Chapter 13

Exponents and Powers

Introduction to Exponents and Powers

Exponents and powers help in writing the large numbers in a shorter form which we can read, understand and compare easily. A number placed in a superscript position to the right of another number or variable indicate repeated multiplication.

The short notation 10^4 stands for the product $10 \times 10 \times 10 \times 10$. Here '10' is called the base and '4' the exponent. The number 10^4 is read as 10 raised to the power of 4 or simply as the fourth power of 10. 10^4 is called the exponential form of 10,000.

a raised to the power *b*

For example:
a^2 indicates $a \times a$
a^3 indicates $a \times a \times a$
In expression 2^5, 2 is called the base and 5 is called the exponent or power.

$32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

i) Express 256 as a power 2.
$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8 = 256$

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

ii) Which one is greater 6^2 or 2^6?

$6^2 = 6 \times 6 = 36$
$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$
So, 2^6 is greater than 6^2.

Laws of exponents

Laws of Exponents

(a) First Law
If a is any non-zero integer (base) and m, n are integers (powers),

then $a^m a^n = a^m \times a^n = a^{m+n}$

for example,

(i) $(2)^5 \times (2)^{10} = (2)^{5+10} = (2)^5$

(ii) Multiply 3^2 and 3^4.

$\Rightarrow 3^2 \times 3^4$
$\Rightarrow 3^{2+4}$
$\Rightarrow 3^6$
$\Rightarrow 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$\Rightarrow 729$

(iii) Multiply -3^2 and -3^3.

$\Rightarrow (-3)^2 \times (-3)^3$
$\Rightarrow (-3)^{2+3}$
$\Rightarrow (-3)_5$
$\Rightarrow (-3) \times (-3) \times (-3) \times (-3) \times (-3)$
$\Rightarrow -243$

(b) Second Law

If a is any non-zero integer (base) and m, n are whole numbers (powers).

If $m > n$, then $a^m \div a^n = a^{m-n}$.

If $m < n$, thenam $\div a^n = \frac{1}{(a)}_{n-m}$.

For example,
(i)$3^6 \div 3^5 = 3^{6-5} = 3^1$

(ii)Divide 36 by 32.

$\Rightarrow 3^6 \div 3^2$
$\Rightarrow 3^{6-2}$
$\Rightarrow 3^4$
$\Rightarrow 3 \times 3 \times 3 \times 3$
$\Rightarrow 81$

(c) Third Law
If a and b are two non-zero integers (base) and m is a whole number (power), then $(am)^n = (a)^{mxn}$

For example,
(i)$(52)^3 = 5^{2\times3} = 5^6$
(ii)Calculate $(3^3)^2$.

$\Rightarrow 3^3 \times 3^3$
$\Rightarrow 3^3 + 3$
$\Rightarrow 3^6$

$\Rightarrow 3 \times 3 \times 3 \times 3 \times 3 \times 3$

$\Rightarrow 729$

iii) Which one is greater $(5^2) \times 3$ or $(5^2)^3$?

$(5^2) \times 3$ means 52 is multiplied by 3
i.e., $5 \times 5 \times 3 = 75$
but $(5^2)^3$ means 5^2 is multiplied by itself 3 times

$5^2 \times 5^2 \times 5^2$

5^{2+2+2}
$5^6 = 15625$
So, $(5^2)^3$ is greater than $(5^2) \times 3$.

(d) Fourth Law
If a and b are two non-zero integers (base) and m is a whole number (power), then $a^m \times b^m = (ab)^m$.

For example,

(i) $(3 \times 5)^4 = 3^4 \times 5^4$

$= (3 \times 3 \times 3 \times 3) \times (5 \times 5 \times 5 \times 5)$
$= 81 \times 625$
$= 50625$

(ii)Multiply 3^3 and 4^3.

$\Rightarrow 3^3 \times 4^3$
$\Rightarrow (3 \times 4)^3$
$\Rightarrow 12^3$
$\Rightarrow 12 \times 12 \times 12$
$\Rightarrow 1728$

(e) Fifth Law

If a and b are two non-zero integer (base) and m is a whole number (power), then $a^m \div b^m = (ab)^m = \frac{a^m}{b^m} = (a \div b)^m$

For example,

(i) $(\frac{2}{3})^4$

$=2^4 \div 3^4$

$=\frac{16}{81}$

(ii)Divide 3^3 by 4^3.

$\Rightarrow 3^3 \div 4^3$

$\Rightarrow (3)^3 \div (4)^3$
$\Rightarrow (34)^3$

(f) Sixth Law
If a is an integer (base) then $a^0 = 1$

For example,
(i) Divide 3^4 by 3^4.

$\Rightarrow 3^4 \div 3^4$

$\Rightarrow \frac{3X3X3X3}{3X3X3X3} = 1$

$\Rightarrow \frac{3^4}{3^4} = 1$

$\Rightarrow 3^{4-4} = 1$

$\Rightarrow 3^0 = 1$

So, we can say that any number (except 0) raised to the power (or

exponent) 0 is 1.

Simplify and write the answer in the exponential form
i) $(3^8 \div 3^5)^5 \div 3^5$

$(3^{8-5})^5 \div 3^5$ (Using the law am ÷ an = am−n)

$= (3^3)^5 \div 3^5$ (Using the law (am)n = amn)

$= (3)^{15} \div 35$

$= 3(^{15-5})$ (Using the law $a^m \div a^n = a^{m-n}$)

$= 3^{10}$

Expressing Numbers in Standard Form

Decimal Number system

Any number can be expressed as a decimal number between 1.0 and 10.0 including 1.0 multiplied by a power of 10.

For example:

1) 5678925 can be written as,

56789 = 50000+6000+700+80+9

= 5x104 + 6x103 + 7x102 + 8x101 + 9x100

2) 14678925 can be written as,

146789 = 100000+40000+6000+700+80+9

= 1x105 + 4x104 + 6x103 + 7x102 + 8x101 + 9x100

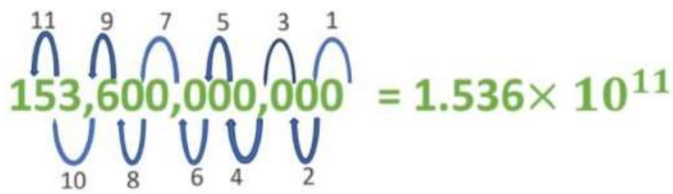
Here, we can see the exponents in first example starts from 4 and go on decreasing till 0.
Similarly, we can see the exponents in first example starts from 5 and go on decreasing till 0.

Expressing numbers in Standard Form

Any number can be expressed as a decimal number between 1.0 and 10.0 including 1.0 multiplied by a power of 10. Such a form of a number is called its standard form.

A number is said to be in the Standard Form, if it is expressed as the product of a number between 1 and 10 and the integral power of 10.

For example,



Decimal is moved 11 places towards left, therefore the power is positive.

Large numbers in Standard Form
Express the following numbers in standard form.
i) 14320000000
ii) 205000000

i) 1432000000 = 1432×10000000

=1.432×1000×10000000

=$1.432 \times 10^3 \times 10^7$

$=1.432 \times 10^{(3+7)}$

$= 1.432 \times 10^{10}$

ii) $205000000 = 205 \times 1000000$

$= 2.05 \times 100 \times 1000000$

$= 2.05 \times 10^{2} \times 106$

$= 2.05 \times 10^{(2+6)}$

$= 2.05 \times 10^{8}$

Express 15360000000 in standard form.

$15360000000 = 1536 \times 10000000$

$= 1.536 \times 1000 \times 10000000$

$= 1.536 \times 10^{3} \times 10^{7}$

$= 1.536 \times 10^{(3+7)}$

$= 1.536 \times 10^{10}$