ISC SEMESTER 2 EXAMINATION
SAMPLE PAPER - 1
MATHEMATICS

Maximum Marks: 40

Time allowed: One and a half hour

*Candidates are allowed an additional **10 minutes** for **only** reading the paper.*

*They must **Not** start writing during this time.*

The Question Paper consists of three sections A, B and C.

*Candidates are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B** **OR** **Section C***

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

Mathematical tables and graph papers are provided.

Section-A

Question 1.

Choose the correct option for the following questions.

(i) $\int_0^a f(x)\,dx = \int_0^\infty f(k-x)\,dx$, then the value of k is :

(a) 0 (b) a (c) $2a$ (d) None of these

(ii) If A and B are two independent events with P(A) = $\frac{3}{5}$ and P(B) = $\frac{4}{9}$, then $P(\overline{A} \cap \overline{B})$ equals :

(a) $\frac{4}{15}$ (b) $\frac{8}{45}$ (c) $\frac{1}{3}$ (d) $\frac{2}{9}$

(iii) $\int \frac{x}{4+x^4}\,dx$ is equal to :

(a) $\frac{1}{4}\tan^{-1} x^2 + c$ (b) $\frac{1}{4}\tan^{-1}\left(\frac{x^2}{2}\right)+c$ (c) $\frac{1}{2}\tan^{-1}\left(\frac{x^2}{2}\right)$ (d) None of these

(iv) Write the order and degree of the differential equation $a^2\frac{d^2y}{dx^2} = \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{1/4}$.

(a) 2, 1 (b) 2, 2 (c) 2, 4 (d) None of these

(v) $\int \frac{(\sin x + \cos x)}{\sqrt{1+\sin 2x}}\,dx =$

(a) $\frac{-x}{2}+c$ (b) $\frac{x}{2}+c$ (c) $x+c$ (d) None of these

(vi) A four-digit number is formed using the digits 1, 2, 3, 5 with no repetition. Find the probability that the number is divisible by 5.

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{24}$ (d) $\frac{1}{6}$

Question 2.

Evaluate : $\int \frac{\cos x}{\sin x + \sqrt{\sin x}}\, dx.$

OR

Evaluate : $\int_0^{2\pi} \sqrt{1 + \sin \frac{x}{2}}\, dx.$

Question 3.

Solve : $dy = (5x - 4y)\, dx$ where $y = 0$ and $x = 0$.

OR

Solve : $\frac{dy}{dx} + y = 1$

Question 4.

Evaluate : $\int_0^{\pi/2} \log \sin x\, dx.$

Question 5.

A problem in mathematics is given to three students A, B and C. The chances of solving it by A, B and C are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. Calculate the probability that the problem will be solved.

OR

Bag A contains 5 white and 4 black balls and Bag B contains 7 white and 6 black balls. One ball is drawn from the bag A and, without noticing its colour, is put in the bag B. If a ball is then drawn from bag B, find the probability that it is black in colour.

Question 6.

Evaluate : $\int_0^{\pi/4} \log (1 + \tan x)\, dx.$

Question 7.

A class consists of 50 students out of which there are 10 girls. In the class, 2 girls and 5 boys are rank holders in an examination. If a student is selected at random from the class and is found to be a rank holder, what is the probability that the student selected is a girl ?

Question 8.

Evaluate : $\int \tan 2x \tan 3x \tan 5x\, dx.$

OR

Evaluate : $\int \frac{\sin (x - \alpha)}{\sin (x + \alpha)}\, dx.$

Section-B

Question 9.

Choose the correct option for the following questions :

(i) The intercepts made on the coordinate axes by the plane $2x + 3y + 6z = 18$ are :

(a) 9, 9, 3 (b) 9, 6, 3 (c) 3, 6, 9 (d) None of these

(ii) The distance between the planes $2x + 2y - z + 2 = 0$ and $4x + 4y - 2z + 5 = 0$ is :

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) 1

Question 10.

Find the equation of the plane through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$.

Question 11.

Find the area bounded by the curves $y = 6x - x^2$ and $y = x^2 - 2x$.

Section-C

Question 12.

Choose the correct option for the following questions.

(i) Find the coefficient of correlation between x and y if the regression line $2x + 5y - 9 = 0$ is y on x and the regression coefficient x on y is -0.7.

(a) -0.529 (b) 0.52 (c) 5.2 (d) -0.40

(ii) Equations of two regression lines are $4x + 3y + 7 = 0$ and $3x + 4y + 8 = 0$. Find mean of x and mean of y.

(a) $\frac{-4}{7}, \frac{11}{7}$ (b) $\frac{4}{7}, \frac{-11}{7}$ (c) $\frac{-4}{7}, \frac{-11}{7}$ (d) $\frac{4}{7}, \frac{11}{7}$

Question 13.

Find b_{xy} and b_{yx} from the following pairs of observations on X and Y :

(1, 2), (2, 3), (3, 5), (4, 6), (5, 4)

Question 14.

A company manufacture two types of products A and B. Each unit of A requires 3 gm of nickel and 1 gm of chromium, while each unit of B requires 1 gm of nickel and 2 gm of chromium. The firm can produce 9 gm of nickel and 8 gm of chromium. The profit is ₹ 40 on each unit of product A and ₹ 50 on each unit of product B. How many units of each type should the company manufacture so as to earn maximize profit ?

Answers

Section-A

Answer 1.

(i) (b) a

Explanation :

From fundamental property of definite integral, we know that,

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

(ii) (d) $\frac{2}{9}$

Explanation :

Given : $P(A) = \frac{3}{5},\ P(B) = \frac{4}{9}$

$$\begin{aligned} P(\overline{A} \cap \overline{B}) &= P(A \cup B)' = 1 - P(A \cup B) \\ &= 1 - \{P(A) + P(B) - P(A \cap B)\} \\ &= 1 - \{P(A) + P(B) - P(A).P(B)\} \\ &= 1 - \left(\frac{3}{5} + \frac{4}{9} - \frac{3}{5} \times \frac{4}{9}\right) \\ &= 1 - \frac{35}{45} \\ &= \frac{10}{45} = \frac{2}{9}. \end{aligned}$$

(iii) (b) $\frac{1}{4}\tan^{-1}\left(\frac{x^2}{2}\right)+c$

Explanation :

Let $$I = \int \frac{x}{4+x^4}\,dx$$

$$= \int \frac{x}{(2)^2+(x^2)^2}\,dx$$

Putting $x^2 = t \Rightarrow 2x\,dx = dt \Rightarrow x\,dx = \frac{dt}{2}$

$\therefore$ $$I = \frac{1}{2}\int \frac{dt}{(2)^2+t^2}$$

$$= \frac{1}{2}\left[\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)\right]+c \qquad \left(\because \int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}\right)$$

$$= \frac{1}{4}\tan^{-1}\left(\frac{x^2}{2}\right)+c \qquad (\because\ t = x^2)$$

(iv) (c) 2, 4

Explanation :

$$a^2\frac{d^2y}{dx^2} = \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{1/4}$$

$$\left(a^2\frac{d^2y}{dx^2}\right)^4 = 1+\left(\frac{dy}{dx}\right)^2$$

$\therefore$ Order of the differential equation is 2.
and degree of the differential equation is 4.

(v) (c) $x + c$

Explanation :

$$\int \frac{(\sin x+\cos x)}{\sqrt{1+\sin 2x}}\,dx = 1$$

Since $1 + \sin 2x = \sin^2 x + \cos^2 x + 2\sin x . \cos x$
$= (\sin x + \cos x)^2$

$$I = \int \frac{(\sin x+\cos x)}{\sqrt{(\sin x+\cos x)^2}}\,dx$$

$$= \int\left(\frac{\sin x+\cos x}{\sin x+\cos x}\right)dx$$

$$= \int dx = x + c$$

(vi) (b) $\frac{1}{4}$

Explanation :

Total four-digit numbers formed by 1, 2, 3 and 5 = 4!

$\therefore$ $n(S) = 4! = 24$

Total number of four digit numbers divisible by 5 = 3!

$\therefore$ $n(E) = 3! = 6$

$\therefore$ Probability that the number is divisible by 5

$$= \frac{n(E)}{n(S)} = \frac{6}{24} = \frac{1}{4}.$$

Answer 2.

$$\int \frac{\cos x}{\sin x + \sqrt{\sin x}}\, dx = \int \frac{\cos x}{\sqrt{\sin x}\,(\sqrt{\sin x} + 1)}\, dx$$

Let $\sqrt{\sin x} = t$

$\Rightarrow$ $\frac{1}{2\sqrt{\sin x}} \cdot \cos x = \frac{dt}{dx}$

$\Rightarrow$ $\frac{\cos x}{\sqrt{\sin x}}\, dx = 2\, dt$

$\therefore$ $\int \frac{\cos x}{\sin x + \sqrt{\sin x}}\, dx = \int \frac{2}{(t+1)}\, dt$

$= 2 \log | t + 1 | + c$

$= 2 \log | \sqrt{\sin x} + 1 | + c.$ **Ans.**

OR

Let $I = \int_0^{2\pi} \sqrt{1 + \sin \frac{x}{2}}\, dx$

Let $\frac{x}{2} = t \Rightarrow \frac{dx}{2} = dt$ or $dx = 2dt$

When $x = 0$, $t = 0$

When $x = 2\pi$, $t = \pi$

$\therefore$ $I = \int_0^{\pi} 2\sqrt{1 + \sin t}\, dt$

$= 2\int_0^{\pi} \sqrt{\cos^2 \frac{t}{2} + \sin^2 \frac{t}{2} + 2 \sin \frac{t}{2} \cos \frac{t}{2}}\, dt$

$= 2\int_0^{\pi} \sqrt{\left(\cos \frac{t}{2} + \sin \frac{t}{2}\right)^2}\, dt$

or $I = 2\int_0^{\pi} \left(\cos \frac{t}{2} + \sin \frac{t}{2}\right) dt$

Let $\frac{t}{2} = u \Rightarrow \frac{dt}{2} = du$ or $dt = 2du$

When $t = 0$, $u = 0$

When $t = \pi$, $u = \frac{\pi}{2}$

$\therefore$ $I = 2\int_0^{\pi/2} (\cos u + \sin u)\,(2du)$

$= 4\int_0^{\pi/2} (\cos u + \sin u)\, du$

$= 4\,[\sin u - \cos u]_0^{\pi/2}$

$= 4\left[\sin \frac{\pi}{2} - \cos \frac{\pi}{2} - \sin 0 + \cos 0\right]$

$= 4[1 - 0 - 0 + 1] = 4 \times 2 = 8.$ **Ans.**

$dy = (5x - 4y)\, dx$

Answer 3.

Given : $dy = (5x - 4y)\, dx$

$$\Rightarrow \quad \frac{dy}{dx} = 5x - 4y$$

$$\Rightarrow \quad \frac{dy}{dx} + 4y = 5x$$

Compare with $\quad \frac{dy}{dx} + Py = Q(x)$

$\therefore$ $\quad P = 4$ and $Q(x) = 5x$

$$\text{I.F.} = e^{\int P dx} = e^{\int 4\, dx} = e^{4x}$$

Solution of differential equation is given by,

$$\text{I.F.} \times y = \int \text{I.F.} \times Q(x)\, dx$$

$$\Rightarrow \quad ye^{4x} = \int e^{4x}.5x\, dx$$

$$\Rightarrow \quad ye^{4x} = 5\left[x\int e^{4x}\, dx - \int\left[\frac{d}{dx}x\int e^{4x}\, dx\right]dx\right]$$

$$\Rightarrow \quad ye^{4x} = 5\left[\frac{xe^{4x}}{4} - \int\frac{e^{4x}}{4}\, dx\right]$$

$$\Rightarrow \quad ye^{4x} = \frac{5xe^{4x}}{4} - \frac{5e^{4x}}{16} + c$$

When $x = 0$, $y = 0$

$$0 = 0 - \frac{5}{16} \times 1 + c$$

$$\Rightarrow \quad c = \frac{5}{16}$$

$$\therefore \quad ye^{4x} = \frac{5xe^{4x}}{4} - \frac{5e^{4x}}{16} + \frac{5}{16}$$

$$\Rightarrow \quad 16y = 20x + 5e^{-4x} - 5.$$ **Ans.**

OR

Given, $\quad \frac{dy}{dx} + y = 1$

$$\Rightarrow \quad \frac{dy}{dx} = 1 - y$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{1}{1-y}$$

$$\Rightarrow \quad dx = \frac{1}{1-y}dy$$

Intergrating both sides, we get

$$\int dx = \int\frac{1}{1-y}dy$$

$$x = -\log|1 - y| + c$$ **Ans.**

Answer 4.

Let $\quad I = \int_0^{\pi/2} \log \sin x\, dx \quad$ ...(i)

Using property, $\quad \int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

$$I = \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right) dx$$

$$= \int_0^{\pi/2} \log \cos x\, dx \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$2I =$

$$\int_0^{\pi/2}[\log(\sin x)+\log(\cos x)]\,dx$$

$$= \int_0^{\pi/2}\log \sin x \cos x\,dx$$

$$= \int_0^{\pi/2}\log \frac{\sin 2x}{2}\,dx$$

$$= \int_0^{\pi/2}(\log \sin 2x - \log 2)\,dx$$

$$= \int_0^{\pi/2}\log \sin 2x\,dx - \int_0^{\pi/2}\log 2\,dx$$

[Putting $t = 2x$, $dt = 2\,dx$]

$$= \frac{1}{2}\int_0^{\pi}\log \sin t\,dt - \log 2\,[x]_0^{\pi/2}$$

$$= \frac{1}{2}.2\int_0^{\pi/2}\log \sin t\,dt - \frac{\pi}{2}\log 2 \quad [\because \sin(\pi - t) = \sin t]$$

$$2I = I - \frac{\pi}{2}\log 2 \qquad \left[\because \int_0^a f(x)\,dx = \int_0^a f(t)dt\right]$$

$$I = -\frac{\pi}{2}\log 2.$$ **Ans.**

Answer 5.

Chances of solving the problem by A, $P(A) = \frac{1}{2}$

$\therefore$ Probability that problem will not be solved by A *i.e.*,

$$P(\overline{A}) = 1 - \frac{1}{2} = \frac{1}{2}$$

Similarly $$P(\overline{B}) = 1 - \frac{1}{3} = \frac{2}{3}$$

and $$P(\overline{C}) = 1 - \frac{1}{4} = \frac{3}{4}$$

$\therefore$ P(Problem will be solved) = 1 – P(None of them can solve the problem)

$$= 1 - P(\overline{A}).P(\overline{B}).P(\overline{C})$$

$$= 1 - \frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

$\therefore$ Probability of the problem will be solved $= \frac{3}{4}\cdot$ **Ans.**

OR

Given :

Bag A contains : 5 White and 4 Black balls

and Bag B contains : 7 White and 6 Black balls

Let X be the event that black ball is drawn from bag B.

Let E_1 be the event that white ball is drawn from bag A.

$\therefore$ $$P(E_1) = \frac{5}{9}$$

Now, probability that black ball is drawn from bag B *i.e.*,

$$P(X/E_1) = \frac{6}{14}$$

Let E_2 be the event that black ball drawn from bag A.

$$\therefore \qquad P(E_2) = \frac{4}{9}$$

Now, probability that black ball is drawn from bag B

$$P(X/E_2) = \frac{7}{14}$$

Now, probability that black ball is drawn from bag B

$$= P(E_1) . P(X/E_1) + P(E_2) . P(X/E_2)$$

$$= \frac{5}{9} \times \frac{6}{14} + \frac{4}{9} \times \frac{7}{14}$$

$$= \frac{30}{126} + \frac{28}{126} = \frac{58}{126}$$

$$\therefore \qquad \text{Required probability} = \frac{29}{63} \cdot$$ **Ans.**

Answer 6.

Let

$$I = \int_0^{\pi/4} \log (1 + \tan x)\, dx$$

$$I = \int_0^{\pi/4} \log \left[1 + \tan \left(\frac{\pi}{4} - x\right)\right] \qquad \left[\because \int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx\right]$$

$$\Rightarrow \quad I = \int_0^{\pi/4} \log \left(1 + \frac{1 - \tan x}{1 + \tan x}\right) dx$$

$$\Rightarrow \quad I = \int_0^{\pi/4} \log \left(\frac{1 + \tan x + 1 - \tan x}{1 + \tan x}\right) dx$$

$$\Rightarrow \quad I = \int_0^{\pi/4} \log \frac{2}{1 + \tan x}\, dx$$

$$\Rightarrow \quad I = \int_0^{\pi/4} \log 2\, dx - \int_0^{\pi/4} \log (1 + \tan x)\, dx$$

$$\Rightarrow \quad I = \int_0^{\pi/4} \log 2\, dx - I$$

$$\Rightarrow \quad 2I = \log 2 \int_0^{\pi/4} dx = \log 2 [x]_0^{\pi/4}$$

$$\Rightarrow \quad 2I = \frac{\pi}{4} \log 2$$

$$\therefore \quad I = \frac{\pi}{8} \log 2.$$ **Ans.**

Answer 7.

Given :

Total number of students = 50

Total number of girls = 10

Total number of boys = 40

Total number of rank holders = 2 girls and 5 boys

Let E_1 be the event to select a boy, E_2 be the event to select a girl and A be the event that selected student is a rank holder.

So

$$P(E_1) = \frac{40}{50} = \frac{4}{5}$$

$$P(E_2) = \frac{10}{50} = \frac{1}{5}$$

$$P(A/E_1) = \frac{5}{40} = \frac{1}{8}$$

$$P(A/E_2) = \frac{2}{10} = \frac{1}{5}$$

$\therefore$ Required probability that selected rank holder is a girl

$$= \frac{P(E_2).\, P(A/E_2)}{P(E_1).\, P(A/E_1) + P(E_2).\, P(A/E_2)}$$

$$= \frac{\frac{1}{5} \times \frac{1}{5}}{\frac{4}{5} \times \frac{1}{8} + \frac{1}{5} \times \frac{1}{5}} = \frac{\frac{1}{25}}{\frac{1}{10} + \frac{1}{25}}$$

$$= \frac{\frac{1}{25}}{\frac{25+10}{250}} = \frac{1}{25} \times \frac{250}{35}$$

$\therefore$ Required probability $= \dfrac{2}{7}$. **Ans.**

Answer 8.

$$\int \tan 2x \tan 3x \tan 5x \, dx$$

$\because$ $5x = 2x + 3x$

$\Rightarrow$ $\tan 5x = \tan(2x + 3x)$

$\Rightarrow$ $\tan 5x = \dfrac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x}$

$\Rightarrow$ $\tan 5x - \tan 5x \tan 2x \tan 3x = \tan 2x + \tan 3x$

$\Rightarrow$ $\tan 5x - \tan 2x - \tan 3x = \tan 5x \tan 2x \tan 3x$

$\therefore$

$$\int \tan 2x \tan 3x \tan 5x \, dx = \int \tan 5x \, dx - \int \tan 2x \, dx - \int \tan 3x \, dx$$

$$= -\frac{1}{5} \log \cos 5x + \frac{1}{2} \log \cos 2x + \frac{1}{3} \log \cos 3x + c$$

$$= \frac{1}{2} \log \cos 2x + \frac{1}{3} \log \cos 3x - \frac{1}{5} \log \cos 5x + c$$ **Ans.**

OR

$$\int \frac{\sin(x - \alpha)}{\sin(x + \alpha)} dx$$

Let $x + \alpha = t \therefore dx = dt$

$\therefore$

$$\int \frac{\sin(x - \alpha)}{\sin(x + \alpha)} dx = \int \frac{\sin(t - 2\alpha)}{\sin t} dt$$

$$= \int \frac{\sin t \cos 2\alpha - \cos t \sin 2\alpha}{\sin t} dt$$

$$= \int \frac{\sin t \cos 2\alpha}{\sin t} dt - \int \frac{\cos t \sin 2\alpha}{\sin t} dt$$

$$= \cos 2\alpha \int dt - \sin 2\alpha \int \cot t \, dt$$

$$= t \cos 2\alpha - \sin 2\alpha \log \sin t + c$$

$$= (x + \alpha) \cos 2\alpha - \sin 2\alpha \log \sin(x + \alpha) + c$$

$$= x \cos 2\alpha - \sin 2\alpha \log \sin(x + \alpha) + c + \alpha \cos 2\alpha$$

$$= x \cos 2\alpha - \sin 2\alpha \log \sin(x + \alpha) + c_1$$ **Ans.**

where $c_1 = c + \alpha \cos 2\alpha$.

Section-B

Answer 9.

(i) (b) 9, 6, 3

Explanation :

Given, $2x + 3y + 6z = 18$

Divide by 18 on both sides, we get

$$\frac{x}{9}+\frac{y}{6}+\frac{z}{3} = 1 \quad \text{...(i)}$$

Compare equation (i) with intercept form $\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$, we get

$a = 9$, $b = 6$ and $c = 3$

(ii) (a) $\frac{1}{2}$

Explanation :

Given : $2x + 2y - z + 2 = 0$

and $4x + 4y - 2z + 5 = 0$

or $2x + 2y - z + 5/2 = 0$

Then we see that these are parallel planes because d.r.'s are same

$\therefore$ Distance between them $= d_2 - d_1$

$$= \frac{5}{2} - 2 = \frac{1}{2}$$

Answer 10.

Let the direction ratios of the plane be $\langle A, B, C\rangle$.

It passes throguh (2, 2, 1).

$$\because \quad A(x-2) + B(y-2) + C(z-1) = 0 \quad \text{...(i)}$$

Plane also passes through (9, 3, 6).

$$\therefore \quad A(9-2) + B(3-2) + C(6-1) = 0$$

$$\Rightarrow \quad 7A + B + 5C = 0 \quad \text{...(ii)}$$

Plane (i) is perpendicular to the plane $2x + 6y + 6z = 9$

$\therefore$ Direction ratios of both the planes are equal.

$$\therefore \quad 2A + 6B + 6C = 0 \quad \text{...(iii)}$$

Solving equations (ii) and (iii) by cross-multiplication method, we get

$$\frac{A}{6-30} = \frac{B}{10-42} = \frac{C}{42-2}$$

$$\Rightarrow \quad \frac{A}{-24} = \frac{B}{-32} = \frac{C}{40}$$

$$\Rightarrow \quad \frac{A}{3} = \frac{B}{4} = \frac{C}{-5}$$

$$\therefore \quad A = 3k, B = 4k \text{ and } C = -5k$$

From equation (i),

$$3k(x-2) + 4k(y-2) - 5k(z-1) = 0$$

$$\Rightarrow \quad 3(x-2) + 4(y-2) - 5(z-1) = 0$$

$$\Rightarrow \quad 3x - 6 + 4y - 8 - 5z + 5 = 0$$

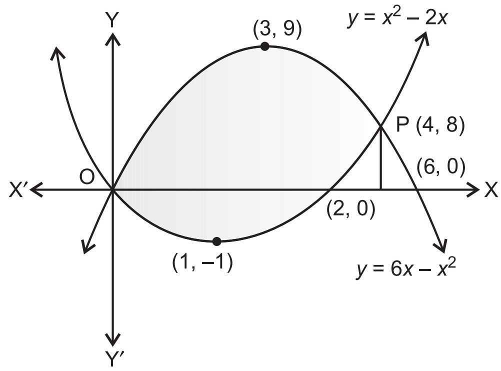
$$\Rightarrow \quad 3x + 4y - 5z - 9 = 0$$

$$\Rightarrow \quad 3x + 4y - 5z = 9.$$ **Ans.**

Answer 11.

Given : $y = 6x - x^2$

$$x^2 - 6x = -y$$
$$x^2 - 6x + 9 = -y + 9$$
$$(x-3)^2 = -(y-9)$$



This represent a parabola with vertex (3, 9).

$y = 6x - x^2$ intersect X-axis at (0, 0) and (6, 0).

Also, $y = x^2 - 2x$

$$x^2 - 2x + 1 = y + 1$$
$$(x-1)^2 = y + 1$$

This represent a parabola with vertex (1, – 1).

$y = x^2 - 2x$ intersect the X-axis at (0, 0) and (2, 0).

The point of intersection of two parabolas is (4, 8).

$$\text{Required shaded area} = \int_0^4 (6x - x^2)\,dx + \int_0^2 (x^2 - 2x)\,dx - \int_2^4 (x^2 - 2x)\,dx$$
$$= \left[3x^2 - \frac{x^3}{3}\right]_0^4 + \left[\frac{x^3}{3} - x^2\right]_0^2 - \left[\frac{x^3}{3} - x^2\right]_2^4$$
$$= \left[3\times16 - \frac{64}{3}\right] + \left[\frac{8}{3} - 4\right] - \left[\left(\frac{64}{3} - 16\right) - \left(\frac{8}{3} - 4\right)\right]$$
$$= \left[48 - \frac{64}{3}\right] + \left[\frac{8-12}{3}\right] - \left[\left(\frac{64-48}{3}\right) - \left(\frac{8-12}{3}\right)\right]$$
$$= \left[\frac{144-64}{3} - \frac{4}{3}\right] - \left[\frac{16}{3} + \frac{4}{3}\right] = \left[\frac{80}{3} - \frac{4}{3}\right] - \frac{20}{3} = \frac{76}{3} - \frac{20}{3}$$
$$= \frac{56}{3} \text{ sq. units}$$

Ans.

Section-C

Answer 12.

(i) (a) – 0.529

Explanation :

Given, $b_{xy} = -0.7$

and regression line y on x is

$$2x + 5y = 9$$
$$\Rightarrow \quad 5y = -2x + 9$$
$$\Rightarrow \quad y = \frac{-2}{5}x + \frac{9}{5}$$
$$\therefore \quad b_{yx} = \frac{-2}{5} = -0.4$$

and, $\rho(x, y)$ = coefficient of correlation between x and y

$$= -\sqrt{b_{xy}.b_{yx}}$$
$$= -\sqrt{(-0.7)(-0.4)}$$
$$\rho(x, y) = -0.529$$

[$\because$ $\rho(x, y)$, b_{xy} and b_{yx} have same sign]

(ii) (c) $\frac{-4}{7}, \frac{-11}{7}$

Explanation :

Given : $4x + 3y + 7 = 0$

and $3x + 4y + 8 = 0$

Solving the two equations, we get

$$\frac{x}{24-28} = \frac{y}{21-32} = \frac{1}{16-9}$$

$$\frac{x}{-4} = \frac{y}{-11} = \frac{1}{7}$$

$\therefore$ $$x = -\frac{4}{7} \text{ and } y = -\frac{11}{7}$$

Hence, mean of x series is $-\frac{4}{7}$ and mean of y series is $-\frac{11}{7}$.

Answer 13.

X	Y	X^2	Y^2	XY
1 2 3 4 5	2 3 5 6 4	1 4 9 16 25	4 9 25 36 16	2 6 15 24 20
$\Sigma X = 15$	$\Sigma Y = 20$	$\Sigma X^2 = 55$	$\Sigma Y^2 = 90$	$\Sigma XY = 67$

Here, N = 5

$\therefore$ $$b_{xy} = \frac{\Sigma XY - \frac{1}{N}\Sigma X \Sigma Y}{\Sigma Y^2 - \frac{1}{N}(\Sigma Y)^2}$$

$\Rightarrow$ $$b_{xy} = \frac{67 - \frac{1}{5} \times 15 \times 20}{90 - \frac{1}{5}(20)^2}$$

$\Rightarrow$ $$b_{xy} = \frac{67-60}{90-80} = \frac{7}{10} = 0.7$$ **Ans.**

and $$b_{yx} = \frac{\Sigma XY - \frac{1}{N}\Sigma X \Sigma Y}{\Sigma X^2 - \frac{1}{N}(\Sigma X)^2} = \frac{67 - \frac{1}{5} \times 15 \times 20}{55 - \frac{1}{5}(15)^2} = \frac{67-60}{55-45}$$

$\therefore$ $$b_{yx} = \frac{7}{10} = 0.7$$ **Ans.**

Answer 14.

Let the company manufactures x units of product A and y units of product B.

Object : To maximise profit $Z = 40x + 50y$

Constraint : $3x + y \leq 9$

$x + 2y \leq 8$

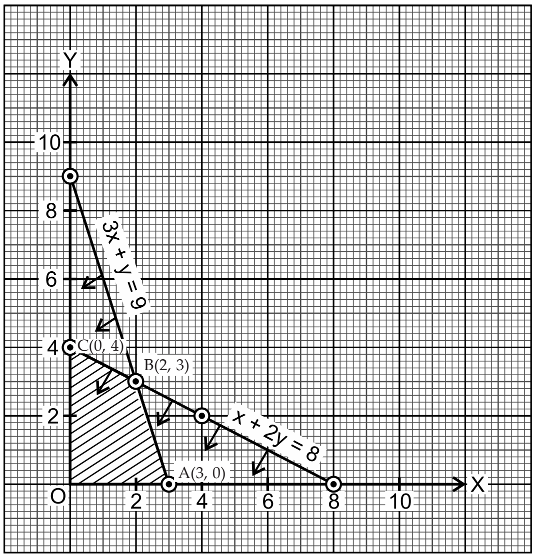
$x \geq 0, y \geq 0$

$3x + y = 9$

x	0	3	2
y	9	0	3

$x + 2y = 8$

x	0	8	4
y	4	0	2



The shaded region is the required feasible region.

Maximum profit $Z = 40x + 50y$

At A(3, 0), $Z = 40 \times 3 + 50 \times 0 =$ ₹ 120

At B(2, 3), $Z = 40 \times 2 + 50 \times 3 =$ ₹ 230

At C(0, 4), $Z = 40 \times 0 + 50 \times 4 =$ ₹ 200

So, the company should manufacture 2 units of product A and 3 units product B to earn maximum profit. **Ans.**

❑❑