected item is produced in factory A_2 and it is defective.

Thus event $D = (A_1 \cap D) \cup (A_2 \cap D)$

Since the events $A_1 \cap D$ and $A_2 \cap D$ are mutually exclusive,

$$P(D) = P(A_1 \cap D) + P(A_2 \cap D)$$

$$= \left[P(A_1) \times P(D/A_1)\right] + \left[P(A_2) \times P(D/A_2)\right]$$

$$= \left[\frac{3}{5} \times \frac{1}{50}\right] + \left[\frac{2}{5} \times \frac{3}{100}\right]$$

$= \frac{3}{250} + \frac{6}{500}$

$= \frac{12}{500}$

$= \frac{3}{125}$

Required probability $= \frac{3}{125}$

Illustration 32 : There are 12 screws in a box of which 4 screws are defective. Two screws are randomly selected one by one without replacement from this box. Find the probability that both the screws selected are defective.

4 screws are defective in the box having 12 screws. Hence, the number of non-defective screws will be 8.

Total number of mutually exclusive, exhaustive and equiprobable outcomes for selecting the first screw are $n = {}^{12}C_1 = 12$.

If A denotes the event that the first screw selected is defective then the number of favourable outcomes of A is $m = {}^{4}C_1 = 4$.

Probability of event A $P(A) = \frac{m}{n} = \frac{4}{12}$

The screws are selected without replacement which means that the first screw is not kept back into the box. Hence, the total number of mutually exclusive, exhaustive and equi-probable outcomes for selecting the second screw is $n = {}^{11}C_1 = 11$.

Let B denote the event that the second screw selected is defective.

The event B occurs under the condition that the event A has occurred. This is the occurrence of event B/A.

Since event A has occurred earlier, there are 3 defective screws in the box.

Hence, the number of favourable outcomes for event B/A is $m = {}^{3}C_1 = 3$.

Probability of B/A $P(B/A) = \frac{m}{n}$

$= \frac{3}{11}$

Now, $A \cap B$ = Event that both the screws are defective

From the law of multiplication of probability,

$P(A \cap B) = P(A) \times P(B/A)$

$= \frac{4}{12} \times \frac{3}{11}$

$= \frac{1}{11}$

Required probability $= \frac{1}{11}$

Illustration 33 : There are 3 boys and 2 girls in a friend-circle. Two persons are randomly selected from this friend-circle one by one with replacement to sing a song. Find the probability that the first person is a boy and the second person is a girl in the two persons selected to sing a song.

The friend-circle consists of 3 boys and 2 girls that is total 5 persons. Two persons are selected one by one with replacement. This means that the person selected first is sent back to the group before selecting the second person. Hence, the events of selecting two persons one by one are independent events. The total number of mutually exclusive, exhaustive and equi-probable outcomes for selecting the first person is $n = {}^5C_1 = 5$.

Event that the first person selected to sing a song is a boy $= A$

The number of favourable outcomes for event A is $m = {}^3C_1 = 3$

Probability of event A $P(A) = \frac{m}{n}$

$= \frac{3}{5}$

The selection is with replacement here. This means that the total number of mutually exclusive, exhaustive and equi-probable outcomes for selecting the second person is $n = {}^5C_1$.

Event that the second person selected to sing a song is a girl $= B$

The number of favourable outcomes of B is $m = {}^2C_1 = 2$

Probability of event B $P(B) = \frac{m}{n}$

$= \frac{2}{5}$

Now, $A \cap B$ = Event that the first boy and the second girl are the two person selected to sing a song. Since the events A and B are independent,

$P(A \cap B) = P(A) \times P(B)$

$= \frac{3}{5} \times \frac{2}{5}$

$= \frac{6}{25}$

Required probability $= \frac{6}{25}$

Illustration 34 : Two balanced dice are thrown simultaneously. Find the probability that at least one of the two dice shows the number 3.

Event that the first die shows number 3 $= A$

Event that the second die shows number 3 $= B$

Event that at least one die shows number 3 $= A \cup B$

The number of favourable outcome for event A is $m=1$

Probability of event A $P(A)=\frac{m}{n}$

$=\frac{1}{6}$

The number of favourable outcomes for event B is $m=1$

Probability of event B $P(B)=\frac{m}{n}$

$=\frac{1}{6}$

Since the events A and B are independent, the events A' and B' are also independent. Moreover, $P(A')=1-P(A)=1-\frac{1}{6}=\frac{5}{6}$ and $P(B')=1-P(B)=1-\frac{1}{6}=\frac{5}{6}$.

Probability of the event that at least one die shows number 3 $=P(A\cup B)$

$=1-P(A'\cap B')$

$=1-\left[P(A')\times P(B')\right]$

$=1-\left[\frac{5}{6}\times\frac{5}{6}\right]$

$=1-\frac{25}{36}$

$=\frac{11}{36}$

Required probability $=\frac{11}{36}$

Illustration 35 : Two cities A and B of different states have rains on 60% and 75% days respectively during the monsoon. For the cities A and B, find the probability that on a certain monsoon day,

(1) both the cities have rains

(2) at least one city has rains

(3) only one city has rains.

Note : The events of rains on a day in these two cities are independent.

Let event A denote that it rains in city A and event B denote that it rains in city B. The given information can be stated as follows :

$P(A)=\frac{60}{100}=\frac{3}{5}$ $\quad\therefore P(A')=1-P(A)=1-\frac{3}{5}=\frac{2}{5}$

$P(B)=\frac{75}{100}=\frac{3}{4}$ $\quad\therefore P(B')=1-P(B)=1-\frac{3}{4}=\frac{1}{4}$

(1) Event that both the cities A and B have rains $A\cap B$

Since the events A and B are independent,

Probability of event $A\cap B$ $\quad P(A\cap B)=P(A)\times P(B)$

$=\frac{3}{5}\times\frac{3}{4}$

$=\frac{9}{20}$

Required probability $=\frac{9}{20}$

(2) Event that at least one of the cities A and B has rains $= A \cup B$

Probability of $A \cup B$ $\quad P(A \cup B) = 1 - P(A' \cap B')$

$$= 1 - [P(A') \times P(B')]$$

$$= 1 - \left[\frac{2}{5} \times \frac{1}{4}\right]$$

$$= 1 - \frac{1}{10}$$

$$= \frac{9}{10}$$

Required probability $= \frac{9}{10}$

(3) Event that only one of cities A and B has rains $= (A \cap B') \cup (A' \cap B)$

If the events A and B are independent then events A and B' as well as A' and B are also independent.

Probability of $(A \cap B') \cup (A' \cap B)$ $\quad = P(A \cap B') + P(A' \cap B)$

$$= [P(A) \times P(B')] + [P(A') \times P(B)]$$

$$= \left[\frac{3}{5} \times \frac{1}{4}\right] + \left[\frac{2}{5} \times \frac{3}{4}\right]$$

$$= \frac{3}{20} + \frac{6}{20}$$

$$= \frac{9}{20}$$

Required probability $= \frac{9}{20}$

Exercise 1.4

1. There are two children in a family. If the first child is a girl then find the probability that both the children in the family are girls.

2. Two six-faced balanced dice are thrown simultaneously. If the sum of numbers on both the dice is more than 7 then find the probability that both the dice show same numbers.

3. Among the various vehicle-owners visiting a petrol pump, 80% vehicle-owners visit to fill petrol in their vehicle and 60% vehicle-owners visit to fill air in their vehicles. 50% vehicle-owners visit to fill air and petrol in their vehicle. Find the probability for the following events :

(1) If a vehicle-owner has come to fill petrol in his vehicle then that vehicle-owner will fill air in his vehicle.

(2) If a vehicle-owner has come to fill air in his vehicle then that vehicle-owner will fill petrol in his vehicle.

4. 80% customers hold saving account and 50% customers hold current account of a nationalised bank. 90% of the customers hold at least one of the saving account and the current account. If one of the account holders randomly selected from this bank holds a current account, find the probability that he holds a saving account.

5. If $P(A)=\frac{2}{3}$, $P(B)=\frac{3}{5}$ and $P(B/A)$ = $\frac{3}{4}$ for two events in the sample space of a random experiment then find $P(A/B)$.

6. If $P(M)=P(F)=\frac{1}{2}$, $P(A/M)=$ $\frac{1}{10}$ and $P(A/F)=\frac{1}{2}$ for events A, M and F then find $P(A\cap M)$ and $P(A\cap F)$.

7. There are 2 gold-coins and 4 silver-coins in a box. The other box contains 3 gold and 5 silver coins. One coin is selected from each box. Find the probability that one of the selected coins is a gold coin and the other is a silver coin.

8. One joint family has 3 sons and 2 daughters whereas the other joint family has 2 sons and 4 daughters. One joint family is selected from two joint families and a child is randomly selected from that family. Find the probability that the selected child is a girl.

9. There are 10 icecream cones in a box of which 3 cones weigh less than the specification and the rest of the 7 cones have the specified weight. Two cones are randomly selected one by one with replacement. Find the probability that both the cones selected weigh less than the specified weight.

10. There are 10 CDs in a CD rack in which 6 are action film CDs and 4 are drama film CDs. Two CDs are randomly selected one by one without replacement from this box. Find the probability that the first selected CD is of action film and the second CD is of drama film.

11. If two balanced dice are thrown then find the probability that

(1) at least one die shows number 5

(2) the first die shows the number 5 or 6 and the other die shows an even number.

12. A problem in Mathematics is given to Tania, Kathan and Kirti to solve. The probabilities of them solving the problem correctly are $\frac{2}{3},\frac{3}{4}$ and $\frac{1}{2}$ repectively. Find the probability that the problem is solved correctly.

13. Person A can hit the target in 3 out of 5 attempts whereas person B can hit the target in 5 out of 6 attempts. If both of them attempt simultaneously, find the probability that the target is hit.

14. Person A speaks truth in 90% cases whereas person B speaks truth in 80% cases. Find the probability that persons A and B differ in stating the same fact.

15. If three events A, B and C of a random experiment are independent events and $P(A)=0.2, P(B)=0.3$ and $P(C)=0.5$ then find $P(A\cup B\cup C)$.

*

1.7 Statistical Definition of Probability

We have seen the mathematical definition of probability earlier. This definition can help to find the probability only in the cases where the outcomes of the sample space of a random experiment are equi-probable and their number is known. But we find several cases in practice where the outcomes of the sample space are infinite and unknown. For example, there are many fish of different types in a huge lake. We have to find the probability of catching a certain type of fish when a fisherman throws net in the lake to catch fish. The mathematical definition of probability can not be used here as the total number of fish in the lake is unknown. Moreover, we come across many cases in practice where the outcomes of the random experiment are not equi-probable. For example, a trader transports certain goods from his godown to his sales centre. The event that these goods safely reach the sales centre and event that it does not safely reach the sales centre are not equi-probable events. It is not possible to evaluate probability using the mathematical definition of probability in such cases. Let us consider another definition of probability, called the statistical definition of probability, which is generally more useful in such situations.

Let us start with an illustration. We have to find the probability that a customer will purchase while visiting a showroom selling ready made garments for a long time. To know this, we should obtain the data about the customers purchasing from this show-room. These data can be obtained by sample inquiry. As the size of the sample increases, we can say that the information from the sample inquiry is more close to the true (population) information. Suppose it is found that 79 customers purchase out of 100 customers in the sample inquiry. When the number of customers in the sample inquiry was 500 then it was found that 403 customers purchased. The data obtained by increasing the sample size (n) are as follows :

Size of the sample (No. of customers visiting the show-room)	**No. of customers purchasing r (Frequency)**	**Proportion of customers purchasing $\frac{r}{n}$ (Expected Frequency)**
100	79	0.79
500	403	0.806
1000	799	0.799
5000	3991	0.7982
10,000	8014	0.8014

It can be seen from the above data that as the size of the sample n increases, the proportion or expected frequency of customers purchasing the ready-made garments takes values close to 0.8. We accept this value as the probability of the event that the customer visiting the show-room will purchase. Thus, the probability is obtained in the form of relative frequency. The definition of porbability based on the relative frequency is called the statistical definition of probability. It is also called the empirical definition. The definition is as stated below :

Suppose a random experiment is repeated n times under identical conditions. If an event A occurs in m trials out of n trials then the relative frequency $\frac{m}{n}$ of event A gives the estimate of the probability of event A, $P(A)$. When the larger and larger value of n is taken, that is when n tends to infinity, the limiting value of $\frac{m}{n}$ is called the probability of event A.

In notation,

$$P(A) = \lim_{n \to \infty} \frac{m}{n}$$

The limiting value of the ratio $\frac{m}{n}$ when n tends to infinite value is denoted by $\lim_{n \to \infty} \frac{m}{n}$. In practice, the relative frequency $\frac{m}{n}$ itself is taken as the probability of event A. Now we shall consider the examples showing the use of the statistical definition of probability.

Illustration 36 : The sample data obtained about marks scored by a large group of candidates appearing for a public examination of 100 marks are given in the following table.

Marks	20 or less	21–40	41–60	61–80	81–100
No. of Candidates	83	162	496	326	124

One candidate is randomly selected from those appearing for the public examination. Find the probability that this candidate has scored :

(1) less than 41 marks

(2) More than 60 marks

(3) Marks from 21 to 80.

The number of candidates selected in the sample is $n = 83 + 162 + 496 + 326 + 124 = 1191$.

(1) Event $A =$ The selected candidate scores less than 41 marks.

$P(A) =$ Relative frequency for the candidates scoring less than 41 marks.

$$= \frac{\text{No. of candidates scoring less than 41 marks}}{\text{Total number of candidates in the sample}} = \frac{m}{n}$$

$m =$ No. of candidates scoring less than 41 marks

$= 83 + 162$

$= 245$

Now, $P(A) = \frac{m}{n}$

$= \frac{245}{1191}$

Required probability $= \frac{245}{1191}$