MATHEMATICAL REASONING

Mathematical Statement : The basic unit involved in mathematical reasoning is a mathematical statement.

Mathematically acceptable : A sentence is called a mathematically acceptable statement if it is either true or false but not both.

Negation of a statement : The denial of a statement is called the negation of the statement. If p is a statement, then the negation of p is also a statement and is denoted by ~p, and read as 'not p'.

Note : While forming the negation of a statement, phrases like "It is not the case" or "It is also false that" are also used.

Compound Statement : A compound statement is a statement which is made up of two or more statements. In this case, each statement is called a component statement.

Rules for the compound statement with "AND"

1. The compound statement with 'And' is true if all its component statements are true.
2. The compound statement with 'And' is false if any of its component statement is false.

Rules for the compound statement with "OR"

Note : "if and only if" ($\Leftrightarrow$)

1. A compound statement with an 'Or' is true when one component statement is true or both the component statements are true.
2. A compound statement with an 'Or' is false when both the component statement are false.

Quantifiers : Quantifiers are phrases like, "There exists" and "for all".

Implications : Implications are "if - then", "only if" and "if and only if".

If p and q is same as the following :

p : a number is a multiple of 9.
q : a number is a multiple of 3.

1. p implies q ($p \xrightarrow{\text{implies}} q$) This says that a number is a multiple of 9 implies that it is a multiple of 3.
2. p is sufficient condition for q. This says that knowing that a number as a multiple of 9 is sufficient to conclude that it is a multiple of 3.
3. p only if q. This says that a no. is a multiple of 9 only if it is a multiple of 3.
4. q is a neccessary condition for p. This says that when a no. is a multiple of 9, it is neccessary a multiple of 3.
5. ~q implies ~p. This says that if a no. is not a multiple of 3, then it is not a multiple of 9.

Contrapositive and converse : Contrapositive and converse are certain other statements which can be formed from a given statement with "if - then".

Validating statements :

Rule 1 : If p and q are mathematical statements, then in order to show that the statement "p and q" is true, the following steps are followed.

Step I Show that the statement p is true.
Step II Show that the statement q is true.

Rule 2. Statements with "or"

If p and q are mathematical statements, then in order to show that the statement "p and q" is true, one must consider the following:

Case I By assuming that p is false, show that q must be true.
Case II By assuming that q is false, show that p must be true.

Rule 3. Statements with "if-then"

In order to prove the statement "if p and q" we need to show that any one of the following case is true.

Case I By assuming that p is false, show that q must be true. (Direct Method)
Case II By assuming that q is false, show that p must be false. (Contrapositive Method)

Rule 4. Statements with "if and only if"

In order to prove the statement "if p if and only if q" we need to show

(i) If p is true, then q is true.
(ii) If q is true, then p is true.

By Contradiction: Here to check whether a statement p is true, we assume that p is not true i.e. $\sim p$ is true. Then we arrive at some result which contradicts our assumption. Therefore, we conclude that p is true.

Counter Example: The method involves giving an example of a situation where the statement is not valid.