

Class XI Session 2024-25
Subject - Applied Mathematics
Sample Question Paper - 7

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there is some internal choice in some questions.
2. Section A has 18 MCQ's and 02 Assertion Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer(VSA) questions of 2 marks each.
4. Section C has 6 Short Answer(SA) questions of 3 marks each.
5. Section D has 4 Long Answer(LA) questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (04 marks each) with sub parts.
7. Internal Choice is provided in 2 questions in Section-B, 2 questions in Section-C, 2 Questions in Section-D. You have to attempt only one alternatives in all such questions.

Section A

1. A bag contains 5 black balls, 4 white balls and 3 red balls. If a ball is selected random wise, the probability that it is black or red ball is [1]
a) $\frac{2}{3}$ b) $\frac{5}{12}$
c) $\frac{1}{3}$ d) $\frac{1}{4}$
2. The mean and standard deviation of 100 items are 50, 5 and that of 150 items are 40, 6 respectively. What is the combined mean of all 250 items? [1]
a) 43 b) 46
c) 44 d) 45
3. A shopkeeper bought a TV from a distributor at a discount of 25% of the listed price of ₹ 32000. The shopkeeper sells the TV to a consumer at the listed price. If the sales are intra-state and the rate of CST is 18%, the selling price of the TV including tax (under GST) by the distributor is [1]
a) ₹ 32000 b) ₹ 28320
c) ₹ 24000 d) ₹ 26160
4. $2^4 = 16$ in logarithmic form is [1]
a) $4 \log 2 = \log 16$ b) $\log_4 16 = 2$
c) $\log_4 2 = 16$ d) $\log_2 16 = 4$

5. For two non empty sets A and B, if $n(A) = m$ and $n(B) = n$ then the total number of relations from set A to set B is [1]
- a) $2^m + 2^n$ b) 2^{mn}
c) mn d) $m + n$
6. $\log(1 \times 2 \times 3)$ is equal to [1]
- a) $\log 3$ b) $\log 1$
c) $\log 2$ d) $\log 1 + \log 2 + \log 3$
7. If A and B are any two events having $P(A \cup B) = \frac{2}{3}$ and $P(\bar{B}) = \frac{1}{2}$, then $P(A \cap \bar{B})$ is: [1]
- a) $\frac{1}{3}$ b) $\frac{2}{3}$
c) $\frac{1}{2}$ d) $\frac{1}{6}$
8. The vertex of the parabola $y^2 = 4a(x - a)$ is [1]
- a) (0, a) b) (a, 0)
c) (0, 0) d) (a, a)
9. In a certain code CAT is written as SATC and DEAF as SEARD, then SING will be written as [1]
- a. TSING
b. TINGS
c. GSING
d. SINGS
- a) TSING b) TINGS
c) SINGS d) GSING
10. The variance of first 5 natural numbers is [1]
- a) 4 b) 3
c) 1 d) 2
11. The value of $5^{\sqrt{\log_5 7}} - 7^{\sqrt{\log_7 5}}$, is: [1]
- a) 0 b) 1
c) $\log 2$ d) -1
12. What does the P stand for in this formula? [1]
Present value = $P \left[\frac{1 - (1+i)^{-n}}{i} \right]$
- a) The present value b) The future value
c) The number of payments d) The fixed payment amount
13. A shopkeeper bought a TV from a distributor at a discount of 25% of the listed price of ₹ 32000. The shopkeeper sells the TV to a consumer at the listed price. If the sales are intra-state and the rate of GST is 18%, the selling price of the TV including tax (under GST) by the distributor is: [1]
- a) ₹ 26160 b) ₹ 32000
c) ₹ 24000 d) ₹ 28320

- ## Section B

21. A is thrice as good a workman as B and so takes 60 days less than B for doing a job. In how many days A and B together will complete the same job? [2]
22. If 'men are very busy' means, '1234', 'Busy person need encouragement' means '4567', 'encouragement is very important' means '3589' and 'Important persons are rare' means, '2680', then [2]
- What is the code for **encouragement**.
 - According to given code, what is the code of **Men need encouragement**?

OR

In a certain language if **LUCKNOW** is coded as **NWEMPQY**, how is **DELHI** coded?

23. A clock gains 4 seconds in 3 minutes and was set right at 9:00 a.m. What time will it show at 11:00 p.m. on the same day? [2]
24. If $xy^2 = 1$, prove that $2\frac{dy}{dx} + y^3 = 0$. [2]

OR

Find the derivative of the given function: $(x^2 + 1)(x - 2)$

25. Convert the decimal number 437 into the binary number. [2]

Section C

26. If a, b, c are in A.P. show that $a\left(\frac{1}{b} + \frac{1}{c}\right), b\left(\frac{1}{c} + \frac{1}{a}\right), c\left(\frac{1}{a} + \frac{1}{b}\right)$ are also in A.P. [3]

OR

Find three numbers in G.P. whose product is 216 and the sum of their products in pairs is 156.

27. Rohit is the husband of Vanshika. Sumita is the sister of Rohit. Anushka is the sister of Vanshika. How Anushka is related to Rohit? [3]
28. Find the domain and range of $f(x) = |2x - 3| - 3$. [3]
29. In what time will ₹ 25000 amount to ₹ 35000 at 6% compounded quarterly? [3]
30. Mr. Rajesh in Bengaluru, Karnataka consumed 159 kL of water in a month. Calculate the water bill for the month. The tariff plan of Bengaluru is as given below: [3]

Units of Consumption (in kL):	up to 8	8-25	25-50	>50
Price per kL consumed :	₹ 7	₹ 11	₹ 25	₹ 45

Meter rent = ₹ 56 per month; Sewerage charges are flat ₹ 14 if the consumption is less than 8 KL and 25% of the consumption charges if the consumption is more than 8 KL.

31. A and B are two sets such that $n(A - B) = 14 + x$, $n(B - A) = 3x$ and $n(A \cap B) = x$. Draw a Venn diagram to illustrate this information. If $n(A) = n(B)$, find [3]
- the value of x
 - $n(A \cup B)$

Section D

32. If $\frac{n!}{2!(n-2)!}$ and $\frac{n!}{4!(n-4)!}$ are in the ratio 2 : 1, find the value of n. [5]

OR

In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

33. For what integers m and n does $\lim_{x \rightarrow 0} f(x)$ and $\lim_{x \rightarrow 1} f(x)$ exist, if $f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \leq x \leq 1 \\ nx^2 + m, & x > 1 \end{cases}$ [5]

34. Using short cut method, find the mean, variance and standard deviation for the data: [5]

Class	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75
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Frequency	64	132	153	140	51
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OR

The mean and standard deviation of some data for the time taken to complete a test are calculated with the following results:

Number of observations = 25, mean = 18.2 seconds, standard deviation = 3.25 seconds.

Further, another set of 15 observations x_1, x_2, \dots, x_{15} , also in seconds, is now available and we have $\sum_{i=1}^{15} x_i = 279$

and $\sum_{i=1}^{15} x_i^2 = 5524$. Calculate the standard deviation based on all 40 observations.

35. Mr. Pandey lives in Lucknow, Uttar Pradesh. The reading of the electric meter of his house is found to be 5678 units. If the previous month's reading was 4803 units and the connected load is 4 kW, calculate his electricity bill for that month. [5]

Tariff plan is given below.

Energy charges

Number of units	0 - 150	151 - 300	301 - 500	> 500
Price per unit (in ₹)	₹ 5.5	₹ 6	₹ 6.5	₹ 7

Fixed charges ₹ 110 per kW/month

Energy tax is 5% of tariff rates

Surcharge is ₹ 0.26 per unit

Section E

36. Read the text carefully and answer the questions: [4]

A market is in the form of a triangle whose vertices are B(-2, 0), C(1, 12). The third vertex A of this triangle lies on the mid point of the line joining the points (2, 1) and (4, 13).



- What will be the coordinates of A?
- Find the slope of the line joining the points B and C?
- Equation of the line joining the points B and C?

OR

Does point A lies on the line BC?

37. Read the text carefully and answer the questions: [4]

In XI standard, teacher was discussing about Spearman's Rank Correlation Coefficient in Statistics. Following points were discussed in the class about the same topic:



When we are finding correlation between two qualitative characteristics, say, beauty and intelligence, we take recourse to using rank correlation coefficient. Rank correlation can also be applied to find the level of agreement (or disagreement) between two judges so far as assessing a qualitative characteristic is concerned. As compared to product moment correlation coefficient, rank correlation is easier to compute, it can also be advocated to get a first hand impression about the correlation between a pair of variables. Spearman's rank correlation coefficient is given by

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

where,

d = difference between ranks of corresponding x and y

n = number of pairs of values (x,y) in the data

- When the rank are repeated, the Spearman's rank correlation coefficient formula is given by

$$r_s = 1 - \frac{6 \left[\sum d^2 + \frac{(m_1^3 - m_1)}{12} + \frac{(m_2^3 - m_2)}{12} + \dots \right]}{n(n^2-1)}$$

where, $m_1, m_2 \dots$, are the number of repetitions of ranks and $\frac{m_1^3 - m_1}{12} \dots$ their corresponding correlation factors.

For example: Ranks obtained by 10 students are given below:

3	4	7	8	9
7	3	4	1	1

- Find the value of $\sum d$?
- Find the value $\sum d^2$?
- What is the value of n^2 in the given data?

OR

What is rank correlation of the given data?

38. **Read the text carefully and answer the questions:**

[4]

Vijay and Ramesh are playing cards. Total number of playing cards are 52 in numbers. Following events happen while playing the game of choosing a card out of 52 cards.



- Vijay draw a card, what is the probability that the drawn card is either red or king?
- Ramesh draw a card, what is the probability that the card drawn is an ace?
- Vijay draw the card again, now what is the probability that the card drawn is a red card?

OR

Read the text carefully and answer the questions:

[4]

A shopkeeper sells three types of flower seeds A_1, A_2 , and A_3 . They are sold as a mixture, where the proportions are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

- Calculate the probability of a randomly chosen seed to germinate.

- (b) Calculate the probability that it is of the type A_2 given that randomly chosen seed does not germinate.
- (c) Calculate the probability that it will not germinate given that the seed is of type A_1 .

Solution

Section A

1. (a) $\frac{2}{3}$

Explanation: $\frac{2}{3}$

We know that the bag contains 5 (black), 4W (white) and 3R (red) balls.

Now,

$$P(B) = \frac{5}{12}$$

$$P(R) = \frac{3}{12}$$

$$P(B \text{ or } R) = P(B) + P(R)$$

$$= \frac{5}{12} + \frac{3}{12}$$

$$= \frac{8}{12} = \frac{2}{3}$$

2.

(c) 44

Explanation: Given that, mean of 100 items, $\bar{x}_{100} = 50$

Mean of 150 items, $\bar{x}_{150} = 40$ and standard deviation of 100 items, $\sigma_{100} = 5$

Standard deviation of 150 items, $\sigma_{150} = 6$

Here, $n_1 = 100$, $\bar{x}_{100} = 50$ and $n_2 = 150$, $\bar{x}_{150} = 40$

$$\therefore \text{Combined mean of all 250 items, } \bar{x}_{250} = \frac{n_1 \cdot \bar{x}_{100} + n_2 \cdot \bar{x}_{150}}{n_1 + n_2}$$

$$= \frac{100 \times 50 + 150 \times 40}{100 + 150} = \frac{5000 + 6000}{250} = \frac{11000}{250} = 44$$

3.

(b) ₹ 28320

Explanation: Listed price = ₹ 32000, discount = 25% of ₹ 32000 = ₹ 8000

\therefore S.P. of distributor = ₹ 32000 - ₹ 8000 = ₹ 24000

CGST = 9% of ₹ 24000 = ₹ 2160

SGST = 9% of ₹ 24000 = ₹ 2160

\therefore S.P. including tax = ₹ 24000 + ₹ 2160 + ₹ 2160 = ₹ 28320

4.

(d) $\log_2 16 = 4$

Explanation: $2^4 = 16$ in logarithmic form.

As we know that

$$\text{if } a^y = x$$

$$\text{then } \log_a x = y$$

$$\therefore \log_2 16 = 4$$

5.

(b) 2^{mn}

Explanation: as $n(A) = m$, $n(B) = n$

$$\Rightarrow n(A \times B) = mn$$

So, number of relations = 2^{mn}

6.

(d) $\log 1 + \log 2 + \log 3$

Explanation: Since, $\log_a(mn) = \log_a m + \log_a n$

One extending this rule for three variables, we get

$$\log_a(mnp) = \log_a m + \log_a n + \log_a p$$

$$\text{Therefore, } \log(1 \times 2 \times 3) = \log 1 + \log 2 + \log 3$$

7.

(d) $\frac{1}{6}$

Explanation: We know that $P(A \cap \bar{B}) = P(A - B) = P(A \cup B) - P(B)$
 $= \frac{2}{3} - \left(1 - \frac{1}{2}\right) = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$.

8.

(b) (a, 0)

Explanation: $y^2 = 4a(x - a)$

Let $Y = y$ and $X = x - a$

For standard parabola $Y^2 = 4aX$ the vertex is (0, 0)

So put $X = 0$ and $Y = 0$.

i.e. put $x - a = 0$ and $y = 0$

So vertex is (a, 0)

9.

(c) SINGS

Explanation: SINGS

10.

(d) 2

Explanation: The variance of first n natural number is $\frac{n^2-1}{12}$

Here $n = 5$

So, Variance = $\frac{5^2-1}{12}$

$= \frac{25-1}{12}$

$= \frac{24}{12}$

$= 2$

11.

(a) 0

Explanation: 0

12.

(d) The fixed payment amount

Explanation: Present value = cash flow $\times \frac{[(1+i)^n - 1]}{i(1+i)^n}$

$= \text{fixed payment amount} \times \left[\frac{1 - (1+i)^{-n}}{i} \right]$

13.

(d) ₹ 28320

Explanation: ₹ 28320

14. (a) $\frac{{}^{13}C_3 \times {}^{13}C_1}{{}^{52}C_4}$

Explanation: Number of ways of drawing 4 cards from 52 cards = ${}^{52}C_4$

In a deck of 52 cards, there are 13 diamonds and 13 spades.

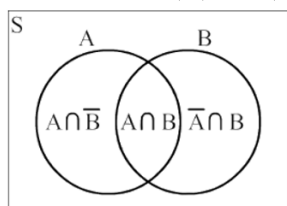
\therefore Number of ways of drawing 3 diamonds and one spade = ${}^{13}C_3 \times {}^{13}C_1$

Thus the probability of obtaining 3 diamonds and one spade = $\frac{{}^{13}C_3 \times {}^{13}C_1}{{}^{52}C_4}$

15.

(a) $P(B) - P(A \cap B)$

Explanation: $P(B) - P(A \cap B)$



From the diagram, we get $A \cap B$ and $\bar{A} \cap B$ are mutually exclusive events such that $(A \cap B) \cup (\bar{A} \cap B) = B$ Therefore by addition theorem of probability we have

$$P(A \cap B) + P(\bar{A} \cap B) = P(B)$$

$$\therefore P(A \cap B) = P(B) - P(\bar{A} \cap B)$$

16. (a) Statement (c) is correct.

Explanation: Time value of money supports the comparison of cash flows recorded at different time period by either compounding or discounting all cash flows to common point of time.

17.

(b) 120

Explanation: Keeping EN together and considering it as one letter, we have to arrange 5 letters at 5 places.

This can be done in ${}^5P_5 = 5! = 120$ ways.

18. (a) $\{(1, 4)\}$

Explanation: $(A - B) = \{1, 2, 4\} - \{2, 4, 5\} = \{1\}$

and $(B - C) = \{2, 4, 5\} - \{2, 5\}$

$= \{4\}$

Now, $(A - B) \times (B - C) = \{1\} \times \{4\}$

$= \{(1, 4)\}$

Since, it is a set so it is written in curly braces.

Therefore, option B is correct.

19. (a) Both A and R are true and R is the correct explanation of A.

Explanation: We know that if each observation is multiplied by a non-zero real number a, then variance is multiplied by a^2 .
 \therefore R is true.

Now, given that variance of 10 observations is 6 and each observation is multiplied by 3. So, new variance is $3^2 \times 6$ i.e. 54.

\therefore A is true and R is the correct explanation of A.

20. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Assertion: Given GP 4, 16, 64, ...

$$\therefore a = 4, r = \frac{16}{4} = 4 > 1$$

$$\therefore S_6 = \frac{4((4)^6 - 1)}{4 - 1} = \frac{4(4095)}{3} = 5460$$

Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

Section B

21. Let A and B separately complete the job in n_A and n_B days respectively. It is given that A is thrice as good a workman as B. This means A takes one third of the time taken by B to complete the job.

$$\therefore n_A = \frac{1}{3} n_B \Rightarrow n_B = 3 n_A \dots(i)$$

It is also given that A takes 60 days less than B to complete the job.

$$\therefore n_A = n_B - 60 \dots(ii)$$

Solving (i) and (ii), we obtain

$$n_A = 30 \text{ and } n_B = 90$$

Suppose A and B together can complete the job in n_{AB} days. Then,

$$\frac{1}{n_{AB}} = \frac{1}{n_A} + \frac{1}{n_B} \Rightarrow \frac{1}{n_{AB}} = \frac{1}{30} + \frac{1}{90} \Rightarrow \frac{1}{n_{AB}} = \frac{4}{90} \Rightarrow n_{AB} = \frac{90}{4} = 22\frac{1}{2}$$

Hence, A and B together can complete the job in $22\frac{1}{2}$ days

22. Given statements and their codes are as follows:

(I) Men are very busy - 1234

(II) Busy persons need encouragement - 4567

(III) Encouragement is very important - 3589

(IV) Important persons are rare - 2680

i. In second (II) and third (III) sentences, common word is 'encouragement' and common code no. is '5'. Hence, number 5 stands for encouragement.

ii. In first (I) and fourth (IV) sentences, the common word is 'are', hence 'are' stands for '2' from first (I) and third (III) sentences, 'very' stand for '3' from first (I) and second (II) sentences, 'busy' stands for '4'. Hence, we get from first sentence 'Men stand for '1' similarly 'needs' stand for '7'. From this, we conclude that 'Men need encouragement' will be coded as '1 5 7'.

OR

Clearly, each letter of the word LUCKNOW is moved two steps forward to obtain the corresponding code

	L	U	C	K	N	O	W
+2	↓	↓	↓	↓	↓	↓	↓
	N	W	E	M	P	Q	Y

So

	D	E	L	H	I
+2	↓	↓	↓	↓	↓
	F	G	N	J	K

∴ 'DELHI' is coded as 'FGNJK'

23. Given that the clock gains 4 seconds in 3 minutes

⇒ it gains $20 \times 4 = 80$ seconds in $20 \times 3 = 60$ minutes i.e. 1 hour

Now, from 9:00 a.m. to 11:00 p.m. on the same day time passed is 14 hours.

So, in 14 hours, the clock will gain 14×80 seconds = 1120 seconds
= 18 min. 40 sec.

Hence, the clock will show 11:18:40 p.m. at 11:00 p.m.

24. Given as $xy^2 = 1$

Differentiating with respect to x,

$$\frac{d}{dx}(xy^2) = \frac{d}{dx}(1)$$

$$\Rightarrow x \frac{d}{dx}(y^2) = y^2 \frac{d}{dx}(x) = 0$$

$$\Rightarrow x(2y) \frac{dy}{dx} + y^2(1) = 0$$

$$\Rightarrow 2xy \frac{dy}{dx} = -y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y^2}{2xy}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{2x}$$

On putting the value of $x = \frac{1}{y^2}$ in above equation

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{2\left(\frac{1}{y^2}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{2\left(\frac{1}{y^2}\right)}$$

$$\Rightarrow 2 \frac{dy}{dx} = -y^3$$

$$\Rightarrow 2 \frac{dy}{dx} + y^3 = 0$$

OR

Let $f(x) = (x^2 + 1)(x - 2) = x^3 - 2x^2 + x - 2$, diff. w.r.t. x, we get

$$f'(x) = 3x^2 - 2 \cdot 2x^1 + 1 - 0 = 3x^2 - 4x + 1$$

Alternatively

$f(x) = (x^2 + 1)(x - 2)$, diff. w.r.t. x, we get

$$f'(x) = (x^2 - 1) \cdot \frac{d}{dx}(x - 2) + (x - 2) \cdot \frac{d}{dx}(x^2 + 1) \text{ (product rule)}$$

$$= (x^2 + 1)(1 - 0) + (x - 2)(2x + 0)$$

$$= x^2 + 1 + 2x^2 - 4x = 3x^2 - 4x + 1$$

25. The given decimal number is 437

2	437	
2	218	1
2	109	0
2	54	1
2	27	0
2	13	1
2	6	1
2	3	0
2	1	1
2	0	1

Hence, the required binary number is 110110101

Section C

26. a, b, c are in A.P.

$$\Rightarrow \frac{a}{abc}, \frac{b}{abc}, \frac{c}{abc} \text{ are in A.P.}$$

[on dividing each term by abc]

$$\Rightarrow \frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab} \text{ are in A.P.}$$

$$\Rightarrow \frac{ab+bc+ca}{bc}, \frac{ab+bc+ca}{ca}, \frac{ab+bc+ca}{ab} \text{ are in A.P.}$$

[on multiplying each term by ab + bc + ca]

$$\Rightarrow \frac{ab+bc+ca}{bc} - 1, \frac{ab+bc+ca}{ca} - 1, \frac{ab+bc+ca}{ab} - 1$$

are also in A.P.

[On adding -1 to each term]

$$\Rightarrow \frac{ab+ac}{bc}, \frac{ab+bc}{ca}, \frac{bc+ca}{ab} \text{ are in A.P.}$$

$$\Rightarrow a \left(\frac{1}{b} + \frac{1}{c} \right), b \left(\frac{1}{c} + \frac{1}{a} \right), c \left(\frac{1}{a} + \frac{1}{b} \right) \text{ are in A.P.}$$

OR

Let three numbers in G.P. be $\frac{a}{r}$, a, ar

\therefore Their product = $\frac{a}{r} \cdot a \cdot ar = 216$ (given)

$$\Rightarrow a^3 = 216 = (6)^3 \Rightarrow a = 6.$$

Also sum of their products in pairs = 156 (given)

$$\Rightarrow \frac{a}{r} \cdot a + a \cdot ar + ar \cdot \frac{a}{r} = 156$$

$$\Rightarrow a^2 \left(\frac{1}{r} + r + 1 \right) = 156$$

$$\Rightarrow 6^2 \cdot \frac{1+r^2+r}{r} = 156$$

$$\Rightarrow 3 \cdot \frac{r^2+r+1}{r} = 13$$

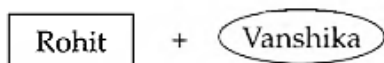
$$\Rightarrow 3r^2 + 3r + 3 = 13r$$

$$\Rightarrow 3r^2 - 10r + 3 = 0$$

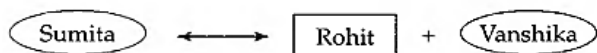
$$\Rightarrow (r-3) \left(r - \frac{1}{3} \right) = 0 \Rightarrow r = 3, \frac{1}{3}$$

When $r = 3$, numbers are 2, 6, 18 and when $r = \frac{1}{3}$, numbers are 18, 6, 2

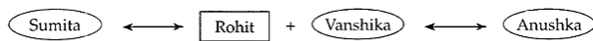
27. Rohit is the husband of Vanshika



Sumita is the sister of Rohit



Anushka is the sister of Vanshika



So Anushka is Rohit's wife's sister

Anushka is the sister-in-law of Rohit.

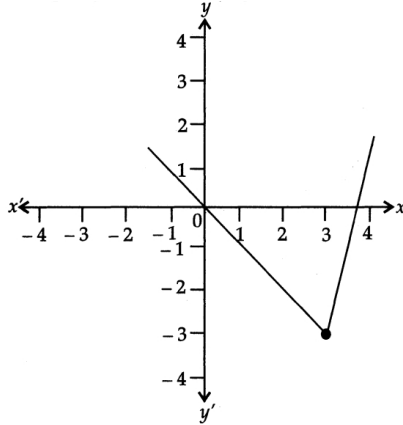
28. Given, $f(x) = |2x - 3| - 3$

The domain of the expression is all real number except where the expression is undefined. In this case, there is not real number that makes the expression undefined.

\therefore Domain of $f = (-\infty, \infty) = \mathbb{R}$

The absolute value of expression has a 'V' shape. The range of a positive absolute value expression starts at its vertex and extends to infinity.

Range of $f = [-3, \infty)$ or $\{y : y \geq -3\}$



29. $A = P \left(1 + \frac{\frac{r}{k}}{100} \right)^{k \times n}$

Here $A = 35000$; $P = 25000$; $r = 6\%$, $n = ?$; $k = 4$

$$35000 = 25000 \left(1 + \frac{\frac{6}{4}}{100} \right)^{4 \times n}$$

$$\frac{35000}{25000} = \left(1 + \frac{3}{200} \right)^{4 \times n}$$

$$\frac{7}{5} = \left(\frac{203}{200} \right)^{4 \times n}$$

$$\frac{7}{5} = (1.015)^{4 \times n}$$

$$(1.4)^{\frac{1}{4}} = (1.015)^n$$

$$\frac{1}{4} \log(1.4) = n \cdot \log(1.015)$$

$$0.0365 = n \cdot (0.00647)$$

$$n = \frac{0.0365}{0.00647}$$

$$n = 5.64 \text{ years}$$

30. Here, the consumption of water is given to be 159 kL

According to the given tariff plan:

$$\text{Water consumption charges} = ₹[(8 \times 7) + (17 \times 11) + (25 \times 25) + (109 \times 45)]$$

$$= ₹(56 + 187 + 625 + 4905)$$

$$= ₹5773$$

Sewerage charges for consumption above 8kL in 25% of the consumption charges

\therefore Sewerage charge = 25% of ₹5773

$$= ₹1443.25$$

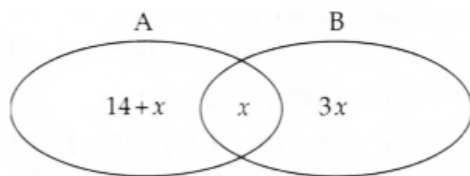
Meter rent = ₹56 per month

\therefore Total Water Bill = Consumption charge + Sewerage charge + Meter rent

$$= ₹5773 + ₹1443.25 + ₹56$$

$$= ₹7272.25$$

31. The adjoining Venn diagram represents the information given in the question.



i. From the Venn diagram, we get

$$n(A) = n(A - B) + n(A \cap B)$$

$$= (14 + x) + x = 14 + 2x \text{ and}$$

$$n(B) = n(B - A) + n(A \cap B)$$

$$= 3x + x = 4x$$

$$\text{But } n(A) = n(B) \text{ (given)}$$

$$\Rightarrow 14 + 2x = 4x \Rightarrow 2x = 14 \Rightarrow x = 7$$

ii. $n(A \cup B) = n(A - B) + n(B - A) + n(A \cap B)$

$$= (14 + x) + 3x + x = 14 + 5x$$

$$= 14 + 5 \times 7 = 14 + 35 = 49$$

Section D

32. We have $\frac{n!}{2!(n-2)!} : \frac{n!}{4!(n-4)!} = 2 : 1$

$$\Rightarrow \frac{n!}{2!(n-2)!} \div \frac{n!}{4!(n-4)!} = \frac{2}{1}$$

$$\Rightarrow \frac{4!(n-4)!}{2!(n-2)!} = 2$$

$$\Rightarrow \frac{4 \times 3 \times 2!}{2!} \times \frac{(n-4)!}{(n-2)(n-3)(n-4)!} = 2$$

$$\Rightarrow n^2 - 5n + 6 = 6$$

$$\Rightarrow n^2 - 5n = 0, \text{ we have } n = 5 \text{ or } n = 0$$

Rejecting $n = 0$, we have $n = 5$.

OR

Total letters of the word MISSISSIPPI = 11

Here M = 1, I = 4, S = 4 and P = 2

$$\therefore \text{Number of permutations} = \frac{11!}{4!4!2!}$$

$$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4! \times 4 \times 3 \times 2 \times 1 \times 2 \times 1} = 34650$$

When the four I's come together then it becomes one letter so total number of letters in the word when all I's come together = 8

$$\therefore \text{Number of Permutations} = \frac{8!}{4!2!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4! \times 2 \times 1} = 840$$

Number of permutations when four I's do not come together = $34650 - 840 = 33810$

33. Given,

$$f(x) = \begin{cases} mx^2 + n, & x < 0 \\ nx + m, & 0 \leq x \leq 1 \\ nx^2 + m, & x > 1 \end{cases}$$

Limit at $x = 0$,

$$\text{LHL} = \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} mx^2 + n$$

$$= \lim_{h \rightarrow 0} m(0 - h)^2 + n$$

[putting $x = 0 - h$ as $x \rightarrow 0$, then $h \rightarrow 0$]

$$= n$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} nx + m$$

$$= \lim_{h \rightarrow 0} n(0 + h) + m = m$$

[putting $x = 0 + h$ as $x \rightarrow 0$, then $h \rightarrow 0$]

As $\lim_{x \rightarrow 0} f(x)$ exists, then

$$\text{LHL} = \text{RHL} \Rightarrow n = m$$

\therefore Limit at $x = 1$

$$\text{Now, LHL} = \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (nx + m)$$

$$= \lim_{h \rightarrow 0} [n(1-h) + m]$$

[putting $x = 1 - h$ as $x \rightarrow 0$, then $h \rightarrow 0$]

$$= n + m$$

Here, $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} f(x)$ exists for all $m, n \in \mathbb{Z}$.

34. From the given information we prepare the following table...

Class	Frequency (f_i)	x_i	$d_i = x_i - 50$	$f_i d_i$	$f_i d_i^2$
25 - 350	64	30	-20	-1280	25600
35 - 45	132	40	-10	-1320	13200
45 - 55	153	50	0	0	0
55 - 65	140	60	10	1400	14000
65 - 75	51	70	20	1020	20400
$\sum f_i = 540$				$\sum f_i d_i = -180$	73200

$$\text{Mean} = A + \frac{\sum f_i d_i}{\sum f_i} \times h$$

$$= 50 + \frac{(-180)}{540} \times 5 = 40$$

Variance (σ^2)

$$\text{Standard deviation } (\sigma) = \sqrt{\frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2} = \sqrt{\frac{73200}{540} - \left(\frac{-180}{540}\right)^2}$$

$$= \sqrt{135.55 - 0.111} = \sqrt{135.44} = 11.64$$

$$\text{Standard deviation } (\sigma) = \sqrt{\text{Variance}} = \sqrt{11.64}$$

OR

Given: Number of observations = 25, mean = 18.2 seconds, standard deviation = 3.25 seconds. Another set of 15 observations x_1 ,

x_2, \dots, x_{15} , also in seconds, is $\sum_{i=1}^{15} x_i = 279$ and $\sum_{i=1}^{15} x_i^2 = 5524$

To find: the standard deviation based on all 40 observations

As per the given criteria,

In first set,

We have Number of observations, $n_1=25$

Mean, $\bar{x}_1 = 18.2$

And standard deviation, $\sigma_1 = 3.25$

And

In second set,

We have Number of observations, $n_2=15$

$\sum_{i=1}^{15} x_i = 279$ and $\sum_{i=1}^{15} x_i^2 = 5524$

For the first set we have

$$\bar{x}_1 = 18.2 = \frac{\sum x_i}{25}$$

$$\sum x_i = 25 \times 18.2 = 455$$

Therefore the standard deviation becomes,

$$\sigma_1^2 = \frac{\sum x_i^2}{25} - (18.2)^2$$

Substituting the values, we get

$$(3.25)^2 = \frac{\sum x_i^2}{25} - 331.24$$

$$\Rightarrow 10.5625 + 331.24 = \frac{\sum x_i^2}{25}$$

$$\Rightarrow \frac{\sum x_i^2}{25} = 341.8025$$

$$\Rightarrow \sum x_i^2 = 25 \times 341.8025 = 8545.06$$

For the combined standard deviation of the 40 observation, $n=40$

And

$$\Rightarrow \sum x_i^2 = 8545.06 + 5524 = 14069.69$$

$$\Rightarrow \sum x_i = 455 + 279 = 734$$

Therefore the standard deviation can be written as,

$$\sigma = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}$$

Substituting the values, we get

Therefore the standard deviation can be written as,

$$\sigma = \sqrt{\frac{14069.69}{40} - \left(\frac{734}{40}\right)^2}$$

$$\sigma = \sqrt{351.7265 - (18.35)^2}$$

$$\sigma = \sqrt{351.7265 - 336.7225}$$

$$\sigma = \sqrt{15.004}$$

$$\sigma = 3.87$$

35. Given current month's reading = 5678 units

Previous month's reading = 4803 units

Number of units consumed = 5678 - 4803 = 875

Energy charges

Units	Price	Amount
0 - 150	₹ 5.5	₹ 825
151 - 300	₹ 6	₹ 900
301 - 500	₹ 6.5	₹ 1300
501 - 875	₹ 7	₹ 2625
	Total	5650

Fixed charges = ₹(110 × 4) = ₹ 440

Surcharge = ₹(0.26 × 875) = ₹ 227.50

Energy tax = 5% of (₹ 5650 + ₹ 440) = 5% of ₹ 6090 = ₹ 304.50

Electricity bill = ₹ 5650 + ₹ 440 + ₹ 227.50 + ₹ 304.50 = ₹ 6622

Section E

36. Read the text carefully and answer the questions:

A market is in the form of a triangle whose vertices are B(-2, 0), C(1, 12). The third vertex A of this triangle lies on the mid point of the line joining the points (2, 1) and (4, 13).



(i) Midpoint of (2, 1) and (4, 13) is given by (3, 7).

(ii) Slope = $\frac{12-0}{1-(-2)} = \frac{12}{3} = 4$

(iii) Equation of BC: = (y - 0) = 4(x + 2)
= y - 2 = 4x

OR

Equation of BC is y - 2 - 4x = 0. Putting coordinates of A (3, 7) in this equation, we get: 7 - 2 - 12 ≠ 0, therefore we can say that point A will not lie on the line BC.

37. Read the text carefully and answer the questions:

In XI standard, teacher was discussing about Spearman's Rank Correlation Coefficient in Statistics. Following points were discussed in the class about the same topic:



When we are finding correlation between two qualitative characteristics, say, beauty and intelligence, we take recourse to using rank correlation coefficient. Rank correlation can also be applied to find the level of agreement (or disagreement) between two judges so far as assessing a qualitative characteristic is concerned. As compared to product moment correlation coefficient, rank correlation is easier to compute, it can also be advocated to get a first hand impression about the correlation between a pair of variables. Spearman's rank correlation coefficient is given by

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

where,

d = difference between ranks of corresponding x and y

n = number of pairs of values (x,y) in the data

- When the rank are repeated, the Spearman's rank correlation coefficient formula is given by

$$r_s = 1 - \frac{6 \left[\sum d^2 + \frac{(m_1^3 - m_1)}{12} + \frac{(m_2^3 - m_2)}{12} + \dots \right]}{n(n^2-1)}$$

where, m_1, m_2, \dots , are the number of repetitions of ranks and $\frac{m_1^3 - m_1}{12}$... their corresponding correction factors.

For example: Ranks obtained by 10 students are given below:

3	4	7	8	9
7	3	4	1	1
Rank of Maths (R_x)	Rank of Physics (R_y)	$d = R_x - R_y$		
10	5	5		
5	6	-1		
6	9	-3		
1	2	-1		
2	8	-6		
3	7	-4		
4	3	1		
7	4	3		
8	1	-2		
9	1	8		
-	-	0		

Therefore, $\sum d = 0$

$$(ii) \sum d^2 = 25 + 1 + 9 + 1 + 36 + 16 + 1 + 9 + 4 + 64$$

$$= 166$$

$$(iii) n^2 = 10^2$$

$$= 100$$

OR

Rank correlation is given by:

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 166}{10(10^2-1)}$$

$$\begin{aligned}
&= 1 - \frac{996}{990} \\
&= \frac{990-996}{990} \\
&= \frac{-6}{990} \\
&= -0.006
\end{aligned}$$

38. Read the text carefully and answer the questions:

Vijay and Ramesh are playing cards. Total number of playing cards are 52 in numbers. Following events happen while playing the game of choosing a card out of 52 cards.



(i) Favourable events are:

Total red cards = 26 (includes two kings)

Two black kings = total cards = 28

Total cards left (favourable) = 52 - 28 = 24

Required probability = $\frac{24}{52} = \frac{4}{13}$

(ii) Total number of aces = 4

Required probability = $\frac{4}{52} = \frac{1}{13}$

(iii) Total number of red cards = 26

Required probability = $\frac{26}{52} = \frac{1}{2}$

OR

Read the text carefully and answer the questions:

A shopkeeper sells three types of flower seeds A_1 , A_2 , and A_3 . They are sold as a mixture, where the proportions are 4 : 4 : 2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

(i) We have: $A_1 : A_2 : A_3 = 4 : 4 : 2$

$$P(A_1) = \frac{4}{10}, P(A_2) = \frac{4}{10}, P(A_3) = \frac{2}{10}$$

where, A_1 , A_2 and A_3 denote three types of flowers seed.

Let 'E' be the event that a seed germinates.

$$\therefore P\left(\frac{E}{A_1}\right) = \frac{45}{100}, P\left(\frac{E}{A_2}\right) = \frac{60}{100} \text{ \& } P\left(\frac{E}{A_3}\right) = \frac{35}{100}$$

$$P\left(\frac{\bar{E}}{A_1}\right) = \frac{55}{100}, P\left(\frac{\bar{E}}{A_2}\right) = \frac{40}{100} \text{ \& } P\left(\frac{\bar{E}}{A_3}\right) = \frac{65}{100}$$

$$P(E) = P(A_1) \cdot P\left(\frac{E}{A_1}\right) + P(A_2) \cdot P\left(\frac{E}{A_2}\right) + P(A_3) \cdot P\left(\frac{E}{A_3}\right)$$

$$= \frac{4}{10} \cdot \frac{45}{100} + \frac{4}{10} \cdot \frac{60}{100} + \frac{2}{10} \cdot \frac{35}{100}$$

$$= \frac{180}{1000} + \frac{240}{1000} + \frac{70}{1000} = 0.49$$

$$(ii) P\left(\frac{A_2}{\bar{E}}\right) = \frac{P(A_2) \cdot P\left(\frac{\bar{E}}{A_2}\right)}{P(A_1) \cdot P\left(\frac{\bar{E}}{A_1}\right) + P(A_2) \cdot P\left(\frac{\bar{E}}{A_2}\right) + P(A_3) \cdot P\left(\frac{\bar{E}}{A_3}\right)}$$

$$= \frac{\frac{4}{10} \cdot \frac{40}{100}}{\frac{4}{10} \cdot \frac{55}{100} + \frac{4}{10} \cdot \frac{40}{100} + \frac{2}{10} \cdot \frac{65}{100}}$$

$$= 0.314$$

$$(iii) P\left(\frac{E}{A_1}\right) = 1 - P\left(\frac{\bar{E}}{A_1}\right)$$

$$= 1 - \frac{45}{100}$$

$$= \frac{55}{100}$$