7

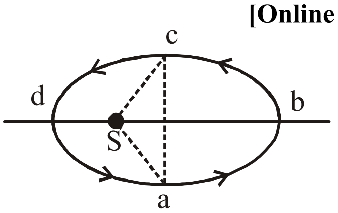
Gravitation

TOPIC 1 Kepler's Laws of Planetary Motion

1. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the center of the Sun, its areal velocity is: **[9 Jan. 2019 I]**

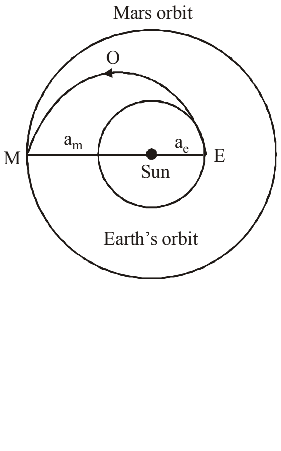
(a) $\frac{L}{m}$ (b) $\frac{4L}{m}$ (c) $\frac{L}{2m}$ (d) $\frac{2L}{m}$

2. Figure shows elliptical path abcd of a planet around the sun S such that the area of triangle csa is $\frac{1}{4}$ the area of the ellipse. (See figure) With db as the semimajor axis, and ca as the semiminor axis. If t_1 is the time taken for planet to go over path abc and t_2 for path taken over cda then: **[Online April 9, 2016]**



(a) $t_1 = 4t_2$ (b) $t_1 = 2t_2$
(c) $t_1 = 3t_2$ (d) $t_1 = t_2$

3. India's Mangalyan was sent to the Mars by launching it into a transfer orbit EOM around the sun. It leaves the earth at E and meets Mars at M. If the semi-major axis of Earth's orbit is $a_e = 1.5 \times 10^{11}$ m, that of Mars orbit $a_m = 2.28 \times 10^{11}$ m, taken Kepler's laws give the estimate of time for Mangalyan to reach Mars from Earth to be close to: **[Online April 9, 2014]**



(a) 500 days (b) 320 days
(c) 260 days (d) 220 days

4. The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become **[2003]**

(a) 10 hours (b) 80 hours
(c) 40 hours (d) 20 hours

TOPIC 2 Newton's Universal Law of Gravitation

5. A straight rod of length L extends from $x = a$ to $x = L + a$. The gravitational force it exerts on point mass 'm' at $x = 0$, if the mass per unit length of the rod is $A + Bx^2$, is given by: **[12 Jan. 2019 I]**

(a) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)-BL\right]$

(b) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)-BL\right]$

(c) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)+BL\right]$

(d) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$

6. Take the mean distance of the moon and the sun from the earth to be 0.4×10^6 km and 150×10^6 km respectively. Their masses are 8×10^{22} kg and 2×10^{30} kg respectively. The radius of the earth is 6400 km. Let ΔF_1 be the difference in the forces exerted by the moon at the nearest and farthest points on the earth and ΔF_2 be the difference in the force exerted by the sun at the nearest and farthest points on the earth. Then, the number closest to $\frac{\Delta F_1}{\Delta F_2}$ is: **[Online April 15, 2018]**

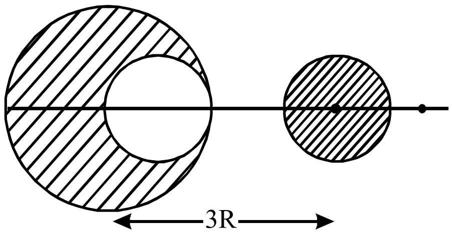
(a) 2 (b) 6 (c) 10^{-2} (d) 0.6

7. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is: **[2014]**

(a) $\sqrt{\frac{GM}{R}}$ (b) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

(c) $\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$ (d) $\frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

8. From a sphere of mass M and radius R, a smaller sphere of radius $\frac{R}{2}$ is carved out such that the cavity made in the original sphere is between its centre and the periphery (See figure). For the configuration in the figure where the distance between the centre of the original sphere and the removed sphere is 3R, the gravitational force between the two sphere is: **[Online April 11, 2014]**



(a) $\frac{41\,GM^2}{3600\,R^2}$ (b) $\frac{41\,GM^2}{450\,R^2}$ (c) $\frac{59\,GM^2}{450\,R^2}$ (d) $\frac{GM^2}{225\,R^2}$

9. Two particles of equal mass '*m*' go around a circle of radius *R* under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is **[2011 RS]**

(a) $\sqrt{\frac{Gm}{4R}}$ (b) $\sqrt{\frac{Gm}{3R}}$ (c) $\sqrt{\frac{Gm}{2R}}$ (d) $\sqrt{\frac{Gm}{R}}$

10. Two spherical bodies of mass *M* and 5*M* & radii *R* & 2*R* respectively are released in free space with initial separation between their centres equal to 12 *R*. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is **[2003]**

(a) 2.5*R* (b) 4.5*R* (c) 7.5*R* (d) 1.5*R*

TOPIC 3 Acceleration due to Gravity

11. The value of acceleration due to gravity is g_1 at a height $h = \frac{R}{2}$ (R = radius of the earth) from the surface of the earth. It is again equal to g_1 and a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$ equals : **[5 Sep. 2020 (I)]**

(a) $\frac{4}{9}$ (b) $\frac{5}{9}$ (c) $\frac{1}{3}$ (d) $\frac{7}{9}$

12. The acceleration due to gravity on the earth's surface at the poles is g and angular velocity of the earth about the axis passing through the pole is ω. An object is weighed at the equator and at a height h above the poles by using a spring balance. If the weights are found to be same, then h is : ($h<<R$, where R is the radius of the earth) **[5 Sep. 2020 (II)]**

(a) $\frac{R^2\omega^2}{2g}$ (b) $\frac{R^2\omega^2}{g}$

(c) $\frac{R^2\omega^2}{4g}$ (d) $\frac{R^2\omega^2}{8g}$

13. The height 'h' at which the weight of a body will be the same as that at the same depth 'h' from the surface of the earth is (Radius of the earth is R and effect of the rotation of the earth is neglected) : **[2 Sep. 2020 (II)]**

(a) $\frac{\sqrt{5}}{2}R-R$ (b) $\frac{R}{2}$

(c) $\frac{\sqrt{5}R-R}{2}$ (d) $\frac{\sqrt{3}R-R}{2}$

14. A box weighs 196 N on a spring balance at the north pole. Its weight recorded on the same balance if it is shifted to the equator is close to (Take $g = 10$ ms^{-2} at the north pole and the radius of the earth = 6400 km): **[7 Jan. 2020 II]**

(a) 195.66 N (b) 194.32 N
(c) 194.66 N (d) 195.32 N

15. The ratio of the weights of a body on the Earth's surface to that on the surface of a planet is 9:4. The mass of the planet is $\frac{1}{9}$th of that of the Earth. If 'R' is the radius of the Earth, what is the radius of the planet ? (Take the planets to have the same mass density). **[12 April 2019 II]**

(a) $\frac{R}{3}$ (b) $\frac{R}{4}$ (c) $\frac{R}{9}$ (d) $\frac{R}{2}$

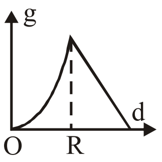
16. The value of acceleration due to gravity at Earth's surface is 9.8 ms^{-2}. The altitude above its surface at which the acceleration due to gravity decreases to 4.9 ms^{-2}, is close to : (Radius of earth = 6.4×10^6 m) **[10 April 2019 I]**

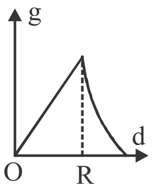
(a) 2.6×10^6 m (b) 6.4×10^6 m
(c) 9.0×10^6 m (d) 1.6×10^6 m

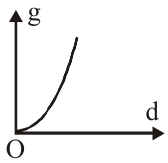
17. Suppose that the angular velocity of rotation of earth is increased. Then, as a consequence. **[Online April 16, 2018]**

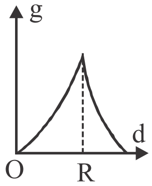
(a) There will be no change in weight anywhere on the earth
(b) Weight of the object, everywhere on the earth, wild decrease
(c) Weight of the object, everywhere on the earth, will increase
(d) Except at poles, weight of the object on the earth will decrease

18. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius): **[2017, Online May 7, 2012]**

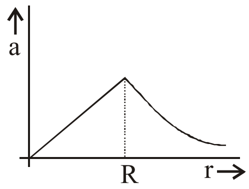
(a) 

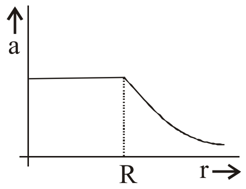
(b) 

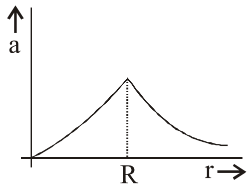
(c) 

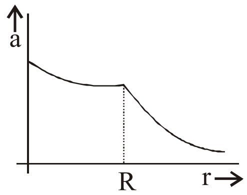
(d) 

19. The mass density of a spherical body is given by $\rho(r) = \frac{k}{r}$ for $r \le R$ and $\rho(r) = 0$ for $r > R$,
where r is the distance from the centre.
The correct graph that describes qualitatively the acceleration, a, of a test particle as a function of r is : **[Online April 9, 2017]**

(a) 

(b) 

(c) 

(d) 

20. If the Earth has no rotational motion, the weight of a person on the equator is W. Determine the speed with which the earth would have to rotate about its axis so that the person at the equator will weight $\frac{3}{4}$W . Radius of the Earth is 6400 km and g = 10 m/s^2. **[Online April 8, 2017]**

(a) 1.1×10^{-3} rad/s (b) 0.83×10^{-3} rad/s
(c) 0.63×10^{-3} rad/s (d) 0.28×10^{-3} rad/s

21. The change in the value of acceleration of earth towards sun, when the moon comes from the position of solar eclipse to the position on the other side of earth in line with sun is:
(mass of the moon = 7.36×10^{22} kg, radius of the moon's orbit = 3.8×10^8 m). **[Online April 22, 2013]**

(a) 6.73×10^{-5} m/s^2 (b) 6.73×10^{-3} m/s^2
(c) 6.73×10^{-2} m/s^2 (d) 6.73×10^{-4} m/s^2

22. Assuming the earth to be a sphere of uniform density, the acceleration due to gravity inside the earth at a distance of r from the centre is proportional to **[Online May 12, 2012]**

(a) r (b) r^{-1} (c) r^2 (d) r^{-2}

23. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth, is **[2009]**

(a) $\frac{R}{\sqrt{2}}$ (b) $R/2$ (c) $\sqrt{2}R$ (d) 2R

24. The change in the value of 'g' at a height 'h' above the surface of the earth is the same as at a depth 'd' below the surface of earth. When both 'd' and 'h' are much smaller than the radius of earth, then which one of the following is correct? **[2005]**

(a) $d = \frac{3h}{2}$ (b) $d = \frac{h}{2}$
(c) $d = h$ (d) $d = 2h$

25. Average density of the earth **[2005]**

(a) is a complex function of g
(b) does not depend on g
(c) is inversely proportional to g
(d) is directly proportional to g

TOPIC 4 Gravitational Field and Potential Energy

26. Two planets have masses M and 16 M and their radii are a and $2a$, respectively. The separation between the centres of the planets is $10a$. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach the surface of smaller planet, the minimum firing speed needed is : **[6 Sep. 2020 (II)]**

(a) $2\sqrt{\frac{GM}{a}}$ (b) $4\sqrt{\frac{GM}{a}}$
(c) $\sqrt{\frac{GM^2}{ma}}$ (d) $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

27. On the x-axis and at a distance x from the origin, the gravitational field due to a mass distribution is given by $\frac{Ax}{(x^2+a^2)^{3/2}}$ in the x-direction. The magnitude of gravitational potential on the x-axis at a distance x, taking its value to be zero at infinity, is : **[4 Sep. 2020 (I)]**

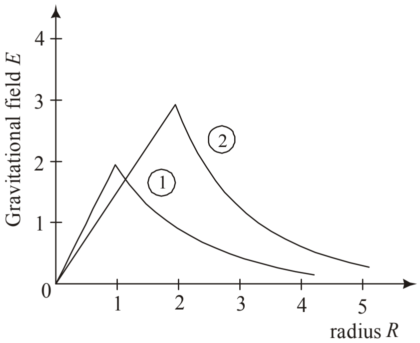
(a) $\frac{A}{(x^2+a^2)^{1/2}}$ (b) $\frac{A}{(x^2+a^2)^{3/2}}$
(c) $A(x^2+a^2)^{1/2}$ (d) $A(x^2+a^2)^{3/2}$

28. The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$. Then the gravitational field is maximum at : **[3 Sep. 2020 (II)]**

(a) $r = \sqrt{\frac{3}{4}}R$ (b) $r = R$

(c) $r = \frac{1}{\sqrt{3}}R$ (d) $r = \sqrt{\frac{5}{9}}R$

29. Consider two solid spheres of radii $R_1 = 1m$, $R_2 = 2m$ and masses M_1 and M_2, respectively. The gravitational field due to sphere ① and ② are shown. The value of $\frac{m_1}{m_2}$ is: **[8 Jan. 2020 I]**



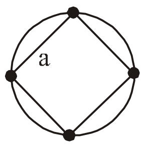
(a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

30. An asteroid is moving directly towards the centre of the earth. When at a distance of 10 R (R is the radius of the earth) from the earths centre, it has a speed of 12 km/s. Neglecting the effect of earths atmosphere, what will be the speed of the asteroid when it hits the surface of the earth (escape velocity from the earth is 11.2 km/ s)? Give your answer to the nearest integer in kilometer/s _____. **[NA 8 Jan. 2020 II]**

31. A solid sphere of mass 'M' and radius 'a' is surrounded by a uniform concentric spherical shell of thickness 2a and mass 2M. The gravitational field at distance '3a' from the centre will be: **[9 April 2019 I]**

(a) $\frac{2GM}{9a^2}$ (b) $\frac{GM}{9a^2}$ (c) $\frac{GM}{3a^2}$ (d) $\frac{2GM}{3a^2}$

32. Four identical particles of mass M are located at the corners of a square of side 'a'. What should be their speed if each of them revolves under the influence of others' gravitational field in a circular orbit circumscribing the square ? **[8 April 2019 I]**



(a) $1.35\sqrt{\frac{GM}{a}}$ (b) $1.16\sqrt{\frac{GM}{a}}$

(c) $1.21\sqrt{\frac{GM}{a}}$ (d) $1.41\sqrt{\frac{GM}{a}}$

33. A test particle is moving in circular orbit in the gravitational field produced by a mass density $r(r) = \frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T: **[8 April 2019 II]**

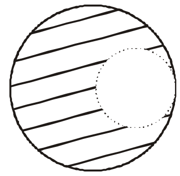
(a) T/R is a constant (b) T^2/R^3 is a constant

(c) T/R^2 is a constant (d) TR is a constant

34. A body of mass m is moving in a circular orbit of radius R about a planet of mass M. At some instant, it splits into two equal masses. The first mass moves in a circular orbit of radius $\frac{R}{2}$, and the other mass, in a circular orbit of radius $\frac{3R}{2}$. The difference between the final and initial total energies is: **[Online April 15, 2018]**

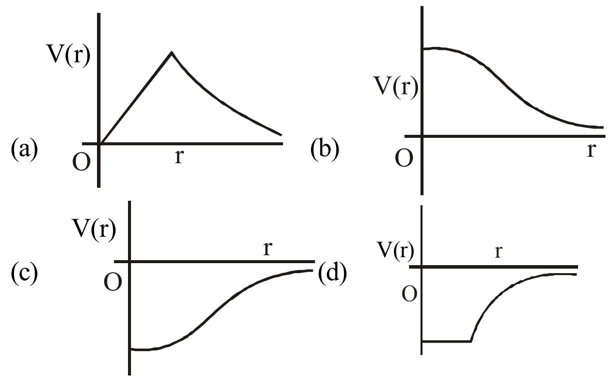
(a) $-\frac{GMm}{2R}$ (b) $+\frac{GMm}{6R}$ (c) $-\frac{GMm}{6R}$ (d) $\frac{GMm}{2R}$

35. From a solid sphere of mass M and radius R, a spherical portion of radius R/2 is removed, as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is :
(G = *gravitational constant*) **[2015]**



(a) $\frac{-2GM}{3R}$ (b) $\frac{-2GM}{R}$ (c) $\frac{-GM}{2R}$ (d) $\frac{-GM}{R}$

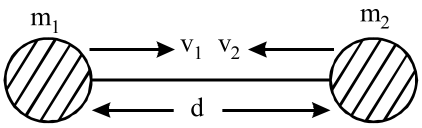
36. Which of the following most closely depicts the correct variation of the gravitational potential V(r) due to a large planet of radius R and uniform mass density ? (figures are not drawn to scale) **[Online April 11, 2015]**



37. The gravitational field in a region is given by $\vec{g} = 5\text{N/kg}\,\hat{i} + 12\text{N/kg}\,\hat{j}$. The change in the gravitational potential energy of a particle of mass 1 kg when it is taken from the origin to a point (7 m, – 3 m) is: **[Online April 19, 2014]**

(a) 71 J (b) $13\sqrt{58}$J (c) – 71 J (d) 1 J

38.



Two hypothetical planets of masses m_1 and m_2 are at rest when they are infinite distance apart. Because of the gravitational force they move towards each other along the line joining their centres. What is their speed when their separation is 'd'? **[Online April 12, 2014]**

(Speed of m_1 is v_1 and that of m_2 is v_2)

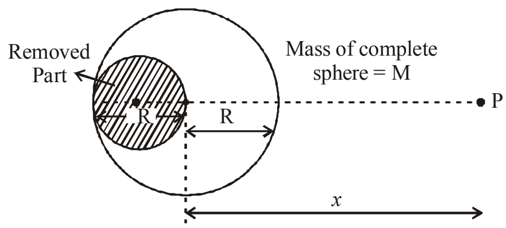
(a) $v_1 = v_2$

(b) $v_1 = m_2\sqrt{\dfrac{2G}{d(m_1+m_2)}}$ $v_2 = m_1\sqrt{\dfrac{2G}{d(m_1+m_2)}}$

(c) $v_1 = m_1\sqrt{\dfrac{2G}{d(m_1+m_2)}}$ $v_2 = m_2\sqrt{\dfrac{2G}{d(m_1+m_2)}}$

(d) $v_1 = m_2\sqrt{\dfrac{2G}{m_1}}$ $v_2 = m_2\sqrt{\dfrac{2G}{m_2}}$

39. The gravitational field, due to the 'left over part' of a uniform sphere (from which a part as shown, has been 'removed out'), at a very far off point, P, located as shown, would be (nearly) : **[Online April 9, 2013]**



(a) $\dfrac{5}{6}\dfrac{GM}{x^2}$ (b) $\dfrac{8}{9}\dfrac{GM}{x^2}$ (c) $\dfrac{7}{8}\dfrac{GM}{x^2}$ (d) $\dfrac{6}{7}\dfrac{GM}{x^2}$

40. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of *g* and *R* (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be **[2012]**

(a) 6.4×10^{11} Joules (b) 6.4×10^{8} Joules
(c) 6.4×10^{9} Joules (d) 6.4×10^{10} Joules

41. A point particle is held on the axis of a ring of mass *m* and radius *r* at a distance *r* from its centre *C*. When released, it reaches *C* under the gravitational attraction of the ring. Its speed at *C* will be **[Online May 26, 2012]**

(a) $\sqrt{\dfrac{2Gm}{r}\left(\sqrt{2}-1\right)}$ (b) $\sqrt{\dfrac{Gm}{r}}$

(c) $\sqrt{\dfrac{2Gm}{r}\left(1-\dfrac{1}{\sqrt{2}}\right)}$ (d) $\sqrt{\dfrac{2Gm}{r}}$

42. Two bodies of masses *m* and 4 *m* are placed at a distance *r*. The gravitational potential at a point on the line joining them where the gravitational field is zero is: **[2011]**

(a) $-\dfrac{4Gm}{r}$ (b) $-\dfrac{6Gm}{r}$ (c) $-\dfrac{9Gm}{r}$ (d) zero

43. This question contains Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements. **[2008]**

Statement-1 : For a mass *M* kept at the centre of a cube of side '*a*', the flux of gravitational field passing through its sides 4 π *GM*. and

Statement-2: If the direction of a field due to a point source is radial and its dependence on the distance '*r*' from the source is given as $\dfrac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement - 2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

44. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them to take the particle far away from the sphere

(you may take $G = 6.67\times 10^{-11}$ Nm^2/kg^2) **[2005]**

(a) 3.33×10^{-10} J (b) 13.34×10^{-10} J
(c) 6.67×10^{-10} J (d) 6.67×10^{-9} J

45. If '*g*' is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass '*m*' raised from the surface of the earth to a height equal to the radius '*R*' of the earth is **[2004]**

(a) $\dfrac{1}{4}mgR$ (b) $\dfrac{1}{2}mgR$ (c) $2\,mgR$ (d) mgR

46. Energy required to move a body of mass *m* from an orbit of radius 2*R* to 3*R* is **[2002]**

(a) $GMm/12R^2$ (b) $GMm/3R^2$
(c) $GMm/8R$ (d) $GMm/6R$.

TOPIC 5 Motion of Satellites, Escape Speed and Orbital Velocity

47. A satellite is in an elliptical orbit around a planet P. It is observed that the velocity of the satellite when it is farthest from the planet is 6 times less than that when it is closest to the planet. The ratio of distances between the satellite and the planet at closest and farthest points is : **[NA 6 Sep. 2020 (I)]**

(a) 1 : 6 (b) 1 : 3 (c) 1 : 2 (d) 3 : 4

48. A body is moving in a low circular orbit about a planet of mass M and radius R. The radius of the orbit can be taken to be R itself. Then the ratio of the speed of this body in the orbit to the escape velocity from the planet is : **[4 Sep. 2020 (II)]**

(a) $\frac{1}{\sqrt{2}}$ (b) 2

(c) 1 (d) $\sqrt{2}$

49. A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e. By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that it become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R. Value of R is : **[3 Sep. 2020 (I)]**

(a) $4R_e$ (b) $2.5R_e$ (c) $3R_e$ (d) $2R_e$

50. The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance 'r' from its centre. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as : **[2 Sep. 2020 (I)]**

(a) $T^2 \propto R$ (b) $T^2 \propto R^3$ (c) $T^2 \propto \frac{1}{R^3}$ (d) $T \propto R$

51. A body A of mass m is moving in a circular orbit of radius R about a planet. Another body B of mass $\frac{m}{2}$ collides with A with a velocity which is half $\left(\frac{\vec{v}}{2}\right)$ the instantaneous velocity $\vec{v}$ or A. The collision is completely inelastic. Then, the combined body: **[9 Jan. 2020 I]**

(a) continues to move in a circular orbit

(b) Escapes from the Planet's Gravitational field

(c) Falls vertically downwards towards the planet

(d) starts moving in an elliptical orbit around the planet

52. The energy required to take a satellite to a height 'h' above Earth surface (radius of Eareth = 6.4×10^3 km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal, is: **[9 Jan. 2019 II]**

(a) 1.6×10^3 km (b) 3.2×10^3 km

(c) 6.4×10^3 km (d) 28×10^4 km

53. Planet A has mass M and radius R. Planet B has half the mass and half the radius of Planet A. If the escape velocities from the Planets A and B are v_A and v_B, respectively, then $\frac{v_A}{v_B} = \frac{n}{4}$. The value of n is : **[9 Jan. 2020 II]**

(a) 4 (b) 1 (c) 2 (d) 3

54. A satellite of mass m is launched vertically upwards with an initial speed u from the surface of the earth. After it reaches height R (R = radius of the earth), it ejects a rocket of mass $\frac{m}{10}$ so that subsequently the satellite moves in a circular orbit. The kinetic energy of the rocket is (G is the gravitational constant; M is the mass of the earth): **[7 Jan. 2020 I]**

(a) $\frac{m}{20}\left(u^2 + \frac{113}{200}\frac{GM}{R}\right)$ (b) $5m\left(u^2 - \frac{119}{200}\frac{GM}{R}\right)$

(c) $\frac{3m}{8}\left(u + \sqrt{\frac{5GM}{6R}}\right)^2$ (d) $\frac{m}{20}\left(u - \sqrt{\frac{2GM}{3R}}\right)^2$

55. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet ? [Given : Mass of Planet = 8×10^{22} kg, Radius of planet = 2×10^6 m, Gravitational constant $G = 6.67 \times 10^{-11}$ Nm^2/kg^2] **[10 April 2019 II]**

(a) 9 (b) 17 (c) 13 (d) 11

56. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon. **[8 April 2019 II]**

(a) $\frac{E}{64}$ (b) $\frac{E}{32}$ (c) $\frac{E}{4}$ (d) $\frac{E}{16}$

57. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass, falling towards the earth collides with the satellite completely in elastically. The speeds of the satellite and the meteorite are the same, Just before the collision. The subsequent motion of the combined body will be **[12 Jan. 2019 I]**

(a) such that it escape to infinity
(b) In an elliptical orbit
(c) in the same circular orbit of radius R
(d) in a circular orbit of a different radius

58. Two satellites, A and B, have masses m and 2m respectively. A is in a circular orbit of radius R, and B is in a circular orbit of radius 2R around the earth. The ratio of their kinetic energies, T_A/T_B, is : **[12 Jan. 2019 II]**

(a) $\frac{1}{2}$ (b) 1
(c) 2 (d) $\sqrt{\frac{1}{2}}$

59. A satellite is revolving in a circular orbit at a height h from the earth surface, such that h << R where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum increase in the speed required so that the satellite could escape from the gravitational field of earth is: **[11 Jan. 2019 I]**

(a) $\sqrt{2gR}$ (b) $\sqrt{gR}$
(c) $\sqrt{\frac{gR}{2}}$ (d) $\sqrt{gR}\left(\sqrt{2}-1\right)$

60. A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is: **[10 Jan. 2019 I]**

(a) $2mv^2$ (b) mv^2
(c) $\frac{1}{2}mv^2$ (d) $\frac{3}{2}mv^2$

61. Two stars of masses 3×10^{31} kg each, and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpen-dicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is: (Take Gravitational constant $G = 66 \times 10^{-11}$ Nm^2 kg^{-2}) **[10 Jan. 2019 II]**

(a) 2.4×10^4 m/s (b) 1.4×10^5 m/s
(c) 3.8×10^4 m/s (d) 2.8×10^5 m/s

62. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; h << R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.) **[2016]**

(a) $\sqrt{gR/2}$ (b) $\sqrt{gR}\left(\sqrt{2}-1\right)$
(c) $\sqrt{2gR}$ (d) $\sqrt{gR}$

63. An astronaut of mass m is working on a satellite orbiting the earth at a distance h from the earth's surface. The radius of the earth is R, while its mass is M. The gravitational pull F_G on the astronaut is : **[Online April 10, 2016]**

(a) Zero since astronaut feels weightless
(b) $\frac{GMm}{(R+h)^2} < F_G < \frac{GMm}{R^2}$
(c) $F_G = \frac{GMm}{(R+h)^2}$
(d) $0 < F_G < \frac{GMm}{R^2}$

64. A very long (length L) cylindrical galaxy is made of uniformly distributed mass and has radius R(R << L). A star outside the galaxy is orbiting the galaxy in a plane perpendicular to the galaxy and passing through its centre. If the time period of star is T and its distance from the galaxy's axis is r, then : **[Online April 10, 2015]**

(a) $T \propto r$ (b) $T \propto \sqrt{r}$
(c) $T \propto r^2$ (d) $T^2 \propto r^3$

65. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R? **[2013]**

(a) $\frac{5GmM}{6R}$ (b) $\frac{2GmM}{3R}$ (c) $\frac{GmM}{2R}$ (d) $\frac{GmM}{2R}$

66. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 km s^{-1}, the escape velocity from the surface of the planet would be **[2008]**

(a) 1.1 km s^{-1} (b) 11 km s^{-1}
(c) 110 km s^{-1} (d) 0.11 km s^{-1}

67. Suppose the gravitational force varies inversely as the nth power of distance. Then the time period of a planet in circular orbit of radius 'R' around the sun will be proportional to **[2004]**

(a) R^n (b) $R^{\left(\frac{n-1}{2}\right)}$
(c) $R^{\left(\frac{n+1}{2}\right)}$ (d) $R^{\left(\frac{n-2}{2}\right)}$

68. The time period of an earth satellite in circular orbit is independent of **[2004]**

(a) both the mass and radius of the orbit
(b) radius of its orbit
(c) the mass of the satellite
(d) neither the mass of the satellite nor the radius of its orbit.

69. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is **[2004]**

(a) $\frac{gR^2}{R+x}$ (b) $\frac{gR}{R-x}$ (c) gx (d) $\left(\frac{gR^2}{R+x}\right)^{1/2}$

70. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45°with the vertical, the escape velocity will be **[2003]**

(a) $11\sqrt{2}$ km / s (b) 22 km/s

(c) 11 km/s (d) $\frac{11}{\sqrt{2}}$ km / s

71. The kinetic energy needed to project a body of mass m from the earth surface (radius R) to infinity is **[2002]**

(a) $mgR/2$ (b) $2mgR$ (c) mgR (d) $mgR/4$.

72. If suddenly the gravitational force of attraction between Earth and a satellite revolving around it becomes zero, then the satellite will **[2002]**

(a) continue to move in its orbit with same velocity

(b) move tangentially to the original orbit in the same velocity

(c) become stationary in its orbit

(d) move towards the earth

73. The escape velocity of a body depends upon mass as **[2002]**

(a) m^0 (b) m^1 (c) m^2 (d) m^3

Hints & Solutions

1. **(c)** Areal velocity; $\frac{dA}{dt}$

$dA = \frac{1}{2}r^2 d\theta$

$\Rightarrow \frac{dA}{dt} = \frac{1}{2}r^2 \frac{d\theta}{dt} = \frac{1}{2}r^2\omega$

Also, $L = mvr = mr^2\omega$

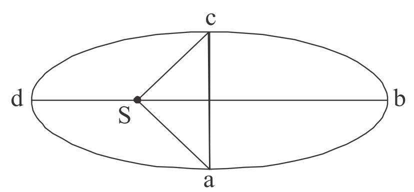
$\therefore \frac{dA}{dt} = \frac{1}{2}\frac{L}{m}$

2. **(c)** Let area of ellipse abcd = x

Area of SabcS $= \frac{x}{2} + \frac{x}{4}$ (i.e., ar of abca + SacS)

(Area of half ellipse + Area of triangle)

$= \frac{3x}{4}$



Area of SadcS $= x - \frac{3x}{4} = \frac{x}{4}$

$\frac{\text{Area of SabcS}}{\text{Area of SadcS}} = \frac{3x/4}{x/4} = \frac{t_1}{t_2}$

$\frac{t_1}{t_2} = 3$ or, $t_1 = 3t_2$

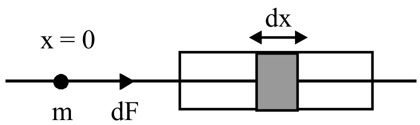
3. **(b)**
4. **(c)** According to Kepler's law of periods $T^2 \propto R^3$

$\therefore \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{R_2}{R_1}\right)^3$

$\Rightarrow T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2} = 5 \times \left[\frac{4R}{R}\right]^{3/2}$

$= 5 \times 2^3 = 40$ hours

5. **(d)** Given $\lambda = (A + Bx^2)$,
Taking small element dm of length dx at a distance x from x = 0



so, $dm = \lambda\, dx$
$dm = (A + Bx^2)dx$

$dF = \frac{Gm\,dm}{x^2}$

$\Rightarrow F = \int_a^{a+L} \frac{Gm}{x^2}(A + Bx^2)dx$

$= Gm\left[-\frac{A}{x} + Bx\right]_a^{a+L}$

$= Gm\left[A\left(\frac{1}{a} - \frac{1}{a+L}\right) + BL\right]$

6. **(a)** As we know, Gravitational force of attraction,

$F = \frac{GMm}{R^2}$

$F_1 = \frac{GM_e m}{r_1^2}$ and $F_2 = \frac{GM_e M_s}{r_2^2}$

$\Delta F_1 = \frac{2GM_e m}{r_1^3}\Delta r_1$ and $\Delta F_2 = \frac{GM_e M_s}{r_2^3}\Delta r_2$

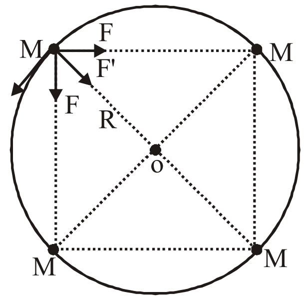
$\frac{\Delta F_1}{\Delta F_2} = \frac{m\Delta r_1}{r_1^3}\frac{r_2^3}{M_s \Delta r_2} = \left(\frac{m}{M_s}\right)\left(\frac{r_2^3}{r_1^3}\right)\left(\frac{\Delta r_1}{\Delta r_2}\right)$

Using $\Delta r_1 = \Delta r_2 = 2\,R_{earth}$; $m = 8 \times 10^{22}$ kg;
$M_s = 2 \times 10^{30}$ kg
$r_1 = 0.4 \times 10^6$ km and $r_2 = 150 \times 10^6$ km

$\frac{\Delta F_1}{\Delta F_2} = \left(\frac{8 \times 10^{22}}{2 \times 10^{30}}\right)\left(\frac{150 \times 10^6}{0.4 \times 10^6}\right)^3 \times 1 \cong 2$

7. **(d)** $2F\cos 45° + F' = \frac{Mv^2}{R}$ (From figure)

Where $F = \frac{GM^2}{(\sqrt{2}R)^2}$ and $F' = \frac{GM^2}{4R^2}$



$\Rightarrow \frac{2 \times GM^2}{\sqrt{2}(R\sqrt{2})^2} + \frac{GM^2}{4R^2} = \frac{Mv^2}{R}$

$\Rightarrow \frac{GM^2}{R}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right] = Mv^2$

$\therefore v = \sqrt{\frac{GM}{R}\left(\frac{\sqrt{2}+4}{4\sqrt{2}}\right)} = \frac{1}{2}\sqrt{\frac{GM}{R}(1 + 2\sqrt{2})}$

8. **(a)** Volume of removed sphere

$$V_{\text{remo}} = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{4}{3}\pi R^3\left(\frac{1}{8}\right)$$

Volume of the sphere (remaining)

$$V_{\text{remain}} = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi R^3\left(\frac{1}{8}\right) = \frac{4}{3}\pi R^3\left(\frac{7}{8}\right)$$

Therefore mass of sphere carved and remaining sphere are at respectively $\frac{1}{8}$ M and $\frac{7}{8}$ M.

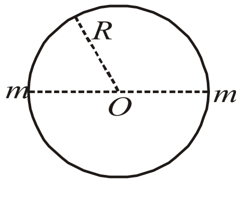
Therefore, gravitational force between these two sphere,

$$F = \frac{GMm}{r^2} = \frac{G\frac{7M}{8}\times\frac{1}{8}M}{(3R)^2} = \frac{7}{64\times 9}\frac{GM^2}{R^2}$$

$$\simeq \frac{41}{3600}\frac{GM^2}{R^2}$$

9. **(a)** As two masses revolve about the common centre of mass O.

$\therefore$ Mutual gravitational attraction = centripetal force



$$\frac{Gm^2}{(2R)^2} = m\omega^2 R$$

$$\Rightarrow \quad \frac{Gm}{4R^3} = \omega^2$$

$$\Rightarrow \quad \omega = \sqrt{\frac{Gm}{4R^3}}$$

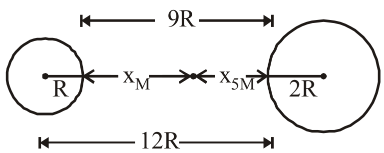
If the velocity of the two particles with respect to the centre of gravity is v then

$$v = \omega R$$

$$v = \sqrt{\frac{Gm}{4R^3}} \times R = \sqrt{\frac{Gm}{4R}}$$

10. **(c)** We know that

Force = mass × acceleration.



The gravitational force acting on both the masses is the same.

$$F_1 = F_2$$

$$ma_1 = ma_2$$

$$\Rightarrow \quad \frac{9M}{95M} = \frac{5M}{M} = 5$$

$$\Rightarrow \quad \frac{9M}{95M} = \frac{1}{5}$$

Let t be the time taken for the two masses to collide and x_{5M}, x_M be the distance travelled by the mass $5M$ and M respectively.

For mass 5*M*

$u = 0,$

$$S = ut + \frac{1}{2}at^2$$

$$\therefore \quad x_{5M} = \frac{1}{2}a_{5M}t^2 \qquad \text{....(ii)}$$

For mass *M*

$u = 0, s = x_M, t = t, a = a_M$

$$\therefore \quad s = ut + \frac{1}{2}at^2$$

$$\Rightarrow x_M = \frac{1}{2}a_M t^2 \qquad \text{...(iii)}$$

Dividing (ii) by (iii)

$$\frac{x_{5M}}{x_M} = \frac{\frac{1}{2}a_{5M}t^2}{\frac{1}{2}a_M t^2} = \frac{a_{5M}}{a_M} = \frac{1}{5} \quad \text{[From (i)]}$$

$$\therefore \quad 5x_{5M} = x_M \qquad \text{....(iv)}$$

From the figure it is clear that

$$x_{5M} + x_M = 9R \qquad \text{....(v)}$$

Where O is the point where the two spheres collide.

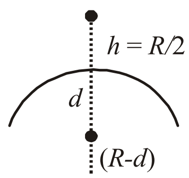
From (iv) and (v)

$$\frac{x_M}{5} + x_M = 9R$$

$$\therefore \quad 6x_M = 45R$$

$$\therefore \quad x_M = \frac{45}{6}R = 7.5R$$

11. **(b)** According to question, $g_h = g_d = g_1$



$$g_h = \frac{GM}{\left(R+\frac{R}{2}\right)^2} \text{ and } g_d = \frac{GM(R-d)}{R^3}$$

$$\frac{GM}{\left(\frac{3R}{2}\right)^2} = \frac{GM(R-d)}{R^3} \Rightarrow \frac{4}{9} = \frac{(R-d)}{R}$$

$$\Rightarrow 4R = 9R - 9d \Rightarrow 5R = 9d$$

$$\therefore \frac{d}{R} = \frac{5}{9}$$

12. **(b)** Value of g at equator, $g_A = g \cdot - R\omega^2$

Value of g at height h above the pole,

$$g_B = g\cdot\left(1 - \frac{2h}{R}\right)$$

As object is weighed equally at the equator and poles, it means g is same at these places.

$g_A = g_B$

$\Rightarrow g - R\omega^2 = g\left(1 - \frac{2h}{R}\right)$

$\Rightarrow R\omega^2 = \frac{2gh}{R} \Rightarrow h = \frac{R^2\omega^2}{2g}$

13. **(c)** The acceleration due to gravity at a height h is given by

$$g = \frac{GM}{(R+h)^2}$$

Here, G = gravitation constant
M = mass of earth

The acceleration due to gravity at depth h is

$$g' = \frac{GM}{R^2}\left(1 - \frac{h}{R}\right)$$

Given, $g = g'$

$\therefore \frac{GM}{(R+h)^2} = \frac{GM}{R^2}\left(1 - \frac{h}{R}\right)$

$\therefore R^3 = (R+h)^2(R-h) = (R^2 + h^2 + 2hR)(R-h)$

$\Rightarrow R^3 = R^3 + h^2R + 2hR^2 - R^2h - h^3 - 2h^2R$

$\Rightarrow h^3 + h^2(2R - R) - R^2h = 0$

$\Rightarrow h^3 + h^2R - R^2h = 0$

$\Rightarrow h^2 + hR - R^2 = 0$

$\Rightarrow h = \frac{-R \pm \sqrt{R^2 + 4(1)R^2}}{2}$

$= \frac{-R + \sqrt{5}R}{2} = \frac{(\sqrt{5}-1)}{2}R$

14. **(d)** Weight at pole, $w = mg = 196\ N$

$\Rightarrow m = 19.6$ kg

Weight at equator, $w' = mg' = m(g - \omega^2 R)$

$$= 19.6\left[10 - \left(\frac{2\pi}{24 \times 3600}\right)^2 \times 6400 \times 10^3\right]\text{N}$$

$$\left(\because \omega = \frac{2\pi}{T}\right)$$

$= 19.6\,[10 - 0.034] = 195.33\ N$

15. **(d)** $\frac{W_e}{W_p} = \frac{mg_e}{mg_p} = \frac{9}{4}$ **or** $\frac{g_e}{g_p} = \frac{9}{4}$

or $\frac{GM/R^2}{G(M/9)/R_p^2} = \frac{9}{4}$

$\therefore$ $\boldsymbol{R_p} = \frac{R}{2}$

16. **(a)** Given

Acceleration due to gravity at a height h from earth's surface is

$g_h = g\left(1 + \frac{h}{R_e}\right)^{-2} \Rightarrow 4.9 = 9.8\left(1 + \frac{h}{R_e}\right)^{-2}$

$\frac{1}{\sqrt{2}} = \left(1 - \frac{h}{R_e}\right)$ [as $h <<< R_e$]

$h = R_e\left(\sqrt{2} - 1\right)$

$h = 6400 \times 0.414\ \text{km} = 2.6 \times 10^6$ m

17. **(d)** With rotation of earth or latitude, acceleration due to gravity vary as $g' = g - \omega^2 R\cos^2\phi$

Where ϕ is latitude, there will be no change in gravity at poles as $\phi = 90°$

At all other points as ω increases g' will decreases hence, weight, W = mg decreases.

18. **(b)** Variation of acceleration due to gravity, g with distance 'd' from centre of the earth

If $d < R, g = \frac{Gm}{R^2}.d$ *i.e.*, $g \propto d$ (straight line)

If $d = R, g_s = \frac{Gm}{R^2}$

If $d > R, g = \frac{Gm}{d^2}$ *i.e.*, $g \propto \frac{1}{d^2}$

19. **(b)** Given that, mass density $\left(\frac{\text{mass}}{\text{volume}}\right)$ of a spherical body $\rho(r) = \frac{k}{r}$

$\frac{M}{V} = \frac{k}{r}$ for inside $r \le R$

$M = \frac{kv}{r}$ ..... (i)

Inside the surface of sphere Intensity

$I = \frac{GMr}{R^3}$ $\because\ I = \frac{F}{m}$

$g_{inside} = \frac{GMr}{R^3}$ or $I = \frac{mg}{m} = g$

$= \frac{G}{R^3}.\frac{kv}{r}.r$ = constant   From eq. (i),

Similarly, $g_{out} = \frac{GM}{r^2}$

Hence, option (2) is correct graph.

20. **(c)** We know, $g' = g - \omega^2 R\cos^2\theta$

$\frac{3g}{4} = g - \omega^2 R$

Given, $g' = \frac{3}{4}g$

$\omega^2 R = \frac{g}{4}$

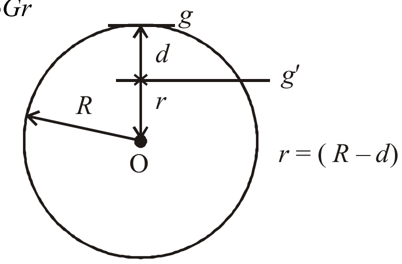
$\omega = \sqrt{\frac{g}{4R}} = \sqrt{\frac{10}{4 \times 6400 \times 10^3}}$

$= \frac{1}{2 \times 8 \times 100} = 0.6 \times 10^{-3}$ rad/s

21. **(a)**

22. **(a)** Acceleration due to gravity at depth d from the surface of the earth or at a distance r from the centre 'O' of the earth $g' = \frac{4}{3}\pi\rho G r$

Hence $g' \propto r$



23. **(d)** On earth's surface $g = \frac{GM}{R^2}$

At height above earth's surface

$$g_h = \frac{GM}{(R+h)^2}$$

$$\therefore \frac{g_n}{g} = \frac{R^2}{(R+h)^2}$$

$$\Rightarrow \frac{g/9}{g} = \left[\frac{R}{R+h}\right]^2$$

$$\Rightarrow \frac{R}{R+h} = \frac{1}{3}$$

$\therefore \quad h = 2R$

24. **(d)** Value of g with altitude is,

$$g_h = g\left[1 - \frac{2h}{R}\right];$$

Value of g at depth d below earth's surface,

$$g_d = g\left[1 - \frac{d}{R}\right]$$

Equating g_h and g_d, we get $d = 2h$

25. **(d)** Value of g on earth's surface,

$$g = \frac{GM}{R^2} = \frac{G\rho \times V}{R^2}$$

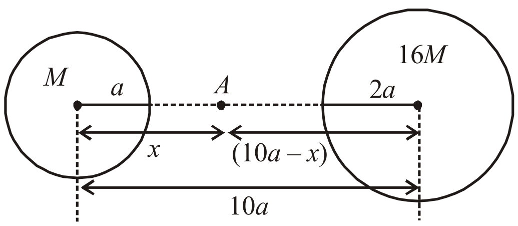
$$\Rightarrow \quad g = \frac{G \times \rho \times \frac{4}{3}\pi R^3}{R^2}$$

$g = \frac{4}{3}\rho\pi G.R$ where $\rho \to$ average density

$$\rho = \left(\frac{3g}{4\pi GR}\right)$$

$\Rightarrow \rho$ is directly proportional to g.

26. **(d)**



Let A be the point where gravitation field of both planets cancel each other i.e. zero.

$$\frac{GM}{x^2} = \frac{G(16M)}{(10a-x)^2}$$

$$\Rightarrow \frac{1}{x} = \frac{4}{(10a-x)} \Rightarrow 4x = 10a - x \Rightarrow x = 2a \qquad \text{... (i)}$$

Using conservation of energy, we have

$$-\frac{GMm}{8a} - \frac{G(16M)m}{2a} + KE = -\frac{GMm}{2a} - \frac{G(16M)m}{8a}$$

$$KE = GMm\left[\frac{1}{8a} + \frac{16}{2a} - \frac{1}{2a} - \frac{16}{8a}\right]$$

$$\Rightarrow KE = GMm\left[\frac{1+64-4-16}{8a}\right]$$

$$\Rightarrow \frac{1}{2}mv^2 = GMm\left[\frac{45}{8a}\right] \Rightarrow v = \sqrt{\frac{90GM}{8a}}$$

$$\Rightarrow v = \frac{3}{2}\sqrt{\frac{5GM}{a}}$$

27. **(a)** Given : Gravitational field,

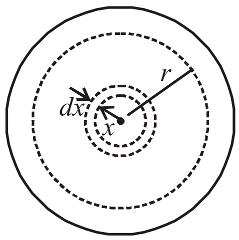
$$E_G = \frac{Ax}{(x^2+a^2)^{3/2}},\ V_\infty = 0$$

$$\int_{V_\infty}^{V_x} dV = -\int_{\infty}^{x} \vec{E}_G \cdot \vec{d}_x$$

$$\Rightarrow V_x - V_\infty = -\int_{\infty}^{x} \frac{Ax}{(x^2+a^2)^{3/2}}dx$$

$$\therefore V_x = \frac{A}{(x^2+a^2)^{1/2}} - 0 = \frac{A}{(x^2+a^2)^{1/2}}$$

28. **(d)**



Mass of small element of planet of radius x and thickness dx.

$$dm = \rho \times 4\pi x^2 dx = \rho_0\left(1 - \frac{x^2}{R^2}\right) \times 4\pi x^2 dx$$

Mass of the planet

$$M = 4\pi\rho_0 \int_0^r \left(x^2 - \frac{x^4}{R^2}\right)dx$$

$$\Rightarrow M = 4\pi\rho_0 \left|\frac{r^3}{3} - \frac{r^5}{5R^2}\right|$$

Gravitational field,

$$E = \frac{GM}{r^2} = \frac{G}{r^2} \times 4\pi\rho_0\left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)$$

$$\Rightarrow E = 4\pi G\rho_0\left(\frac{r}{3} - \frac{r^3}{5R^2}\right)$$

E is maximum when $\frac{dE}{dr} = 0$

$$\Rightarrow \frac{dE}{dr} = 4\pi G\rho_0\left(\frac{1}{3} - \frac{3r^2}{5R^2}\right) = 0$$

$$\Rightarrow r = \frac{\sqrt{5}}{3}R$$

29. (b) Gravitation field at the surface

$$E = \frac{Gm}{r^2}$$

$$\therefore E_1 = \frac{Gm_1}{r_1^2} \text{ and } E_2 = \frac{Gm_2}{r_2^2}$$

From the diagram given in question,

$\frac{E_1}{E_2} = \frac{2}{3}$ ($r_1 = 1$m, $R_2 = 2m$ given)

$$\therefore \frac{E_1}{E_2} = \left(\frac{r_2}{r_1}\right)^2\left(\frac{m_1}{m_2}\right) \Rightarrow \frac{2}{3} = \left(\frac{2}{1}\right)^2\left(\frac{m_1}{m_2}\right)$$

$$\Rightarrow \left(\frac{m_1}{m_2}\right) = \frac{1}{6}$$

30. (16.00)

Using law of conservation of energy

Total energy at height 10 R = total energy at earth

$$-\frac{GM_E m}{10R} + \frac{1}{2}mV_0^2 = -\frac{GM_E m}{R} + \frac{1}{2}mV^2$$

$$\left[\because \text{Gravitational potential energy} = -\frac{GMm}{r}\right]$$

$$\Rightarrow \frac{GM_E}{R}\left(1 - \frac{1}{10}\right) + \frac{V_0^2}{2} = \frac{V^2}{2}$$

$$\Rightarrow V^2 = V_0^2 + \frac{9}{5}gR$$

$$\Rightarrow V = \sqrt{V_0^2 + \frac{9}{5}gR} \approx 16 \ km/s$$

[$\because V_0 = 12$ km/s given]

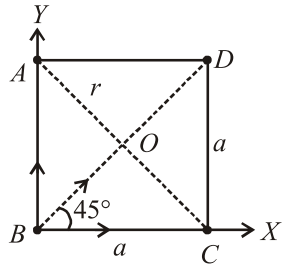
31. (c) $E_g = \frac{GM}{(3a)^2} + \frac{G(2M)}{(3a)^2} = \frac{GM}{3a^2}$

32. (b) $AC = a\sqrt{2} \quad \because r = \frac{AC}{2} = \frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$

Resultant force on the body

$$B = \frac{GM^2}{a^2}\hat{i} + \frac{GM^2}{a^2}\hat{j} + \frac{GM^2}{(a\sqrt{2})^2}(\cos 45°\hat{i} + \sin 45°\hat{j})$$

$$\Rightarrow |F| = \frac{GM^2}{a^2}(\sqrt{2}) + \frac{GM^2}{2a^2}$$



$\frac{Mv^2}{r}$ = Resultant force towards centre

$$\therefore \frac{Mv^2}{\left(\frac{a}{\sqrt{2}}\right)} = \frac{GM^2}{a^2}\left(\sqrt{2} + \frac{1}{2}\right)$$

$$\Rightarrow v^2 = \frac{GM}{a}\left(1 + \frac{1}{2\sqrt{2}}\right)$$

$$\Rightarrow v = \sqrt{\frac{GM}{a}\left(1 + \frac{1}{2\sqrt{2}}\right)} = 1.16\sqrt{\frac{GM}{a}}$$

33. (a) $F = \frac{GMm}{r} = \int a\frac{\rho(dV)m}{r^2}$

$$= mG\int_0^R \frac{k}{r^2}\frac{4\pi r^2 dr}{r^2}$$

$$= -4\pi kGm\left(\frac{1}{r}\right)_0^R$$

$$= -\frac{4\pi kGm}{R}$$

Using Newton's second law, we have

$$\frac{mv_0^2}{R} = \frac{4\pi kGm}{R}$$

or $v_0 = C$ (const.)

Time period, $T = \frac{2\pi R}{v_0} = \frac{2\pi R}{C}$

or $= \frac{T}{R}$ = constant.

34. (c) Initial gravitational potential energy, $E_i = -\frac{GMm}{2R}$

Final gravitational potential energy,

$$E_f = -\frac{GMm/2}{2\left(\frac{R}{2}\right)} - \frac{GMm/2}{2\left(\frac{3R}{2}\right)} = -\frac{GMm}{2R} - \frac{GMm}{6R}$$

$$= -\frac{4GMm}{6R} = -\frac{2GMm}{3R}$$

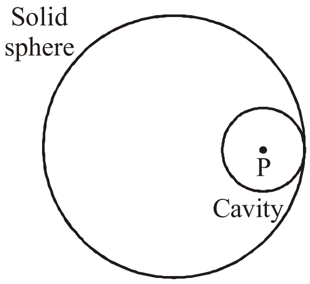
$\therefore$ Difference between initial and final energy,

$$E_f - E_i = \frac{GMm}{R}\left(-\frac{2}{3} + \frac{1}{2}\right) = -\frac{GMm}{6R}$$

35. **(d)** Due to complete solid sphere, potential at point P

$$V_{sphere} = \frac{-GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$$

$$= \frac{-GM}{2R^3}\left(\frac{11R^2}{4}\right) = -11\frac{GM}{8R}$$



Due to cavity part potential at point P

$$V_{cavity} = -\frac{3}{2}\frac{\frac{GM}{8}}{\frac{R}{2}} = -\frac{3GM}{8R}$$

So potential at the centre of cavity

$= V_{sphere} - V_{cavity}$

$$= -\frac{11GM}{8R} - \left(-\frac{3}{8}\frac{GM}{R}\right) = \frac{-GM}{R}$$

36. **(c)** As, $V = -\dfrac{GM}{2R^3}(3R^2 - r^2)$

Graph (c) most closely depicts the correct variation of $v(r)$.

37. **(d)** Gravitational field, $I = \left(5\hat{i} + 12\hat{j}\right)$ N/kg

$$I = -\frac{dv}{dr}$$

$$v = -\left[\int_0^x I_x dx + \int_0^y I_y dy\right]$$

$$= -\left[I_x.x + I_y.y\right]$$

$$= -\left[5(7-0) + 12(-3-0)\right]$$

$$= -\left[35 + (-36)\right] = 1\ J/kg$$

i.e., change in gravitational potential 1 J/kg.

Hence change in gravitational potential energy 1 J

38. **(b)** We choose reference point, infinity, where total energy of the system is zero.

So, initial energy of the system = 0

$$\text{Final energy} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d}$$

From conservation of energy,

Initial energy = Final energy

$$\therefore\ 0 = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 - \frac{Gm_1m_2}{d}$$

$$\text{or}\quad \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1v_2^2 = \frac{Gm_1m_2}{d}\quad ...(1)$$

By conservation of linear momentum

$$m_1v_1 + m_2v_2 = 0 \text{ or } \frac{v_1}{v_2} = -\frac{m_2}{m_1} \Rightarrow v_2 = -\frac{m_1}{m_2}v_1$$

Putting value of v_2 in equation (1), we get

$$m_1v_1^2 + m_2\left(-\frac{m_1v_1}{m_2}\right)^2 = \frac{2Gm_1m_2}{d}$$

$$\frac{m_1m_2v_1^2 + m_1^2v_1^2}{m_2} = \frac{2Gm_1m_2}{d}$$

$$v_1 = \sqrt{\frac{2Gm_2^2}{d(m_1+m_2)}} = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$$

Similarly $v_2 = -m_1\sqrt{\dfrac{2G}{d(m_1+m_2)}}$

39. **(c)** Let mass of smaller sphere (which has to be removed) is m

Radius $= \dfrac{R}{2}$ (from figure)

$$\frac{M}{\frac{4}{3}\pi R^3} = \frac{m}{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}$$

$$\Rightarrow m = \frac{M}{8}$$

Mass of the left over part of the sphere

$$M' = M - \frac{M}{8} = \frac{7}{8}M$$

Therefore gravitational field due to the left over part of the sphere

$$= \frac{GM'}{x^2} = \frac{7}{8}\frac{GM}{x^2}$$

40. **(d)** The work done to launch the spaceship

$$W = -\int_R^\infty \vec{F}\cdot\vec{dr} = -\int_R^\infty \frac{GMm}{r^2}dr$$

$$W = +\frac{GMm}{R} \quad \ldots(i)$$

The force of attraction of the earth on the spaceship, when it was on the earth's surface

$$F = \frac{GMm}{R^2}$$

$$\Rightarrow\ mg = \frac{GMm}{R^2} \Rightarrow g = \frac{GM}{R^2} \quad \ldots(ii)$$

Substituting the value of g in (i) we get

$$W = \frac{gR^2m}{R}$$

$\Rightarrow\ W = mgR$

$\Rightarrow W = 1000 \times 10 \times 6400 \times 10^3$

$= 6.4 \times 10^{10}$ Joule

41. **(c)** Let 'M' be the mass of the particle

Now, $E_{initial} = E_{final}$

i.e. $\frac{GMm}{\sqrt{2}r} + 0 = \frac{GMm}{r} + \frac{1}{2}MV^2$

or, $\frac{1}{2}MV^2 = \frac{GMm}{r}\left[1 - \frac{1}{\sqrt{2}}\right]$

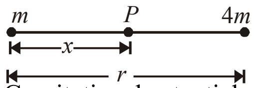
$\Rightarrow \frac{1}{2}V^2 = \frac{Gm}{r}\left[1 - \frac{1}{\sqrt{2}}\right]$

or, $V = \sqrt{\frac{2Gm}{r}\left(1 - \frac{1}{\sqrt{2}}\right)}$

42. **(c)** Let P be the point where gravitational field is zero.

$\therefore \frac{Gm}{x^2} = \frac{4Gm}{(r-x)^2}$

$\Rightarrow \frac{1}{x} = \frac{2}{r-x} \Rightarrow r - x = 2x \qquad \Rightarrow x = \frac{r}{3}$



Gravitational potential at P,

$V = -\frac{Gm}{\frac{r}{3}} - \frac{4Gm}{\frac{2r}{3}} = -\frac{9Gm}{r}$

43. **(b)** Gravitational field, $E = -\frac{GM}{r^2}$

Flux, $\phi = \int \vec{E_g} \cdot \vec{dS} = |E \cdot 4\pi r^2| = -4\pi GM$

where, M = mass enclosed in the closed surface

This relationship is valid when $|\vec{E}_g| \mu \frac{1}{r^2}$.

44. **(c)** Initial P.E. $U_i = -\frac{GMm}{R}$

When the particle is far away from the sphere, the P.E. of the system is zero.

$\therefore \quad U_f = 0$

$W = \Delta U = U_f - U_i = 0 - \left[\frac{-GMm}{R}\right]$

$W = \frac{6.67 \times 10^{-11} \times 100}{0.1} \times \frac{10}{1000} \qquad = 6.67 \times 10^{-10}$ J

45. **(b)** On earth's surface potential energy,

$U = \frac{GmM}{R}$

At a height R from the earth's surface, $P.E.$ of system = $-\frac{GmM}{2R}$

$\therefore \Delta U = \frac{-GmM}{2R} + \frac{GmM}{R};$

$\Rightarrow \Delta U = \frac{GmM}{2R}$

Now $\frac{GM}{R^2} = g; \therefore \frac{GM}{R} = gR \qquad \therefore \Delta U = \frac{1}{2}mgR$

46. **(d)** Gravitational potential energy of mass m in an orbit of radius R

$u = -\frac{GMm}{R}$

Energy required = potential energy at 3R – potential energy a 2R

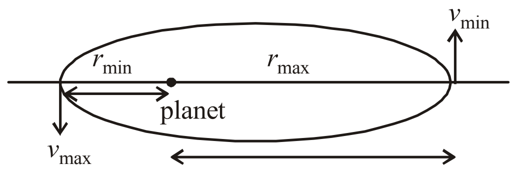
$= \frac{-GMm}{3R} - \left(\frac{-GMm}{2R}\right)$

$= \frac{-GMm}{3R} + \frac{GMm}{2R}$

$= \frac{-2GMm + 3GMm}{6R} = \frac{GMm}{6R}$

47. **(a)** By angular momentum conservation

$r_{min}v_{max} = mr_{max}v_{min}$



Given, $v_{min} = \frac{v_{max}}{6}$

$\therefore \frac{r_{min}}{r_{max}} = \frac{v_{min}}{v_{max}} = \frac{1}{6}$

48. **(a)** Orbital speed of the body when it revolves very close to the surface of planet

$V_0 = \sqrt{\frac{GM}{R}} \qquad$ ...(i)

Here, G = gravitational constant

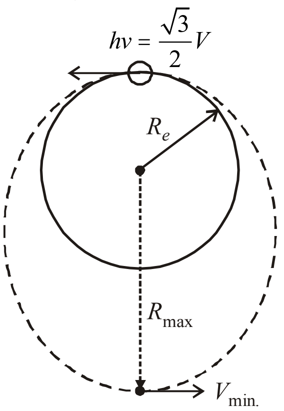
Escape speed from the surface of planet

$V_e = \sqrt{\frac{2GM}{R}} \qquad$ ...(ii)

Dividing (i) by (ii), we have

$\frac{V_0}{V_e} = \frac{\sqrt{\frac{GM}{R}}}{\sqrt{\frac{2GM}{R}}} = \frac{1}{\sqrt{2}}$

49. **(c)**



Orbital velocity, $V_0 = \sqrt{\dfrac{GM}{R_e}}$

From energy conversation,

$$-\frac{GMm}{R_e} + \frac{1}{2}m\left(\sqrt{\frac{3}{2}}V\right)^2 = \frac{GMm}{R_{max}} + \frac{1}{2}mV_{min}^2 \qquad ...(1)$$

From angular momentum conversation

$$\sqrt{\frac{3}{2}}VR_e = V_{min}R_{max} \qquad ...(2)$$

Solving equation (1) and (2) we get,

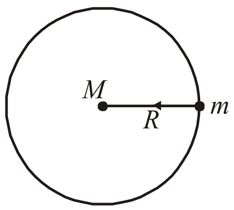
$R_{max} = 3R_e$

50. **(a)** According to question, mass density of a spherical galaxy varies as $\dfrac{k}{r}$.

Mass, $M = \int \rho dV$

$$\Rightarrow M = \int_0^{r=R_0} \frac{k}{r} 4\pi r^2\, dr$$

$$\Rightarrow M = 4\pi k \int_0^{R_0} r\, dr$$



or, $M = \dfrac{4\pi k R_0^2}{2} = 2\pi k R^2$

$$F_G = \frac{GMm}{R_0^2} = m\omega_0^2 R \ (= F_C)$$

$$\Rightarrow \frac{G\dfrac{4\pi k R^2}{2}}{R^2} = \omega_0^2 R \Rightarrow \omega_0 = \sqrt{\frac{2\pi KG}{R}} \quad \left(\because \omega = \frac{2\pi}{T}\right)$$

$$\therefore T = \frac{2\pi}{\omega_0} = \frac{2\pi\sqrt{R}}{\sqrt{2\pi KG}} = \sqrt{\frac{2\pi R}{KG}} \Rightarrow T^2 = \frac{2\pi R}{KG}$$

$\because 2\pi$, K and G are constants

$\therefore T^2 \propto R.$

51. **(d)** From law of conservation of momentum, $\vec{p}_i = \vec{p}_f$

$m_1u_1 + m_2u_2 = MV_f$

$$\Rightarrow v_f = \frac{\left(mv + \dfrac{mv}{4}\right)}{\dfrac{3m}{2}} = \frac{5v}{6}$$

Clearly, $v_f < v_i$ $\therefore$ Path will be elliptical

52. **(b)** K.E. of satellite is zero at earth surface and at height h from energy conservation

$U_{surface} + E_1 = U_h$

$$-\frac{GM_em}{R_e} + E_1 = -\frac{GM_em}{(Re+h)}$$

$$\Rightarrow E_1 = GM_em\left(\frac{1}{R_e} - \frac{1}{R_e+h}\right) \Rightarrow E_1 = \frac{GM_em}{(R_e+h)} \times \frac{h}{R_e}$$

Gravitational attraction

$$F_G = ma_c = \frac{mv^2}{(R_e+h)} = \frac{GM_em}{(R_e+h)^2}$$

$$mv^2 = \frac{GM_em}{(R_e+h)}$$

$$E_2 = \frac{mv^2}{2} = \frac{GM_em}{2(R_e+h)}$$

$E_1 = E_2$

Clearly, $\dfrac{h}{R_e} = \dfrac{1}{2} \Rightarrow h = \dfrac{R_e}{2} = 3200\text{km}$

53. **(a)** Escape velocity of the planet A is $V_A = \sqrt{\dfrac{2GM_A}{R_A}}$

where M_A and R_A be the mass and radius of the planet A.

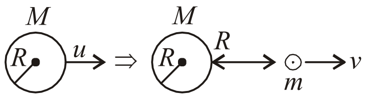
According to given problem

$$M_B = \frac{M_A}{2}, \ R_B = \frac{R_A}{2}$$

$$\therefore \ V_B = \sqrt{\frac{2G\dfrac{M_A}{2}}{\dfrac{R_A}{2}}} \ \therefore \ \frac{V_A}{V_B} = \sqrt{\frac{\dfrac{2GM_A}{R_A}}{\dfrac{2GM_A/2}{R_A/2}}} = \frac{n}{4} = 1$$

$\Rightarrow \ n = 4$

54. **(b)**



$$\frac{1}{2}mu^2 + \frac{-GMm}{R} = \frac{1}{2}mv^2 + \frac{-GMm}{2R}$$

$$\Rightarrow \frac{1}{2}m(v^2 - u^2) = \frac{-GMm}{2R}$$

$$\Rightarrow V = \sqrt{V = u^2 - \frac{GM}{R}} \qquad ...(i)$$

$$v_0 = \sqrt{\frac{GM}{2R}} \quad \therefore \ v_{rad} = \frac{m \times v}{\left(\dfrac{m}{10}\right)} = 10\,v$$

Ejecting a rocket of mass $\dfrac{m}{10}$

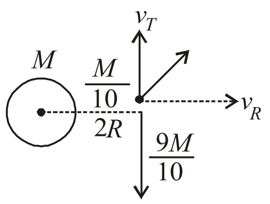
$$\therefore \ \frac{9m}{10} \times \sqrt{\frac{GM}{2R}} = \frac{m}{10} \times v_\tau \Rightarrow V_\tau^2 = 81\frac{GM}{2R}$$

Kinetic energy of rocket,

$$KE_{rocket}=\frac{1}{2}\frac{M}{10}\left(V_T^2+V_r^2\right)$$

$$=\frac{1}{2}\times\frac{m}{10}\times\left((u^2-\frac{GM}{R})100+81\frac{GM}{R}\right)$$

$$=\frac{m}{20}\times 100\left(u^2-\frac{GM}{R}+\frac{81}{200}\frac{GM}{R}\right)$$



$$=5m\left(u^2-\frac{119}{200}\frac{GM}{R}\right)$$

55. (d) Time period of revolution of satellite,

$$T=\frac{2\pi r}{v}$$

$$v=\sqrt{\frac{GM}{r}}$$

$$\therefore T=2\pi r\sqrt{\frac{r}{GM}}=2\pi\sqrt{\frac{r^3}{GM}}$$

Substituting the values, we get

$$T=2\pi\sqrt{\frac{(202)^3\times 10^{12}}{6.67\times 10^{-11}\times 8\times 10^{22}}}\text{ sec}$$

$T=7812.2$ s

$T\simeq 2.17$ hr $\Rightarrow$ 11 revolutions.

56. (d) Escape velocity,

$$v_c=\sqrt{\frac{2GM}{R}}=\sqrt{\frac{2G\rho V}{R}}$$

$$=\sqrt{\frac{2GS\times 4\pi R^3}{R}}=\sqrt{\frac{8}{3}\pi\rho GR^2}$$

For moon, $v_c'=\sqrt{\frac{8}{3}\pi\rho GR_m^2}$

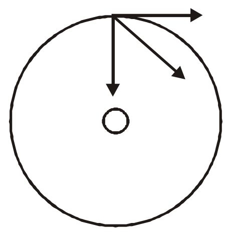
Given, $\frac{4}{3}\pi R^3=64\times\frac{4}{3}\pi R_m^3$ or $R_m=\frac{R}{4}$

$$\therefore\quad v_e'=\sqrt{\frac{8}{3}\pi\rho G\left(\frac{R}{4}\right)^2}=\frac{v_c}{4}$$

$$\frac{E}{E'}=\frac{\frac{1}{2}mv_e^2}{\frac{1}{2}mv_e'^2}=\frac{v_e^2}{v_c'^2}=\frac{v_e}{\left(\frac{v_e}{4}\right)}=16$$

or $E'=\frac{E}{16}$

57. (b) $m v\hat{i}+m v\hat{j}=2m\vec{v}$



$$\Rightarrow\ \vec{v}=\frac{v}{2}\hat{i}+\frac{v}{2}\hat{j}$$

$$\Rightarrow \vec{v}=\sqrt{\left(\frac{v}{2}\right)^2+\left(\frac{v}{2}\right)^2}=\frac{v}{\sqrt{2}}$$

$$=\frac{1}{\sqrt{2}}\times\sqrt{\frac{GM}{R}}$$

58. (b) Orbital, velocity, $v=\sqrt{\frac{GM}{r}}$

Kinetic energy of satellite A,

$$T_A=\frac{1}{2}m_A V_A^2$$

Kinetic energy of satellite B,

$$T_B=\frac{1}{2}m_B V_B^2$$

$$\Rightarrow\frac{T_A}{T_B}=\frac{m\times\frac{GM}{R}}{2m\times\frac{GM}{2R}}=1$$

59. (d) For a satellite orbiting close to the earth, orbital velocity is given by

$$v_0=\sqrt{g(R+h)}\approx\sqrt{gR}$$

Escape velocity (v_e) is

$$v_e=\sqrt{2g(R+h)}\approx\sqrt{2gR}\quad[\because h<<R]$$

$$\Delta v=v_e-v_0=(\sqrt{2}-1)\sqrt{gR}$$

60. (b) At height r from center of earth, orbital velocity

$$v=\sqrt{\frac{GM}{r}}$$

By principle of energy conservation

$$\text{KE of 'm'}+\left(-\frac{GMm}{r}\right)=0+0$$

($\because$ At infinity, PE = KE = 0)

$$\text{or KE of 'm'}=\frac{GMm}{r}=\left(\sqrt{\frac{GM}{r}}\right)^2 m=mv^2$$

61. (d) Let M is mass of star m is mass of meteroite

By energy convervation between 0 and ∞.

$$-\frac{GMm}{r}+\frac{-GMm}{r}+\frac{1}{2}mV_{esc}^2=0+0$$

$$\therefore v=\sqrt{\frac{4GM}{r}}=\sqrt{\frac{4\times 6.67\times 10^{-11}\times 3\times 10^{31}}{10^{11}}}$$

$$\simeq 2.8\times 10^5\text{ m/s}$$

62. **(b)** For h << R, the orbital velocity is $\sqrt{gR}$

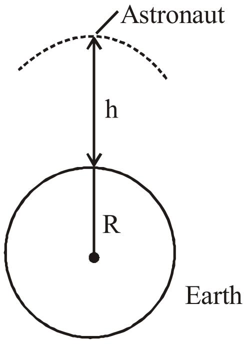
Escape velocity = $\sqrt{2gR}$

$\therefore$ The minimum increase in its orbital velocity

$= \sqrt{2gR} - \sqrt{gR} = \sqrt{gR}\,(\sqrt{2}-1)$

63. **(c)** According to universal law of Gravitation,

Gravitational force $F = \dfrac{GMm}{(R+h)^2}$



64. **(a)** $F = \dfrac{2GM}{Lr}m$ or, $\dfrac{mv^2}{r} = \dfrac{2GM}{Lr}m$

$$mr\left(\frac{2\pi}{T}\right)^2 = \frac{2GMm}{Lr}\left[\because v = r\omega \text{ and } \omega = \frac{2\pi}{T}\right]$$

$\Rightarrow \quad T \propto r$

65. **(a)** As we know,

Gravitational potential energy $= \dfrac{-GMm}{r}$

and orbital velocity, $v_0 = \sqrt{GM/R+h}$

$$E_f = \frac{1}{2}mv_0^2 - \frac{GMm}{3R} = \frac{1}{2}m\frac{GM}{3R} - \frac{GMm}{3R}$$

$$= \frac{GMm}{3R}\left(\frac{1}{2}-1\right) = \frac{-GMm}{6R}$$

$$E_i = \frac{-GMm}{R} + K$$

$E_i = E_f$

Therefore minimum required energy, $K = \dfrac{5GMm}{6R}$

66. **(c)** Escape velocity on earth,

$$v_e = \sqrt{\frac{2GM_e}{R_e}} = 11 \text{ km} s^{-1}$$

$$\therefore \quad \frac{(v_e)_p}{(v_e)_e} = \frac{\sqrt{\dfrac{2GM_p}{R_p}}}{\sqrt{\dfrac{2GM_e}{R_e}}} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}}$$

$$= \sqrt{\frac{10M_e}{M_e} \times \frac{R_e}{R_e/10}} = 10$$

$$\therefore \ (v_e)_p = 10 \times (v_e)_e = 10 \times 11 = 110 \ km/s$$

67. **(c)** Gravitational force, $F = KR^{-n}$

This force provides the centripetal force $MR\omega^2$ to the planet at height h above earth's surface.

$\therefore \quad F = KR^{-n} = MR\omega^2$

$\Rightarrow \quad \omega^2 = KR^{-(n+1)}$

$\Rightarrow \quad \omega = KR^{\frac{-(n+1)}{2}}$

$\dfrac{2\pi}{T} \propto R^{\frac{-(n+1)}{2}}$

$\therefore \ T \propto R^{\frac{+(n+1)}{2}}$

68. **(c)** Time period of satellite is given by

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$$

Where $R + h$ = radius of orbit of satellite

M = mass of earth.

Time period is independent of mass of satellite.

69. **(d)** Gravitational force provides the necessary centripetal force.

$\therefore$ Centripetal force on a satellite = Gravitational force

$$\therefore \quad \frac{mv^2}{(R+x)} = \frac{GmM}{(R+x)^2} \text{ also } g = \frac{GM}{R^2}$$

$$\therefore \frac{mv^2}{(R+x)} = m\left(\frac{GM}{R^2}\right)\frac{R^2}{(R+x)^2}\frac{n!}{r!(n-r)!}$$

$$\therefore \frac{mv^2}{(R+x)} = mg\frac{R^2}{(R+x)^2}$$

$$\therefore v^2 = \frac{gR^2}{R+x} \Rightarrow v = \left(\frac{gR^2}{R+x}\right)^{1/2}$$

70. **(c)** $v_e = \sqrt{2gR}$

Clearly escape velocity does not depend on the angle at which the body is projected.

71. **(c)** $K.E = \dfrac{1}{2}mv_e^2$

Here v_e = escape velocity is independent of mass of the body

Escape velocity, $v_e = \sqrt{2gR}$

Substituting value of v_e in above equation we get

$$K.E = \frac{1}{2}m \times 2gR = mgR$$

72. **(b)** Due to inertia of motion it will move tangentially to the original orbit with the same velocity.

73. **(a)** Escape velocity, $v_e = \sqrt{2gR} = \sqrt{\dfrac{2GM}{R}}$

$\Rightarrow v_e \propto m^0$

Where M, R are the mass and radius of the planet respectively. Clearly, escape velocity is independent of mass of the body