4. Factorisation

Exercise 4.1

1. Question

Resolve in to factors:

(i) $x^2 + xy$

(ii) $3x^2 - 6x$

(iii) $(1.6)a^2 - (0.8)a$

(iv) $5 - 10m - 20n$

Answer

(i) $x \times x + x \times y$

taking x in common,

$= x(x + y)$

(ii) $3 \times x \times x - 6 \times x$

taking 3 x in common,

$= 3x(x-2)$

(iii) $2 \times 0.8 \times a \times a - 0.8 \times a$

taking 0.8a in common,

$= 0.8a(2a-1)$

(iv) $5 - 5 \times 2 \times m - 5 \times 4 \times n$

taking 5 in common,

$= 5(1-2m-4n)$

2 A. Question

Factorise:

$a^2 + ax + ab + bx$

Answer

= a(a + x) + b(a + x)

= (a + b)(a + x)

2 B. Question

Factorise:

3ac + 7bc – 3ad – 7bd

Answer

= c(3a + 7b)–d(3a + 7b)

= (c–d)(3a + 7b)

2 C. Question

Factorise:

3xy – 6zy – 3xt + 6zt

Answer

= 3y(x–2z)–3t(x–2z)

= (3y–3t)(x–2z)

= 3(y–t)(x–2z)

2 D. Question

Factorise:

$y^3 – 3y^2 + 2y – 6 – xy + 3x$

Answer

$= y^3 + 2y–xy–3y^2 + 3x–6$

$= y(y^2 + 2–x)–3(y^2 + 2–x)$

$= (y–3)(y^2 + 2–x)$

3 A. Question

Factorise:

$4a^2 – 25$

Answer

Using the identity,

$x^2–y^2 = (x + y)(x–y)$

so, $(2a)^2-(5)^2$

$= (2a-5)(2a + 5)$

3 B. Question

Factorise:

$$x^9 - \frac{9}{16}$$

Answer

Using the identity,

$x^2-y^2 = (x + y)(x-y)$

so, $x^2-\left(\frac{3}{4}\right)^2$

$= \left(x - \frac{3}{4}\right)\left(x + \frac{3}{4}\right)$

3 C. Question

Factorise:

$x^4 - y^4$

Answer

Using the identity,

$x^2-y^2 = (x + y)(x-y)$

so, $(x^2)^2-(y^2)^2$

$= (x^2-y^2)(x^2 + y^2)$

$= (x-y)(x + y)(x^2 + y^2)$

3 D. Question

Factorise:

$$\left(7\frac{3}{10}\right)^2 - \left(2\frac{1}{10}\right)^2$$

Answer

: Using the identity,

$x^2 - y^2 = (x + y)(x - y)$

so, $\left(\frac{73}{10}\right)^2 - \left(\frac{21}{10}\right)^2$

$= \left(\frac{73}{10} - \frac{21}{10}\right)\left(\frac{73}{10} + \frac{21}{10}\right)$

$= \left(\frac{73 - 21}{10}\right)\left(\frac{73 + 21}{10}\right)$

$= \left(\frac{52}{10}\right)\left(\frac{94}{10}\right)$

$= \frac{52 \times 94}{100}$

$= \frac{4888}{1000}$

$= \frac{1222}{5}$

3 E. Question

Factorise:

$(0.7)^2 - (0.3)^2$

Answer

: Using the identity,

$x^2 - y^2 = (x + y)(x - y)$

so, $0.7^2 - 0.3^2 = (0.7 - 0.3)(0.7 + 0.3)$

$= 0.4 \times 1.0$

$= 0.4$

3 F. Question

Factorise:

$(5a - 2b)^2 - (2a - b)^2$

Answer

: Using the identity,

$x^2 - y^2 = (x + y)(x - y)$

so, $(5a - 2b)^2 - (2a - b)^2 = (5a-2b-2a + b)(5a-2b + 2a-b)$

$= (3a-b)(7a-3b)$

$= 21a^2-9ab-7ab + 3b^2$

$= 21a^2-16ab + 3b^2$

Exercise 4.2

1 A. Question

In the following, you are given the product pq and the sum p + q. Determine p and q:

pq = 18 and p + q = 11

Answer

as, p + q = 11

$\Rightarrow p = 11-q$

putting the value of q in other equation,

$\Rightarrow pq = 18$

$\Rightarrow (11-q)q = 18$

$\Rightarrow 11q-q^2 = 18$

$\Rightarrow q^2-11q + 18 = 0$

$\Rightarrow q^2-2q-9q + 18 = 0$

$\Rightarrow q(q-2)-9(q-2) = 0$

$\Rightarrow (q-2)(q-9) = 0$

So, q = 2 & q = 9

As, p = 11–q

Thus, p = 11–2 = 9 & p = 11–9 = 2

1 B. Question

In the following, you are given the product pq and the sum p + q. Determine p and q:

pq = 32 and p + q = –12

Answer

As, p + q = –12

$\Rightarrow p = -12-q$

putting the value of q in other equation,

$\Rightarrow pq = 32$

$\Rightarrow (-12-q)q = 32$

$\Rightarrow -12q-q^2 = 32$

$\Rightarrow q^2 + 12q + 32 = 0$

$\Rightarrow q^2 + 8q + 4q + 32 = 0$

$\Rightarrow q(q + 8) + 4(q + 4) = 0$

$\Rightarrow (q + 4)(q + 8) = 0$

so, $q = -4$ & $q = -8$

as, $p = -12-q$

thus, $p = -12 + 4$ & $p = -12 + 8$

$p = -8$ & $p = -4$

1 C. Question

In the following, you are given the product pq and the sum p + q. Determine p and q:

$pq = -24$ and $p + q = 2$

Answer

As, $p + q = 2$

$\Rightarrow p = 2-q$

putting the value of q in other equation,

$\Rightarrow pq = -24$

$\Rightarrow (2-q)q = -24$

$\Rightarrow 2q-q^2 = -24$

$\Rightarrow q^2-2q-24 = 0$

$\Rightarrow q^2-6q + 4q-24 = 0$

$\Rightarrow q(q-6) + 4(q-6) = 0$

$\Rightarrow (q + 4)(q-6) = 0$

so, q = –4 & q = 6

as, p = 2–4

thus, p = 2 + 4 & p = 2–6

p = 6 & p = –4

1 D. Question

In the following, you are given the product pq and the sum p + q. Determine p and q:

pq = –12 and p + q = 11

Answer

As, p + q = 11

⇒p = 11–q

putting the value of q in other equation, ⇒

⇒ pq = –12

⇒ (11–q)q = –12

⇒$11q–q^2$ = –12

⇒q^2–11q–12 = 0

⇒q^2 + q–12q–12 = 0

⇒q(q + 1)–12(q + 1) = 0

⇒ (q–12)(q + 1) = 0

So. q = 12 & q = –1

As, p = 11–q

thus, p = 11–12 & p = 11 + 1

p = –1 & p = 12

1 E. Question

In the following, you are given the product pq and the sum p + q. Determine p and q:

pq = –6 and p + q = –5

Answer

p + q = –5

$\Rightarrow p = -5-q$

putting the value of q in other equation,

$\Rightarrow pq = -6$

$\Rightarrow (-5-q)q = -6$

$\Rightarrow -5q-q^2 = -6$

$\Rightarrow q^2 + 5q-6 = 0$

$\Rightarrow q^2 + 6q-q-6 = 0$

$\Rightarrow q(q + 6)-1(q + 6) = 0$

$\Rightarrow (q-1)(q + 6) = 0$

so, $q = 1$ & $q = -6$

as $p = -5-q$

thus, $p = -5-1$ & $p = -5 + 6$

$p = -6$ & $p = 1$

1 F. Question

In the following, you are given the product pq and the sum p + q. Determine p and q:

$pq = -44$ and $p + q = -7$

Answer

$p + q = -7$

$\Rightarrow p = -7-q$

putting the value of q in other equation,

$\Rightarrow pq = -44$

$\Rightarrow (-7-q)q = -44$

$\Rightarrow -7q-q^2 = -44$

$\Rightarrow q^2 + 7q-44 = 0$

$\Rightarrow q^2 + 11q-4q-44 = 0$

$\Rightarrow q(q + 11)-4(q + 11) = 0$

$\Rightarrow (q-4)(q + 11) = 0$

so, q = 4 & q = −11

as, p = −7−q

thus, p = −7−4 & p = −7 + 11

p = −11 & p = 4

2 A. Question

Factorise:

$x^2 + 6x + 8$

Answer

$x^2 + 4x + 2x + 8$

= x(x + 4) + 2(x + 4)

= (x + 2)(x + 4)

2 B. Question

Factorise:

$x^2 + 4x + 3$

Answer

$x^2 + x + 3x + 3$

= x(x + 1) + 3(x + 1)

= (x + 1)(x + 3)

2 C. Question

Factorise:

$a^2 + 5a + 6$

Answer

$a^2 + 2a + 3a + 6$

= a(a + 2) + 3(a + 2)

= (a + 3)(a + 2)

2 D. Question

Factorise:

$a^2 - 5a + 6$

Answer

$a^2 - 2a - 3a + 6$

$= a(a-2) - 3(a-2)$

$= (a-3)(a-2)$

2 E. Question

Factorise:

$a^2 - 3a - 40$

Answer

$a^2 - 8a + 5a - 40$

$= a(a-4) + 5(a-4)$

$= (a + 5)(a-4)$

2 F. Question

Factorise:

$x^2 - x - 72$

Answer

$x^2 - 9x + 8x - 72$

$= x(x-9) + 8(x-9)$

$= (x + 8)(x-9)$

3 A. Question

Factorise:

$x^2 + 14x + 49$

Answer

using $(x + y)^2 = x^2 + 2xy + y^2$

$= x^2 + 2 \times 7 \times x + 7 \times 7$

$= (x + 7)^2$

$= (x + 7)(x + 7)$

3 B. Question

Factorise:

$4x^2 + 4x + 1$

Answer

using $(x + y)^2 = x^2 + 2xy + y^2$

$= (2x)^2 + 2 \times 2x \times 1 + 1^2$

$= (2x + 1)^2$

$= (2x + 1)(2x + 1)$

3 C. Question

Factorise:

$a^2 - 10a + 25$

Answer

using $(x-y)^2 = x^2-2xy + y^2$

$= a^2-2 \times a \times 5 + 5^2$

$= (a-5)^2$

$= (a-5)(a-5)$

3 D. Question

Factorise:

$2x^2-24x + 72$

Answer

using $(x-y)^2 = x^2-2xy + y^2$

$= 2[x^2-12x + 36]$

$= 2[x^2-2 \times x \times 6 + 6^2]$

$= 2(x-6)^2$

$= 2(x-6)(x-6)$

3 E. Question

Factorise:

$p^2 - 24p + 144$

Answer

using $(x-y)^2 = x^2-2xy + y^2$

$= p^2-2 \times p \times 12 + 12^2$

$= (p-12)^2$

$= (p-12)(p-12)$

3 F. Question

Factorise:

$x^3 - 12x^2 + 36x$

Answer

using $(x-y)^2 = x^2-2xy + y^2$

$= x[x^2-12x + 36]$

$= x[x^2-2 \times x \times 6 + 6^2]$

$= x(x-6)^2$

$= x(x-6)(x-6)$

Additional Problems 4

1 A. Question

4a + 12b is equal to

A. 4a

B. 12b

C. 4(a + 3b)

D. 3a

Answer

$\Rightarrow$ 4a + 12b

$\Rightarrow$ 4(a + 3b)

1 B. Question

The product of two numbers is positive and their sum negative only when

A. both are positive

B. both are negative

C. one positive the other negative

D. one of them equal to zero

Answer

The product of two numbers is positive, when either both the numbers are positive or both the numbers are negative.

Sum of two positive numbers is positive and the sum of two negative numbers is negative.

∴ The product of two numbers is positive and their sum negative only when both are negative.

1 C. Question

Factorising $x^2 + 6x + 8$, we get

A. (x + 1)(x + 8)

B. (x + 6)(x + 2)

C. (x + 10)(x – 2)

D. (x + 4)(x + 2)

Answer

$\Rightarrow x^2 + 6x + 8$

$\Rightarrow x^2 + 4x + 2x + 8$

$\Rightarrow x(x + 4) + 2(x + 4)$

$\Rightarrow (x + 4)(x + 2)$

1 D. Question

The denominator of an algebraic fraction should not be

A. 1

B. 0

C. 4

D. 7

Answer

The denominator of a algebraic function should not be 0.

∵ When denominator is 0, the fraction becomes undefined.

1 E. Question

If the sum of two integers is –2 and their product is –24, the numbers are

A. 6 and 4

B. –6 and 4

C. –6 and –4

D. 6 and –4

Answer

Let, the two integers = x and y

According to problem,

$\Rightarrow x + y = -2$ ……. (1)

and

$\Rightarrow xy = -24$ ……. (2)

$\therefore (x + y)^2 = (-2)^2$

$\Rightarrow (x - y)^2 + 4xy = 4$

$\Rightarrow (x - y)^2 + 4 \times (-24) = 4$

$\Rightarrow (x - y)^2 = 4 + 96 = 100$

$\Rightarrow x - y = \pm 10$

Case 1.

$x - y = 10$ …… (a) and $x + y = -2$ …… (b)

From (a) + (b) we get,

$\Rightarrow 2x = 8$

$\Rightarrow x = 4$

$\therefore 4 - y = 10$ [from (a)]

$\Rightarrow y = 4 - 10$

$\Rightarrow y = -6$

Case 2.

$x - y = -10$ …… (c) and $x + y = -2$ …… (d)

From (c) + (d) we get,

⇒ 2x = - 12

⇒ x = - 6

∴ -6 – y = - 10

⇒ y = 10 – 6

⇒ y = 4

∴ The numbers are 4 and – 6.

1 F. Question

The difference $(0.7)^2 - (0.3)^2$ simplifies to

A. 0.4

B. 0.04

C. 0.49

D. 0.56

Answer

$a^2 - b^2 = (a + b)(a - b)$

⇒ $(0.7)^2 - (0.3)^2$⇒ (0.7 + 0.3)(0.7 – 0.3)

⇒ 1 × 0.4

⇒ 0.4

2. Question

Factorise the following:

(i) $x^2 + 6x + 9$

(ii) $1 - 8x + 16x^2$

(iii) $4x^2 - 81y^2$

(iv) $4a^2 + 4ab + b^2$

(v) $a^2b^2 + c^2d^2 - a^2c^2 - b^2d^2$.

Answer

(i) ⇒ $x^2 + 6x + 9$

⇒ $x^2 + 2 \times 3 \times x + 3^2$

$\Rightarrow (x + 3)^2$

(ii) $\Rightarrow 1 - 8x + 16x^2$

$\Rightarrow 1^2 - 2 \times 1 \times 4x + (4x)^2$

$\Rightarrow (1 - 4x)^2$

(iii) $\Rightarrow 4x^2 - 81y^2$

$\Rightarrow (2x)^2 - (9y)^2$ $a^2 - b^2 = (a + b)(a - b)$

$\Rightarrow (2x + 9y)(2x - 9y)$

(iv) $\Rightarrow 4a^2 + 4ab + b^2$

$\Rightarrow (2a)^2 + 2 \times 2a \times b + b^2$

$\Rightarrow (2a + b)^2$

(v) $\Rightarrow a^2b^2 + c^2d^2 - a^2c^2 - b^2d^2$

$\Rightarrow (a^2b^2 - a^2c^2) - (c^2d^2 - b^2d^2)$

$\Rightarrow a^2(b^2 - c^2) - d^2(b^2 - c^2)$

$\Rightarrow (b^2 - c^2)(a^2 - d^2)$

3 A. Question

Foctorise the following:

$x^2 + 7x + 12$

Answer

$\Rightarrow x^2 + 7x + 12$

$\Rightarrow x^2 + 4x + 3x + 12$

$\Rightarrow x(x + 4) + 3(x + 4)$

$\Rightarrow (x + 4)(x + 3)$

3 B. Question

Foctorise the following:

$x^2 + x - 12$

Answer

$\Rightarrow x^2 + x - 12$

$\Rightarrow x^2 + 4x - 3x - 12$

$\Rightarrow x(x + 4) - 3(x + 4)$

$\Rightarrow (x + 4)(x - 3)$

3 C. Question

Foctorise the following:

$x^2 - 3x - 18$

Answer

$\Rightarrow x^2 - 3x - 18$

$\Rightarrow x^2 - 6x + 3x - 18$

$\Rightarrow x(x - 6) + 3(x - 6)$

$\Rightarrow (x - 6)(x + 3)$

3 D. Question

Foctorise the following:

$x^2 + 4x - 21$

Answer

$\Rightarrow x^2 + 4x - 21$

$\Rightarrow x^2 + 7x - 3x - 21$

$\Rightarrow x(x + 7) - 3(x + 7)$

$\Rightarrow (x + 7)(x - 3)$

3 E. Question

Foctorise the following:

$x^2 - 4x - 192$

Answer

$\Rightarrow x^2 - 4x - 192$

$\Rightarrow x^2 - 16x + 12x - 192$

$\Rightarrow x(x - 16) + 12(x - 16)$

$\Rightarrow (x - 16)(x + 12)$

3 F. Question

Foctorise the following:

$x^4 - 5x^2 + 4$

Answer

$\Rightarrow x^4 - 5x^2 + 4$

$\Rightarrow x^4 - 4x^2 - x^2 + 4$

$\Rightarrow x^2(x^2 - 4) - (x^2 - 4)$

$\Rightarrow (x^2 - 4)(x^2 - 1)$

$\Rightarrow (x^2 - 2^2)(x^2 - 1^2)$ $a^2 - b^2 = (a + b)(a - b)$

$\Rightarrow (x - 2)(x + 2)(x - 1)(x + 1)$

3 G. Question

Foctorise the following:

$x^4 - 13x^2y^2 + 36y^4$.

Answer

$\Rightarrow x^4 - 13x^2y^2 + 36y^4$

$\Rightarrow x^4 - 4x^2 y^2 - 9x^2y^2 + 36y^4$

$\Rightarrow x^2(x^2 - 4y^2) - 9y^2(x^2 - 4y^2)$

$\Rightarrow (x^2 - 4y^2)(x^2 - 9y^2)$

$\Rightarrow \{x^2 - (2y)^2\}\{x^2 - (3y)^2\}$

$\Rightarrow (x - 2y)(x + 2y)(x - 3y)(x + 3y)$ Since, $a^2 - b^2 = (a + b)(a - b)$

4 A. Question

Foctorise the following:

$2x^2 + 7x + 6$

Answer

$\Rightarrow 2x^2 + 7x + 6$

$\Rightarrow 2x^2 + 4x + 3x + 6$

$\Rightarrow 2x(x + 2) + 3(x + 2)$

$\Rightarrow (x + 2)(2x + 3)$

4 B. Question

Foctorise the following:

$3x^2 – 17x + 20$

Answer

$\Rightarrow 3x^2 – 17x + 20$

$\Rightarrow 3x^2 - 12x - 5x + 20$

$\Rightarrow 3x(x - 4) - 5(x - 4)$

$\Rightarrow (x - 4)(3x - 5)$

4 C. Question

Foctorise the following:

$6x^2 – 5x – 14$

Answer

$\Rightarrow 6x^2 - 5x – 14$

$\Rightarrow 6x^2 - 12x + 7x – 14$

$\Rightarrow 6x(x – 2) + 7(x – 2)$

$\Rightarrow (x - 2)(6x + 7)$

4 D. Question

Foctorise the following:

$4x^2 + 12xy + 5y^2$

Answer

$\Rightarrow 4x^2 + 12xy + 5y^2$

$\Rightarrow 4x^2 + 10xy + 2xy + 5y^2$

$\Rightarrow 2x(2x + 5y) + y(2x + 5y)$

$\Rightarrow (2x + 5y)(2x + y)$

4 E. Question

Foctorise the following:

$4x^4 - 5x^2 + 1$.

Answer

$\Rightarrow 4x^4 - 5x^2 + 1$

$\Rightarrow 4x^4 - 4x^2 - x^2 + 1$

$\Rightarrow 4x^2(x^2 - 1) - (x^2 - 1)$

$\Rightarrow (x^2 - 1)(4x^2 - 1)$

$\Rightarrow (x - 1)(x + 1)(2x - 1)(2x + 1)$

5 A. Question

Factorise the following:

$x^8 - y^8$

Answer

$\Rightarrow x^8 - y^8$

$\Rightarrow (x^4)^2 - (y^4)^2$ $a^2 - b^2 = (a + b)(a - b)$

$\Rightarrow (x^4 + y^4)(x^4 - y^4)$

$\Rightarrow (x^4 + y^4)\{(x^2)^2 - (y^2)^2\}$

$\Rightarrow (x^4 + y^4)(x^2 + y^2)(x^2 - y^2)$

$\Rightarrow (x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$

5 B. Question

Factorise the following:

$ax^4 - ax^{12}$

Answer

$\Rightarrow ax^4 - ax^{12}$

$\Rightarrow ax^4(1 - x^8)$

$\Rightarrow ax^4\{1 - (x^4)^2\}$

$\Rightarrow ax^4(1 + x^4)(1 - x^4)$ $a^2 - b^2 = (a + b)(a - b)$

$\Rightarrow ax^4(1 + x^4)\{1 - (x^2)^2\}$

$\Rightarrow ax^4 (1 + x^4)(1 + x^2)(1 - x^2)$

$\Rightarrow ax^4 (1 + x^4)(1 + x^2)(1 + x)(1 - x)$

5 C. Question

Factorise the following:

$x + x^2 + 1$

Answer

$\Rightarrow x^2 + x^4 + 1$

$\Rightarrow x^4 + 2x^2 + 1 - x^2$

$\Rightarrow x^4 + 2 \times x^2 \times 1 + 1^2 - x^2$

$\Rightarrow (x^2 + 1)^2 - x^2$

$\Rightarrow (x^2 + 1 + x)(x^2 + 1 - x)$ Since, $a^2 - b^2 = (a + b)(a - b)$

5 D. Question

Factorise the following:

$x^4 + 5x^2 + 9$.

Answer

$\Rightarrow x^4 + 5x^2 + 9$

$\Rightarrow x^4 + 6x^2 + 9 - x^2$

$\Rightarrow (x^2)^2 + 2 \times 3 \times x^2 + 3^2 - x^2$

$\Rightarrow (x^2 + 3)^2 - x^2$

$\Rightarrow (x^2 + 3 + x)\ (x^2 + 3 - x)$ Since, $a^2 - b^2 = (a + b)(a - b)$

6. Question

Factorise $x^4 + 4y^4$. Use this to prove that $2011^4 + 64$ is a composite number.

Answer

$\Rightarrow x^4 + 4y^4$

$\Rightarrow (x^2)^2 + (2y^2)^2$

$\Rightarrow (x^2 + 2y^2)^2 - 2 \times x^2 \times 2y^2$

$\Rightarrow (x^2 + 2y^2)^2 - 4x^2y^2\ (a^2 - b^2) = (a + b)(a - b)$

$\Rightarrow (x^2 + 2y^2)^2 - (2xy)^2$

$\Rightarrow (x^2 + 2y^2 + 2xy)(x^2 + 2y^2 - 2xy)$

Now we get,

$\Rightarrow 2011^4 + 64$

$\Rightarrow 2011^4 + 4 \times 2^4$

$\Rightarrow (2011^2)^2 + (2\times2^2)^2$

$\Rightarrow (2011^2 + 4^2)^2 - 2\times 2011^2 \times 2 \times 2^2$

$\Rightarrow (2011^2 + 4^2)^2 - 4 \times 2011^2 \times 2^2$

$\Rightarrow (2011^2 + 4^2)^2 - (2\times2011\times2)^2\ (a^2 - b^2 = (a + b)(a - b))$

$\Rightarrow (2011^2 + 4^2 + 2\times2011\times2)(2011^2 + 4^2 - 2\times2011\times2)$

$\Rightarrow (4044121 + 16 + 8044)(4044121 + 16 - 8044)$

$\Rightarrow 4052181 \times 4036093$